RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Mark Schemes for the Units

January 2010

3895-8/7895-8/MS/10J

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MARK SCHEMES FOR THE UNITS

4751 (C1) Introduction to Advanced Mathematics

1	$[a =]2c^2 - b$ www o.e.	**3**	**M1** for each of 3 complete correct steps, ft from previous error if equivalent difficulty
2	$5x-3<2x+10$ $3x<13$ $x<\frac{13}{3}$ o.e.	**M1** **M1** **M1**	condone '=' used for first two Ms **M0** for just $5x - 3 < 2(x + 5)$ or $-13 < -3x$ or ft or ft; isw further simplification of 13/3; M0 for just $x < 4.3$
3 (i)	(4, 0)	**1**	allow $y = 0$, $x = 4$ bod **B1** for $x = 4$ but do not isw: **0** for (0, 4) seen **0** for (4, 0) and (0, 10) **both** given (choice) unless (4, 0) clearly identified as the x-axis intercept
3 (ii)	$5x + 2(5 - x) = 20$ o.e. (10/3, 5/3) www isw	**M1** **A2**	for subst or for multn to make coeffts same and appropriate addn/subtn; condone one error or **A1** for $x = 10/3$ and **A1** for $y = 5/3$ o.e. isw; condone 3.33 or better and 1.67 or better **A1** for (3.3, 1.7)
4 (i)	translation by $\begin{pmatrix}-4\\0\end{pmatrix}$ or 4 [units] to left	**B1** **B1**	**0** for shift/move or 4 units in negative x direction o.e.
4 (ii)	sketch of parabola right way up and with minimum on negative y-axis min at (0, −4) and graph through −2 and 2 on x-axis	**B1** **B1**	mark intent for both marks must be labelled or shown nearby
5 (i)	$\frac{1}{12}$ or $\pm\frac{1}{12}$	**2**	**M1** for $\frac{1}{144^{\frac{1}{2}}}$ o.e. or for $\sqrt{144}=12$ soi
5 (ii)	denominator = 18 numerator = $5-\sqrt{7}+4\left(5+\sqrt{7}\right)$ $= 25+3\sqrt{7}$ as final answer	**B1** **M1** **A1**	**B0** if 36 after addition for **M1**, allow in separate fractions allow **B3** for $\frac{25+3\sqrt{7}}{18}$ as final answer www

6 (i)	cubic correct way up and with two turning pts touching x-axis at −1, and through it at 2.5 and no other intersections y- axis intersection at −5	**B1** **B1** **B1**	 intns must be shown labelled or worked out nearby
6 (ii)	$2x^3 - x^2 - 8x - 5$	**2**	**B1** for 3 terms correct or **M1** for correct expansion of product of two of the given factors
7	attempt at f(−3) $-27 + 18 - 15 + k = 6$ $k = 30$	**M1** **A1** **A1**	or **M1** for long division by $(x + 3)$ as far as obtaining $x^2 - x$ and **A1** for obtaining remainder as $k - 24$ (but see below) equating coefficients method: **M2** for $(x + 3)(x^2 - x + 8)$ [+6] o.e. (from inspection or division) eg M2 for obtaining $x^2 - x + 8$ as quotient in division
8	$x^3 + 15x + \frac{75}{x} + \frac{125}{x^3}$ www isw or $x^3 + 15x + 75x^{-1} + 125x^{-3}$ www isw	**4**	**B1** for **both** of x^3 **and** $\frac{125}{x^3}$ or $125x^{-3}$ isw and **M1** for 1 3 3 1 soi; **A1** for **each** of $15x$ **and** $\frac{75}{x}$ or $75x^{-1}$ isw **or** **SC2** for completely correct unsimplified answer

9	$x^2 - 5x + 7 = 3x - 10$ $x^2 - 8x + 17$ [= 0] o.e or $y^2 - 4y + 13$ [= 0] o.e use of $b^2 - 4ac$ with numbers subst (condone one error in substitution) (may be in quadratic formula) $b^2 - 4ac = 64 - 68$ or -4 cao [or $16 - 52$ or -36 if y used] [< 0] so no [real] roots [so line and curve do not intersect]	**M1** **M1** **M1** **A1** **A1**	or attempt to subst $(y + 10)/3$ for x condone one error; allow **M1** for $x^2 - 8x = -17$ [oe for y] only if they go on to completing square method or $(x - 4)^2 = 16 - 17$ or $(x - 4)^2 + 1 = 0$ (condone one error) or $(x - 4)^2 = -1$ or $x = 4 \pm \sqrt{-1}$ [or $(y - 2)^2 = -9$ or $y = 2 \pm \sqrt{-9}$] or conclusion from comp. square; needs to be explicit correct conclusion and correct ft; allow '< 0 so no intersection' o.e.; allow '−4 so no roots' etc allow A2 for full argument from sum of two squares = 0; A1 for weaker correct conclusion some may use the condition $b^2 < 4ac$ for no real roots; allow equivalent marks, with first A1 for 64 < 68 o.e.
10 (i)	grad CD = $\frac{5-3}{3-(-1)}\left[=\frac{2}{4} \text{ o.e.}\right]$ isw grad AB = $\frac{3-(-1)}{6-(-2)}$ or $\frac{4}{8}$ isw same gradient so parallel www	**M1** **M1** **A1**	NB needs to be obtained independently of grad AB must be explicit conclusion mentioning 'same gradient' or 'parallel' if M0, allow **B1** for 'parallel lines have same gradient' o.e.
10 (ii)	[BC^2=] $3^2 + 2^2$ [BC^2 =] 13 showing $AD^2 = 1^2 + 4^2$ [=17] [$\neq BC^2$] isw	**M1** **A1** **A1**	accept $(6 - 3)^2 + (3 - 5)^2$ o.e. or [BC =] $\sqrt{13}$ or [AD =] $\sqrt{17}$ or equivalent marks for finding AD or AD^2 first alt method: showing AC ≠ BD – mark equivalently

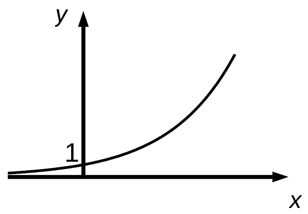
10 (iii)	[BD eqn is] $y = 3$ eqn of AC is $y - 5 = 6/5 \times (x - 3)$ o.e [$y = 1.2x + 1.4$ o.e.] M is (4/3, 3) o.e. isw	**M1** **M2** **A1**	eg allow for 'at M, $y = 3$' or for 3 subst in eqn of AC or **M1** for grad AC = 6/5 o.e. (accept unsimplified) and M1 for using their grad of AC with coords of A(−2, −1) or C (3, 5) in eqn of line or **M1** for 'stepping' method to reach M allow : at M, $x = 16/12$ o.e. [eg =4/3] isw A0 for 1.3 without a fraction answer seen
10 (iv)	midpt of BD = (5/2, 3) or equivalent simplified form cao midpt AC = (1/2, 2) or equivalent simplified form cao or 'M is 2/3 of way from A to C' conclusion 'neither diagonal bisects the other'	**M1** **M1** **A1**	or showing BM ≠ MD oe [BM = 14/3, MD = 7/3] or showing AM ≠ MC or $AM^2 \neq MC^2$ in these methods A1 is dependent on coords of M having been obtained in part (iii) or in this part; the coordinates of M need not be correct; it is also dependent on midpts of both AC and BD attempted, at least one correct alt method: show that mid point of BD does not lie on AC (M1) and vice-versa (M1), A1 for both and conclusion

11 (i)	centre C' = (3, −2) radius 5	**1** **1**	 0 for ±5 or −5
11 (ii)	showing $(6-3)^2 + (-6+2)^2 = 25$ showing that $\overrightarrow{AC'} = \overrightarrow{C'B} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ o.e.	**B1** **B2**	interim step needed **or B1** each for two of**:** showing midpoint of AB = (3, −2); showing B (0, 2) is on circle; showing AB = 10 **or B2** for showing midpoint of AB = (3, −2) and saying this is centre of circle **or B1** for finding eqn of AB as $y = -4/3\, x + 2$ o.e. and **B1** for finding one of its intersections with the circle is (0, 2) **or B1** for showing C′B = 5 and **B1** for showing AB = 10 or that AC′ and BC′ have the same gradient **or B1** for showing that AC′ and BC′ have the same gradient and B1 for showing that B (0, 2) is on the circle
11 (iii)	grad AC' or AB = −4/3 o.e. grad tgt = −1/their AC' grad $y-(-6) = $ their $m(x-6)$ o.e. $y = 0.75x - 10.5$ o.e. isw	**M1** **M1** **M1** **A1**	or ft from their C', must be evaluated may be seen in eqn for tgt; allow M2 for grad tgt = ¾ oe soi as first step or **M1** for $y = $ their $m \times x + c$ then subst (6, −6) eg **A1** for $4y = 3x - 42$ allow **B4** for correct equation www isw
11 (iv)	centre C is at (12, −14) cao circle is $(x-12)^2 + (y+14)^2 = 100$	**B2** **B1**	B1 for each coord ft their C if at least one coord correct

