GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Mark Schemes for the Units

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MARK SCHEME FOR THE UNITS

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4751 (C1) Introduction to Advanced Mathematics

Section A

1	$[v =][\pm]\sqrt{\frac{2E}{m}}$ www	3	M2 for $v^2 = \frac{2E}{m}$ or for $[v =][\pm]\sqrt{\frac{E}{\frac{1}{2}m}}$ or M1 for a correct constructive first step and M1 for $v = [\pm]\sqrt{k}$ ft their $v^2 = k$; if M0 then SC1 for $\surd E/ \frac{1}{2} m$ or $\surd 2E/m$ etc	3
2	$\frac{3x-4}{x+1}$ or $3-\frac{7}{x+1}$ www as final answer	3	M1 for $(3x - 4)(x - 1)$ and M1 for $(x + 1)(x - 1)$	3
3	(i) 1	1		
	(ii) 1/64 www	3	M1 for dealing correctly with each of reciprocal, square root and cubing (allow 3 only for 1/64) eg M2 for 64 or −64 or $1/\surd 4096$ or $\frac{1}{4}^3$ or M1 for $1/16^{3/2}$ or 4^3 or -4^3 or 4^{-3} etc	4
4	$6x + 2(2x - 5) = 7$	M1	for subst or multn of eqns so one pair of coeffts equal (condone one error)	
	$10x = 17$	M1	simplification (condone one error) or appropriate addn/subtn to eliminate variable	
	$x = 1.7$ o.e. isw	A1	allow as separate or coordinates as requested	
	$y = -1.6$ o.e .isw	A1	graphical soln: M0	4
5	(i) −4/5 or −0.8 o.e.	2	M1 for 4/5 or 4/−5 or 0.8 or −4.8/6 or correct method using two points on the line (at least one correct) (may be graphical) or for $-0.8x$ o.e.	
	(ii) (15, 0) or 15 found www	3	M1 for y = their (i) $x + 12$ o.e. or $4x + 5y = k$ and (0, 12) subst and M1 for using $y = 0$ eg $-12 = -0.8x$ or ft their eqn <u>or</u> M1 for given line goes through (0, 4.8) and (6, 0) and M1 for 6 × 12/4.8 graphical soln: allow M1 for correct required line drawn and M1 for answer within 2mm of (15, 0)	5

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6	f(2) used $2^3 + 2k + 7 = 3$ $k = -6$	M1 M1 A1	or division by $x - 2$ as far as $x^2 + 2x$ obtained correctly or remainder 3 = 2(4 + k) + 7 o.e. 2nd M1 dep on first	3
7	(i) 56 (ii) −7 or ft from −their (i)/8	2 2	M1 for $\frac{8\times7\times6}{3\times2\times1}$ or more simplified M1 for 7 or ft their (i)/8 or for 56 × $(-1/2)^3$ o.e. or ft; condone x^3 in answer or in M1 expression; 0 in qn for just Pascal's triangle seen	4
8	(i) 5√3 (ii) common denominator = (5 − √2)(5 + √2) =23 numerator = 10	2 M1 A1 B1	M1 for √48 = 4√3 allow M1A1 for $\frac{5-\sqrt{2}}{23}+\frac{5+\sqrt{2}}{23}$ allow 3 only for 10/23	5
9	(i) $n = 2m$ $3n^2 + 6n = 12m^2 + 12m$ or $= 12m(m + 1)$ (ii) showing false when n is odd e.g. $3n^2 + 6n$ = odd + even = odd	M1 M2 B2	or any attempt at generalising; M0 for just trying numbers or M1 for $3n^2 + 6n = 3n(n + 2)$ = 3 × even × even and M1 for explaining that 4 is a factor of even × even or M1 for 12 is a factor of 6n when n is even and M1 for 4 is a factor of n^2 so 12 is a factor of $3n^2$ or $3n(n + 2)$ =3 × odd × odd = odd or counterexample showing not always true; M1 for false with partial explanation or incorrect calculation	5

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Section B

10	**i**	correct graph with clear asymptote $x = 2$ (though need not be marked)	G2	G1 for one branch correct; condone (0, − ½) not shown SC1 for both sections of graph shifted two to left		
		(0, − ½) shown	G1	allow seen calculated	3	
	ii	11/5 or 2.2 o.e. isw	2	M1 for correct first step	2	
	iii	$x = \frac{1}{x-2}$	M1	or equivs with *y*s		
		$x(x - 2) = 1$ o.e.	M1			
		$x^2 - 2x - 1$ [= 0]; ft their equiv eqn	M1	or $(x - 1)^2 - 1 = 1$ o.e.		
		attempt at quadratic formula	M1	or $(x - 1) = \pm\sqrt{2}$ (condone one error)		
		$1 \pm\sqrt{2}$ cao	A1			
		position of points shown	B1	on their curve with $y = x$ (line drawn or $y = x$ indicated by both coords); condone intent of diagonal line with gradient approx 1through origin as $y = x$ if unlabelled	6	11
11	**i**	$(x - 2.5)^2$ o.e.	M1			
		$- 2.5^2 + 8$	M1	for clear attempt at -2.5^2		
		$(x - 2.5)^2 + 7/4$ o.e.	A1	allow M2A0 for $(x - 2.5) + 7/4$ o.e. with no $(x - 2.5)^2$ seen		
		min $y = 7/4$ o.e. [so above x axis] or commenting $(x - 2.5)^2 \geq 0$	B1	ft, dep on $(x - a)^2 + b$ with b positive; condone starting again, showing $b^2 - 4ac < 0$ or using calculus	4	
	ii	correct symmetrical quadratic shape	G1			
		8 marked as intercept on y axis	G1	or (0, 8) seen in table		
		tp (5/2, 7/4) o.e. or ft from (i)	G1		3	
	iii	$x^2 - 5x - 6$ seen or used	M1	or $(x - 2.5)^2$ [> or =] 12.25 or ft $14 - b$		
		−1 and 6 obtained	M1	also implies first M1		
		$x < -1$ and $x > 6$ isw or ft their solns	M1	if M0, allow B1 for one of $x < -1$ and $x > 6$	3	
	iv	min = (2.5, − 8.25) or ft from (i)	M1	or M1 for other clear comment re translated 10 down and A1 for referring to min in (i) or graph in (ii); or M1 for correct method for solving $x^2 - 5x - 2 = 0$ or using $b^2 - 4ac$ with this and A1 for showing real solns eg $b^2 - 4ac = 33$; allow M1A0 for valid comment but error in −8.25 ft; allow M1 for showing y can be neg eg (0, −2) found and A1 for correct conclusion		
		so yes, crosses	A1		2	12

12	**i**	$(x - 4)^2 - 16 + (y - 2)^2 - 4 = 9$ o.e.	M2	M1 for one completing square or for $(x - 4)^2$ or $(y - 2)^2$ expanded correctly or starting with $(x - 4)^2 + (y - 2)^2 = r^2$: M1 for correct expn of at least one bracket and M1 for $9 + 20 = r^2$ o.e.	
		rad = $\sqrt{29}$	B1	or using $x^2 - 2gx + y^2 - 2fy + c = 0$ M1 for using centre is (g, f) [must be quoted] and M1 for $r^2 = g^2 + f^2 - c$	3
	ii	$4^2 + 2^2$ o.e = 20 which is less than 29	M1 A1	allow 2 for showing circle crosses x axis at −1 and 9 or equiv for y (or showing one positive; one negative); 0 for graphical solutions (often using A and B from (iii) to draw circle)	2
	iii	showing midpt of AB = (4, 2) and	2	in each method, two things need to be established. Allow M1 for the concept of what should be shown and A1 for correct completion with method shown	
		showing AB = $2\sqrt{29}$ or showing AC or BC = $\sqrt{29}$ or that A or B lie on circle	2		
		or showing both A and B lie on circle (or AC = BC = $\sqrt{29}$), and	2	allow M1A0 for AB just shown as $\sqrt{116}$ not $2\sqrt{29}$	
		showing AB = $2\sqrt{29}$ or that C is midpt of AB or that C is on AB or that gradients of AB and AC are the same or equiv.	2	allow M1A0 for stating mid point of AB = (4,2) without working/method shown NB showing AB = $2\sqrt{29}$ and C lies on AB is not sufficient – earns 2 marks only	
		or showing C is on AB and	2		
		showing both A and B are on circle or AC = BC = $\sqrt{29}$	2	if M0, allow SC2 for accurate graph of circle drawn with compasses and AB joined with ruled line through C.	4
	iv	grad AC or AB or BC = −5/2 o.e.	M1	may be seen in (iii) but only allow this M1 if they go on to use in this part	
		grad tgt = −1/their grad AC	M1	allow for $m_1m_2=-1$ used	
		tgt is $y - 7 =$ their $m(x - 2)$ o.e.	M1	eg $y =$ their $mx + c$ then (2, 7) subst; M0 if grad AC used	
		$y = 2/5\,x + 31/5$ o.e.	A1	condone $y = 2/5x + c$ and $c = 31/5$ o.e.	4

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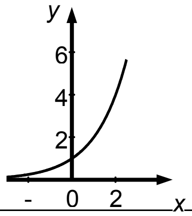
4752 (C2) Concepts for Advanced Mathematics

Section A

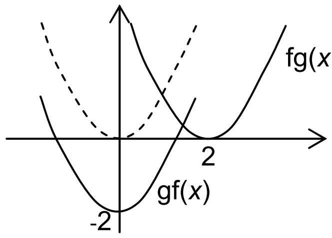
1	$40x^3$	2	-1 if extra term	2
2	(i) 3	1		
	(ii) 141	2	M1 for 9 × (1 + 2 + 3 + 4 + 5) + 1 + 2 +3	3
3	right angled triangle with 1 and 2 on correct sides	M1	or M1 for sinθ = ½cosθ and M1 for substituting in $\sin^2\theta + \cos^2\theta = 1$	
	Pythagoras used to obtain hyp = √5	M1	E1 for sufficient working	
	$\cos\theta = \frac{a}{h} = \frac{2}{\sqrt{5}}$	A1		3
4	(i)line along $y = 6$ with V (1, 6), (2, 2), (3, 6)	2	1 for two points correct	
	(ii) line along $y = 3$ with V (−2,3), (−1,1), (0,3)	2	1 for two points correct	4
5	$2x^6 + \frac{3}{4}x^{\frac{4}{3}} + 7x + c$	5	1 for $2x^6$; 2 for $\frac{3}{4}x^{\frac{4}{3}}$ or 1 for other $kx^{\frac{4}{3}}$; 1 for $7x$; 1 for $+c$	5
6	(i) correct sine shape through O	1		
	amplitude of 1 and period 2π shown	1		
	(ii) $7\pi/6$ and $11\pi/6$	3	B2 for one of these; 1 for $-\pi/6$ found	5
7	(i) 60	2	M1 for $2^2 + 2^3 + 2^4 + 2^5$ o.e.	
	(ii) −6	1		
	(iii) 	1 1	Correct in both quadrants Through (0, 1) shown dep.	5
8	$r = 1/3$ s.o.i.	2	1 mark for ar = 18 *and* $ar^3 = 2$ s.o.i.	
	$a = 54$ or ft 18 ÷ their r	M1		
	$S = \frac{a}{1-r}$ used with $-1 < r < 1$	M1		
	S = 81 c.a.o.	A1		5
9	(i) 0.23 c.a.o.	1		
	(ii) 0.1 or 1/10	1	10^{-1} not sufficient	
	(iii) $4(3x + 2)$ or $12x + 8$	1		
	(iv) [y =] 10^{3x+2} o.e.	1		4

Section B

10	**i**	$h = 120/x^2$	B1		
		$A = 2x^2 + 4xh$ o.e.	M1		
		completion to given answer	A1	at least one interim step shown	3
	ii	$A' = 4x - 480/x^2$ o.e.	2	1 for kx^{-2} o.e. included	
		$A'' = 4 + 960 / x^3$	2	ft their A' only if kx^{-2} seen ; 1 if one error	4
	iii	use of $A' = 0$	M1		
		$x = \sqrt[3]{120}$ or 4.9(3..)	A1		
		Test using A' or A'' to confirm minimum	T1		
		Substitution of their x in A	M1	Dependent on previous M1	
		A = 145.9 to 146	A1		5
11	**iA**	$BC^2 = 348^2 + 302^2 - 2 \times 348 \times 302 \times \cos 72°$	M2	M1 for recognisable attempt at Cosine Rule	
		BC = 383.86…	A1	to 3 sf or more	
		1033.86…[m] or ft 650 + their BC	1	accept to 3 sf or more	4
	iB	$\frac{\sin B}{302} = \frac{\sin 72}{\text{their } BC}$	M1	Cosine Rule acceptable or Sine Rule to find C	
		B = 48.4..	A1		
		355 − their B o.e.	M1	or 247 + their C	
		answer in range 306 to307	A1		4
	ii	Arc length PQ = $\frac{224}{360}\times 2\pi\times 120$ o.e. or 469.1... to 3 sf or more	M2	M1 for $\frac{136}{360}\times 2\pi\times 120$	
		QP = 222.5…to 3 sf or more	B1		
		answer in range 690 to 692 [m]	A1		4
12	**iA**	$x^4 = 8x$	M1		
		(2, 16) c.a.o.	A1		
		PQ = 16 and completion to show ½ × 2 × 16 = 16	A1	NB answer 16 given	3
	iB	$x^5/5$	M1		
		evaluating their integral at their co-ord of P and zero [or 32/5 o.e.]	M1	ft only if integral attempted, not for x^4 or differentiation	
		9.6 o.e.	A1	c.a.o.	3
	iiA	$6x^2h^2 + 4xh^3 + h^4$	2	B1 for two terms correct.	2
	iiB	$4x^3 + 6x^2h + 4xh^2 + h^3$	2	B1 for three terms correct	2
	iiC	$4x^3$	1		1
	iiD	gradient of [tangent to] curve	1		1

4753 (C3) Methods for Advanced Mathematics

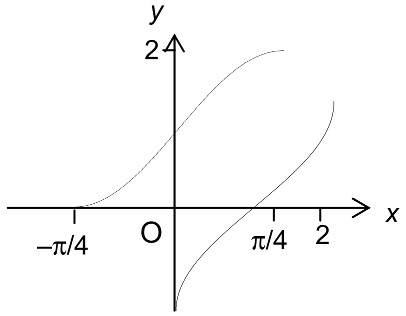
Section A

1 $y=(1+6x^2)^{1/3}$ $\Rightarrow \frac{dy}{dx}=\frac{1}{3}(1+6x^2)^{-2/3}.12x$ $=4x(1+6x^2)^{-2/3}$	M1 B1 A1 A1 [4]	chain rule used $\frac{1}{3}u^{-2/3}$ ×12*x* cao (must resolve 1/3 × 12) Mark final answer
2 (i) fg(*x*) = f(*x* – 2) = (*x* – 2)^2 gf(*x*) = g(*x*^{2}) = *x*^{2} – 2.	M1 A1 A1 [3]	forming a composite function mark final answer If fg and gf the wrong way round, M1A0A0
(ii) 	B1ft B1ft [2]	fg – must have (2, 0)labelled (or inferable from scale). Condone no *y*-intercept, unless wrong gf – must have (0, –2) labelled (or inferable from scale) Condone no x-intercepts, unless wrong Allow ft only if fg and gf are correct but wrong way round.
3 (i) When *n* = 1, 10 000 = *A* e^b when *n* = 2, 16 000 = *A* e^{2b} $\Rightarrow \frac{16000}{10000}=\frac{Ae^{2b}}{Ae^b}=e^b$ $\Rightarrow$ e^b = 1.6 $\Rightarrow$ *b* = ln 1.6 = 0.470 *A* = 10000/1.6 = 6250.	B1 B1 M1 E1 B1 B1 [6]	soi soi eliminating *A* (do not allow verification) SCB2 if initial 'B's are missing, and ratio of years = 1.6 = e^b ln 1.6 or 0.47 or better (mark final answer) cao – allow recovery from inexact *b*'s
(ii) When *n* = 20, *P* = 6250×e$^{0.470×20}$ = £75,550,000	M1 A1 [2]	substituting *n* = 20 into their equation with their *A* and *b* Allow answers from £75 000 000 to £76 000 000.
4 (i) 5 = *k*/100 ⇒ *k* = 500*	E1 [1]	NB answer given
(ii) $\frac{dP}{dV}=-500V^{-2}=-\frac{500}{V^2}$	M1 A1 [2]	(–1)V^{-2} o.e. – allow –*k*/V^2
(iii) $\frac{dP}{dt}=\frac{dP}{dV}.\frac{dV}{dt}$ When *V* = 100, d*P/dV* = –500/10000 = –0.05 d*V*/d*t* = 10 $\Rightarrow$ d*P*/d*t* = –0.05 × 10 = –0.5 So P is decreasing at 0.5 Atm/s	M1 B1ft B1 A1 [4]	chain rule (any correct version) (soi) (soi) –0.5 cao

5(i) $p = 2$, $2^p - 1 = 3$, prime $p = 3$, $2^p - 1 = 7$, prime $p = 5$, $2^p - 1 = 31$, prime $p = 7$, $2^p - 1 = 127$, prime	M1 E1 [2]	Testing at least one prime testing all 4 primes (correctly) Must comment on answers being prime (allow ticks) Testing $p = 1$ is E0
(ii) $23\times 89 = 2047 = 2^{11} - 1$ 11 is prime, 2047 is not So statement is false.	M1 E1 [2]	$2^{11} - 1$ must state or imply that 11 is prime ($p = 11$ is sufficient)
6 (i) $e^{2y} = x^2 + y$ $\Rightarrow$ $2e^{2y}\frac{dy}{dx} = 2x + \frac{dy}{dx}$ $\Rightarrow$ $(2e^{2y} - 1)\frac{dy}{dx} = 2x$ $\Rightarrow$ $\frac{dy}{dx} = \frac{2x}{2e^{2y} - 1}$ *	M1 A1 M1 E1 [4]	Implicit differentiation – allow one slip (but with dy/dx both sides) collecting terms
(ii) Gradient is infinite when $2e^{2y} - 1 = 0$ $\Rightarrow$ $e^{2y} = ½$ $\Rightarrow$ $2y = \ln ½$ $\Rightarrow$ $y = ½ \ln ½ = –0.347$ (3 s.f.) $x^2 = e^{2y} – y = ½ – (–0.347)$ $= 0.8465$ $\Rightarrow$ $x = 0.920$	M1 A1 M1 A1 [4]	 must be to 3 s.f. substituting their y and solving for x cao – must be to 3 s.f., but penalise accuracy once only.

