4751 (C1) Introduction to Advanced Mathematics

1	$[a =]2c^2 - b$ www o.e.	**3**	**M1** for each of 3 complete correct steps, ft from previous error if equivalent difficulty
2	$5x - 3 < 2x + 10$ $3x < 13$ $x < \frac{13}{3}$ o.e.	**M1** **M1** **M1**	condone '=' used for first two Ms **M0** for just $5x - 3 < 2(x + 5)$ or $-13 < -3x$ or ft or ft; isw further simplification of 13/3; M0 for just $x < 4.3$
3 (i)	(4, 0)	**1**	allow $y = 0$, $x = 4$ bod **B1** for $x = 4$ but do not isw: **0** for (0, 4) seen **0** for (4, 0) and (0, 10) **both** given (choice) unless (4, 0) clearly identified as the x-axis intercept
3 (ii)	$5x + 2(5 - x) = 20$ o.e. (10/3, 5/3) www isw	**M1** **A2**	for subst or for multn to make coeffts same and appropriate addn/subtn; condone one error or **A1** for $x = 10/3$ and **A1** for $y = 5/3$ o.e. isw; condone 3.33 or better and 1.67 or better **A1** for (3.3, 1.7)
4 (i)	translation by $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ or 4 [units] to left	**B1** **B1**	**0** for shift/move or 4 units in negative x direction o.e.
4 (ii)	sketch of parabola right way up and with minimum on negative y-axis min at (0, −4) and graph through −2 and 2 on x-axis	**B1** **B1**	mark intent for both marks must be labelled or shown nearby
5 (i)	$\frac{1}{12}$ or $\pm\frac{1}{12}$	**2**	**M1** for $\frac{1}{144^{\frac{1}{2}}}$ o.e. or for $\sqrt{144} = 12$ soi
5 (ii)	denominator = 18 numerator = $5 - \sqrt{7} + 4\left(5 + \sqrt{7}\right)$ = $25 + 3\sqrt{7}$ as final answer	**B1** **M1** **A1**	**B0** if 36 after addition for **M1**, allow in separate fractions allow **B3** for $\frac{25 + 3\sqrt{7}}{18}$ as final answer www

6 (i)	cubic correct way up and with two turning pts touching x-axis at −1, and through it at 2.5 and no other intersections y- axis intersection at −5	**B1** **B1** **B1**	 intns must be shown labelled or worked out nearby
6 (ii)	$2x^3 - x^2 - 8x - 5$	**2**	**B1** for 3 terms correct or **M1** for correct expansion of product of two of the given factors
7	attempt at f(−3) $-27 + 18 - 15 + k = 6$ $k = 30$	**M1** **A1** **A1**	or **M1** for long division by $(x + 3)$ as far as obtaining $x^2 - x$ and **A1** for obtaining remainder as $k - 24$ (but see below) equating coefficients method: **M2** for $(x + 3)(x^2 - x + 8)$ [+6] o.e. (from inspection or division) eg M2 for obtaining $x^2 - x + 8$ as quotient in division
8	$x^3 + 15x + \frac{75}{x} + \frac{125}{x^3}$ www isw or $x^3 + 15x + 75x^{-1} + 125x^{-3}$ www isw	**4**	**B1** for **both** of x^3 **and** $\frac{125}{x^3}$ or $125x^{-3}$ isw and **M1** for 1 3 3 1 soi; **A1** for **each** of $15x$ **and** $\frac{75}{x}$ or $75x^{-1}$ isw **or** **SC2** for completely correct unsimplified answer

9	$x^2 - 5x + 7 = 3x - 10$	**M1**	or attempt to subst $(y + 10)/3$ for x
	$x^2 - 8x + 17$ [= 0] o.e or $y^2 - 4y + 13$ [= 0] o.e	**M1**	condone one error; allow **M1** for $x^2 - 8x = -17$ [oe for y] only if they go on to completing square method
	use of $b^2 - 4ac$ with numbers subst (condone one error in substitution) (may be in quadratic formula)	**M1**	or $(x - 4)^2 = 16 - 17$ or $(x - 4)^2 + 1 = 0$ (condone one error)
	$b^2 - 4ac = 64 - 68$ or -4 cao [or 16 – 52 or -36 if y used]	**A1**	or $(x - 4)^2 = -1$ or $x = 4 \pm \sqrt{-1}$ [or $(y - 2)^2 = -9$ or $y = 2 \pm \sqrt{-9}$]
	[< 0] so no [real] roots [so line and curve do not intersect]	**A1**	or conclusion from comp. square; needs to be explicit correct conclusion and correct ft; allow '< 0 so no intersection' o.e.; allow '-4 so no roots' etc allow A2 for full argument from sum of two squares = 0; A1 for weaker correct conclusion some may use the condition $b^2 < 4ac$ for no real roots; allow equivalent marks, with first A1 for 64 < 68 o.e.
10 (i)	grad CD = $\frac{5-3}{3-(-1)}\left[=\frac{2}{4}\text{ o.e.}\right]$ isw	**M1**	NB needs to be obtained independently of grad AB
	grad AB = $\frac{3-(-1)}{6-(-2)}$ or $\frac{4}{8}$ isw	**M1**	
	same gradient so parallel www	**A1**	must be explicit conclusion mentioning 'same gradient' or 'parallel' if M0, allow **B1** for 'parallel lines have same gradient' o.e.
10 (ii)	[BC^2=] $3^2 + 2^2$ [BC^2 =] 13 showing $AD^2 = 1^2 + 4^2$ [=17] [$\neq BC^2$] isw	**M1** **A1** **A1**	accept $(6 - 3)^2 + (3 - 5)^2$ o.e. or [BC =] $\sqrt{13}$ or [AD =] $\sqrt{17}$ or equivalent marks for finding AD or AD^2 first alt method: showing AC $\neq$ BD – mark equivalently