12 (i)	10	**1**	
12 (ii)	[x =] 5 or ft their (i) ÷ 2 ht = 5[m] cao	**1** **1**	not necessarily ft from (i) eg they may start again with calculus to get $x = 5$
12 (iii)	$d = 7/2$ o.e. [y =] $1/5 \times 3.5 \times (10 - 3.5)$ o.e. or ft = 91/20 o.e. cao isw	**M1** **M1** **A1**	or ft their (ii) − 1.5 or their (i) ÷ 2 − 1.5 o.e. or $7 - 1/5 \times 3.5^2$ or ft or showing $y - 4 = 11/20$ o.e. cao
12 (iv)	$4.5 = 1/5 \times x(10 - x)$ o.e. $22.5 = x(10 - x)$ o.e. $2x^2 - 20x + 45$ [= 0] o.e. eg $x^2 - 10x + 22.5$ [=0] or $(x - 5)^2 = 2.5$ $[x =]\dfrac{20 \pm \sqrt{40}}{4}$ or $5 \pm \dfrac{1}{2}\sqrt{10}$ o.e. width = $\sqrt{10}$ o.e. eg $2\sqrt{2.5}$ cao	**M1** **M1** **A1** **M1** **A1**	 eg $4.5 = x(2 - 0.2x)$ etc cao; accept versions with fractional coefficients of x^2, isw or $x - 5 = [\pm]\sqrt{2.5}$ o.e.; ft their quadratic eqn provided at least M1 gained already; condone one error in formula or substitution; need not be simplified or be real accept simple equivalents only

4752 (C2) Concepts for Advanced Mathematics

1		$\frac{1}{2} x^2 + 3x^{-1} + c$ o.e.	3	1 for each term	3
2	**(i)** **(ii)**	5 with valid method 165 www	1 2	eg sequence has period of 4 nos. M1 for 13× (1 + 3 + 5 + 3) + 1 + 3 +5 or for 14 × (1 + 3 + 5 + 3) − 3	 3
3		rt angled triangle with √2 on one side and 3 on hyp Pythag. used to obtain remaining side = √7 $\tan\theta = \frac{opp}{adj} = \frac{\sqrt{2}}{\sqrt{7}}$ o.e.	1 1 1	or M1 for $\cos^2\theta = 1 - \sin^2\theta$ used A1 for $\cos\theta = \frac{\sqrt{7}}{\sqrt{9}}$ A1 for $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{2}}{\sqrt{7}}$ o.e.	3
4		radius = 6.5 [cm]	3	M1 for $\frac{1}{2} \times r^2 \times 0.4$ [= 8.45] o.e. and M1 for $r^2 = \frac{169}{4}$ o.e. [= 42.25]	3
5	**(i)** **(ii)**	sketch of correct shape with P (−0.5,2) Q (0,4) and R (2,2) sketch of correct shape with P (−1,0.5) Q (0,1) and R (4,0.5)	2 2	1 if Q and one other are correct 1 if Q and one other are correct	 4
6	**(i)** **(ii)**	205 $\frac{25}{3}$ o.e.	3 2	M1 for AP identified with d = 4 and M1 for 5 + 50 d used M1 for $r = \frac{2}{5}$ o.e.	5
7	**(i)** **(ii)**	$\frac{\sin A}{5.6} = \frac{\sin 79}{8.4}$ s.o.i. [A =] 40.87 to 41 [BC^2 =] $5.6^2 + 7.8^2 - 2 \times 5.6 \times 7.8 \times$ cos ("180-79") = 108.8 to 108.9 [BC =] 10.4(…)	M1 A1 M1 A1 A1		5
8		$y' = 3x^{-\frac{1}{2}}$ ¾ when $x = 16$ $y = 24$ when $x = 16$ y − their 24 = their ¾ (x − 16) $y - 24 = \frac{3}{4}(x - 16)$ o.e.	M1 A1 B1 M1 A1	condone if unsimplified dependent on $\frac{dy}{dx}$ used for m	5

9	**(i)**		G1 DG1	for curve of correct shape in both quadrants must go through (0, 1) shown	
	(ii)	$2x + 1 = \frac{\log 10}{\log 13}$ o.e. $[x =]$ 0.55	M1 A2	or M1 for $2x + 1 = \log_3 10$ A1 for other versions of 0.547…or 0.548	5
10	**(i)**	$3x^2 - 6x - 9$ use of their $y' = 0$ $x = -1$ $x = 3$ valid method for determining nature of turning point max at $x = -1$ and min at $x = 3$	M1 M1 A1 A1 M1 A1	 c.a.o.	6
	(ii)	$x(x^2 - 3x - 9)$ $\frac{3 \pm \sqrt{45}}{2}$ or $(x - \frac{3}{2})^2 = 9 + \frac{9}{4}$ $0, \frac{3}{2} \pm \frac{\sqrt{45}}{2}$ o.e.	M1 M1 A1		3
	(iii)	sketch of cubic with two turning points correct way up x-intercepts – negative, 0, positive shown	G1 DG1		2
11	**(i)**	47.625 [m^2] to 3 sf or more, with correct method shown	4	M3 for $\frac{1.5}{2} \times (2.3 + 2 + 2[2.7 + 3.3 + 4 + 4.8 + 5.2 + 5.2 + 4.4])$	4
	(ii)	43.05	2	M1 for $1.5 \times (2.3+2.7+3.3+4+4.8+5.2+4.4+2)$	2
	(iii)	$-0.013x^4/4 + 0.16x^3/3 - 0.082x^2/2 + 2.4x$ o.e. their integral evaluated at $x = 12$ (and 0) only 47.6 to 47.7	M2 M1 A1	M1 for three terms correct dep on integration attempted	4
	(iv)	5.30.. found compared with 5.2 s.o.i.	1 D1		2
12	**(i)**	$\log P = \log a + bt$ www comparison with $y = mx + c$ s.o.i. intercept = $\log_{10} a$	1 1 1	 must be with correct equation dependent on correct equation	3
	(ii)	[2.12, 2.21], 2.32, 2.44, 2.57, 2.69 plots ft ruled line of best fit	1 1 1	 Between (10, 2.08) and (10, 2.12)	3

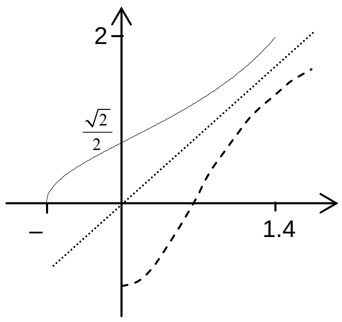
	(iii)	$0.0100 \leq m < 0.0125$	B2	M1 for $\frac{y-\text{step}}{x-\text{step}}$	
		$a = 10^c$ or $\log a = c$	B1	$1.96 \leq c \leq 2.02$	
		$P = 10^c \times 10^{mt}$ or 10^{mt+c}	B1	f.t. their m and a	4
	(iv)	use of $t = 105$ 1.0 – 2.0 billion approx unreliable since extrapolation o.e.	B1 B1 E1		3