Section B

7(i) $y = 2x\ln(1+x)$ $\Rightarrow \quad \frac{dy}{dx} = \frac{2x}{1+x} + 2\ln(1+x)$ When $x = 0$, $dy/dx = 0 + 2\ln 1 = 0$ $\Rightarrow$ origin is a stationary point.	M1 B1 A1 E1 [4]	product rule $d/dx(\ln(1+x)) = 1/(1+x)$ soi www (i.e. from correct derivative)
(ii) $\frac{d^2y}{dx^2} = \frac{(1+x).2 - 2x.1}{(1+x)^2} + \frac{2}{1+x}$ $= \frac{2}{(1+x)^2} + \frac{2}{1+x}$ When $x = 0$, $d^2y/dx^2 = 2 + 2 = 4 > 0$ $\Rightarrow$ (0, 0) is a min point	M1 A1ft A1 M1 E1 [5]	Quotient or product rule on their $2x/(1+x)$ correctly applied to their $2x/(1+x)$ o.e., e.g. $\frac{4+2x}{(1+x)^2}$ cao substituting $x = 0$ into their d^2y/dx^2 www – dep previous A1
(iii) Let $u = 1 + x \Rightarrow du = dx$ $\Rightarrow \int \frac{x^2}{1+x}dx = \int \frac{(u-1)^2}{u}du$ $= \int \frac{(u^2 - 2u + 1)}{u}du$ $= \int (u - 2 + \frac{1}{u})du$ * $\Rightarrow \int_0^1 \frac{x^2}{1+x}dx = \int_1^2 (u - 2 + \frac{1}{u})du$ $= \left[\frac{1}{2}u^2 - 2u + \ln u\right]_1^2$ $= 2 - 4 + \ln 2 - (½ - 2 + \ln 1)$ $= \ln 2 - ½$	M1 E1 B1 B1 M1 A1 [6]	$\frac{(u-1)^2}{u}$ www (but condone du omitted except in final answer) changing limits (or substituting back for x and using 0 and 1) $\left[\frac{1}{2}u^2 - 2u + \ln u\right]$ substituting limits (consistent with u or x) cao
(iv) $A = \int_0^1 2x\ln(1+x)dx$ Parts: $u = \ln(1+x)$, $du/dx = 1/(1+x)$ $dv/dx = 2x \Rightarrow v = x^2$ $= \left[x^2\ln(1+x)\right]_0^1 - \int_0^1 \frac{x^2}{1+x}dx$ $= \ln 2 - \ln 2 + ½ = ½$	M1 A1 M1 A1 [4]	soi substituting their $\ln 2 - ½$ for $\int_0^1 \frac{x^2}{1+x}dx$ cao

8 (i) Stretch in x-direction s.f. ½ translation in y-direction 1 unit up	M1 A1 M1 A1 [4]	(in either order) – allow 'contraction' dep 'stretch' allow 'move', 'shift', etc – direction can be inferred from $\begin{pmatrix}0\\1\end{pmatrix}$ or $\begin{pmatrix}0\\1\end{pmatrix}$ dep 'translation'. $\begin{pmatrix}0\\1\end{pmatrix}$ alone is M1 A0
(ii) $A=\int_{-\pi/4}^{\pi/4}(1+\sin 2x)dx$ $=\left[x-\frac{1}{2}\cos 2x\right]_{-\pi/4}^{\pi/4}$ $= \pi/4 - ½ \cos \pi/2 + \pi/4 + ½ \cos (-\pi/2)$ $= \pi/2$	M1 B1 M1 A1 [4]	correct integral and limits. Condone dx missing; limits may be implied from subsequent working. substituting their limits (if zero lower limit used, must show evidence of substitution) or 1.57 or better – cao (www)
(iii) $y = 1 + \sin 2x$ $\Rightarrow$ $dy/dx = 2\cos 2x$ When $x = 0$, $dy/dx = 2$ So gradient at (0, 1) on f(x) is 2 $\Rightarrow$ gradient at (1, 0) on $f^{-1}(x) = ½$	M1 A1 A1ft B1ft [4]	differentiating – allow 1 error (but not $x + 2\cos 2x$) If 1, then must show evidence of using reciprocal, e.g. 1/1
(iv) Domain is $0 \le x \le 2$. 	B1 M1 A1 [3]	Allow 0 to 2, but not $0 < x < 2$ or y instead of x clear attempt to reflect in $y = x$ correct domain indicated (0 to 2), and reasonable shape
(v) $y = 1 + \sin 2x \quad x \leftrightarrow y$ $x = 1 + \sin 2y$ $\Rightarrow$ $\sin 2y = x - 1$ $\Rightarrow$ $2y = \arcsin(x - 1)$ $\Rightarrow$ $y = ½ \arcsin(x - 1)$	M1 A1 [2]	or $\sin 2x = y - 1$ cao

4754 (C4) Applications of Advanced Mathematics

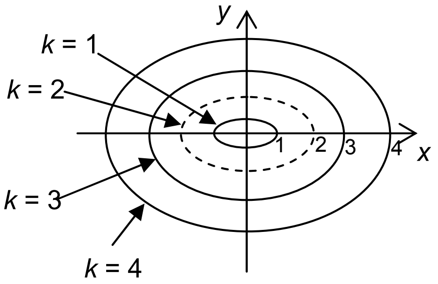
Section A

1 $3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$ $= R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 3\ ,\ R\sin\alpha = 4$ $\Rightarrow R^2 = 3^2 + 4^2 = 25 \Rightarrow R = 5$ $\tan\alpha = 4/3 \Rightarrow \alpha = 0.9273$ $5\cos(\theta - 0.9273) = 2$ $\Rightarrow \cos(\theta - 0.9273) = 2/5$ $\theta - 0.9273 = 1.1593, -1.1593$ $\Rightarrow \theta = 2.087, -0.232$	M1 B1 M1A1 M1 A1 A1 [7]	 $R = 5$ cwo and no others in the range
2(i) $(1-2x)^{-\frac{1}{2}} = 1 - \frac{1}{2}(-2x) + \frac{(-\frac{1}{2}).(-\frac{3}{2})}{2!}(-2x)^2 + \ldots$ $= 1 + x + \frac{3}{2}x^2 + \ldots$ Valid for $-1 < -2x < 1 \Rightarrow -½ < x < ½$	M1 A1 A1 M1 A1 [5]	binomial expansion with $p = -½$ correct expression cao
(ii) $\frac{1+2x}{\sqrt{1-2x}} = (1+2x)(1+x+\frac{3}{2}x^2+\ldots)$ $= 1 + x + \frac{3}{2}x^2 + 2x + 2x^2 + \ldots$ $= 1 + 3x + \frac{7}{2}x^2 + \ldots$	M1 A1ft A1 [3]	substitituting their $1+x+\frac{3}{2}x^2+\ldots$ and expanding cao
3 $V = \int_1^2 \pi x^2 dy$ $y = 1 + x^2 \Rightarrow x^2 = y - 1$ $\Rightarrow V = \int_1^2 \pi(y-1)dy$ $= \pi\left[\frac{1}{2}y^2 - y\right]_1^2$ $= \pi(2 - 2 - ½ + 1)$ $= ½\pi$	B1 M1 B1 M1 A1 [5]	 $\left[\frac{1}{2}y^2 - y\right]$ substituting limits into integrand

4(i) $\sin(\theta + 45°) = \cos\theta$ $\Rightarrow \sin\theta\cos 45 + \cos\theta\sin 45 = \cos\theta$ $\Rightarrow (1/\surd 2)\sin\theta + (1/\surd 2)\cos\theta = \cos\theta$ $\Rightarrow \sin\theta + \cos\theta = \surd 2\cos\theta$ $\Rightarrow \sin\theta = (\surd 2 - 1)\cos\theta$ $\Rightarrow \frac{\sin\theta}{\cos\theta} = \tan\theta = \surd 2 - 1$ *	M1 B1 A1 M1 E1 [5]	compound angle formula sin 45 = 1/√2, cos 45 = 1/√2 collecting terms
(ii) $\tan\theta = \surd 2 - 1$ $\Rightarrow \theta = 22.5°,$ $202.5°$	B1 B1 [2]	and no others in the range
5 $\frac{4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$ $= \frac{A(x^2+4)+(Bx+C)x}{x(x^2+4)}$ $\Rightarrow 4 = A(x^2+4)+(Bx+C)x$ $x = 0 \Rightarrow 4 = 4A \Rightarrow A = 1$ coefft of x^2: $0 = A + B \Rightarrow B = -1$ coeffts of x: $0 = C$ $\Rightarrow \frac{4}{x(x^2+4)} = \frac{1}{x} - \frac{x}{x^2+4}$	M1 M1 B1 DM1 A1 A1 [6]	correct partial fractions A=1 Substitution or equating coeffts B= –1 C= 0
6 $\text{cosec}\,\theta = 3$ $\Rightarrow \sin\theta = 1/3$ $\Rightarrow \theta = 19.47°,$ $160.53°$	M1 A1 A1 [3]	and no others in the range

Section B

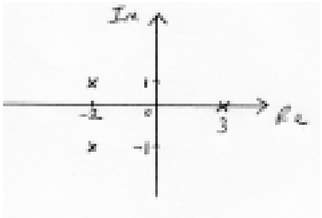
7(i) $\overrightarrow{CD} = \begin{pmatrix} -6 \\ 6 \\ 24 \end{pmatrix}$ $\overrightarrow{CB} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix}$.	B1 B1 [2]	
(ii) $\sqrt{(-6)^2 + 6^2 + 24^2}$ = 25.46 cm	M1 A1 [2]	
(iii) $\overrightarrow{CD}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 6 \\ 24 \end{pmatrix}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = -24 + 0 + 24 = 0$ $\overrightarrow{CB}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = 0 + 0 + 0 = 0$ $\Rightarrow$ plane BCDE is $4x + z = c$ At C (say) $4 \times 15 + 0 = c \Rightarrow c = 60$ $\Rightarrow$ plane BCDE is $4x + z = 60$	M1 A1 B1 M1 A1 [5]	using scalar product or other equivalent methods
(iv) OG: $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 6 \\ 24 \end{pmatrix}$ AF: $\mathbf{r} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -6 \\ 24 \end{pmatrix}$ At (5, 10, 40) , $3\lambda = 5 \Rightarrow \lambda = 5/3$ $\Rightarrow 6\lambda = 10$, $24\lambda = 40$, so consistent. At (5, 10, 40) , $3\mu = 5 \Rightarrow \mu = 5/3$ $\Rightarrow 20 - 6\mu = 10$, $24\mu = 40$, so consistent. So lines meet at (5, 10, 40)*	B1 B1 M1 E1 E1 [5]	evaluating parameter and checking consistency. [or other methods, e.g. solving]
(v) h=40 POABC: $V = 1/3 \times 20 \times 15 \times 40$ $= 4000$ cm^3. PDEFG: $V = 1/3 \times 8 \times 6 \times (40-24)$ $= 256$ cm^3 $\Rightarrow$ vol of ornament = 4000 − 256 = 3744 cm^3	B1 M1 A1 A1 [4]	soi 1/3 x w x d x h used for either –condone one error both volumes correct cao

8(i) $\cos\theta = \frac{x}{k}, \sin\theta = \frac{2y}{k}$ $\cos^2\theta + \sin^2\theta = 1$ $\Rightarrow \left(\frac{x}{k}\right)^2 + \left(\frac{2y}{k}\right)^2 = 1$ $\Rightarrow \frac{x^2}{k^2} + \frac{4y^2}{k^2} = 1$ $\Rightarrow x^2 + 4y^2 = k^2$ *	M1 M1 E1 [3]	Used substitution
(ii) $\frac{dx}{d\theta} = -k\sin\theta, \frac{dy}{d\theta} = \frac{1}{2}k\cos\theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{\frac{1}{2}k\cos\theta}{k\sin\theta}$ $= -\frac{1}{2}\cot\theta$ $-\frac{x}{4y} = -\frac{2k\cos\theta}{4k\sin\theta} = -\frac{1}{2}\cot\theta = \frac{dy}{dx}$	M1 A1 E1	oe
or, by differentiating implicitly $2x + 8y\, dy/dx = 0$ $\Rightarrow dy/dx = -2x/8y = -x/4y$*	M1 A1 E1 [3]	
(iii) $k = 2$	B1 [1]	
(iv) 	B1 B1 B1 [3]	1 correct curve –shape and position 2 or more curves correct shape- in concentric form all 3 curves correct
(v) grad of stream path = –1/grad of contour $\Rightarrow \frac{dy}{dx} = -\frac{1}{(-x/4y)} = \frac{4y}{x}$ *	M1 E1 [2]	
(vi) $\frac{dy}{dx} = \frac{4y}{x} \Rightarrow \int \frac{dy}{y} = \int \frac{4dx}{x}$ $\Rightarrow \ln y = 4\ln x + c = \ln e^c x^4$ $\Rightarrow y = Ax^4$ where $A = e^c$. When $x = 2, y = 1 \Rightarrow 1 = 16A \Rightarrow A = 1/16$ $\Rightarrow y = x^4/16$ *	M1 A1 M1 M1 A1 E1 [6]	Separating variables $\ln y = 4\ln x\ (+c)$ antilogging correctly (at any stage) substituting $x = 2, y = 1$ evaluating a correct constant www

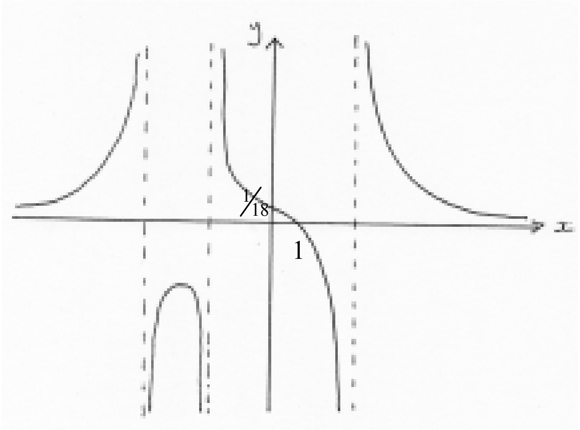
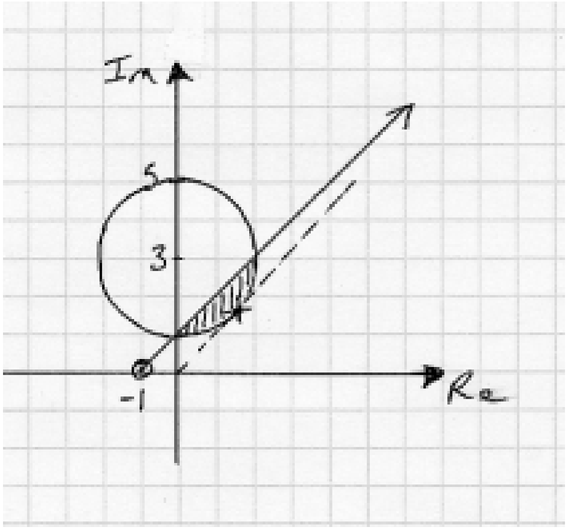
Paper B Comprehension 4754 (C4)

1	4, 1, 5, 6, 11, 17	B1 B1	for 11 and 17 for 1 and 4
2	Even, odd, odd, even, odd, odd recurs 100th term is therefore even	M1 A1	for reason www
3	$\phi^6 = (3\phi+2)+(5\phi+3) = 8\phi+5$	B1	
4	$1-EH = 1-CG = 1-(\phi-1)$ $= 2-\phi = 2-\left(\frac{1+\sqrt{5}}{2}\right)$ $= \frac{3-\sqrt{5}}{2}$	M1 A1 A1	oe
5	(i)Gradients $-\frac{1}{\phi}$ and $\frac{1}{\phi-1}$ (ii) Product of gradients: $-\frac{1}{\phi}\times\frac{1}{\phi-1} = -\frac{1}{\phi^2-\phi}$ $= -\frac{1}{1} = -1$	B1 B1 M1 E1	
6	$\frac{\phi+1}{2\phi-1} = \frac{\frac{1+\sqrt{5}}{2}+1}{1+\sqrt{5}-1}$ $= \frac{3+\sqrt{5}}{2\sqrt{5}}$ $= \frac{(3+\sqrt{5})\sqrt{5}}{2\sqrt{5}\times\sqrt{5}} = \frac{3\sqrt{5}+5}{10}$	M1 A1 E1	
7	$a+(a+d) = a+2d \quad \Rightarrow \quad a=d$ $(a+d)+(a+2d) = a+3d \quad \Rightarrow \quad a=0$ $a=d=0$ *	M1 M1 E1 [18]	

4755 (FP1) Further Concepts for Advanced Mathematics

Qu	Answer	Mark	Comment
Section A			
1(i)	$\mathbf{BA} = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ -4 & 14 \end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply c.a.o.
1(ii)	$\det \mathbf{BA} = (6\times 14) - (-4\times 0) = 84$ $3\times 84 = 252$ square units	M1 A1 A1(ft) **[3]**	Attempt to calculate any determinant c.a.o. Correct area
2(i)	$\alpha^2 = (-3+4\mathrm{j})(-3+4\mathrm{j}) = (-7-24\mathrm{j})$	M1 A1 **[2]**	Attempt to multiply with use of $\mathrm{j}^2 = -1$ c.a.o.
2(ii)	$\lvert\alpha\rvert = 5$	B1	
	$\arg\alpha = \pi - \arctan\frac{4}{3}$ = 2.21 (2d.p.) (or 126.87°)	B1	Accept 2.2 or 127°
	$\alpha = 5(\cos 2.21 + \mathrm{j}\sin 2.21)$	B1(ft) **[3]**	Accept degrees and $(r,\ \theta)$ form s.c. lose 1 mark only if α^2 used throughout (ii)
3(i)	$3^3 + 3^2 - 7\times 3 - 15 = 0$	B1	Showing 3 satisfies the equation (may be implied)
	$z^3 + z^2 - 7z - 15 = (z-3)(z^2+4z+5)$	M1 A1	Valid attempt to factorise Correct quadratic factor
	$z = \dfrac{-4\pm\sqrt{16-20}}{2} = -2\pm\mathrm{j}$	M1	Use of quadratic formula, or other valid method
	So other roots are $-2+\mathrm{j}$ and $-2-\mathrm{j}$	A1 **[5]**	One mark for both c.a.o.
3(ii)		B2 **[2]**	Minus 1 for each error ft provided conjugate imaginary roots