10 (iii)	[BD eqn is] $y = 3$	**M1**	eg allow for 'at M, $y = 3$' or for 3 subst in eqn of AC
	eqn of AC is $y - 5 = 6/5 \times (x - 3)$ o.e [$y = 1.2x + 1.4$ o.e.]	**M2**	or **M1** for grad AC = 6/5 o.e. (accept unsimplified) and M1 for using their grad of AC with coords of A(−2, −1) or C (3, 5) in eqn of line or **M1** for 'stepping' method to reach M
	M is (4/3, 3) o.e. isw	**A1**	allow : at M, $x = 16/12$ o.e. [eg =4/3] isw A0 for 1.3 without a fraction answer seen
10 (iv)	midpt of BD = (5/2, 3) or equivalent simplified form cao	**M1**	or showing BM ≠ MD oe [BM = 14/3, MD = 7/3]
	midpt AC = (1/2, 2) or equivalent simplified form cao or 'M is 2/3 of way from A to C'	**M1**	or showing AM ≠ MC or $AM^2 \neq MC^2$
	conclusion 'neither diagonal bisects the other'	**A1**	in these methods A1 is dependent on coords of M having been obtained in part (iii) or in this part; the coordinates of M need not be correct; it is also dependent on midpts of both AC and BD attempted, at least one correct alt method: show that mid point of BD does not lie on AC (M1) and vice-versa (M1), A1 for both and conclusion

11 (i)	centre C' = (3, −2) radius 5	**1** **1**	 0 for ±5 or −5
11 (ii)	showing $(6-3)^2 + (-6+2)^2 = 25$ showing that $\overrightarrow{AC'} = \overrightarrow{C'B} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ o.e.	**B1** **B2**	interim step needed **or B1** each for two of**:** showing midpoint of AB = (3, −2); showing B (0, 2) is on circle; showing AB = 10 **or B2** for showing midpoint of AB = (3, −2) and saying this is centre of circle **or B1** for finding eqn of AB as $y = -4/3\,x + 2$ o.e. and **B1** for finding one of its intersections with the circle is (0, 2) **or B1** for showing C′B = 5 and **B1** for showing AB = 10 or that AC′ and BC′ have the same gradient **or B1** for showing that AC′ and BC′ have the same gradient and B1 for showing that B (0, 2) is on the circle
11 (iii)	grad AC' or AB = −4/3 o.e. grad tgt = −1/their AC' grad $y - (-6) = \text{their } m(x - 6)$ o.e. $y = 0.75x - 10.5$ o.e. isw	**M1** **M1** **M1** **A1**	or ft from their C', must be evaluated may be seen in eqn for tgt; allow M2 for grad tgt = ¾ oe soi as first step or **M1** for $y = \text{their } m \times x + c$ then subst (6, −6) eg **A1** for $4y = 3x - 42$ allow **B4** for correct equation www isw
11 (iv)	centre C is at (12, −14) cao circle is $(x-12)^2 + (y+14)^2 = 100$	**B2** **B1**	B1 for each coord ft their C if at least one coord correct

12 (i)	10	**1**	
12 (ii)	[x =] 5 or ft their (i) ÷ 2 ht = 5[m] cao	**1** **1**	not necessarily ft from (i) eg they may start again with calculus to get $x = 5$
12 (iii)	$d = 7/2$ o.e. [y =] $1/5 \times 3.5 \times (10 - 3.5)$ o.e. or ft = 91/20 o.e. cao isw	**M1** **M1** **A1**	or ft their (ii) − 1.5 or their (i) ÷ 2 − 1.5 o.e. or $7 - 1/5 \times 3.5^2$ or ft or showing $y - 4 = 11/20$ o.e. cao
12 (iv)	$4.5 = 1/5 \times x(10 - x)$ o.e. $22.5 = x(10 - x)$ o.e. $2x^2 - 20x + 45$ [= 0] o.e. eg $x^2 - 10x + 22.5$ [=0] or $(x-5)^2 = 2.5$ $[x =]\dfrac{20 \pm \sqrt{40}}{4}$ or $5 \pm \dfrac{1}{2}\sqrt{10}$ o.e. width = $\sqrt{10}$ o.e. eg $2\sqrt{2.5}$ cao	**M1** **M1** **A1** **M1** **A1**	 eg $4.5 = x(2 - 0.2x)$ etc cao; accept versions with fractional coefficients of x^2, isw or $x - 5 = [\pm]\sqrt{2.5}$ o.e.; ft their quadratic eqn provided at least M1 gained already; condone one error in formula or substitution; need not be simplified or be real accept simple equivalents only