4753 (C3) Methods for Advanced Mathematics

1 $e^{2x} - 5e^x = 0$ $\Rightarrow$ $e^x(e^x - 5) = 0$ $\Rightarrow$ $e^x = 5$ $\Rightarrow$ $x = \ln 5$ or 1.6094	M1 M1 A1 A1 [4]	factoring out e^x or dividing $e^{2x} = 5e^x$ by e^x $e^{2x} / e^x = e^x$ ln 5 or 1.61 or better, mark final answer –1 for additional solutions, e.g. $x = 0$
or $\ln(e^{2x}) = \ln(5e^x)$ $\Rightarrow$ $2x = \ln 5 + x$ $\Rightarrow$ $x = \ln 5$ or 1.6094	M1 A1 A1 A1 [4]	taking lns on $e^{2x} = 5e^x$ $2x$, $\ln 5 + x$ ln 5 or 1.61 or better, mark final answer –1 for additional solutions, e.g. $x = 0$
2 (i) When $t = 0$, $T = 100$ $\Rightarrow$ $100 = 20 + b$ $\Rightarrow$ $b = 80$ When $t = 5$, $T = 60$ $\Rightarrow$ $60 = 20 + 80e^{-5k}$ $\Rightarrow$ $e^{-5k} = ½$ $\Rightarrow$ $k = \ln 2 / 5 = 0.139$	 M1 A1 M1 A1 [4]	 substituting $t = 0$, $T = 100$ cao substituting $t = 5$, $T = 60$ 1/5 ln 2 or 0.14 or better
(ii) $50 = 20 + 80e^{-kt}$ $\Rightarrow$ $e^{-kt} = 3/8$ $\Rightarrow$ $t = \ln(8/3) / k = 7.075$ mins	M1 A1 [2]	Re-arranging and taking lns correctly – ft their b and k answers in range 7 to 7.1
3(i) $\frac{dy}{dx} = \frac{1}{3}(1+3x^2)^{-2/3}.6x$ $= 2x(1+3x^2)^{-2/3}$	M1 B1 A1 [3]	chain rule $1/3\ u^{-2/3}$ or $\frac{1}{3}(1+3x^2)^{-2/3}$ o.e but must be '2' (not 6/3) mark final answer
(ii) $3y^2 \frac{dy}{dx} = 6x$ $\Rightarrow$ $dy/dx = 6x/3y^2$ $= \frac{2x}{(1+3x^2)^{2/3}} = 2x(1+3x^2)^{-2/3}$	M1 A1 A1 E1 [4]	$3y^2 \frac{dy}{dx}$ $= 6x$ if deriving $2x(1+3x^2)^{-2/3}$, needs a step of working

4(i) $\int_0^1 \frac{2x}{x^2+1}\,dx = \left[\ln(x^2+1)\right]_0^1$ $= \ln 2$	M2 A1 [3]	$[\ln(x^2 + 1)]$ cao (must be exact)
or let $u = x^2 + 1$, $du = 2x\,dx$ $\Rightarrow$ $\int_0^1 \frac{2x}{x^2+1}\,dx = \int_1^2 \frac{1}{u}\,du$ $= [\ln u]_1^2$ $= \ln 2$	M1 A1 A1 [3]	$\int \frac{1}{u}\,du$ or $\left[\ln(1+x^2)\right]_0^1$ with correct limits cao (must be exact)
(ii) $\int_0^1 \frac{2x}{x+1}\,dx = \int_0^1 \frac{2x+2-2}{x+1}\,dx = \int_0^1 (2 - \frac{2}{x+1})\,dx$ $= [2x - 2\ln(x+1)]_0^1$ $= 2 - 2\ln 2$	M1 A1, A1 A1 A1 [5]	dividing by $(x + 1)$ 2, $-2/(x+1)$
or $\int_0^1 \frac{2x}{x+1}\,dx$ let $u = x + 1$, $\Rightarrow du = dx$ $= \int_1^2 \frac{2(u-1)}{u}\,du$ $= \int_1^2 (2 - \frac{2}{u})\,du$ $= [2u - 2\ln u]_1^2$ $= 4 - 2\ln 2 - (2 - 2\ln 1)$ $= 2 - 2\ln 2$	M1 B1 M1 A1 A1 [5]	substituting $u = x + 1$ and $du = dx$ (or $du/dx = 1$) *and* correct limits used for u or x $2(u - 1)/u$ dividing through by u $2u - 2\ln u$ allow ft on $(u - 1)/u$ (i.e. with 2 omitted) o.e. cao (must be exact)
5 (i) $a = 0$, $b = 3$, $c = 2$	B(2,1,0)	or $a = 0$, $b = -3$, $c = -2$
(ii) $a = 1$, $b = -1$, $c = 1$ *or* $a = 1$, $b = 1$, $c = -1$	B(2,1,0) [4]	
6 $f(-x) = -f(x)$, $g(-x) = g(x)$ $g\,f(-x) = g\,[-f(x)]$ $= g\,f(x)$ $\Rightarrow$ g f is even	B1B1 M1 E1 [4]	condone f and g interchanged forming $gf(-x)$ or $gf(x)$ and using $f(-x) = -f(x)$ www
7 Let $\arcsin x = \theta$ $\Rightarrow$ $x = \sin\theta$ $\theta = \arccos y \Rightarrow y = \cos\theta$ $\sin^2\theta + \cos^2\theta = 1$ $\Rightarrow$ $x^2 + y^2 = 1$	M1 M1 E1 [3]	

8(i) At P, $x \cos 3x = 0$ $\Rightarrow$ $\cos 3x = 0$ $\Rightarrow$ $3x = \pi/2, 3\pi/2$ $\Rightarrow$ $x = \pi/6, \pi/2$ So P is $(\pi/6, 0)$ and Q is $(\pi/2, 0)$	M1 M1 A1 A1 [4]	or verification $3x = \pi/2, (3\pi/2\ldots)$ dep both Ms condone degrees here
(ii) $\frac{dy}{dx} = -3x\sin 3x + \cos 3x$ At P, $\frac{dy}{dx} = -\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2} = -\frac{\pi}{2}$ At TPs $\frac{dy}{dx} = -3x\sin 3x + \cos 3x = 0$ $\Rightarrow$ $\cos 3x = 3x \sin 3x$ $\Rightarrow$ $1 = 3x \sin 3x / \cos 3x = 3x \tan 3x$ $\Rightarrow$ $x \tan 3x = 1/3$ *	M1 B1 A1 M1 A1cao M1 E1 [7]	Product rule $d/dx (\cos 3x) = -3 \sin 3x$ cao (so for $dy/dx = -3x\sin 3x$ allow B1) mark final answer substituting their $-\pi/6$ (must be rads) $-\pi/2$ $dy/dx = 0$ and $\sin 3x / \cos 3x = \tan 3x$ used www
(iii) $A = \int_0^{\pi/6} x\cos 3x\,dx$ Parts with $u = x$, $dv/dx = \cos 3x$ $du/dx = 1$, $v = 1/3 \sin 3x$ $\Rightarrow$ $A = \left[\frac{1}{3}x\sin 3x\right]_0^{\frac{\pi}{6}} - \int_0^{\pi/6}\frac{1}{3}\sin 3x\,dx$ $= \left[\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x\right]_0^{\frac{\pi}{6}}$ $= \frac{\pi}{18} - \frac{1}{9}$	B1 M1 A1 A1 M1dep A1 cao [6]	Correct integral and limits (soi) – ft their P, but must be in radians can be without limits dep previous A1. substituting correct limits, dep 1st M1: ft their P provided in radians o.e. but must be exact

9(i) $f'(x)=\dfrac{(x^2+1)4x-(2x^2-1)2x}{(x^2+1)^2}$ $=\dfrac{4x^3+4x-4x^3+2x}{(x^2+1)^2}=\dfrac{6x}{(x^2+1)^2}$ * When $x > 0$, $6x > 0$ and $(x^2 + 1)^2 > 0$ $\Rightarrow$ $f'(x) > 0$	M1 A1 E1 M1 E1 [5]	Quotient or product rule correct expression www attempt to show or solve f'(x) > 0 numerator > 0 and denominator > 0 or, if solving, $6x > 0 \Rightarrow x > 0$
(ii) $f(2) = \dfrac{8-1}{4+1}=1\dfrac{2}{5}$ Range is $-1 \le y \le 1\dfrac{2}{5}$	B1 B1 [2]	 must be ≤, y or f(x)
(iii) f'(x) max when f''(x) = 0 $\Rightarrow$ $6 - 18x^2 = 0$ $\Rightarrow$ $x^2 = 1/3$, $x = 1/\sqrt{3}$ $\Rightarrow$ $f'(x)=\dfrac{6/\sqrt{3}}{(1\frac{1}{3})^2}=\dfrac{6}{\sqrt{3}}.\dfrac{9}{16}=\dfrac{9\sqrt{3}}{8}=1.95$	M1 A1 M1 A1 [4]	 (±)$1/\sqrt{3}$ oe (0.577 or better) substituting $1/\sqrt{3}$ into f'(x) $9\sqrt{3}/8$ o.e. or 1.95 or better (1.948557..)
(iv) Domain is $-1 < x < 1\dfrac{2}{5}$ Range is $0 \le y \le 2$ 	B1 B1 M1 A1 cao [4]	ft their 1.4 but not $x \ge -1$ or $0 \le g(x) \le 2$ (not f) Reasonable reflection in $y = x$ from (–1, 0) to (1.4, 2), through (0, $\sqrt{2}/2$) allow omission of one of –1, 1.4, 2, $\sqrt{2}/2$
(v) $y=\dfrac{2x^2-1}{x^2+1}$ $x \leftrightarrow y$ $x=\dfrac{2y^2-1}{y^2+1}$ $\Rightarrow$ $xy^2 + x = 2y^2 - 1$ $\Rightarrow$ $x + 1 = 2y^2 - xy^2 = y^2(2 - x)$ $\Rightarrow$ $y^2=\dfrac{x+1}{2-x}$ $\Rightarrow$ $y=\sqrt{\dfrac{x+1}{2-x}}$ *	 M1 M1 M1 E1 [4]	(could start from g) Attempt to invert clearing fractions collecting terms in y^2 and factorising www