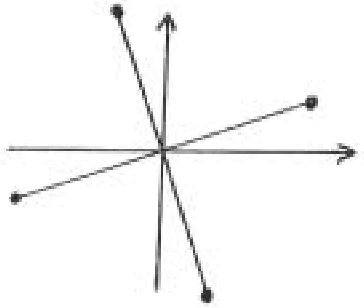
4	$\sum_{r=1}^{n}\left[(r+1)(r-2)\right] = \sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r - 2n$	M1	Attempt to split sum up
	$= \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - 2n$	A2	Minus one each error
	$= \frac{1}{6}n\left[(n+1)(2n+1) - 3(n+1) - 12\right]$	M1	Attempt to factorise
	$= \frac{1}{6}n\left(2n^2 + 3n + 1 - 3n - 3 - 12\right)$	M1	Collecting terms
	$= \frac{1}{6}n\left(2n^2 - 14\right)$		
	$= \frac{1}{3}n\left(n^2 - 7\right)$	A1 **[6]**	All correct
5(i)	$p = -3,\ r = 7$	B2 **[2]**	One mark for each s.c. B1 if *b* and *d* used instead of *p* and *r*
5(ii)	$q = \alpha\beta + \alpha\gamma + \beta\gamma$	B1	
	$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$ $= (\alpha + \beta + \gamma)^2 - 2q$ $\Rightarrow 13 = 3^2 - 2q$	M1	Attempt to find *q* using $\alpha^2 + \beta^2 + \gamma^2$ and $\alpha + \beta + \gamma$, but not $\alpha\beta\gamma$
	$\Rightarrow q = -2$	A1 **[3]**	c.a.o.
6(i)	$a_2 = 7 \times 7 - 3 = 46$	M1	Use of inductive definition
	$a_3 = 7 \times 46 - 3 = 319$	A1 **[2]**	c.a.o.
6(ii)	When *n* = 1, $\frac{13 \times 7^0 + 1}{2} = 7$, so true for *n* = 1	B1	Correct use of part (i) (may be implied)
	Assume true for *n* = *k* $a_k = \frac{13 \times 7^{k-1} + 1}{2}$	E1	Assuming true for *k*
	$\Rightarrow a_{k+1} = 7 \times \frac{13 \times 7^{k-1} + 1}{2} - 3$ $= \frac{13 \times 7^k + 7}{2} - 3$ $= \frac{13 \times 7^k + 7 - 6}{2}$	M1	Attempt to use $a_{k+1} = 7a_k - 3$
	$= \frac{13 \times 7^k + 1}{2}$	A1	Correct simplification
	But this is the given result with *k* + 1 replacing *k*. Therefore if it is true for *k* it is true for *k* + 1.	E1	Dependent on A1 and previous E1
	Since it is true for *k* = 1, it is true for *k* = 1, 2, 3 and so true for all positive integers.	E1 **[6]**	Dependent on B1 and previous E1
			Section A Total: 36

Section B			
7(i)	$(1,\ 0)$ and $(0,\ \frac{1}{18})$	B1 B1 **[2]**	
7(ii)	x = 2, $x=-3,\ x=\frac{-3}{2}, y=0$	B4 **[4]**	Minus 1 for each error
7(iii)		B1 B1 **[2]**	Correct approaches to vertical asymptotes Through clearly marked $(1,\ 0)$ and $(0,\ \frac{1}{18})$
7(iv)	$x<-3$, $x>2$ $\frac{-3}{2}<x\le 1$	B1 B2 **[3]**	 B1 for $\frac{-3}{2}<x<1$, or $\frac{-3}{2}\le x\le 1$
8(i)		B3 B3 **[6]**	Circle, B1; radius 2, B1; centre 3j, B1 Half line, B1; from -1, B1; $\frac{\pi}{4}$ to *x*-axis, B1
8(ii)		B2(ft) **[2]**	Correct region between their circle and half line indicated s.c. B1 for interior of circle
	Sketch should clearly show the radius and centre of the circle and the starting point and angle of the half-line.	M1 A1(ft)	Tangent from origin to circle Correct point placed by eye where tangent from origin meets circle
8(iii)	$\arg z=\frac{\pi}{2}-\arcsin\frac{2}{3}$ = 0.84 (2d.p.)	M1 A1 **[4]**	Attempt to use right angled triangle c.a.o. Accept 48.20° (2d.p.)

9(i)	$(-3,\ -3)$	B1 **[1]**	
9(ii)	$(x,\ x)$	B1 B1 **[2]**	
9(iii)	$\begin{pmatrix}1 & 0\\1 & 0\end{pmatrix}$	B3 **[3]**	Minus 1 each error to min of 0
9(iv)	Rotation through $\frac{\pi}{2}$ anticlockwise about the origin	B1 B1 **[2]**	Rotation and angle (accept 90°) Centre and sense
9(v)	$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\times\begin{pmatrix}1 & 0\\1 & 0\end{pmatrix}=\begin{pmatrix}-1 & 0\\1 & 0\end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply using their **T** in correct order c.a.o.
9(vi)	$\begin{pmatrix}-1 & 0\\1 & 0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-x\\x\end{pmatrix}$ So (-*x*, *x*) Line *y* = -*x*	M1 A1(ft) A1 **[3]**	May be implied c.a.o. from correct matrix

4756 (FP2) Further Methods for Advanced Mathematics

1(a)	Area is $\int_0^{\pi} \frac{1}{2}a^2(1-\cos 2\theta)^2\,\mathrm{d}\theta$ $= \int_0^{\pi} \frac{1}{2}a^2\left(1-2\cos 2\theta+\frac{1}{2}(1+\cos 4\theta)\right)\mathrm{d}\theta$ $=\frac{1}{2}a^2\left[\frac{3}{2}\theta-\sin 2\theta+\frac{1}{8}\sin 4\theta\right]_0^{\pi}$ $=\frac{3}{4}\pi a^2$	M1 A1 B1 B1B1B1 ft A1 **7**	For $\int(1-\cos 2\theta)^2\,\mathrm{d}\theta$ Correct integral expression including limits (may be implied by later work) For $\cos^2 2\theta=\frac{1}{2}(1+\cos 4\theta)$ Integrating $a+b\cos 2\theta+c\cos 4\theta$ *[Max B2 if answer incorrect and no mark has previously been lost]*
(b)(i)	$\mathrm{f}'(x)=\frac{1}{1+(\sqrt{3}+x)^2}$ $\mathrm{f}''(x)=\frac{-2(\sqrt{3}+x)}{\left(1+(\sqrt{3}+x)^2\right)^2}$	M1 A1 M1 A1 **4**	Applying $\frac{\mathrm{d}}{\mathrm{d}u}\arctan u=\frac{1}{1+u^2}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sec^2 y}$ Applying chain (or quotient) rule
(ii)	$\mathrm{f}(0)=\frac{1}{3}\pi$ $\mathrm{f}'(0)=\frac{1}{4},\quad \mathrm{f}''(0)=-\frac{1}{8}\sqrt{3}$ $\arctan(\sqrt{3}+x)=\frac{1}{3}\pi+\frac{1}{4}x-\frac{1}{16}\sqrt{3}\,x^2+\ldots$	B1 M1 A1A1 ft **4**	Stated; or appearing in series *Accept* 1.05 Evaluating $\mathrm{f}'(0)$ or $\mathrm{f}''(0)$ For $\frac{1}{4}x$ and $-\frac{1}{16}\sqrt{3}\,x^2$ *ft provided coefficients are non-zero*
(iii)	$\int_{-h}^{h}(\frac{1}{3}\pi x+\frac{1}{4}x^2-\frac{1}{16}\sqrt{3}\,x^3+\ldots)\,\mathrm{d}x$ $=\left[\frac{1}{6}\pi x^2+\frac{1}{12}x^3-\frac{1}{64}\sqrt{3}\,x^4+\ldots\right]_{-h}^{h}$ $\approx(\frac{1}{6}\pi h^2+\frac{1}{12}h^3-\frac{1}{64}\sqrt{3}\,h^4)$ $-(\frac{1}{6}\pi h^2-\frac{1}{12}h^3-\frac{1}{64}\sqrt{3}\,h^4)$ $=\frac{1}{6}h^3$	M1 A1 ft A1 ag **3**	Integrating (award if *x* is missed) for $\frac{1}{12}x^3$ Allow ft from $a+\frac{1}{4}x+cx^2$ provided that $a\neq 0$ Condone a proof which neglects h^4

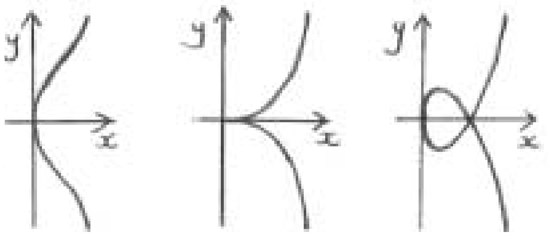
2(a)	4th roots of $16\mathrm{j}=16\mathrm{e}^{\frac{1}{2}\pi\mathrm{j}}$ are $r\mathrm{e}^{\mathrm{j}\theta}$ where		
	$r=2$	B1	*Accept* $16^{\frac{1}{4}}$
	$\theta=\frac{1}{8}\pi$	B1	
	$\theta=\frac{\pi}{8}+\frac{2k\pi}{4}$	M1	Implied by at least two correct (ft) further values
	$\theta=-\frac{7}{8}\pi,\ -\frac{3}{8}\pi,\ \frac{5}{8}\pi$	A1	or stating $k=-2,\ -1,\ (0),\ 1$
		M1 A1 **6**	Points at vertices of a square centre O *or* 3 correct points (ft) *or* 1 point in each quadrant
(b)(i)	$(1-2\mathrm{e}^{\mathrm{j}\theta})(1-2\mathrm{e}^{-\mathrm{j}\theta})=1-2\mathrm{e}^{\mathrm{j}\theta}-2\mathrm{e}^{-\mathrm{j}\theta}+4$	M1 A1	For $\mathrm{e}^{\mathrm{j}\theta}\mathrm{e}^{-\mathrm{j}\theta}=1$
	$=5-2(\mathrm{e}^{\mathrm{j}\theta}+\mathrm{e}^{-\mathrm{j}\theta})$		
	$=5-4\cos\theta$	A1 ag **3**	
	OR		
	$(1-2\cos\theta-2\mathrm{j}\sin\theta)(1-2\cos\theta+2\mathrm{j}\sin\theta)$ M1		
	$=(1-2\cos\theta)^2+4\sin^2\theta$ A1		
	$=1-4\cos\theta+4(\cos^2\theta+\sin^2\theta)$		
	$=5-4\cos\theta$ A1		
(ii)	$C+\mathrm{j}S=2\mathrm{e}^{\mathrm{j}\theta}+4\mathrm{e}^{2\mathrm{j}\theta}+8\mathrm{e}^{3\mathrm{j}\theta}+\ldots+2^n\ \mathrm{e}^{n\mathrm{j}\theta}$	M1	Obtaining a geometric series
	$=\dfrac{2\mathrm{e}^{\mathrm{j}\theta}\left(1-(2\mathrm{e}^{\mathrm{j}\theta})^n\right)}{1-2\mathrm{e}^{\mathrm{j}\theta}}$	M1 A1	Summing (M0 for sum to infinity)
	$=\dfrac{2\mathrm{e}^{\mathrm{j}\theta}(1-2^n\mathrm{e}^{n\mathrm{j}\theta})(1-2\mathrm{e}^{-\mathrm{j}\theta})}{(1-2\mathrm{e}^{\mathrm{j}\theta})(1-2\mathrm{e}^{-\mathrm{j}\theta})}$	M1	
	$=\dfrac{2\mathrm{e}^{\mathrm{j}\theta}-4-2^{n+1}\mathrm{e}^{(n+1)\mathrm{j}\theta}+2^{n+2}\mathrm{e}^{n\mathrm{j}\theta}}{5-4\cos\theta}$	A2	Give A1 for two correct terms in numerator
	$C=\dfrac{2\cos\theta-4-2^{n+1}\cos(n+1)\theta+2^{n+2}\cos n\theta}{5-4\cos\theta}$	M1 A1 ag	Equating real (or imaginary) parts
	$S=\dfrac{2\sin\theta-2^{n+1}\sin(n+1)\theta+2^{n+2}\sin n\theta}{5-4\cos\theta}$	A1 **9**	

3 (i)	Characteristic equation is $(7-\lambda)(-1-\lambda)+12=0$ $\lambda^2-6\lambda+5=0$ $\lambda=1,\ 5$ When $\lambda=1$, $\begin{pmatrix}7 & 3\\ -4 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}$ $7x+3y=x$ $-4x-y=y$ $y=-2x$, eigenvector is $\begin{pmatrix}1\\ -2\end{pmatrix}$ When $\lambda=5$, $\begin{pmatrix}7 & 3\\ -4 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=5\begin{pmatrix}x\\ y\end{pmatrix}$ $7x+3y=5x$ $-4x-y=5y$ $y=-\frac{2}{3}x$, eigenvector is $\begin{pmatrix}3\\ -2\end{pmatrix}$	M1 A1A1 M1 M1 A1 M1 A1 **8**	or $\begin{pmatrix}6 & 3\\ -4 & -2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ *can be awarded for either eigenvalue* Equation relating *x* and *y* or any (non-zero) multiple *SR* $(\mathbf{M}-\lambda\mathbf{I})\mathbf{x}=\lambda\mathbf{x}$ can earn M1A1A1M0M1A0M1A0
(ii)	$\mathbf{P}=\begin{pmatrix}1 & 3\\ -2 & -2\end{pmatrix}$ $\mathbf{D}=\begin{pmatrix}1 & 0\\ 0 & 5\end{pmatrix}$	B1 ft B1 ft **2**	B0 if **P** is singular For B2, the order must be consistent

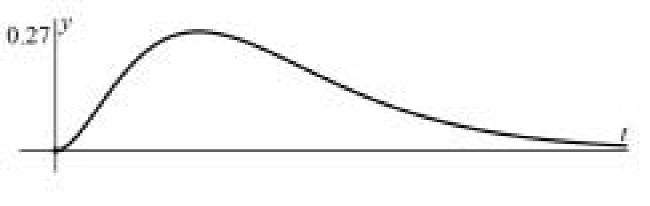
(iii)	$\mathbf{M}=\mathbf{P}\,\mathbf{D}\,\mathbf{P}^{-1}$	M1	*May be implied*
	$\mathbf{M}^n=\mathbf{P}\,\mathbf{D}^n\,\mathbf{P}^{-1}$	M1	
	$=\mathbf{P}\begin{pmatrix}1&0\\0&5^n\end{pmatrix}\mathbf{P}^{-1}$	A1 ft	*Dependent on M1M1*
	$=\begin{pmatrix}1&3\\-2&-2\end{pmatrix}\begin{pmatrix}1&0\\0&5^n\end{pmatrix}\frac{1}{4}\begin{pmatrix}-2&-3\\2&1\end{pmatrix}$	B1 ft	For $\mathbf{P}^{-1}$
	$=\begin{pmatrix}1&3\times5^n\\-2&-2\times5^n\end{pmatrix}\frac{1}{4}\begin{pmatrix}-2&-3\\2&1\end{pmatrix}$		or $\begin{pmatrix}1&3\\-2&-2\end{pmatrix}\frac{1}{4}\begin{pmatrix}-2&-3\\2\times5^n&5^n\end{pmatrix}$
	$=\frac{1}{4}\begin{pmatrix}-2+6\times5^n&-3+3\times5^n\\4-4\times5^n&6-2\times5^n\end{pmatrix}$	M1	Obtaining at least one element in a product of three matrices
	$a=-\frac{1}{2}+\frac{3}{2}\times5^n$		
	$b=-\frac{3}{4}+\frac{3}{4}\times5^n$	A1 ag	
	$c=1-5^n$ $\quad d=\frac{3}{2}-\frac{1}{2}\times5^n$	A2 **8**	Give A1 for one of *b, c, d* correct *SR* If $\mathbf{M}^n=\mathbf{P}^{-1}\,\mathbf{D}^n\,\mathbf{P}$ is used, max marks are M0M1A0B1M1A0A1 (*d* should be correct) *SR* If their **P** is singular, max marks are M1M1A1B0M0

4 (i)	$\frac{1}{2}(e^x+e^{-x})=k$	M1	or $\cosh x+\sinh x=e^x$
	$e^{2x}-2k\,e^x+1=0$	M1	or $k\pm\sqrt{k^2-1}=e^x$
	$e^x=\dfrac{2k\pm\sqrt{4k^2-4}}{2}=k\pm\sqrt{k^2-1}$		
	$x=\ln(k+\sqrt{k^2-1})$ or $\ln(k-\sqrt{k^2-1})$	A1	One value sufficient
	$(k+\sqrt{k^2-1})(k-\sqrt{k^2-1})=k^2-(k^2-1)=1$ $\ln(k-\sqrt{k^2-1})=\ln(\dfrac{1}{k+\sqrt{k^2-1}})=-\ln(k+\sqrt{k^2-1})$	M1	or $\cosh x$ is an even function *(or equivalent)*
	$x=\pm\ln(k+\sqrt{k^2-1})$	A1 ag **5**	
(ii)		M1	For arcosh *or* $\ln(\lambda x+\sqrt{\lambda^2x^2-...})$ or any cosh substitution
	$\displaystyle\int_1^2\frac{1}{\sqrt{4x^2-1}}\,dx=\left[\frac{1}{2}\text{arcosh}\,2x\right]_1^2$	A1	For $\text{arcosh}\,2x$ or $2x=\cosh u$ or $\ln(2x+\sqrt{4x^2-1})$ or $\ln(x+\sqrt{x^2-\frac{1}{4}})$
		A1	For $\frac{1}{2}$ or $\int\frac{1}{2}du$
	$=\frac{1}{2}(\text{arcosh}\,4-\text{arcosh}\,2)$	M1	
	$=\frac{1}{2}\left(\ln(4+\sqrt{15})-\ln(2+\sqrt{3})\right)$	A1 **5**	Exact numerical logarithmic form
(iii)	$6\sinh x-2\sinh x\cosh x=0$	M1	
	$\cosh x=3$ (or $\sinh x=0$)	M1	Obtaining a value for $\cosh x$
	$x=0$	B1	
	$x=\pm\ln(3+\sqrt{8})$	A1 **4**	or $x=\ln(3\pm\sqrt{8})$
	OR $e^{4x}-6e^{3x}+6e^x-1=0$		
	$(e^{2x}-1)(e^{2x}-6e^x+1)=0$ M2		or $(e^x-e^{-x})(e^x+e^{-x}-6)=0$
	$x=0$ B1		
	$x=\ln(3\pm\sqrt{8})$ A1		
(iv)	$\frac{dy}{dx}=6\cosh x-2\cosh 2x$	B1	
	If $\frac{dy}{dx}=5$ then $6\cosh x-2(2\cosh^2x-1)=5$ $4\cosh^2x-6\cosh x+3=0$	M1	Using $\cosh 2x=2\cosh^2x-1$
	Discriminant $D=6^2-4\times4\times3=-12$	M1	Considering *D*, or completing square, or considering turning point
	Since $D<0$ there are no solutions	A1 **4**	