4754 (C4) Applications of Advanced Mathematics

1		$\frac{1+2x}{(1-2x)^2}=(1+2x)(1-2x)^{-2}$ $=(1+2x)[1+(-2)(-2x)+\frac{(-2)(-3)}{1.2}(-2x)^2+...]$ $=(1+2x)[1+4x+12x^2+...]$ $=1+4x+12x^2+2x+8x^2+...$ $=1+6x+20x^2+...$ Valid for $-1<-2x<1$ $\Rightarrow -½<x<½$	M1 A1 A1 M1 A1 M1 A1 **[7]**	binomial expansion power -2 unsimplified,correct sufficient terms
2		$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$ $\Rightarrow \cot 2\theta=\frac{1}{\tan 2\theta}=\frac{1-\tan^2\theta}{2\tan\theta}$ * $\cot 2\theta = 1+\tan\theta$ $\Rightarrow \frac{1-\tan^2\theta}{2\tan\theta}=1+\tan\theta$ $\Rightarrow 1-\tan^2\theta=2\tan\theta+2\tan^2\theta$ $\Rightarrow 3\tan^2\theta+2\tan\theta-1=0$ $\Rightarrow (3\tan\theta-1)(\tan\theta+1)=0$ $\Rightarrow \tan\theta=1/3,\ \theta=18.43°,\ 198.43°$ or $\tan\theta=-1,\ \theta=135°,\ 315°$	M1 E1 M1 M1 A3,2,1, 0 **[7]**	oe eg converting either side into a one line fraction(s) involving sin θ and cos θ. quadratic = 0 factorising or solving 18.43°, 198.43°, 135°, 315° –1 extra solutions in the range
3	**(i)**	$\frac{dy}{dt}=\frac{(1+t).2-2t.1}{(1+t)^2}=\frac{2}{(1+t)^2}$ $\frac{dx}{dt}=2e^{2t}$ $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$ $\Rightarrow \frac{dy}{dx}=\frac{\frac{2}{(1+t)^2}}{2e^{2t}}=\frac{1}{e^{2t}(1+t)^2}$ $t=0\Rightarrow dy/dx=1$	M1A1 B1 M1 A1 B1ft **[6]**	

	(ii)	$2t = \ln x \Rightarrow t = ½ \ln x$ $\Rightarrow \quad y = \dfrac{\ln x}{1+\frac{1}{2}\ln x} = \dfrac{2\ln x}{2+\ln x}$	M1 A1 **[2]**	or t in terms of y
4	**(i)**	$\overrightarrow{AB} = \begin{pmatrix}-2\\-1\\-1\end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix}-1\\-11\\3\end{pmatrix}$	B1 B1 **[2]**	
	(ii)	$\mathbf{n}.\overrightarrow{AB} = \begin{pmatrix}2\\-1\\-3\end{pmatrix}.\begin{pmatrix}-2\\-1\\-1\end{pmatrix} = -4+1+3 = 0$ $\mathbf{n}.\overrightarrow{AC} = \begin{pmatrix}2\\-1\\-3\end{pmatrix}.\begin{pmatrix}-1\\-11\\3\end{pmatrix} = -2+11-9 = 0$ $\Rightarrow$ plane is $2x - y - 3z = d$ $x = 1, y = 3, z = -2 \Rightarrow d = 2 - 3 + 6 = 5$ $\Rightarrow$ plane is $2x - y - 3z = 5$	M1 E1 E1 M1 A1 **[5]**	scalar product
5	**(i)**	$x = -5 + 3\lambda = 1 \Rightarrow \lambda = 2$ $y = 3 + 2 \times 0 = 3$ $z = 4 - 2 = 2$, so (1, 3, 2) lies on 1st line. $x = -1 + 2\mu = 1 \Rightarrow \mu = 1$ $y = 4 - 1 = 3$ $z = 2 + 0 = 2$, so (1, 3, 2) lies on 2nd line.	M1 E1 E1 **[3]**	finding λ or μ verifying two other coordinates for line 1 verifying two other coordinates for line 2
	(ii)	Angle between $\begin{pmatrix}3\\0\\-1\end{pmatrix}$ and $\begin{pmatrix}2\\-1\\0\end{pmatrix}$ $\cos\theta = \dfrac{3\times2+0\times(-1)+(-1)\times0}{\sqrt{10}\sqrt{5}}$ $= 0.8485\ldots$ $\Rightarrow \quad \theta = 31.9°$	M1 M1 A1 A1 **[4]**	direction vectors only allow M1 for any vectors or 0.558 radians

6	**(i)**	BAC = 120 – 90 – (90 – θ) = θ – 60 $\Rightarrow$ BC = b sin(θ – 60) CD = AE = a sin θ $\Rightarrow$ h = BC + CD = a sin θ + b sin (θ – 60°) *	B1 M1 E1 **[3]**	
	(ii)	h = a sin θ + b sin (θ – 60°) = a sin θ + b (sinθ cos 60 – cosθ sin 60) = a sin θ + ½ b sin θ – √3/2 b cos θ $= (a+\frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta *$	M1 M1 E1 **[3]**	corr compound angle formula sin 60 = √3/2, cos 60 = ½ used
	(iii)	OB horizontal when h = 0 $\Rightarrow$ $(a+\frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta = 0$ $\Rightarrow$ $(a+\frac{1}{2}b)\sin\theta = \frac{\sqrt{3}}{2}b\cos\theta$ $\Rightarrow$ $\frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{3}}{2}b}{a+\frac{1}{2}b}$ $\Rightarrow$ $\tan\theta = \frac{\sqrt{3}b}{2a+b} *$	M1 M1 E1 **[3]**	 $\frac{\sin\theta}{\cos\theta} = \tan\theta$
	(iv)	$2\sin\theta - \sqrt{3}\cos\theta = R\sin(\theta-\alpha)$ $= R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$ $\Rightarrow$ R cos α = 2, R sin α = √3 $\Rightarrow$ $R^2 = 2^2 + (\sqrt{3})^2 = 7$, $R = \sqrt{7} = 2.646$ m tan α = √3/2, α = 40.9° So h = √7 sin(θ – 40.9°) $\Rightarrow$ $h_{max} = \sqrt{7} = 2.646$ m when θ – 40.9° = 90° $\Rightarrow$ θ = 130.9°	 M1 B1 M1A1 B1ft M1 A1 **[7]**	

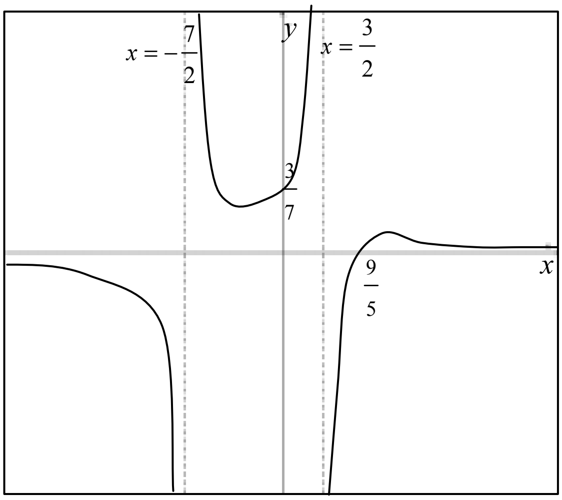
7	(i)	$\frac{\mathrm{d}x}{\mathrm{d}t} = -1(1+\mathrm{e}^{-t})^{-2}.-\mathrm{e}^{-t}$ $= \frac{\mathrm{e}^{-t}}{(1+\mathrm{e}^{-t})^2}$ $1-x = 1-\frac{1}{1+\mathrm{e}^{-t}}$ $1-x = \frac{1+\mathrm{e}^{-t}-1}{1+\mathrm{e}^{-t}} = \frac{\mathrm{e}^{-t}}{1+\mathrm{e}^{-t}}$ $\Rightarrow \quad x(1-x) = \frac{1}{1+\mathrm{e}^{-t}}\frac{\mathrm{e}^{-t}}{1+\mathrm{e}^{-t}} = \frac{\mathrm{e}^{-t}}{(1+\mathrm{e}^{-t})^2}$ $\Rightarrow \quad \frac{dx}{dt} = x(1-x)$ When $t = 0$, $x = \frac{1}{1+\mathrm{e}^0} = 0.5$	M1 A1 M1 A1 E1 B1 **[6]**	chain rule substituting for $x(1-x)$ $1-x = \frac{1+\mathrm{e}^{-t}-1}{1+\mathrm{e}^{-t}} = \frac{\mathrm{e}^{-t}}{1+\mathrm{e}^{-t}}$ [OR,M1 A1 for solving differential equation for *t,* B1 use of initial condition, M1 A1 making *x* the subject, E1 required form]
	(ii)	$\frac{1}{(1+\mathrm{e}^{-t})} = \frac{3}{4}$ $\Rightarrow \quad \mathrm{e}^{-t} = 1/3$ $\Rightarrow \quad t = -\ln 1/3 = 1.10$ years	M1 M1 A1 **[3]**	 correct log rules
	(iii)	$\frac{1}{x^2(1-x)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{1-x}$ $\Rightarrow \quad 1 = A(1-x) + Bx(1-x) + Cx^2$ $x = 0 \Rightarrow A = 1$ $x = 1 \Rightarrow C = 1$ coefft of x^2: $0 = -B + C \Rightarrow B = 1$	M1 M1 B(2,1,0) **[4]**	clearing fractions substituting or equating coeffs for A,B or C $A = 1$, $B = 1$, $C = 1$ www
	(iv)	$\int \frac{\mathrm{d}x}{x^2(1-x)}\mathrm{d}x = \int \mathrm{d}t$ $\Rightarrow \quad t = \int (\frac{1}{x^2} + \frac{1}{x} + \frac{1}{1-x})\mathrm{d}x$ $= -1/x + \ln x - \ln(1-x) + c$ When $t = 0$, $x = ½ \Rightarrow 0 = -2 + \ln ½ - \ln ½ + c$ $\Rightarrow \quad c = 2.$ $\Rightarrow \quad t = -1/x + \ln x - \ln(1-x) + 2$ $= 2 + \ln\frac{x}{1-x} - \frac{1}{x}$ *	M1 B1 B1 M1 E1 **[5]**	separating variables $-1/x + \dots$ $\ln x - \ln(1-x)$ ft their A,B,C substituting initial conditions
	(v)	$t = 2 + \ln\frac{3/4}{1-3/4} - \frac{1}{3/4} = \ln 3 + \frac{2}{3} = 1.77$ yrs	M1A1 **[2]**	

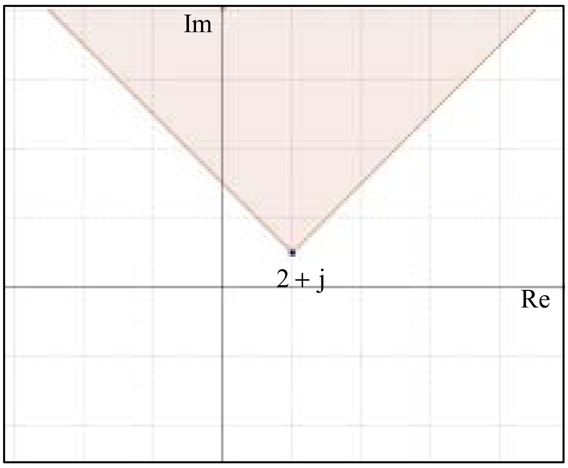
1	15	B1	
2	THE MATHEMATICIAN	B1	
3	M H X I Q 3 or 4 correct – award 1 mark	B2	
4	Two from Ciphertext N has high frequency E would then correspond to ciphertext R which also has high frequency T would then correspond to ciphertext G which also has high frequency A is preceded by a string of six letters displaying low frequency	 B1 B1	 oe oe
5	The length of the keyword is a factor of both 84 and 40. The <u>only</u> common factors of 84 and 40 are 1,2 and 4 (and a keyword of length 1 can be dismissed in this context)	M1 E1	
6	Longer strings to analyse so letter frequency more transparent. Or there are fewer 2-letter keywords to check	B2	
7	OQH DRR EBG One or two accurate – award 1 mark	B2	
8 (i) **(ii)** **(iii)**	Evidence of intermediate H FACE Evidence of intermediate HCEG – award 2 marks Evidence of accurate application of one of the two decoding ciphers - award 1 mark $800 = (3\times 266)+2$; the second row gives T so plaintext is R	B1 B3 M1 A1	 Use of second row

4755 (FP1) Further Concepts for Advanced Mathematics

1	$\alpha\beta = (-3+\mathrm{j})(5-2\mathrm{j}) = -13+11\mathrm{j}$	M1 A1 **[2]**	Use of $\mathrm{j}^2 = -1$
	$\frac{\alpha}{\beta} = \frac{-3+\mathrm{j}}{5-2\mathrm{j}} = \frac{(-3+\mathrm{j})(5+2\mathrm{j})}{29} = \frac{-17}{29} - \frac{1}{29}\mathrm{j}$	M1 A1 A1 **[3]**	Use of conjugate 29 in denominator All correct
2 (i)	**AB** is impossible $\mathbf{CA} = (50)$ $\mathbf{B}+\mathbf{D} = \begin{pmatrix} 3 & 1 \\ 6 & -2 \end{pmatrix}$ $\mathbf{AC} = \begin{pmatrix} 20 & 4 & 32 \\ -10 & -2 & -16 \\ 20 & 4 & 32 \end{pmatrix}$	B1 B1 B1 B2 **[5]**	–1 each error
(ii)	$\mathbf{DB} = \begin{pmatrix} -2 & 0 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} -10 & -2 \\ 22 & 1 \end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply in correct order c.a.o.
3	$\alpha+\beta+\gamma = a-d+a+a+d = \frac{12}{4} \Rightarrow a = 1$	M1 A1	Valid attempt to use sum of roots $a = 1$, c.a.o.
	$(a-d)a(a+d) = \frac{3}{4} \Rightarrow d = \pm\frac{1}{2}$	M1	Valid attempt to use product of roots
	So the roots are $\frac{1}{2}$, 1 and $\frac{3}{2}$	A1	All three roots
	$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{k}{4} = \frac{11}{4} \Rightarrow k = 11$	M1 A1 **[6]**	Valid attempt to use $\alpha\beta+\alpha\gamma+\beta\gamma$, or to multiply out factors, or to substitute a root $k = 11$ c.a.o.