	OR Gradient $g = 6\cosh x - 2\cosh 2x$ B1 $g' = 6\sinh x - 4\sinh 2x = 2\sinh x(3 - 4\cosh x)$ $= 0$ when $x = 0$ (only) M1 $g'' = 6\cosh x - 8\cosh 2x = -2$ when $x = 0$ M1 Max value $g = 4$ when $x = 0$ So *g* is never equal to 5 A1		Final A1 requires a complete proof showing this is the only turning point

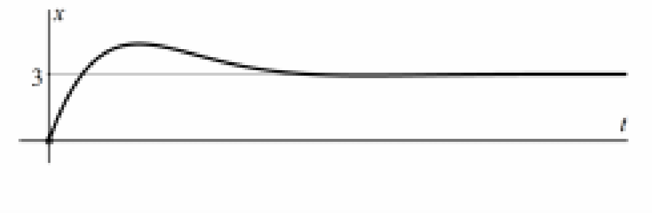
5 (i)	$\lambda=-1$ $\lambda=0$ $\lambda=1$  cusp loop	B1B1B1 B1B1 **5**	Two different features (cusp, loop, asymptote) correctly identified
(ii)	$x=1$	B1 **1**	
(iii)	Intersects itself when $y=0$ $t=(\pm)\sqrt{\lambda}$ $\left(\frac{\lambda}{1+\lambda},\ 0\right)$	M1 A1 A1 **3**	
(iv)	$\frac{\mathrm{d}y}{\mathrm{d}t}=3t^2-\lambda=0$ $t=\pm\sqrt{\frac{\lambda}{3}}$ $x=\frac{\lambda/3}{1+\lambda/3}=\frac{\lambda}{3+\lambda}$ $y=\pm\left((\frac{\lambda}{3})^{\frac{3}{2}}-\lambda(\frac{\lambda}{3})^{\frac{1}{2}}\right)$ $=\pm\lambda^{\frac{3}{2}}\left(\frac{1}{3\sqrt{3}}-\frac{1}{\sqrt{3}}\right)=\pm\lambda^{\frac{3}{2}}\left(-\frac{2}{3\sqrt{3}}\right)$ $=\pm\sqrt{\frac{4\lambda^3}{27}}$	M1 A1 ag M1 A1 ag **4**	One value sufficient
(v)	From asymptote, $a=8$ From intersection point, $\frac{a\lambda}{1+\lambda}=2$ $\lambda=\frac{1}{3}$ From maximum point, $b\sqrt{\frac{4\lambda^3}{27}}=2$ $b=27$	B1 M1 A1 M1 A1 **5**	

4758 Differential Equations

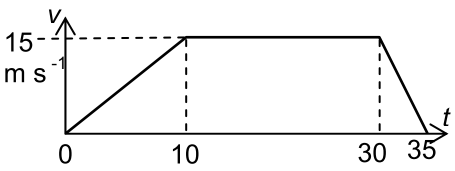
1(i)	$\alpha^2+2\alpha+1=0$	M1	Auxiliary equation	
	$\alpha=-1$ (repeated)	A1		
	CF $y=(A+Bt)\mathrm{e}^{-t}$	F1	CF for their roots	
	PI $y=a$	B1	Constant PI	
	in DE $\Rightarrow y=2$	B1	PI correct	
	$y=2+(A+Bt)\mathrm{e}^{-t}$	F1	Their PI + CF (with two arbitrary constants)	
	$t=0, y=0 \Rightarrow 0=2+A \Rightarrow A=-2$	M1	Condition on *y*	
	$\dot{y}=(B-A-Bt)\mathrm{e}^{-t}$	M1	Differentiate (product rule)	
	$t=0, \dot{y}=0 \Rightarrow 0=B-A \Rightarrow B=-2$	M1	Condition on $\dot{y}$	
	$y=2-(2+2t)\mathrm{e}^{-t}$	A1		10
(ii)	Both terms in CF hence will give zero if substituted in LHS	E1		
	PI $y=bt^2\,\mathrm{e}^{-t}$	B1		
	$\dot{y}=\left(2bt-bt^2\right)\mathrm{e}^{-t}, \ddot{y}=\left(2b-4bt+bt^2\right)\mathrm{e}^{-t}$			
	in DE $\Rightarrow \left(2b-4bt+bt^2+2\left(2bt-bt^2\right)+bt^2\right)\mathrm{e}^{-t}=\mathrm{e}^{-t}$	M1	Differentiate twice and substitute	
	$\Rightarrow b=\frac{1}{2}$	A1	PI correct	
	$y=\left(C+Dt+\frac{1}{2}t^2\right)\mathrm{e}^{-t}$	F1	Their PI + CF (with two arbitrary constants)	
	$t=0, y=0 \Rightarrow 0=C$	M1	Condition on *y*	
	$\dot{y}=\left(D+t-C-Dt-\frac{1}{2}t^2\right)\mathrm{e}^{-t}$			
	$t=0, \dot{y}=0 \Rightarrow 0=D-C \Rightarrow D=0$	M1	Condition on $\dot{y}$	
	$y=\frac{1}{2}t^2\,\mathrm{e}^{-t}$	A1		8
(iii)	$t>0 \Rightarrow \frac{1}{2}t^2>0$ and $\mathrm{e}^{-t}>0 \Rightarrow y>0$	E1		
	$\dot{y}=\left(t-\frac{1}{2}t^2\right)\mathrm{e}^{-t}$ so $\dot{y}=0 \Leftrightarrow t-\frac{1}{2}t^2=0 \Leftrightarrow t=0$ or 2	M1	Solve $\dot{y}=0$	
	Maximum at $t=2, y=2\,\mathrm{e}^{-2}$	A1	Maximum value of *y*	
		B1 B1 B1	Starts at origin Maximum at their value of *y* $y>0$	6

2(i)	$\frac{\mathrm{d}v}{\mathrm{d}t}+\frac{3}{1+t}v=g-\frac{3}{1+t}$	M1	Rearrange
	$I=\exp\left(\int\frac{3}{1+t}\,\mathrm{d}t\right)=\mathrm{e}^{3\ln(1+t)}=(1+t)^3$	M1 A1 A1	Attempt integrating factor Correct Simplified
	$(1+t)^3\frac{\mathrm{d}v}{\mathrm{d}t}+3(1+t)^2v=g(1+t)^3-3(1+t)^2$	F1	Multiply DE by their *I*
	$\frac{\mathrm{d}}{\mathrm{d}t}\left((1+t)^3v\right)=g(1+t)^3-3(1+t)^2$		
	$(1+t)^3v=\int\left(g(1+t)^3-3(1+t)^2\right)\mathrm{d}x$	M1	Integrate
	$=\frac{1}{4}g(1+t)^4-(1+t)^3+A$	A1	RHS
	$v=\frac{1}{4}g(1+t)-1+A(1+t)^{-3}$	F1	Divide by their *I* (must also divide constant)
	$t=0,v=0\Rightarrow 0=\frac{1}{4}g-1+A$	M1	Use condition
	$v=\frac{1}{4}g(1+t)-1+\left(1-\frac{1}{4}g\right)(1+t)^{-3}$	E1	Convincingly shown
			10
(ii)	$(1+t)\frac{\mathrm{d}v}{\mathrm{d}t}+5v=(1+t)g$	M1	Rearrange
	$\frac{\mathrm{d}v}{\mathrm{d}t}+\frac{5}{1+t}v=g$		
	$I=\exp\left(\int\frac{5}{1+t}\,\mathrm{d}t\right)=\mathrm{e}^{5\ln(1+t)}=(1+t)^5$	M1 A1	Attempt integrating factor Simplified
	$(1+t)^5\frac{\mathrm{d}v}{\mathrm{d}t}+5(1+t)^4v=g(1+t)^5$	F1	Multiply DE by their *I*
	$\frac{\mathrm{d}}{\mathrm{d}t}\left((1+t)^5v\right)=g(1+t)^5$		
	$(1+t)^5v=\int g(1+t)^5\,\mathrm{d}x$	M1	Integrate
	$=\frac{1}{6}g(1+t)^6+B$	A1	RHS
	$v=\frac{1}{6}g(1+t)+B(1+t)^{-5}$	F1	Divide by their *I* (must also divide constant)
	$t=0,v=0\Rightarrow 0=\frac{1}{6}g+B$	M1	Use condition
	$v=\frac{1}{6}g\left(1+t-(1+t)^{-5}\right)$	F1	Follow a non-trivial GS
			9
(iii)	First model: $\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{1}{4}g-3\left(1-\frac{1}{4}g\right)(1+t)^{-4}$	M1	Find acceleration
	As $t\to\infty,(1+t)^{-4}\to 0$	B1	Identify term(s) → 0 in their solution for either model
	Hence acceleration tends to $\frac{1}{4}g$	A1	
	Second model $\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{1}{6}g\left(1+5(1+t)^{-6}\right)$	M1	Find acceleration
	Hence acceleration tends to $\frac{1}{6}g$	A1	
			5

3(i)	$P = A\mathrm{e}^{0.5t}$	M1	Any valid method	
	$t = 0, P = 2000 \Rightarrow A = 2000$	M1	Use condition	
	$P = 2000\,\mathrm{e}^{0.5t}$	A1		
				3
(ii)	CF $P = A\mathrm{e}^{0.5t}$	F1	Correct or follows (i)	
	PI $P = a\cos 2t + b\sin 2t$	B1		
	$\dot{P} = -2a\sin 2t + 2b\cos 2t$	M1	Differentiate	
	$-2a\sin 2t + 2b\cos 2t = 0.5(a\cos 2t + b\sin 2t) + 170\sin 2t$	M1	Substitute	
	$-2a = 0.5b + 170$	M1	Compare coefficients	
	$2b = 0.5a$	M1	Solve	
	solving $\Rightarrow a = -80, b = -20$	A1		
	GS $P = A\mathrm{e}^{0.5t} - 80\cos 2t - 20\sin 2t$	F1	Their PI + CF (with one arbitrary constant)	
				8
(iii)	$t = 0, P = 2000 \Rightarrow A = 2080$	M1	Use condition	
	$P = 2080\,\mathrm{e}^{0.5t} - 80\cos 2t - 20\sin 2t$	F1	Follow a non-trivial GS	
				2
(iv)	t P $\dot{P}$	M1	Use of algorithm	
	0 2000 1000	A1	2100	
	0.1 2100 1082.58	A1	1082.5…	
	0.2 2208	A1	2208	
				4
(v)	*(A)* Limiting value $\Rightarrow \dot{P} = 0$	M1	Set $\dot{P} = 0$	
	$\Rightarrow P\left(1 - \frac{P}{12\,000}\right)^{\frac{1}{2}} = 0$	M1	Solve	
	(as limit non-zero) limiting value = 12000	A1		
				3
	(B) Growth rate max when			
	$\mathrm{f}(P) = P\left(1 - \frac{P}{12\,000}\right)^{\frac{1}{2}}$ max	M1	Recognise expression to maximise	
	$\mathrm{f}'(P) = \left(1 - \frac{P}{12\,000}\right)^{\frac{1}{2}} - \frac{1}{2\times 12000}P\left(1 - \frac{P}{12\,000}\right)^{-\frac{1}{2}}$	M1	Reasonable attempt at derivative	
	$\mathrm{f}'(P) = 0 \Leftrightarrow \left(1 - \frac{P}{12\,000}\right) - \frac{1}{2\times 12000}P = 0$	M1	Set derivative to zero	
	$\Leftrightarrow P = 8000$	A1		
				4

4(i)	$\ddot{x} = -3\dot{x} + \dot{y}$	M1	Differentiate first equation	
	$= -3\dot{x} + (-5x + y + 15)$	M1	Substitute for $\dot{y}$	
	$y = 3x - 9 + \dot{x}$	M1	*y* in terms of $x, \dot{x}$	
	$\ddot{x} = -3\dot{x} - 5x + (3x - 9 + \dot{x}) + 15$	M1	Substitute for *y*	
	$\ddot{x} + 2\dot{x} + 2x = 6$	E1		
				5
(ii)	$\lambda^2 + 2\lambda + 2 = 0$	M1	Auxiliary equation	
	$\lambda = -1 \pm j$	A1		
	CF $x = \mathrm{e}^{-t}(A\cos t + B\sin t)$	M1	CF for complex roots	
		F1	CF for their roots	
	PI $x = a$	B1	Constant PI	
	$2a = 6 \Rightarrow a = 3$	B1	PI correct	
	GS $x = 3 + \mathrm{e}^{-t}(A\cos t + B\sin t)$	F1	Their CF + PI (with two arbitrary constants)	
				7
(iii)	$y = 3x - 9 + \dot{x}$	M1	*y* in terms of $x, \dot{x}$	
	$= 9 + 3\mathrm{e}^{-t}(A\cos t + B\sin t) - 9$ $-\mathrm{e}^{-t}(A\cos t + B\sin t) + \mathrm{e}^{-t}(-A\sin t + B\cos t)$	M1	Differentiate *x* and substitute	
	$y = \mathrm{e}^{-t}((2A + B)\cos t + (2B - A)\sin t)$	A1	Constants must correspond with those in *x*	
				3
(iv)	$0 = 3 + A \Rightarrow A = -3$	M1	Condition on *x*	
	$0 = 2A + B \Rightarrow B = 6$	M1	Condition on *y*	
	$x = 3 + 3\mathrm{e}^{-t}(2\sin t - \cos t)$	F1	Follow their GS	
	$y = 15\mathrm{e}^{-t}\sin t$	F1	Follow their GS	
				4
(v)	 	B1	Sketch of *x* starts at origin	
		B1	Asymptote *x* = 3	
	 	B1	Sketch of *y* starts at origin	
		B1	Decaying oscillations (may decay rapidly)	
		B1	Asymptote *y* = 0	
				5

4761 Mechanics 1

Q 1		Mark	Comment	Sub
(i)		B1 B1 B1	Acc and dec shown as straight lines Horizontal straight section All correct with *v* and times marked and at least one axis labelled. Accept (*t*, *v*) or (*v*, *t*) used.	3
(ii)	Distance is found from the area area is $\frac{1}{2}\times10\times15+20\times15+\frac{1}{2}\times5\times15$ (or $\frac{1}{2}\times(20+35)\times15$) = 412.5 so distance is 412.5 m	M1 A1 A1	At least one area attempted or equivalent *uvast* attempted over one appropriate interval. Award for at least two areas (or equivalent) correct Allow if a trapezium used and only 1 substitution error. FT **their** diagram. cao (Accept 410 or better accuracy)	3
		6		
2 (i)	$\begin{pmatrix}6\\9\end{pmatrix}=1.5\mathbf{a}$ giving $\mathbf{a}=\begin{pmatrix}4\\6\end{pmatrix}$ so $\begin{pmatrix}4\\6\end{pmatrix}$ m s^{-2}	M1 A1	Use of N2L with an attempt to find **a**. Condone spurious notation. Must be a vector in proper form. Penalise only once in paper.	2
(ii)	Angle is $\arctan\left(\frac{6}{4}\right)$ = 56.309… so 56.3° (3 s. f.)	M1 F1	Use of arctan with **their** $\frac{6}{4}$ or $\frac{4}{6}$ or equiv. May use **F**. FT **their a** provided both cpts are +ve and non-zero.	2
(iii)	Using $\mathbf{s}\ = t\mathbf{u}+0.5t^2\mathbf{a}$ we have $\mathbf{s}=2\begin{pmatrix}-2\\3\end{pmatrix}+0.5\times4\begin{pmatrix}4\\6\end{pmatrix}$ so $\begin{pmatrix}4\\18\end{pmatrix}$ m	M1 A1 A1 7	Appropriate single ***uvast*** (or equivalent sequence of ***uvast***). If integration used twice condone omission of **r**(0) but not **v**(0). FT **their a** only cao. isw for magnitude subsequently found. Vector must be in proper form (penalise only once in paper).	3

Q 3		**Mark**	**Comment**	**Sub**
(i)	$m\times 9.8 = 58.8$ so m = 6	M1 A1	$T = mg$. Condone sign error. cao. CWO.	2
(ii)	Resolve → $58.8\cos 40 - F = 0$ F = 45.043… so 45.0 N (3 s. f.)	M1 B1 A1	Resolving **their** tension. Accept $s \leftrightarrow c$. Condone sign errors but not extra forces. (**their** T) × $\cos 40$ (or equivalent) seen Accept ± 45 only.	3
(iii)	Resolve ↑ $R + 58.8\sin 40 - 15\times 9.8 = 0$ R = 109.204… so 109 N (3 s. f.)	M1 A1 A1	Resolving **their** tension. All forces present. No extra forces. Accept $s \leftrightarrow c$. Condone errors in sign. All correct cao	3
		8		
Q 4		**Mark**	**Comment**	**Sub**
(i)	Resultant is $\begin{pmatrix}4\\1\\2\end{pmatrix}+\begin{pmatrix}-6\\2\\4\end{pmatrix}=\begin{pmatrix}-2\\3\\6\end{pmatrix}$ Magnitude is $\sqrt{(-2)^2+3^2+6^2} = \sqrt{49} = 7$ N	M1 A1 M1 F1	Adding the vectors. Condone spurious notation. Vector must be in proper form (penalise only once in the paper). Accept clear components. Pythagoras on **their** 3 component vector. Allow e.g. – 2² for (– 2)² even if evaluated as – 4. FT **their** resultant.	4
(ii)	**F** + 2**G** + **H** = **0** So **H** = - 2**G** - **F** = $-\begin{pmatrix}-12\\4\\8\end{pmatrix}-\begin{pmatrix}4\\1\\2\end{pmatrix}$ = $\begin{pmatrix}8\\-5\\-10\end{pmatrix}$	M1 A1 A1	Either **F** + 2**G** + **H** = **0** or **F** + 2**G** = **H** Must see attempt at **H** = – 2 **G** – **F** cao. Vector must be in proper form (penalise only once in the paper).	3
		7		