4	$\mathbf{MM}^{-1} = \frac{1}{k}\begin{pmatrix} 4 & 0 & 1 \\ -6 & 1 & 1 \\ 5 & 2 & 5 \end{pmatrix}\begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}$	M1	Attempt to consider $\mathbf{MM}^{-1}$ or $\mathbf{M}^{-1}\mathbf{M}$ (may be implied)
	$= \frac{1}{k}\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} \Rightarrow k = 5$	A1 **[2]**	c.a.o.
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}\begin{pmatrix} 9 \\ 32 \\ 81 \end{pmatrix}$	M1	Attempt to pre-multiply by $\mathbf{M}^{-1}$
		M1	Attempt to multiply matrices
	$\frac{1}{5}\begin{pmatrix} -3 & -2 & 1 \\ -35 & -15 & 10 \\ 17 & 8 & -4 \end{pmatrix}\begin{pmatrix} 9 \\ 32 \\ 81 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -10 \\ 15 \\ 85 \end{pmatrix}$	A1	Correct
	$\Rightarrow x = -2,\ y = 3,\ z = 17$	A1 **[4]**	All 3 correct s.c. B1 if matrices not used
5	$\sum_{r=1}^{n}(r+2)(r-3) = \sum_{r=1}^{n}(r^2 - r - 6)$		
	$= \sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r - 6n$	M1	Separate into 3 sums
	$= \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - 6n$	A2	–1 each error
	$= \frac{1}{6}n[(n+1)(2n+1) - 3(n+1) - 36]$	M1	Valid attempt to factorise (with n as a factor)
	$= \frac{1}{6}n(2n^2 - 38) = \frac{1}{3}n(n^2 - 19)$	A1 A1 **[6]**	Correct expression c.a.o. Complete, convincing argument
6	When $n = 1$, $\frac{n(n+1)(n+2)}{3} = 2$,	B1	
	so true for $n = 1$ Assume true for $n = k$ $2 + 6 + ... + k(k+1) = \frac{k(k+1)(k+2)}{3}$	E1	Assume true for k
	$\Rightarrow 2 + 6 + ... + (k+1)(k+2)$ $= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$	M1	Add $(k+1)$th term to both sides
	$= \frac{1}{3}(k+1)(k+2)(k+3)$ $= \frac{(k+1)((k+1)+1)((k+1)+2)}{3}$	A1	c.a.o. with correct simplification
	But this is the given result with $k + 1$ replacing k. Therefore if it is true for $n = k$ it is true for $n = k + 1$.	E1	Dependent on A1 and previous E1
	Since it is true for $n = 1$, it is true for $n = 1, 2, 3$ and so true for all positive integers.	E1 **[6]**	Dependent on B1 and previous E1

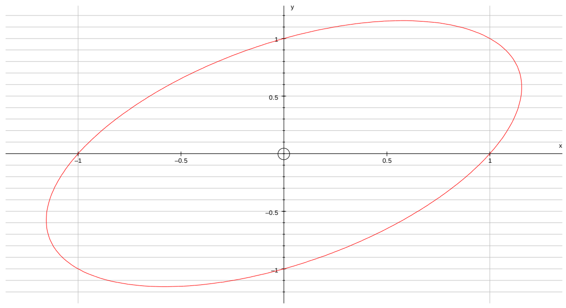
7 (i)	$x=\frac{-7}{2},\ x=\frac{3}{2},\ y=0$	B1 B1 B1 **[3]**	
(ii)	Large positive x, $y \to 0^+$ (e.g. consider $x = 100$) Large negative x, $y \to 0^-$ (e.g. consider $x = -100$)	B1 B1 M1 **[3]**	Evidence of method
(iii)		B1 B1 B1 **[3]**	Intercepts correct and labelled LH and central branches correct RH branch correct, with clear maximum
(iv)	$x<-\frac{7}{2}$ or $\frac{3}{2}<x\le\frac{9}{5}$	B1 B2 **[3]**	 Award B1 if only error relates to inclusive/exclusive inequalities

8(a) (i)	$\lvert z-(2+6\mathrm{j})\rvert = 4$	B1 B1 B1 **[3]**	2 + 6j seen (expression in z) = 4 Correct equation
(ii)	$\lvert z-(2+6\mathrm{j})\rvert < 4$ and $\lvert z-(3+7\mathrm{j})\rvert > 1$	B1 B1 B1 **[3]**	$\lvert z-(2+6\mathrm{j})\rvert < 4$ $\lvert z-(3+7\mathrm{j})\rvert > 1$ (allow errors in inequality signs) Both inequalities correct
(b)(i)		B1 B1 B1 **[3]**	Any straight line through 2 + j Both correct half lines Region between their two half lines indicated
(ii)	$43+47\mathrm{j}-(2+\mathrm{j}) = 41+46\mathrm{j}$ $\arg(41+46\mathrm{j}) = \arctan\left(\frac{46}{41}\right) = 0.843$ $\frac{\pi}{4} < 0.843 < \frac{3\pi}{4}$ so $43+47\mathrm{j}$ does fall within the region	M1 A1 E1 **[3]**	Attempt to calculate argument, or other valid method such as comparison with $y = x - 1$ Correct Justified

9 (i)	$\frac{2}{r}-\frac{3}{r+1}+\frac{1}{r+2}$ $=\frac{2(r+1)(r+2)-3r(r+2)+r(r+1)}{r(r+1)(r+2)}$ $=\frac{2r^2+6r+4-3r^2-6r+r^2+r}{r(r+1)(r+2)}=\frac{4+r}{r(r+1)(r+2)}$	M1 A1 **[2]**	Attempt a common denominator Convincingly shown
(ii)	$\sum_{r=1}^{n}\frac{4+r}{r(r+1)(r+2)}=\sum_{r=1}^{n}\left[\frac{2}{r}-\frac{3}{r+1}+\frac{1}{r+2}\right]$ $=\left(\frac{2}{1}-\frac{3}{2}+\frac{1}{3}\right)+\left(\frac{2}{2}-\frac{3}{3}+\frac{1}{4}\right)+\left(\frac{2}{3}-\frac{3}{4}+\frac{1}{5}\right)+..$ $..+\left(\frac{2}{n-1}-\frac{3}{n}+\frac{1}{n+1}\right)+\left(\frac{2}{n}-\frac{3}{n+1}+\frac{1}{n+2}\right)$ $=\frac{2}{1}-\frac{3}{2}+\frac{2}{2}+\frac{1}{n+1}-\frac{3}{n+1}+\frac{1}{n+2}$ $=\frac{3}{2}-\frac{2}{n+1}+\frac{1}{n+2}$ as required	M1 M1 A2 M1 A1 **[6]**	Use of the given result (may be implied) Terms in full (at least first and one other) At least 3 consecutive terms correct, -1 each error Attempt to cancel, including algebraic terms Convincingly shown
(iii)	$\frac{3}{2}$	B1 **[1]**	
(iv)	$\sum_{r=50}^{100}\frac{4+r}{r(r+1)(r+2)}$ $=\sum_{r=1}^{100}\frac{4+r}{r(r+1)(r+2)}-\sum_{r=1}^{49}\frac{4+r}{r(r+1)(r+2)}$ $=\left(\frac{3}{2}-\frac{2}{101}+\frac{1}{102}\right)-\left(\frac{3}{2}-\frac{2}{50}+\frac{1}{51}\right)$ $=0.0104$ (3s.f.)	M1 M1 A1 **[3]**	Splitting into two parts Use of result from (ii) c.a.o.

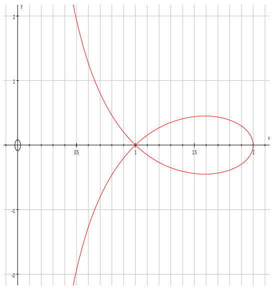
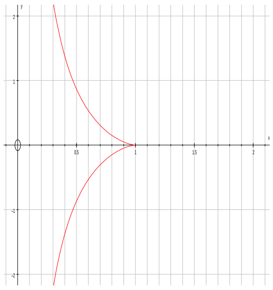
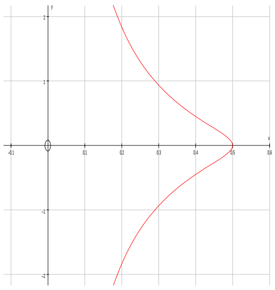
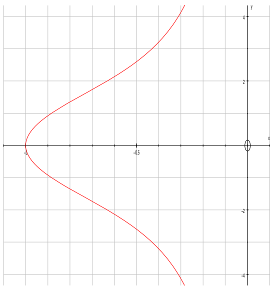
4756 (FP2) Further Methods for Advanced Mathematics

1 (a)	$y = \arctan\sqrt{x}$ $u = \sqrt{x}$, $y = \arctan u$ $\Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}, \frac{dy}{du} = \frac{1}{1+u^2}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{1+u^2} \times \frac{1}{2\sqrt{x}}$ $= \frac{1}{1+x} \times \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}(x+1)}$	M1 A1 A1	Using Chain Rule Correct derivative in any form Correct derivative in terms of x
	OR $\tan y = \sqrt{x}$ $\Rightarrow \sec^2 y \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$ M1A1 $\sec^2 y = 1 + \tan^2 y = 1 + x$ $\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}(x+1)}$ A1		Rearranging for $\sqrt{x}$ or x and differentiating implicitly
	$\Rightarrow \int_0^1 \frac{1}{\sqrt{x}(x+1)} dx = \left[2\arctan\sqrt{x}\right]_0^1$ $= 2 \arctan 1 - 2 \arctan 0$ $= 2 \times \frac{\pi}{4} = \frac{\pi}{2}$	M1 A1 A1 (ag)	Integral in form $k \arctan\sqrt{x}$ $k = 2$
(b)(i)	$x = r\cos\theta, y = r\sin\theta, x^2 + y^2 = r^2$ $x^2 + y^2 = xy + 1$ $\Rightarrow r^2 = r^2\cos\theta\sin\theta + 1$ $\Rightarrow r^2 = \frac{1}{2}r^2\sin 2\theta + 1$ $\Rightarrow 2r^2 = r^2\sin 2\theta + 2$ $\Rightarrow r^2(2 - \sin 2\theta) = 2$ $\Rightarrow r^2 = \frac{2}{2 - \sin 2\theta}$	M1 A1 A1 A1 (ag) **4**	Using at least one of these LHS RHS Clearly obtained *SR*: $x = r\sin\theta, y = r\cos\theta$ used M1A1A0A0 max.
(ii)	Max r is $\sqrt{2}$ Occurs when $\sin 2\theta = 1$ $\Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$ Min $r = \sqrt{\frac{2}{3}}$ Occurs when $\sin 2\theta = -1$ $\Rightarrow \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$	B1 M1 A1 B1 M1 A1 **6**	 Attempting to solve Both. Accept degrees. A0 if extras in range $\frac{\sqrt{6}}{3}$ Attempting to solve (must be -1) Both. Accept degrees. A0 if extras in range

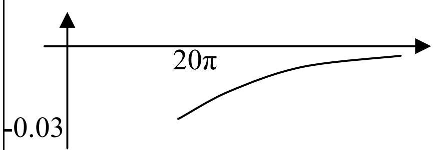
(iii)		G1 G1 **2**	Closed curve, roughly elliptical, with no points or dents Major axis along $y = x$ **18**
2 (a)	$\cos 5\theta + \mathrm{j}\sin 5\theta = (\cos\theta + \mathrm{j}\sin\theta)^5$ $= \cos^5\theta + 5\cos^4\theta\,\mathrm{j}\sin\theta + 10\cos^3\theta\,\mathrm{j}^2\sin^2\theta$ $+ 10\cos^2\theta\,\mathrm{j}^3\sin^3\theta + 5\cos\theta\,\mathrm{j}^4\sin^4\theta + \mathrm{j}^5\sin^5\theta$ $= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta + \mathrm{j}(\ldots)$ $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ $= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	M1 M1 A1 M1 M1 A1 **6**	Using de Moivre Using binomial theorem appropriately Correct real part. Must evaluate powers of j Equating real parts Replacing $\sin^2\theta$ by $1 - \cos^2\theta$ $a = 16$, $b = -20$, $c = 5$
(b)	$C + \mathrm{j}S$ $= e^{\mathrm{j}\theta} + e^{\mathrm{j}\left(\theta + \frac{2\pi}{n}\right)} + \ldots + e^{\mathrm{j}\left(\theta + \frac{(2n-2)\pi}{n}\right)}$ This is a G.P. $a = e^{\mathrm{j}\theta}$, $r = e^{\mathrm{j}\frac{2\pi}{n}}$ Sum $= \dfrac{e^{\mathrm{j}\theta}\left(1 - \left(e^{\mathrm{j}\frac{2\pi}{n}}\right)^n\right)}{1 - e^{\mathrm{j}\frac{2\pi}{n}}}$ Numerator $= e^{\mathrm{j}\theta}\left(1 - e^{2\pi\mathrm{j}}\right)$ and $e^{2\pi\mathrm{j}} = 1$ so sum = 0 $\Rightarrow \quad C = 0$ and $S = 0$	M1 A1 M1 A1 A1 E1 E1 **7**	Forming series $C + \mathrm{j}S$ as exponentials Need not see whole series Attempting to sum finite or infinite G.P. Correct a, r used or stated, and n terms Must see j Convincing explanation that sum = 0 $C = S = 0$. Dep. on previous E1 Both E marks dep. on 5 marks above
(c)	$e^t \approx 1 + t + \frac{1}{2}t^2$ $\dfrac{t}{e^t - 1} \approx \dfrac{t}{t + \frac{1}{2}t^2}$ $\dfrac{t}{t + \frac{1}{2}t^2} = \dfrac{1}{1 + \frac{1}{2}t} = \left(1 + \frac{1}{2}t\right)^{-1} = 1 - \frac{1}{2}t + \ldots$	B1 M1 A1 M1	Ignore terms in higher powers Substituting Maclaurin series Suitable manipulation and use of binomial theorem
	OR $\dfrac{1}{1 + \frac{1}{2}t} = \dfrac{1}{1 + \frac{1}{2}t} \times \dfrac{1 - \frac{1}{2}t}{1 - \frac{1}{2}t} = \dfrac{1 - \frac{1}{2}t}{1 - \frac{1}{4}t^2}$ M1		
	Hence $\dfrac{t}{e^t - 1} \approx 1 - \frac{1}{2}t$	A1 (ag)	
	OR $\left(e^t - 1\right)\left(1 - \frac{1}{2}t\right) = \left(t + \frac{1}{2}t^2 + \ldots\right)\left(1 - \frac{1}{2}t\right)$ M1 A1 $\approx t +$ terms in t^3 M1 $\Rightarrow \dfrac{t}{e^t - 1} \approx 1 - \frac{1}{2}t$ A1		Substituting Maclaurin series Correct expression Multiplying out Convincing explanation
		5	**18**

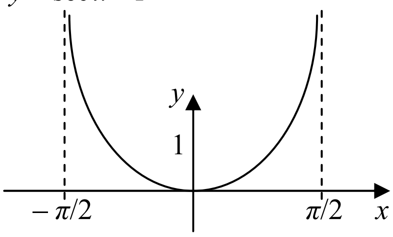
3 (i)	$\mathbf{M}^{-1} = \frac{1}{4-a}\begin{pmatrix} 2 & -2-2a & 2+a \\ 2 & 2-3a & 2a-2 \\ -1 & 5 & -3 \end{pmatrix}$	M1 A1 M1 A1 M1	Evaluating determinant $4 - a$ Finding at least four cofactors Six signed cofactors correct Transposing and dividing by det
	When $a = -1$, $\mathbf{M}^{-1} = \frac{1}{5}\begin{pmatrix} 2 & 0 & 1 \\ 2 & 5 & -4 \\ -1 & 5 & -3 \end{pmatrix}$	A1	$\mathbf{M}^{-1}$ correct (in terms of a) and result for $a = -1$ stated *SR*: After 0 scored, SC1 for $\mathbf{M}^{-1}$ when $a = -1$, obtained correctly with some working
		6	
(ii)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 & 0 & 1 \\ 2 & 5 & -4 \\ -1 & 5 & -3 \end{pmatrix}\begin{pmatrix} -2 \\ b \\ 1 \end{pmatrix}$	M2	Attempting to multiply $(-2 \;\; b \;\; 1)^{\mathrm{T}}$ by given matrix (M0 if wrong order)
		M1	Multiplying out
	$\Rightarrow \quad x = -\frac{3}{5},\ y = b - \frac{8}{5},\ z = b - \frac{1}{5}$	A2	A1 for one correct
	OR $4x + y = b - 4$ $x - y = 1 - b$ o.e. M1		Eliminating one unknown in 2 ways Or e.g. $3x + z = b - 2$, $5x = -3$ Or e.g. $3y - 4z = -b - 4$, $5y - 5z = -7$
	M1		Solve to obtain one value. Dep. on M1 above
	$\Rightarrow \quad x = -\frac{3}{5}$ A1		One unknown correct After M0, SC1 for value of x
	M1		Finding the other two unknowns
	$\Rightarrow \quad y = b - \frac{8}{5},\ z = b - \frac{1}{5}$ A1		Both correct
		5	
(iii)	e.g. $3x - 3y = 2b + 2$ $5x - 5y = 4$	M1 A1A1	Eliminating one unknown in 2 ways Two correct equations Or e.g. $3x + 6z = b - 2$, $5x + 10z = -3$ Or e.g. $3y + 6z = -b - 4$, $5y + 10z = -7$
	Consistent if $\frac{2b+2}{3} = \frac{4}{5}$	M1	Attempting to find b
	$\Rightarrow \quad b = \frac{1}{5}$	A1	
	Solution is a line	B2	
		7	**18**