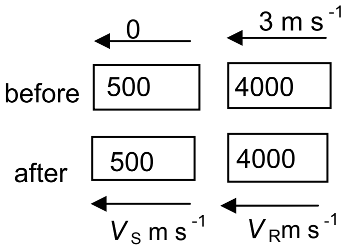
Q 5		**Mark**	**Comment**	**Sub**
	$a = 12 - 6t$	M1 A1	Differentiation, at least one term correct.	
	a = 0 gives t = 2	F1	Follow **their** *a*	
	$x = \int (2 + 12t - 3t^2)\mathrm{d}x$	M1	Integration indefinite or definite, at least one term correct.	
	$2t + 6t^2 - t^3 + C$	A1	Correct. Need not be simplified. Allow as definite integral. Ignore *C* or limits	
	x = 3 when t = 0	M1	Allow x = ± 3 or argue it is $\int_0^2$ from A then ± 3	
	so $3 = C$ and			
	$x = 2t + 6t^2 - t^3 + 3$	A1	Award if seen WWW or $x = 2t + 6t^2 - t^3$ seen with +3 added later.	
	$x(2) = 4 + 24 - 8 + 3 = 23$ m	B1	FT **their** *t* and **their** *x* if obtained by integration but not if -3 obtained instead of +3. [If 20 m seen WWW for displacement award SC6] [Award SC1 for position if constant acceleration used for displacement and then +3 applied]	8
		8		

Q 6		Mark	**Comment**	**Sub**
(i)	$3.5 = 0.5 + 1.5T$	M1	Suitable *uvast*, condone sign errors.	
	so *T* = 2 so 2 s	A1	cao	
	$s = \frac{3.5+0.5}{2} \times 2$	M1	Suitable *uvast*, condone sign errors.	
	so *s* = 4 so 4 m	F1	FT **their** *T*. [If *s* found first then it is cao. In this case when finding *T*, FT **their** *s*, if used.]	4
(ii) **(A)**	N2L ↓ : $80 \times 9.8 - T = 80 \times 1.5$	M1	Use of N2L. Allow weight omitted and use of *F* = *mga* Condone errors in sign but do not allow extra forces.	
		B1	weight correct (seen in (A) or (B))	
	T = 664 so 664 N	A1	cao	
(B)	N2L ↓ : $80 \times 9.8 - T = 80 \times (-1.5)$	M1	N2L with all forces and using *F* = *ma*. Condone errors in sign but do not allow extra forces.	
	T = 904 so 904 N	A1	cao [Accept 904 N seen for M1 A1]	5
(iii)	N2L ↑ : $2500 - 80 \times 9.8 - 116 = 80a$	M1	Use of N2L with *F* = *ma*. Allow 1 force missing. No extra forces. Condone errors in sign.	
		A1		
	a = 20 so 20 m s^{-2} upwards.	A1	± 20, accept direction wrong or omitted	
		A1	upwards made clear (accept diagram)	4
(iv)	N2L ↑ on equipment: $80 - 10 \times 9.8 = 10a$	M1	Use of N2L on equipment. All forces. *F = ma.* No extra forces. Allow sign errors.	
	a = -1.8	A1	Allow ± 1.8	
	N2L ↑ **either** all: $T - (80+10) \times 9.8 - 116 = 90 \times (-1.8)$ **or** on man: *T* – (80×9.8) – 116 – 80 = 80×(– 1.8)	M1	N2L for system or for man alone. Forces correct (with no extras); accept sign errors; **their** ± 1.8 used	
	T = 836 so 836 N	A1	cao [NB The answer 836 N is independent of the value taken for *g* and hence may be obtained if all weights are omitted.]	4
		17		

Q 7		Mark	**Comment**	**Sub**
(i)	Horiz $21t = 60$ so $\frac{20}{7}$ s (2.8571…) **either** $0 = u - 9.8\times\frac{20}{7}$ **or** $-u = u - 9.8\times\left(\frac{40}{7}\right)$ **or** $40 = u\times\frac{20}{7} - 4.9\left(\frac{20}{7}\right)^2$ so *u* = 28 so 28 m s $^{-1}$	M1 A1 M1 E1	Use of horizontal components and *a* = 0 or $s = vt - 0.5at^2$ with *v* = 0. Any form acceptable. Allow M1 A1 for answer seen WW. [If $s = ut + 0.5at^2$ and *u* = 0 used without justification award M1 A0] [If *u* = 28 *assumed* to find time then award SC1] Use of $v = u + at$ (or $v^2 = u^2 + 2as$) with *v* = 0. **or** Use of *v* = *u* + *at* with *v* = – *u* and appropriate *t.* **or** Use of $s = ut + 0.5at^2$ with *s* = 40 and appropriate *t* Condone sign errors and, where appropriate, $u \leftrightarrow v$. Accept signs not clear but not errors. Enough working must be given for 28 to be properly shown. [NB *u* = 28 may be found first and used to find time]	4
(ii)	$y = 28t - 0.5\times 9.8t^2$	E1	*Clear & convincing* use of *g* = – 9.8 in $s = ut + 0.5at^2$ or $s = vt - 0.5at^2$ **NB: AG**	1
(iii)	Start from same height with same (zero) vertical speed at same time, same acceleration Distance apart is $0.75\times 21t = 15.75t$	E1 M1 A1	For two of these reasons 0.75×21*t* seen **or** 21*t* and 5.25*t* both seen with intention to subtract. Need simplification - LHS alone insufficient. CWO.	3
(iv) (A)	**either** Time is $\frac{20}{7}$ s by symmetry so $15.75\times\frac{20}{7} = 45$ so 45 m **or** Hit ground at same time. By symmetry one travels 60 m so the other travels 15 m in this time ($\frac{1}{4}$ speed) so 45 m.	B1 B1 B1 B1	Symmetry or *uvast* FT their (iii) with $t = \frac{20}{7}$ [SC1 if 90 m seen]	2
(B)	see next page			

Q7	continued			
(B)			[SC1 if **either** and **or** methods mixed to give $\pm 30 = 28t - 4.9t^2$ or $\pm 10 = 4.9t^2$]	
	either Time to fall is $40 - 10 = 0.5 \times 9.8 \times t^2$	M1	Considering time from explosion with u = 0. Condone sign errors.	
		A1	LHS. Allow ±30	
		A1	All correct	
	t = 2.47435…	A1	cao	
	need $15.75 \times 2.47435.. = 38.971..$ so 39.0 (3sf)	F1	FT **their** (iii) only.	
	or Need time so $10 = 28t - 4.9t^2$	M1	Equating $28t - 4.9t^2 = \pm 10$	
	$4.9t^2 - 28t + 10 = 0$	M1*	Dep. Attempt to solve quadratic by a method that could give two roots.	
	so $t = \frac{28 \pm \sqrt{28^2 - 4 \times 4.9 \times 10}}{9.8}$			
	so 0.382784… or 5.33150…	A1	Larger root correct to at least 2 s. f. Both method marks may be implied from two correct roots alone (to at least 1 s. f.). [SC1 for either root seen WW]	
	Time required is 5.33150… $-\frac{20}{7}$ = 2.47435..	M1		
	need $15.75 \times 2.47435.. = 38.971..$ so 39.0 (3sf)	F1	FT **their** (iii) only.	5
(v)	Horiz $(x =)\ 21t$	B1		
	Elim t between $x = 21t$ and $y = 28t - 4.9t^2$	M1	Intention must be clear, with some attempt made.	
	so $y = 28\left(\frac{x}{21}\right) - 4.9\left(\frac{x}{21}\right)^2$	A1	t completely and correctly eliminated from their expression for x and correct y. Only accept wrong notation if subsequently explicitly given correct value e.g. $\frac{x^2}{21}$ *seen* as $\frac{x^2}{441}$.	
	so $y = \frac{4x}{3} - \frac{0.1x^2}{9} = \frac{1}{90}\left(120x - x^2\right)$	E1	Some simplification must be shown. [SC2 for 3 points shown to be on the curve. Award more only if it is made clear that (a) trajectory is a parabola (b) 3 points define a parabola]	4
		19		

4762 Mechanics 2

Q1		Mark	Comment	Sub
(a) (i)	**either**			
	In direction of the force			
	I = *Ft* = *mv*	M1	Use of *Ft* = *mv*	
	so $1500 \times 8 = 4000v$	A1		
	giving *v* = 3 so 3 m s^{-1}	A1		
	or			
	N2L gives $a = \frac{1500}{4000}$	M1	Appropriate use of N2L **and** *uvast*	
	v = 0 + $\frac{1500}{4000} \times 8$	A1		
	giving *v* = 3 so 3 m s^{-1}	A1		3
(ii)				
	PCLM $12000 = 4000V_R + 500V_S$	M1	Appropriate use of PCLM	
	so $24 = 8V_R + V_S$	A1	Any form	
	NEL $\frac{V_S - V_R}{0-3} = -0.2$	M1	Appropriate use of NEL	
	so $V_S - V_R = 0.6$	A1	Any form	
	Solving			
	$V_R = 2.6,\ \ V_S = 3.2$	A1	Either value	
	so ram 2.6 m s^{-1} and stone 3.2 m s^{-1}	F1		6
(iii)	$0.5 \times 4000 \times 3^2 - 0.5 \times 4000 \times 2.6^2 - 0.5 \times 500 \times 3.2^2$	M1	Change in KE. Accept two terms	
		B1	Any relevant KE term correct (FT their speeds)	
	= 1920 J	A1	cao	3
(b)	see over			

1		Mark	Comment	Sub
(b) (i)	72**i** N s $8(9\cos 60\mathbf{i}+9\sin 60\mathbf{j})$ = ($36\mathbf{i}+36\sqrt{3}\mathbf{j}$) N s	B1 E1	Neglect units but must include direction Evidence of use of 8 kg , 9 m s^{-1} and 60°	2
(ii)	$72\mathbf{i}+(36\mathbf{i}+36\sqrt{3}\mathbf{j})=12(u\mathbf{i}+v\mathbf{j})$ Equating components 72 + 36 = 12*u* so *u* = 9 $36\sqrt{3}=12v$ so $v=3\sqrt{3}$	M1 M1 A1	PCLM. Must be momenta both sides Both	3
(iii)	**either** $4\times18=8\times9$ so equal momenta so 60/2 = 30° **or** $\arctan\left(\frac{3\sqrt{3}}{9}\right)=\arctan\left(\frac{1}{\sqrt{3}}\right)=30°$	M1 A1 M1 A1	Must be clear statements cao FT **their** *u* and *v*. cao	2
		19		

Q 2		Mark	Comment	Sub
(i) (A)	$0.5\times80\times3^2=360$ J	M1 A1	Use of KE	2
(B)	$360=F\times12$ so *F* = 30 so 30 N	M1 F1	*W* = *Fd* attempted FT **their** WD	2
(ii)	Using the WE equation $0.5\times80\times10^2-0.5\times80\times4^2$ $=80\times9.8\times h-1600$ *h* = 6.32653… so 6.33 (3 s. f.)	M1 M1 B1 A1 A1	Attempt to use the WE equation. Condone one missing term Δ KE attempted 1600 with correct sign All terms present and correct (neglect signs) cao	5
(iii) (A)	We have driving force *F* = 40 so 200 = 40*v* and *v* = 5 so 5 m s^{-1}	B1 M1 A1	May be implied Use of *P* = *Fv*	3
(B)	From N2L, force required to give accn is $F-40=80\times2$ so *F* = 200 *P* = 200×0.5 = 100 so 100 W	M1 A1 A1 M1 A1	Use of N2L with all terms present (neglect signs) All terms correct correct use of P = Fv cao	5
		17		

Q 3		Mark	Comment	Sub
(i)	For $\bar{z}$ $(2\times20\times100+2\times50\times120)\bar{z}$ $=2\times2000\times50+2\times6000\times60$ so $\bar{z}$ = 57.5 and $\bar{y}$ = 0	M1 B1 B1 A1 B1	Method for c.m. Total mass of 16000 (or equivalent) At least one term correct NB This result is given below. NB This result is given below. Statement (or proof) required. N.B. If incorrect axes specified, award max 4/5	5
(ii)	$\bar{y}$ and $\bar{z}$ are not changed with the folding For $\bar{x}$ $100\times120\times0+2\times20\times100\times10=16000\bar{x}$ so $\bar{x}=\frac{40000}{16000}=2.5$	E1 M1 B1 E1	A statement, calculation or diagram required. Method for the c.m. with the folding Use of the 10 Clearly shown	4
(iii)	Moments about AH. Normal reaction acts through this line c.w. $P\times120-72\times(20-2.5)=0$ so *P* = 10.5	M1 B1 B1 A1 A1	 May be implied by diagram or statement $20-2.5$ or equivalent All correct cao	5
(iv)	$F_{\max}=\mu R$ so $F_{\max}=72\mu$ For slipping before tipping we require $72\mu<10.5$ so $\mu<0.1458333...\ (7/48)$	M1 A1 M1 A1	Allow $F=\mu R$ Must have clear indication that this is max F Accept $\le$. Accept **their** $F_{\max}$ and *R*. cao	4
		18		

Q 4		Mark	**Comment**	**Sub**
(i)	Centre of CE is 0.5 m from D	B1	Used below correctly	
	a.c. moment about D	M1	Use of **their** 0.5	
	$2200 \times 0.5 = 1100$ so 1100 N m	E1	0.5 must be clearly established.	
	c.w moments about D $R \times 2.75 - 1100 = 0$	M1	Use of moments about D in an equation	
		B1	Use of 1100 and 2.75 or equiv	
	R = 400 so 400 N	A1		6
(ii)	c.w moments about D $W \times 1.5 - 1100 - 440 \times 2.75 = 0$	M1	Moments of all relevant forces attempted	
		A1	All correct	
	so *W* = 1540	E1	Some working shown	3
(iii) (*A*)	c.w. moments about D	M1	Moments equation. Allow one missing term; there must be some attempt at resolution.	
	$1.5 \times 1540 \cos 20 - 1.75T$ $-1100 \cos 20 - 400 \times 2.75 \cos 20 = 0$	M1	At least one res attempt with correct length Allow sin ↔ cos	
		A1	Any two of the terms have cos 20 correctly used (or equiv)	
		B1	1.75 *T*	
		A1	All correct	
	T = 59.0663… so 59.1 N (3 s. f.)	A1	cao Accept no direction given	6
(iii) (*B*)	**either** Angle required is at 70° to the normal to CE	B1		
	so $T_1 \cos 70 = 59.0663...$	M1		
	so $T_1 = 172.698...$ so 173 N (3 s.f.)	A1	FT (iii) (A)	
	or $400 \cos 20 \times 2.75 + 1100 \cos 20$ $= 1540 \cos 20 \times 1.5 - T \sin 20 \times 1.75$	M1	Moments attempted with all terms present	
		A1	All correct (neglect signs)	
	T = 172.698… so 173 N (3s.f.)	A1	FT(iii)(A)	3
		18		

4763 Mechanics 3

1(a)(i)	[Force] $= \mathrm{M\,L\,T^{-2}}$ [Density] $= \mathrm{M\,L^{-3}}$	B1 B1 **2**	
(ii)	$[E]=\dfrac{[F][l_0]}{[A][l-l_0]}=\dfrac{(\mathrm{M\,L\,T^{-2}})(\mathrm{L})}{(\mathrm{L^2})(\mathrm{L})}$ $= \mathrm{M\,L^{-1}\,T^{-2}}$	B1 M1 A1 **3**	for $[A]=\mathrm{L}^2$ Obtaining the dimensions of *E*
(iii)	$\mathrm{T}=\mathrm{L}^{\alpha}(\mathrm{M\,L^{-3}})^{\beta}(\mathrm{M\,L^{-1}\,T^{-2}})^{\gamma}$ $-2\gamma=1,\quad \beta+\gamma=0$ $\gamma=-\frac{1}{2}$ $\beta=\frac{1}{2}$ $\alpha-3\beta-\gamma=0$ $\alpha=1$	B1 cao F1 M1 A1 A1 **5**	Obtaining equation involving α,β,γ
(b)	$AP=1.7\text{ m}$ $F=T\cos\theta$ $R+T\sin\theta=5\times 9.8$ $T\cos\theta=0.4(49-T\sin\theta)$ $\frac{8}{17}T=0.4(49-\frac{15}{17}T)$ $T=23.8$ $T=k(1.7-1.5)$ Stiffness is $119\ \mathrm{N\,m^{-1}}$	B1 M1 M1 M1 A1 A1 M1 A1 **8**	Resolving in any direction Resolving in another direction *(M1 for resolving requires no force omitted, with attempt to resolve all appropriate forces)* Using $F=0.4R$ to obtain an equation involving just one force (or *k*) Correct equation *Allow* $T\cos 61.9$ *etc* or $R=28$ or $F=11.2$ *May be implied* *Allow M1 for* $T=\frac{\lambda}{1.5}\times 0.2$ If $R=49$ is assumed, max marks are B1M1M0M0A0A0M1A0

2(a)(i)	$0.1+0.01\times 9.8=0.01\times\frac{u^2}{0.55}$ Speed is $3.3\ \mathrm{m\,s^{-1}}$	M1 A1 A1 **3**	Using acceleration $u^2/0.55$
(ii)	$\tfrac{1}{2}m(v^2-u^2)=mg(2\times 0.55-0.15)$ $\tfrac{1}{2}(v^2-3.3^2)=9.8\times 0.95$ $v^2=29.51$ $R-mg\cos\theta=m\frac{v^2}{a}$ $R-0.01\times 9.8\times\frac{0.4}{0.55}=0.01\times\frac{29.51}{0.55}$ Normal reaction is 0.608 N	M1 A1 M1 A1 A1 **5**	Using conservation of energy *(ft is* $v^2=u^2+18.62$*)* Forces and acceleration towards centre *(ft is* $\frac{u^2+22.54}{55}$ *)*
(b)(i)	$T=0.8r\omega^2$ $T=\frac{160}{2}(r-2)$ $\omega^2=\frac{80(r-2)}{0.8r}=\frac{100(r-2)}{r}$ $\omega^2=100-\frac{200}{r}<100,\quad \text{so}\quad \omega<10$	B1 B1 E1 E1 **4**	
(ii)	$\mathrm{EE}=\frac{1}{2}\times\frac{160}{2}\times(r-2)^2=40(r-2)^2$ $\mathrm{KE}=\tfrac{1}{2}m(r\omega)^2$ $=\tfrac{1}{2}\times 0.8\times r^2\times\frac{100(r-2)}{r}$ $=40r(r-2)$ Since $r>r-2$, $40r(r-2)>40(r-2)^2$ i.e. KE > EE	B1 M1 A1 E1 **4**	Use of $\tfrac{1}{2}mv^2$ with $v=r\omega$ From fully correct working only
(iii)	When $\omega=6$, $36=\frac{100(r-2)}{r}$ $r=3.125$ $T=80(r-2)=80(3.125-2)$ Tension is 90 N	M1 M1 A1 cao **3**	Obtaining *r*

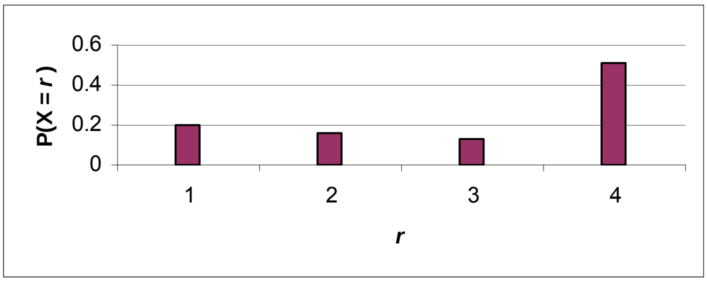
3 (i)	$\frac{\mathrm{d}x}{\mathrm{d}t} = A\omega\cos\omega t - B\omega\sin\omega t$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$ $= -\omega^2(A\sin\omega t + B\cos\omega t) = -\omega^2 x$	B1 B1 ft E1 **3**	*Must follow from their* $\dot{x}$ Fully correct completion *SR* For $\dot{x} = -A\omega\cos\omega t + B\omega\sin\omega t$ $\ddot{x} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$ award B0B1E0
(ii)	$B = 2$ $A\omega = -1.44$ $-B\omega^2 = -0.18$ *or* $-0.18 = -\omega^2(2)$ $\omega = 0.3,\quad A = -4.8$	B1 M1 A1 cao M1 A1 cao A1 cao **6**	Using $\frac{\mathrm{d}x}{\mathrm{d}t} = -1.44$ when $t = 0$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -0.18$ when $t = 0$ (or $x = 2$)
(iii)	Period is $\frac{2\pi}{\omega} = \frac{2\pi}{0.3} = 20.94 = 20.9$ s (3 sf) Amplitude is $\sqrt{A^2 + B^2} = \sqrt{4.8^2 + 2^2}$ $= 5.2$ m	E1 M1 A1 **3**	or $1.44^2 = 0.3^2(a^2 - 2^2)$
(iv)	$x = -4.8\sin 0.3t + 2\cos 0.3t$ $v = -1.44\cos 0.3t - 0.6\sin 0.3t$ When $t = 12,\ x = 0.3306 \quad (v = 1.56)$ When $t = 24,\ x = -2.5929 \quad (v = -1.35)$ Distance travelled is $(5.2 - 0.3306) + 5.2 + 2.5929$ $= 12.7$ m	M1 A1 M1 M1 A1 **5**	Finding *x* when $t = 12$ and $t = 24$ Both displacements correct Considering change of direction Correct method for distance ft from their A, B, ω and amplitude: *Third M1 requires the method to be comparable to the correct one* *A1A1 both require* $\omega \approx 0.3,\ A \neq 0,\ B \neq 0$ *Note* ft from $A = +4.8$ is $x_{12} = -3.92\ (v < 0) \quad x_{24} = 5.03\ (v > 0)$ Distance is $(5.2 - 3.92) + 5.2 + 5.03$ $= 11.5$

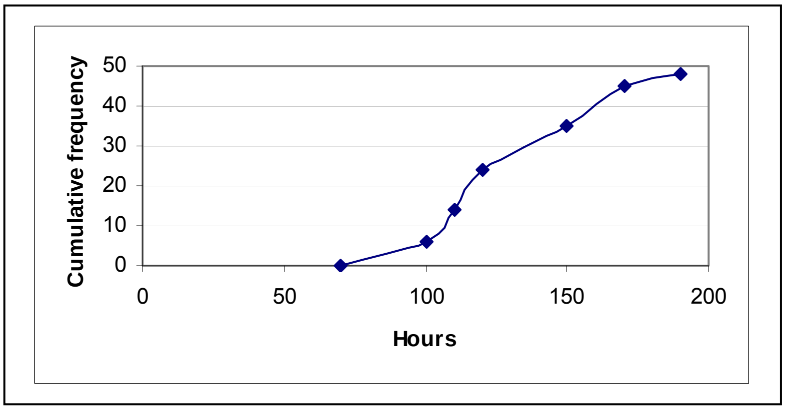
4 (i)	$V=\int_1^8 \pi\,(x^{-\frac{1}{3}})^2\,\mathrm{d}x$	M1	*π may be omitted throughout*
	$=\pi\left[\,3x^{\frac{1}{3}}\,\right]_1^8=3\pi$	A1	
	$V\bar{x}=\int_1^8 \pi\,x\,(x^{-\frac{1}{3}})^2\,\mathrm{d}x$	M1	
	$=\pi\left[\,\tfrac{3}{4}x^{\frac{4}{3}}\,\right]_1^8=\frac{45}{4}\pi$	A1	
	$\bar{x}=\frac{\frac{45}{4}\pi}{3\pi}$	M1	*Dependent on previous M1M1*
	$=\frac{15}{4}=3.75$	A1	
		6	
(ii)	$A=\int_1^8 x^{-\frac{1}{3}}\,\mathrm{d}x$	M1	
	$=\left[\,\tfrac{3}{2}x^{\frac{2}{3}}\,\right]_1^8=\frac{9}{2}=4.5$	A1	
	$A\bar{x}=\int_1^8 x\,(x^{-\frac{1}{3}})\,\mathrm{d}x$	M1	
	$=\left[\,\tfrac{3}{5}x^{\frac{5}{3}}\,\right]_1^8=\frac{93}{5}=18.6$	A1	
	$\bar{x}=\frac{18.6}{4.5}=\frac{62}{15}\quad(\approx 4.13)$	A1	
	$A\bar{y}=\int_1^8 \tfrac{1}{2}(x^{-\frac{1}{3}})^2\,\mathrm{d}x$	M1	If $\frac{1}{2}$ omitted, award M1A0A0
	$=\left[\,\tfrac{3}{2}x^{\frac{1}{3}}\,\right]_1^8=\frac{3}{2}=1.5$	A1	
	$\bar{y}=\frac{1.5}{4.5}=\frac{1}{3}$	A1	
		8	

(iii)	$(1)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}+(3.5)\begin{pmatrix}4.5\\ 0.25\end{pmatrix}=(4.5)\begin{pmatrix}62/15\\ 1/3\end{pmatrix}=\begin{pmatrix}18.6\\ 1.5\end{pmatrix}$ $\bar{x}=2.85$ $\bar{y}=0.625$	M1 M1 A1 A1 **4**	Attempt formula for CM of composite body (one coordinate sufficient) Formulae for both coordinates; signs must now be correct, but areas (1 and 3.5) may be wrong. ft only if $1<\bar{x}<8$ ft only if $0.5<\bar{y}<1$ *Other methods:* M1A1 for $\bar{x}$ M1A1 for $\bar{y}$ *(In each case, M1 requires a complete and correct method leading to a numerical value)*

4766 Statistics 1

Q1 **(i)**	Mode = 7 Median = 12.5	B1 **cao** B1 **cao**	**2**
(ii)	Positive or positively skewed	E1	**1**
(iii)	(*A*) Median (*B*) There is a large outlier or possible outlier of 58 / figure of 58. Just 'outlier' on its own without reference to either 58 or large scores E0 Accept the large outlier affects the mean (more) E1	E1 cao E1indep	**2**
(iv)	There are 14.75 × 28 = 413 messages So total cost = 413 × 10 pence = £41.30	M1 for 14.75 × 28 but 413 can also imply the mark A1**cao**	**2**
		TOTAL	**7**
Q2 **(i)**	$\begin{pmatrix}4\\3\end{pmatrix}\times 3! = 4\times 6 = 24$ codes or $^4P_3$ = 24 (M2 for $^4P_3$) *Or* 4 × 3 × 2 = 24	M1 for 4 M1 for ×6 A1	**3**
(ii)	4^3 = 64 codes	M1 for 4^3 A1 **cao**	**2**
		TOTAL	**5**
Q3 **(i)**	Probability = 0.3 × 0.8 = 0.24	M1 for 0.8 from (1-0.2) A1	**2**
(ii)	*Either:* P(*A*U*B*) = P(*A*) + P(*B*) – P(*A*∩*B*) = 0.3 + 0.2 – 0.3 × 0.2 = 0.5 – 0.06 = 0.44 *Or:* P(*A*U*B*) = 0.7 × 0.2 + 0.3 × 0.8 + 0.3 × 0.2 = 0.14 + 0.24 + 0.06 = 0.44 *Or:* P(*A*U*B*) = 1 – P(*A'* ∩*B'*) = 1 – 0.7 × 0.8 = 1 – 0.56 = 0.44	M1 for adding 0.3 and 0.2 M1 for **subtraction** of (0.3 × 0.2) A1 **cao** M1 either of first terms M1 for last term A1 M1 for 0.7 × 0.8 or 0.56 M1 for complete method as seen A1	**3**
(iii)	P(*A*|*B*) = $\frac{P(A\cap B)}{P(B)} = \frac{0.06}{0.44} = \frac{6}{44} = 0.136$	M1 for numerator of their 0.06 only M1 for 'their 0.44' in denominator A1 FT (must be valid p)	**3**
		TOTAL	**8**

Q4 **(i)**	E(X) = $1\times0.2+2\times0.16+3\times0.128+4\times0.512=2.952$ Division by 4 or other spurious value at end loses A mark E(X^2) = $1\times0.2+4\times0.16+9\times0.128+16\times0.512=10.184$ Var(X) = $10.184 - 2.952^2 = 1.47$ (to 3 s.f.)	M1 for Σ *rp* (at least 3 terms correct) A1 cao M1 for Σ x^2p *at least 3 terms correct* M1 for E(X^2) – E(X) ^{2} Provided ans > 0 A1 FT their E(X) but not a wrong E(X^2)	**5**
(ii)	Expected cost = 2.952 × £45000 = £133000 (3sf)	B1 FT (no extra multiples / divisors introduced at this stage)	**1**
(iii)		G1 labelled linear scales G1 height of lines	**2**
		TOTAL	**8**
Q5 **(i)**	Impossible because the competition would have finished as soon as Sophie had won the first 2 matches	E1	**1**
(ii)	**SS, JSS, JSJSS**	B1, B1, B1 (-1 each error or omission)	**3**
(iii)	$0.7^2 + 0.3 \times 0.7^2 + 0.7 \times 0.3 \times 0.7^2 = 0.7399$ or 0.74(0) { 0.49 + 0.147 + 0.1029 = 0.7399}	M1 for any correct term M1 for any other correct term M1 for sum of all three correct terms A1 cao	**4**
		TOTAL	**8**

<table>
<tr><td></td><td>Section B</td><td></td><td></td></tr>
<tr><td>Q6
(i)</td><td>Mean = $\frac{180.6}{12}$ = 15.05 or 15.1
S_{xx} = $3107.56 - \frac{180.6^2}{12}$ or 3107.56 – 12(their 15.05)^2 =
(389.53)
s = $\sqrt{\frac{389.53}{11}}$ = 5.95 or better
NB Accept answers seen without working (from calculator)</td><td>B1 for mean
M1 for attempt at S_{xx}
A1 cao</td><td>3</td></tr>
<tr><td>(ii)</td><td>$\bar{x} + 2s$ = 15.05 + 2 × 5.95 = 26.95
$\bar{x} - 2s$ = 15.05 – 2 × 5.95 = 3.15
So no outliers</td><td>M1 for attempt at either
M1 for both
A1 for limits and conclusion FT their mean and sd</td><td>3</td></tr>
<tr><td>(iii)</td><td>New mean = 1.8 × 15.05 + 32 = 59.1
New s = 1.8 × 5.95 = 10.7</td><td>B1FT
M1 A1FT</td><td>3</td></tr>
<tr><td>(iv)</td><td>New York has a higher mean or ‘ is on average’ higher (oe)
New York has greater spread /range /variation or SD (oe)</td><td>E1FT using ^{0}F ($\bar{x}$ dep)
E1FT using ^{0}F (σ dep)</td><td>2</td></tr>
<tr><td>(v)
(vi)</td><td>
<table>
<tr><td>Upper bound</td><td>(70)</td><td>100</td><td>110</td><td>120</td><td>150</td><td>170</td><td>190</td></tr>
<tr><td>Cumulative frequency</td><td>(0)</td><td>6</td><td>14</td><td>24</td><td>35</td><td>45</td><td>48</td></tr>
</table>


NB all G marks dep on attempt at cumulative frequencies.
NB All G marks dep on attempt at cumulative frequencies
Line on graph at cf = 43.2(soi) or used
90th percentile = 166</td><td>B1 for all correct cumulative frequencies (may be implied from graph). Ignore cf of 0 at this stage
G1 for linear scales (linear from 70 to 190) ignore x < 70 vertical: 0 to 50 but not beyond 100 (no inequality scales)
G1 for labels
G1 for points plotted as (UCB, their cf). Ignore (70,0) at this stage. No mid – point or LCB plots.
G1 for joining all of ‘their points’(line or smooth curve) AND now including (70,0)
M1 for use of 43.2
A1FT but dep on 3rd G mark earned</td><td>5
2</td></tr>
<tr><td></td><td></td><td>TOTAL</td><td>18</td></tr>
</table>

Q7 **(i)**	$X \sim$ B(12, 0.05) $(\boldsymbol{A})\quad \text{P}(\boldsymbol{X}=1) = \binom{12}{1} \times 0.05 \times 0.95^{11} = 0.3413$	M1 0.05×0.95^{11} M1 $\binom{12}{1} \times pq^{11}$ (p+q) = 1 A1 cao	**3**
	OR from tables $\quad 0.8816 - 0.5404 = 0.3412$	OR: M1 for 0.8816 seen and M1 for subtraction of 0.5404 A1 cao	**2**
	$(\boldsymbol{B})\quad \text{P}(\boldsymbol{X} \geq 2) = 1 - 0.8816 = 0.1184$	M1 for 1 – P(X ≤ 1) A1 cao	**2**
	$(\boldsymbol{C})\quad$ Expected number E($\boldsymbol{X}$) $= \boldsymbol{np} = 12 \times 0.05 = 0.6$	M1 for 12×0.05 A1 cao (= 0.6 seen)	
(ii)	*Either*: $1 - 0.95^n \leq ⅓$ $0.95^n \geq ⅔$ $n \leq \log ⅔ / \log 0.95$, so $n \leq 7.90$ Maximum n = 7	M1 for equation in n M1 for use of logs A1 cao	
	Or: (using tables with p = 0.05): n = 7 leads to P(X ≥ 1) = 1 - P(X = 0) = 1 – 0.6983 = 0.3017 (< ⅓) or 0.6983 (> 2/3) n = 8 leads to P(X ≥ 1) = 1 - P(X = 0) = 1 – 0.6634 = 0.3366 (> ⅓) or 0.6634 (< 2/3) Maximum n = 7 (total accuracy needed for tables)	M1indep M1indep A1 cao dep on both M's	**3**
	Or: (using trial and improvement): $1 - 0.95^7 = 0.3017$ (< ⅓) or $0.95^7 = 0.6983$ (> 2/3) $1 - 0.95^8 = 0.3366$ (> ⅓) or $0.96^8 = 0.6634$ (< 2/3) Maximum n = 7 (3 sf accuracy for calculations)	M1indep (as above) M1indep (as above) A1 cao dep on both M's	
	NOTE: n = 7 unsupported scores SC1 only		
(iii)	Let $X \sim$ B(60, p) Let p = probability of a bag being faulty H_0: $p = 0.05$ H_1: $p < 0.05$	B1 for definition of p B1 for H_0 B1 for H_1	**8**
	P($X \leq 1$) = $0.95^{60} + 60 \times 0.05 \times 0.95^{59} = 0.1916 > 10\%$	M1 A1 for probability M1 for comparison	
	So not enough evidence to reject H_0	A1	
	Conclude that there is not enough evidence to indicate that the new process reduces the failure rate or scientist incorrect/ wrong.	E1	
		TOTAL	**18**

4767 Statistics 2

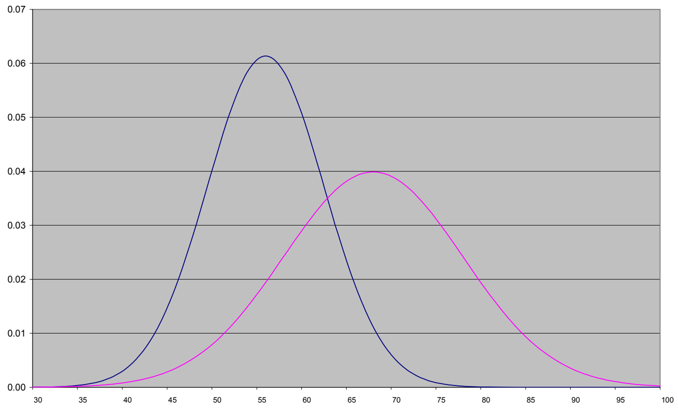
Question 1

(i)	x is independent, y is dependent since the values of x are chosen by the student but the values of y are dependent on x	B1 E1 dep E1 dep	**3**
(ii)	$\bar{x} = 2.5$, $\bar{y} = 80.63$ $b = \frac{Sxy}{Sxx} = \frac{2530.3 - 30 \times 967.6/12}{90 - 30^2/12} = \frac{111.3}{15} = 7.42$ OR $b = \frac{2530.3/12 - 2.50 \times 80.63}{90/12 - 2.50^2} = \frac{9.275}{1.25} = 7.42$ Hence least squares regression line is: $y - \bar{y} = b(x - \bar{x})$ $\Rightarrow y - 80.63 = 7.42(x - 2.5)$ $\Rightarrow y = 7.42x + 62.08$	B1 for $\bar{x}$ and $\bar{y}$ used (SOI) M1 for attempt at gradient (b) A1 for 7.42 **cao** M1 for equation of line A1 FT (b>0) for complete equation	**5**
(iii)	*(A)* For $x = 1.2$, predicted growth $= 7.42 \times 1.2 + 62.08 = 71.0$ *(B)* For $x = 4.3$, predicted growth $= 7.42 \times 4.3 + 62.08 = 94.0$ Valid relevant comments relating to the predictions such as : Comment re interpolation/extrapolation Comment relating to the fact that $x = 4.3$ is only just beyond the existing data. Comment relating to size of residuals near each predicted value (need not use word 'residual')	M1 for at least one prediction attempted. A1 for both answers (FT their equation if b>0) E1 (first comment) E1 (second comment)	**4**
(iv)	$x = 3 \Rightarrow$ predicted $y = 7.42 \times 3 + 62.08 = 84.3$ Residual $= 80 - 84.3 = -4.3$	M1 for prediction M1 for subtraction A1 FT (b>0)	**3**
(v)	This point is a long way from the regression line. The line may be valid for the range used in the experiment but then the relationship may break down for higher concentrations, or the relationship may be non linear.	E1 E1 for valid in range E1 for *either* 'may break down' *or* 'could be non linear' *or* other relevant comment	**3**
			18

Question 2

(i)	Binomial (94,0.1)	B1 for binomial B1 dep for parameters	**2**
(ii)	*n* is large and *p* is small	B1, B1 Allow appropriate numerical ranges	**2**
(iii)	$\lambda = 94 \times 0.1 = 9.4$ *(A)* $\mathrm{P}(\mathrm{X} = 4) = \mathrm{e}^{-9.4}\,\dfrac{9.4^4}{4!} = 0.0269$ (3 s.f.) or from tables = 0.0429 − 0.0160 = 0.0269 ***cao*** *(B)* Using tables*:* $\mathrm{P}(X \geq 4) = 1 - \mathrm{P}(X \leq 3)$ = 1 − 0.0160 = 0.9840 ***cao***	B1 for mean M1 for calculation or use of tables A1 M1 for attempt to find $\mathrm{P}(X \geq 4)$ A1 **cao**	**5**
(iv)	P(sufficient rooms throughout August) $= 0.9840^{31} = 0.6065$	M1 A1 FT	**2**
(v)	(*A*) 31 × 94 = 2914 Binomial (2914,0.1) (*B*)Use Normal approx with $\mu = np = 2914 \times 0.1 = 291.4$ $\sigma^2 = npq = 2914 \times 0.1 \times 0.9 = 262.26$ $\mathrm{P}(X \leq 300.5) = \mathrm{P}\left(Z \leq \dfrac{300.5 - 291.4}{\sqrt{262.26}}\right)$ $= \mathrm{P}(Z \leq 0.5619) = \Phi(0.5619) = 0.7130$	B1 for binomial B1 dep, for parameters B1 B1 B1 for continuity corr. M1 for probability using correct tail A1 **cao,** (but FT wrong or omitted CC)	**2** **5**
			18

Question 3

(i)	$X \sim \mathrm{N}(56, 6.5^2)$ $\mathrm{P}(52.5 < X < 57.5) = \mathrm{P}\left(\frac{52.5-56}{6.5} < Z < \frac{57.5-56}{6.5}\right)$ $= \mathrm{P}(-0.538 < Z < 0.231)$ $= \Phi(0.231) - (1 - \boxed{\Phi}(0.538))$ $= 0.5914 - (1 - 0.7046)$ $= 0.5914 - 0.2954$ = 0.2960 (4 s.f.) *or* 0.296 (to 3 s.f.)	M1 for standardizing A1 for -0.538 and 0.231 M1 for prob. with tables and correct structure A1 CAO (min 3 s.f., to include use of difference column)	**4**
(ii)	$\mathrm{P}(\text{5-year-old} < 62) = \mathrm{P}\left(Z < \frac{62-56}{6.5}\right)$ $= \Phi(0.923) = 0.8220$ $\mathrm{P}(\text{young adult} < 62) = \mathrm{P}\left(Z < \frac{62-68}{10}\right)$ $= \Phi(-0.6) = 1 - 0.7257 = 0.2743$ P(One over, one under) $= 0.8220 \times 0.7257 + 0.1780 \times 0.2743$ $= 0.645$	B1 for 0.8220 or 0.1780 B1 for 0.2743 or 0.7257 M1 for either product M1 for sum of both products A1 CAO	**5**
(iii)		G1 for shape G1 for means, shown explicitly or by scale G1 for lower max height in young adults G1 for greater variance in young adults	**4**
(iv)	$Y \sim \mathrm{N}(82, \sigma^2)$ From tables $\Phi^{-1}(0.88) = 1.175$ $\frac{62-82}{\sigma} = -1.175$ $-20 = -1.175\,\sigma$ $\sigma = 17.0$	B1 for 1.175 seen M1 for equation in σ with z-value M1 for correct handling of LH tail A1 **cao**	**4**
			17

Question 4

<table>
<tr>
<td>(i)</td>
<td>
H_0: no association between sex and subject;
H_1: some association between sex and subject;
<table>
<tr><td>OBS</td><td>Maths</td><td>English</td><td>Both</td><td>Neither</td><td>Row sum</td></tr>
<tr><td>Male</td><td>38</td><td>19</td><td>6</td><td>32</td><td>95</td></tr>
<tr><td>Female</td><td>42</td><td>55</td><td>9</td><td>49</td><td>155</td></tr>
<tr><td>Col sum</td><td>80</td><td>74</td><td>15</td><td>81</td><td>250</td></tr>
</table>
<table>
<tr><td>EXP</td><td>Maths</td><td>English</td><td>Both</td><td>Neither</td><td>Row sum</td></tr>
<tr><td>Male</td><td>30.40</td><td>28.12</td><td>5.70</td><td>30.78</td><td>95</td></tr>
<tr><td>Female</td><td>49.60</td><td>45.88</td><td>9.30</td><td>50.22</td><td>155</td></tr>
<tr><td>Col sum</td><td>80</td><td>74</td><td>15</td><td>81</td><td>250</td></tr>
</table>
<table>
<tr><td>CONT</td><td>Maths</td><td>English</td><td>Both</td><td>Neither</td></tr>
<tr><td>Male</td><td>1.900</td><td>2.958</td><td>0.016</td><td>0.048</td></tr>
<tr><td>Female</td><td>1.165</td><td>1.813</td><td>0.010</td><td>0.030</td></tr>
</table>
$X^2 = 7.94$

Refer to χ_3^2
Critical value at 5% level = 7.815
Result is significant
There is evidence to suggest that there is some association between sex and subject choice.
NB if H_0 H_1 reversed, or ‘correlation’ mentioned, do not award first B1 or final E1
</td>
<td>
B1

M1 A2 for expected values
(allow A1 for at least one row or column correct)

M1 for valid attempt at $(O-E)^2/E$
A1
NB These M1 A1 marks cannot be implied by a correct final value of X^2

M1 for summation
A1 cao for X^2

B1 for 3 deg of f
B1 CAO for cv

B1

E1
</td>
<td>1

7

4</td>
</tr>
<tr>
<td>(ii)</td>
<td>
H_0: $\mu = 67.4$; H_1: $\mu > 67.4$
Where μ denotes the mean score of the population of students taught with the new method.

Test statistic = $\frac{68.3 - 67.4}{8.9/\sqrt{12}} = \frac{0.9}{2.57}$
= 0.35

10% level 1 tailed critical value of z = 1.282
0.35 < 1.282 so not significant.
There is insufficient evidence to reject H_0
There is insufficient evidence to conclude that the mean score is increased by the new teaching method.
</td>
<td>
B1 for both correct

B1 for definition of μ

M1

A1 cao

B1 for 1.282

M1 for comparison

A1 for conclusion in words and in context
</td>
<td>7</td>
</tr>
<tr><td></td><td></td><td></td><td>19</td></tr>
</table>

4768 Statistics 3

<table>
<tr><td>Q1
(a)</td><td>$P(T > t) = \frac{k}{t^2}$, $t \ge 1$,</td><td></td><td></td><td></td></tr>
<tr><td>(i)</td><td>F(t) = P(T < t) = 1 – P(T > t)
$\therefore F(t) = 1 - \frac{k}{t^2}$</td><td>M1</td><td>Use of 1 – P(…).</td><td></td></tr>
<tr><td></td><td>F(1) = 0
$\therefore 1 - \frac{k}{1^2} = 0$</td><td>M1</td><td></td><td></td></tr>
<tr><td></td><td>∴ k = 1</td><td>A1</td><td>Beware: answer given.</td><td>3</td></tr>
<tr><td>(ii)</td><td>$f(t) = \frac{\mathrm{d}\,F(t)}{\mathrm{d}t}$</td><td>M1</td><td>Attempt to differentiate c’s cdf.</td><td></td></tr>
<tr><td></td><td>$= \frac{2}{t^3}$</td><td>A1</td><td>(For $t \ge 1$, but condone absence of this.) Ft c’s cdf provided answer sensible.</td><td>2</td></tr>
<tr><td>(iii)</td><td>$\mu = \int_1^\infty t f(t) \mathrm{d}t = \int_1^\infty \frac{2}{t^2} \mathrm{d}t$</td><td>M1</td><td>Correct form of integral for the mean, with correct limits. Ft c’s pdf.</td><td></td></tr>
<tr><td></td><td>$= \left[\frac{-2}{t}\right]_1^\infty$</td><td>A1</td><td>Correctly integrated. Ft c’s pdf.</td><td></td></tr>
<tr><td></td><td>$= 0 - (-2) = 2$</td><td>A1</td><td>Correct use of limits leading to correct value. Ft c’s pdf provided answer sensible.</td><td>3</td></tr>
<tr><td>(b)</td><td>H_0: m = 5.4
H_1: $m \neq 5.4$</td><td>B1</td><td>Both hypotheses. Hypotheses in words only must include “population”.</td><td></td></tr>
<tr><td></td><td>where m is the population median time for the task.</td><td>B1</td><td>For adequate verbal definition.</td><td></td></tr>
<tr><td></td><td><table>
<tr><th>Times</th><th>− 5.4</th><th>Rank of |diff|</th></tr>
<tr><td>6.4</td><td>1.0</td><td>8</td></tr>
<tr><td>5.9</td><td>0.5</td><td>5</td></tr>
<tr><td>5.0</td><td>−0.4</td><td>4</td></tr>
<tr><td>6.2</td><td>0.8</td><td>7</td></tr>
<tr><td>6.8</td><td>1.4</td><td>10</td></tr>
<tr><td>6.0</td><td>0.6</td><td>6</td></tr>
<tr><td>5.2</td><td>−0.2</td><td>2</td></tr>
<tr><td>6.5</td><td>1.1</td><td>9</td></tr>
<tr><td>5.7</td><td>0.3</td><td>3</td></tr>
<tr><td>5.3</td><td>−0.1</td><td>1</td></tr>
</table></td><td>M1
M1
A1</td><td>for subtracting 5.4.
for ranks.
FT if ranks wrong.</td><td></td></tr>
<tr><td></td><td>W_- = 1 +2 + 4 = 7 (or W_+ = 3+5+6+7+8+9+10 = 48)</td><td>B1</td><td></td><td></td></tr>
<tr><td></td><td>Refer to tables of Wilcoxon single sample (/paired) statistic for n = 10.</td><td>M1</td><td>No ft from here if wrong.</td><td></td></tr>
<tr><td></td><td>Lower (or upper if 48 used) double-tailed 5% point is 8 (or 47 if 48 used).</td><td>A1</td><td>i.e. a 2-tail test. No ft from here if wrong.</td><td></td></tr>
<tr><td></td><td>Result is significant.</td><td>A1</td><td>ft only c’s test statistic.</td><td></td></tr>
<tr><td></td><td>Seems that the median time is no longer as previously thought.</td><td>A1</td><td>ft only c’s test statistic.</td><td>10</td></tr>
</table>

Q2	$X \sim \text{N}(260,\ \sigma = 24)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.	
(i)	P (X <300) = P$(Z < \frac{300-260}{24} = 1.6667)$ = 0.9522	M1 A1 A1	For standardising. Award once, here or elsewhere.	3
(ii)	$Y \sim \text{N}(260 \times 0.6 = 156,$ $24^2 \times 0.6^2 = 207.36$ $\text{P}(Y > 175) = \text{P}(Z > \frac{175-156}{14.4} = 1.3194)$ $= 1 - 0.9063 = 0.0937$	B1 B1 A1	Mean. Variance. Accept sd (= 14.4). c.a.o.	3
(iii)	$Y_1 + Y_2 + Y_3 + Y_4 \sim \text{N}(624,$ $829.44)$ $\text{P}(\text{this} < 600) = \text{P}(Z < \frac{600-624}{28.8} = -0.8333)$ $= 1 - 0.7976 = 0.2024$	B1 B1 A1	Mean. Ft mean of (ii). Variance. Accept sd (= 28.8). Ft variance of (ii). c.a.o.	3
(iv)	Require w such that $0.975 = \text{P}(\text{above} > w) = \text{P}\left(Z > \frac{w-624}{28.8}\right)$ $= \text{P}(Z > -1.96)$ $\therefore w - 624 = 28.8 \times -1.96 \Rightarrow w = 567.5(52)$	M1 B1 A1	Formulation of requirement. − 1.96 Ft parameters of (iii).	3
(v)	$On \sim \text{N}(150,\ \sigma = 18)$ $X_1 + X_2 + X_3 + On_1 + On_2 \sim \text{N}(1080,$ $2376)$ $\text{P}(\text{this} > 1000) = \text{P}(Z > \frac{1000-1080}{48.744} = -1.6412)$ $= 0.9496$	 B1 B1 A1	 Mean. Variance. Accept sd (= 48.744). c.a.o.	3
(vi)	Given $\bar{x} = 252.4$ $s_{n-1} = 24.6$ CI is given by $252.4 \pm 2.576 \times \frac{24.6}{\sqrt{100}}$ = 252.4 ± 6.33(6) = (246.0(63), 258.7(36))	M1 B1 A1	Correct use of 252.4 and $24.6/\sqrt{100}$. For 2.576. c.a.o. Must be expressed as an interval.	3
				18

Q3				
(i)	A t test should be used because the sample is small, the population variance is unknown, the background population is Normal	E1 E1 E1		3
(ii)	H_0: $\mu = 380$ H_1: $\mu < 380$	B1	Both hypotheses. Hypotheses in words only must include "population".	
	where μ is the mean temperature in the chamber.	B1	For adequate verbal definition. Allow absence of "population" if correct notation μ is used, but do NOT allow " $\bar{X} = \ldots$ " or similar unless $\bar{X}$ is clearly and explicitly stated to be a <u>population</u> mean.	
	$\bar{x} = 373.825 \qquad s_{n-1} = 9.368$	B1	s_n = 8.969 but do <u>NOT</u> allow this here or in construction of test statistic, but FT from there.	
	Test statistic is $\dfrac{373.825 - 380}{\frac{9.368}{\sqrt{12}}}$	M1	Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: 380 + (c's – 1.796) $\times \frac{9{\cdot}368}{\sqrt{12}}$ (= 375.143) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ – (c's –1.796) $\times \frac{9{\cdot}368}{\sqrt{12}}$ (= 378.681) for comparison with 380.)	
	= –2.283(359).	A1	c.a.o. but ft from here in any case if wrong. Use of 380 – $\bar{x}$ scores M1A0, but ft.	
	Refer to t_{11}.	M1	No ft from here if wrong.	
	Single-tailed 5% point is –1.796.	A1	Must be minus 1.796 unless absolute values are being compared. No ft from here if wrong.	
	Significant.	A1	ft only c's test statistic.	
	Seems mean temperature in the chamber has fallen.	A1	ft only c's test statistic.	9
(iii)	CI is given by 373.825 ±	M1		
	2·201	B1		
	$\times \dfrac{9.368}{\sqrt{12}}$	M1		
	= 373.825 ± 5.952= (367.87(3), 379.77(7))	A1	c.a.o. Must be expressed as an interval. ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_{11} is OK.	4
(iv)	Advantage: greater certainty.	E1	Or equivalents.	
	Disadvantage: less precision.	E1		2 18

Q4				
(a) **(i)**	$\bar{x} = \frac{1125}{500} = 2.25$ For binomial E(X) = $n \times p$ $\therefore \hat{p} = \frac{2.25}{5} = 0.45$	B1 M1 A1	 Use of mean of binomial distribution. May be implicit. Beware: answer given.	3
(ii)				

f_o	32	110	154	125	63	16
f_e (calc)	25.164	102.944	168.455	137.827	56.384	9.226
f_e (tables)	25.15	102.95	168.45	137.85	56.35	9.25

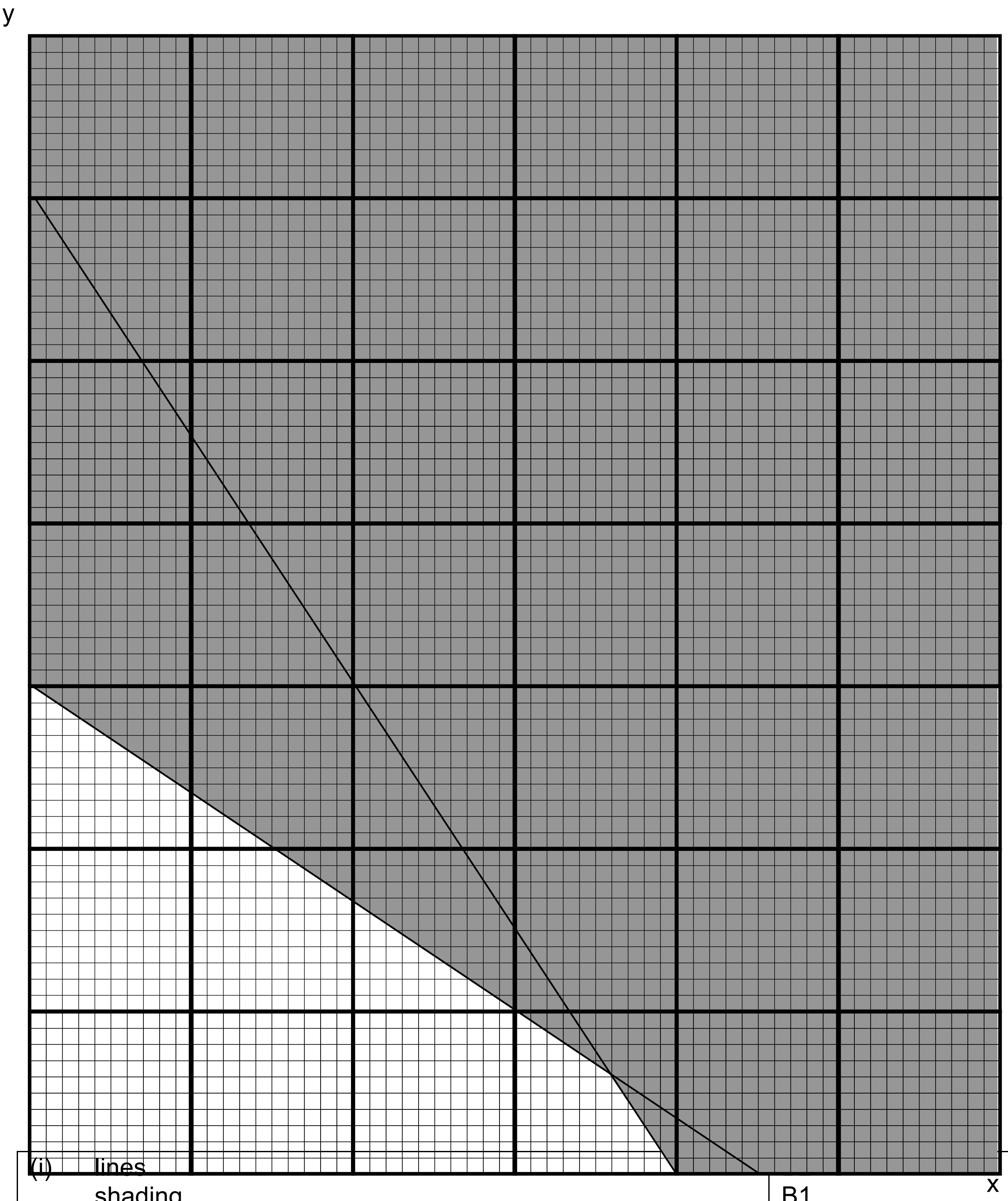
		M1	Calculation of expected frequencies.	
		A1	All correct.	
	X^2 = 1.8571 + 0.4836 + 1.2404 + 1.1938 + 0.7763 + 4.9737	M1	Or using tables: 1.8657 + 0.4828 + 1.2396 + 1.1978 + 0.7848 + 4.9257	
	= 10.52(49)	A1	c.a.o. Or using tables: 10.49(64)	
	Refer to χ_4^2.	M1	Allow correct df (= cells – 2) from wrongly grouped or ungrouped table, and FT. Otherwise, no FT if wrong.	
	Upper 5% point is 9.488.	A1	No ft from here if wrong.	
	Significant.	A1	ft only c's test statistic.	
	Suggests binomial model does not fit.	A1	ft only c's test statistic.	
	The model appears to overestimate in the middle and to underestimate at the tails. The biggest discrepancy is at $X = 5$.	E1 E1	Accept also any other sensible comment e.g. at 2.5% significance, the result would NOT have been significant.	
	A binomial model assumes all trials are independent with a constant probability of "success". It seems unlikely that there will be independence within families and/or that p will be the same for all families.	E2	(E2, 1, 0) Any sensible comment which addresses independence and constant p.	12
(b)	She should try to choose a simple random sample which would involve establishing a sampling frame and using some form of random number generator.	E1 E1 E1	Allow sensible discussion of practical limitations of choosing a random sample. Allow other sensible suggestions. E.g systematic sample - choosing every tenth family; stratified sample - by the number of girls in a family.	3
				18

4771 Decision Mathematics 1

1

(i)	6 routes M→A→I→T→Pi→C M→A→I→T→Pi→R→C M→A→I→T→Pi→H→R→C M→V→I→T→Pi→C M→V→I→T→Pi→R→C M→V→I→T→Pi→H→R→C	B1 B1	
(ii)	6 routes M→A→I→Pa→Pi→C M→A→I→Pa→Pi→R→C M→A→I→Pa→Pi→H→R→C M→V→I→Pa→Pi→C M→V→I→Pa→Pi→R→C M→V→I→Pa→Pi→H→R→C	B1 B1	
(iii)	$\begin{tikzcd} M \arrow[r] \arrow[dr] & V \arrow[r] & I \arrow[r] & Pa \arrow[r] & Pi \arrow[r] \arrow[dr] & H \arrow[r] & R \arrow[r] & Pi \arrow[r] & C \\ & A \arrow[ur] & & & & R \arrow[r] & H \arrow[ur] & & \end{tikzcd}$	B1	
(iv)	e.g. P→T→I→V→M→A→I→Pa→P→H→R→C→P→R	M1 A2	ends at R (–1 each error/omission)

2.



(i)	lines		
	shading	B1	
	(3.6, 0.6)	B1	graph or sim. eqns
	25.8 at (3.6, 0.6) versus 21 and 24 (or profit line)	M1 A1	
(ii)	25 at (3, 1)	B1 B1	

3.

y = 2008 c = 2008/100 = 20 n = 2008 – 19 x (2008/19) = 2008 – 19 x (105) = 13 k = 3/25 = 0 i = 20 – 5 – 20 / 3 + 19 × 13 + 15 = 271 i = 1 i = 1 – 0 = 1 j = 2008 + 502 + 1 + 2 – 20 + 5 = 2498 j = 6 p = – 5 m = 3 d = 23 So 23rd March	B1 B1 B1 B1 B1 B1 B1 B1

4.

(i) e.g. 0–3→brown
4–7→blue
8–9→green

M1
A1 proportions OK
A1 efficient

(ii) e.g. 0–1→brown
2–5→blue
6–7→green
8–9→reject

M1 some rejected
A2 proportions OK (–1 each error)
A1 efficient

(iii) e.g.

Eye colours

Parent 1	brown	brown	brown	blue
Parent 2	brown	blue	brown	blue
Offspring	brown	brown	brown	brown

brown	green	blue	green	brown	brown
brown	blue	brown	green	brown	green
brown	blue	brown	green	brown	blue

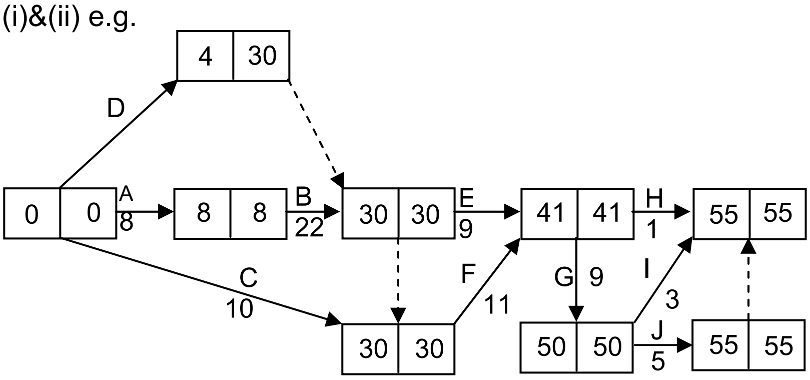
B1 br/br→br (4 times)
B1 br/gr→bl
B1 gr/gr→gr

M1 br/bl rule
A1 application
A1 application

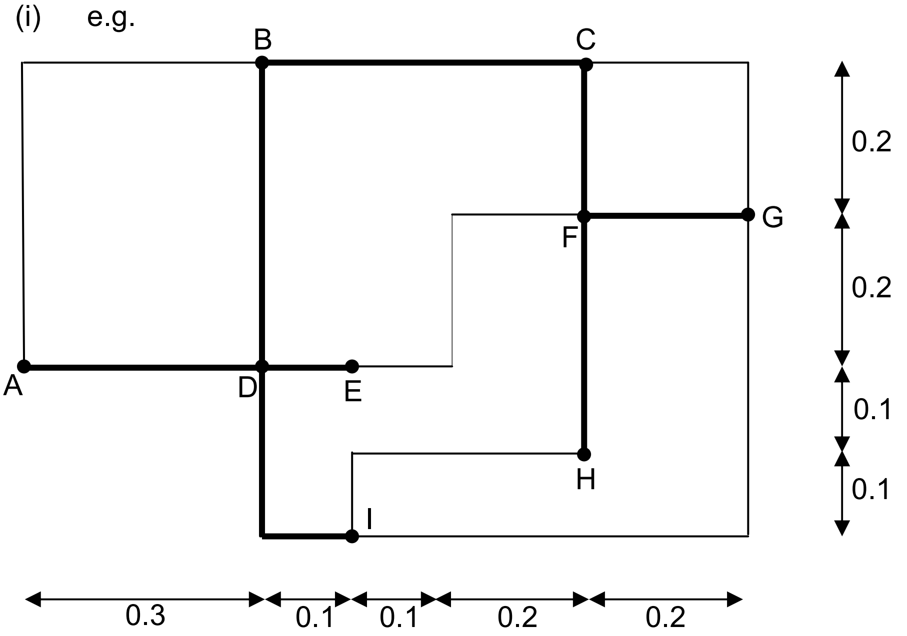
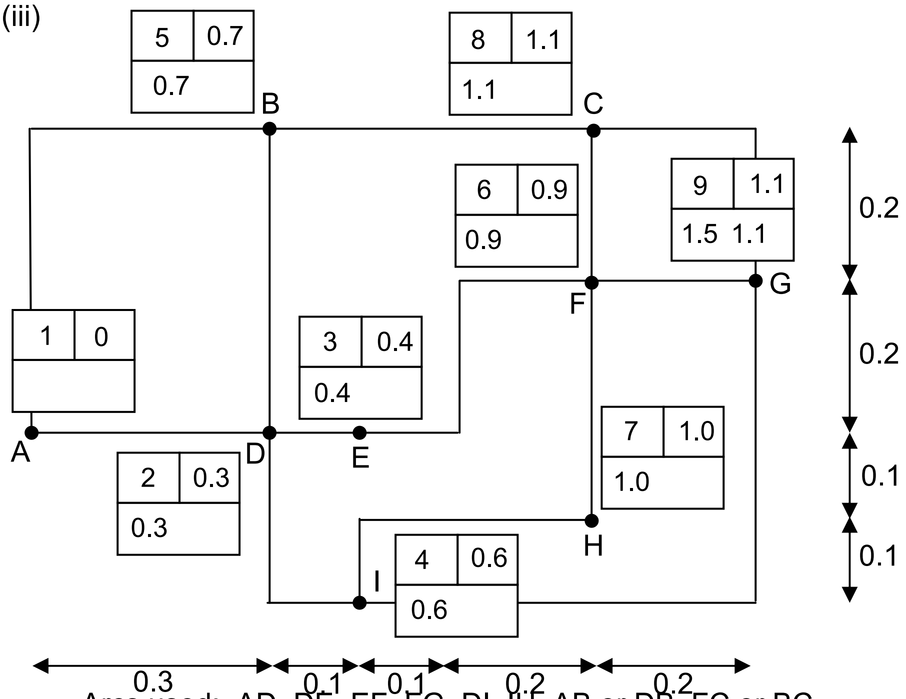
B1 bl/bl application

M1 gr/bl rule
A1 application

5.

(i)&(ii) e.g. 	M1 sca (activity on arc) A1 dummy activities + E and F A1 A, B, C, D A1 G, H, I, J M1 forward pass A1 M1 backward pass A1
time – 55 weeks	B1 cao
critical – A; B; F; G; J	B1 cao
(iii) 50 weeks (49 weeks if G crashed rather than H)	B1
(iv) E – 1 week F – 3 weeks J – 2 weeks (G − 1 week, if crashed)	M1 A3
(v) £115000 (£121000)	A1

6.

(i) e.g.  Total length = 2.2 km	M1 connecting tree A1 DE A1 FC, FG A1 AD, DI, FH A1 2 of length 0.4 M1 A1
(ii) Prim: connect in nearest to connected set Kruskal: Shortest arc s.t. no cycles	M1 name A1 description
(iii)  Arcs used: AD, DE, EF, FG, DI, IH, AB or DB, FC or BC Total length = 2.7 km (AB&FC) or 2.9 km (AB&BC) or 2.4 km (DB&FC) or 2.6 km (DB&BC)	M1 Dijkstra A1 working values (see vertex G) A1 order of labelling A1 labels M1 arcs counted A1 only once A1

4776 Numerical Methods

1

x	2	3	root = (2 x 0.03 - 3 x 0.24) / (0.03 - 0.24)	**[M1A1]**
f(x)	0.24	0.03	= 3.142857	**[A1]**

Eg: graph showing turning point at x = 3 with root some way to the left or the right. **[G2]**

[TOTAL 5]

2

x	f(x)				
0	1				
1	0.333333	T1 =	0.666667		**[M1A1]**
0.5	0.477592	M =	0.477592		**[M1A1]**
		hence	T2 = (T1 + M)/2 =	0.572129	**[M1A1]**
		and	S = (T1 + 2*M)/3 =	0.540617	**[M1A1]**

[TOTAL 8]

3

x	0	1	3		
f(x)	2	2.57	3.85	3 terms:	**[M1]**
				form:	**[M1]**
				use x=2:	**[M1]**

f(2) = 2(2-1)(2-3)/(0-1)(0-3) + 2.57(2-0)(2-3)/(1-0)(1-3) + 3.85(2-0)(2-1)/(3-0)(3-1) **[A1A1A1]**

= 3.186667 (3.19) **[A1]**

[TOTAL 7]

4

x	1.5	2		
x^3(2-x)-1	0.6875	-1	change of sign, so root *(may be implied)*	**[M1A1]**

a	**b**	**x**	**x^3(2-x)-1**	**mpe**	
1.5	2	1.75	0.339844	0.25	**[M1A1]**
1.75	2	1.875	-0.17603	0.125	**[A1]**
1.75	1.875	**1.8125**		0.0625	**[A1]**

4 further iterations reqd: mpe 0.0325, 0.015625, 0.0078125, 0.00390625 **[M1A1]**

[TOTAL 8]

5

Sketch showing curve, tangent, chord, h. Makes clear that tangent and chord have substantially different gradients. **[G3]**

h	0	0.1	0.01	0.001	
g(2 + *h*)	3.61	3.849	3.633	3.612	
est g '(2)		2.39	2.3	2	**[M1A1A1A1]**

Clear loss of significant figures as h is reduced **[E1]**

[TOTAL 8]

6

(i)

x	f(x)	Δf	$\Delta^2 f$	$\Delta^3 f$
3	1			
4	3	2		
5	-1	-4	-6	
6	-10	-9	-5	1

[M1A1A1]

quadratic = 1 + 2(x-3) - 6(x-3)(x-4)/2 **[M1A1]**
= 1 + 2x-6 - 3x^2+21x-36 **[A1]**
= -3x^2 +23x -41 **[A1]**

q'(x) = -6x + 23 = 0 at x = 23/6 (= 3.833...) **[M1A1]**

q(x) = 0 at x = 4.847(127); *also at 2.81954 - not reqd.* **[M1A1]**

q(6) = -11 (or point out that the second differences not constant) **[A1]**
[subtotal 12]

(ii) cubic est = 1 + 2(4.5-3) - 6(4.5-3)(4.5-4)/2 + 1(4.5-3)(4.5-4)(4.5-5)/6 **[M1A1A1]**
= 1.6875 **[A1]**

S = 1.5/3 (1 + 4x1.6875 -10) = -1.125 **[M1A1]**
[subtotal 6]

[TOTAL 18]

---

7 (i) mpe 0.000 000 5 **[B1]**
mpre 0.000 000 5 / 2.506 628
= 1.99 x 10^{-7} **[M1A1]**
[subtotal 3]

(ii) mpe 1000 x 0.000 000 5 = 0.000 5 **[M1A1]**
In practice the positive and negative errors will tend to cancel out **[E1]**
[subtotal 3]

(iii) mpe 1000 x 0.000 001 = 0.001 **[M1A1]**
In practice 1000 x 0.000 000 5 = 0.000 5 **[M1A1]**
because average error in chopping will be 0.000 000 5 **[E1]**
[subtotal 5]

(iv) L to R: 1 (or 1.000 000) **[B1]**
R to L: 1.000 001 **[B1]**
L to R requires 8 sf, (R to L doesn't) **[E1]**
[subtotal 3]

(v) Reverse order more accurate **[E1]**
as that way allows the very small terms at the end of the series
to contribute to the sum. **[E1]**

The spreadsheet is likely to work to greater accuracy **[E1]**
The spreadsheet works to more sf than are displayed **[E1]**
[subtotal 4]

[TOTAL 18]

Grade Thresholds

Advanced GCE (Subject) (Aggregation Code(s))
January 2008 Examination Series

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	54	46	38	31	24	0
4752	Raw	72	55	48	41	34	28	0
4753	Raw	72	57	50	43	36	28	0
4753/02	Raw	18	15	13	11	9	8	0
4754	Raw	90	77	68	59	50	41	0
4755	Raw	72	55	47	39	32	25	0
4756	Raw	72	59	51	44	37	30	0
4758	Raw	72	62	54	46	38	30	0
4758/02	Raw	18	15	13	11	9	8	0
4761	Raw	72	60	52	44	37	30	0
4762	Raw	72	61	53	45	37	30	0
4763	Raw	72	58	51	44	37	30	0
4766/ G241	Raw	72	56	49	42	35	28	0
4767	Raw	72	62	54	46	38	31	0
4768	Raw	72	54	47	40	33	27	0
4771	Raw	72	60	53	46	39	33	0
4776	Raw	72	58	50	42	35	27	0
4776/02	Raw	18	14	12	10	8	7	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895-7898	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	25.5	50.0	75.5	85.9	95.3	100	106
7896	42.9	85.7	85.7	85.7	85.7	100	7
7897							0
7898							0
3895	22.7	40.7	59.3	77.8	94.8	100	383
3896	80	80	95	95	100	100	20
3897	0	100	100	100	100	100	1
3898	56.4	76.9	87.2	97.4	97.4	100	39

556 candidates aggregated this series

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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