4 (i)	$\sinh x = \frac{e^x - e^{-x}}{2} \Rightarrow \sinh^2 x = \frac{\left(e^x - e^{-x}\right)^2}{4}$		
	$= \frac{e^{2x} - 2 + e^{-2x}}{4}$	B1	$e^{2x} - 2 + e^{-2x}$
	$\Rightarrow\ 2\sinh^2 x + 1 = \frac{e^{2x} - 2 + e^{-2x}}{2} + 1$		
	$= \frac{e^{2x} + e^{-2x}}{2} = \cosh 2x$	B1	Correct completion
	$\Rightarrow\ 2\sinh 2x = 4\sinh x\cosh x$	B1	Both correct derivatives
	$\Rightarrow\ \sinh 2x = 2\sinh x\cosh x$	B1	Correct completion
		4	
(ii)	$2\cosh 2x + 3\sinh x = 3$		
	$\Rightarrow\ 2(1 + 2\sinh^2 x) + 3\sinh x = 3$	M1	Using identity
	$\Rightarrow\ 4\sinh^2 x + 3\sinh x - 1 = 0$	A1	Correct quadratic
	$\Rightarrow\ (4\sinh x - 1)(\sinh x + 1) = 0$	M1	Solving quadratic
	$\Rightarrow\ \sinh x = ¼, -1$	A1	Both
		M1	Use of arsinh $x = \ln(x + \sqrt{x^2+1})$ o.e. Must obtain at least one value of x
	$\Rightarrow\ x = \text{arsinh}(¼) = \ln\left(\frac{1+\sqrt{17}}{4}\right)$	A1	Must evaluate $\sqrt{x^2+1}$
	$x = \text{arsinh}(-1) = \ln(-1+\sqrt{2})$	A1	
	OR $2e^{4x} + 3e^{3x} - 6e^{2x} - 3e^x + 2 = 0$		
	$\Rightarrow\ \left(2e^{2x} - e^x - 2\right)\left(e^{2x} + 2e^x - 1\right) = 0$ M1A1		Factorising quartic
	$\Rightarrow\ e^x = \frac{1\pm\sqrt{17}}{4}$ or $-1\pm\sqrt{2}$ M1A1		Solving either quadratic
	$\Rightarrow\ x = \ln\left(\frac{1+\sqrt{17}}{4}\right)$ or $\ln(-1+\sqrt{2})$ M1A1A1		Using ln (dependent on first M1)
		7	
(iii)	$\cosh t = \frac{5}{4} \Rightarrow \frac{e^t + e^{-t}}{2} = \frac{5}{4}$		
	$\Rightarrow\ 2e^{2t} - 5e^t + 2 = 0$	M1	Forming quadratic in e^t
	$\Rightarrow\ \left(2e^t - 1\right)\left(e^t - 2\right) = 0$	M1	Solving quadratic
	$\Rightarrow\ e^t = \frac{1}{2}, 2$	A1	
	$\Rightarrow\ t = \pm\ln 2$	A1 (ag)	Convincing working
	$\int_4^5 \frac{1}{\sqrt{x^2-16}}\,dx = \left[\text{arcosh}\,\frac{x}{4}\right]_4^5$	B1	
	$= \text{arcosh}\,\frac{5}{4} - \text{arcosh}\,1$	M1	Substituting limits
	$= \ln 2$	A1	A0 for $\pm\ln 2$
	OR $\int_4^5 \frac{1}{\sqrt{x^2-16}}\,dx = \left[\ln\left(x + \sqrt{x^2-16}\right)\right]_4^5$ B1		
	$= \ln 8 - \ln 4$ M1		Substituting limits
	$= \ln 2$ A1		
		7	**18**

5 (i)	Horz. projection of QP = $k\cos\theta$ Vert. projection of QP = $k\sin\theta$ Subtract OQ = $\tan\theta$	B1 B1 B1 **3**	 Clearly obtained
(ii)	$k = 2$  $k = 1$  $k = ½$  $k = -1$ 	G1 G1 G1 G1 **4**	Loop Cusp
(iii)(A) **(B)** **(C)**	for all k, y axis is an asymptote $k = 1$ $k > 1$	B1 B1 B1 **3**	Both
(iv)	Crosses itself at (1, 0) $k = 2 \Rightarrow \cos\theta = ½ \Rightarrow \theta = 60°$ $\Rightarrow$ curve crosses itself at 120°	 M1 A1 **2**	 Obtaining a value of θ Accept 240°
(v)	$y = 8\sin\theta - \tan\theta$ $\Rightarrow \dfrac{dy}{d\theta} = 8\cos\theta - \sec^2\theta$ $\Rightarrow 8\cos\theta - \dfrac{1}{\cos^2\theta} = 0$ at highest point $\Rightarrow \cos^3\theta = \dfrac{1}{8} \Rightarrow \cos\theta = \pm\dfrac{1}{2} \Rightarrow \theta = 60°$ at top $\Rightarrow x = 4$ $y = 3\sqrt{3}$	M1 A1 A1 **3**	Complete method giving θ Both
(vi)	RHS $= \dfrac{(k\cos\theta - 1)^2}{k^2\cos^2\theta}(k^2 - k^2\cos^2\theta)$ $= \dfrac{(k\cos\theta - 1)^2}{k^2\cos^2\theta} \times k^2\sin^2\theta$ $= (k\cos\theta - 1)^2\tan^2\theta$ $= ((k\cos\theta - 1)\tan\theta)^2$ $= (k\sin\theta - \tan\theta)^2 =$ LHS	M1 M1 E1 **3**	Expressing one side in terms of θ Using trig identities **18**

4758 Differential Equations

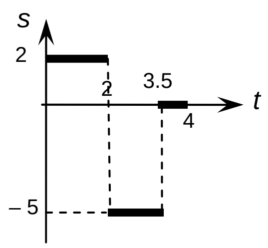
1(i)	$\alpha^2 + 6\alpha + 9 = 0$	**M1**	Auxiliary equation	
	$\alpha = -3$ (repeated)	**A1**		
	$y = e^{-3t}(A + Bt)$	**F1**	CF for their roots	
	PI $y = a\sin t + b\cos t$	**B1**		
	$\dot{y} = a\cos t - b\sin t$			
	$\ddot{y} = -a\sin t - b\cos t$			
	$-a\sin t - b\cos t + 6(a\cos t - b\sin t)$ $+9(a\sin t + b\cos t) = 0.5\sin t$	**M1**	Differentiate twice and substitute	
	$8a - 6b = 0.5$	**M1**	Compare coefficients	
	$8b + 6a = 0$	**M1**	Solve	
	Solving gives $a = 0.04, b = -0.03$	**A1**		
	GS $y = e^{-3t}(A + Bt) + 0.04\sin t - 0.03\cos t$	**F1**	PI + CF with two arbitrary constants	
				9
(ii)	$t = 0, y = 0 \Rightarrow A = 0.03$	**M1**	Use condition	
	$\dot{y} = e^{-3t}(B - 3A - 3Bt) + 0.04\cos t + 0.03\sin t$	**M1**	Differentiate	
		F1	Follows their GS	
	$t = 0, \dot{y} = 0 \Rightarrow 0 = B - 3A + 0.04$	**M1**	Use condition	
	$y = 0.01\left(e^{-3t}(3 + 5t) + 4\sin t - 3\cos t\right)$	**A1**	Cao	
				5
(iii)	For large t, the particle oscillates	**B1**	Oscillates	
	With amplitude constant (≈ 0.05)	**B1**	Amplitude approximately constant	
				2
(iv)	$t = 20\pi \Rightarrow e^{-60\pi}$ very small	**M1**		
	$y \approx -0.03$	**A1**		
	$\dot{y} \approx 0.04$	**A1**		
				3
(v)	$y = e^{-3t}(C + Dt)$	**M1**	CF of correct type or same type as in (i)	
		A1	Must use new arbitrary constants	
		**B1√**	$y \approx -0.03$ at $t = 20\pi$	
		B1	Gradient at 20π consistent with (iv)	
		B1	Shape consistent	
				5

2(a)(i)	$I = \exp\int -\tan x \,\mathrm{d}x$ $= \exp(-\ln \sec x)$ $= (\sec x)^{-1} = \cos x$ $\cos x \frac{\mathrm{d}y}{\mathrm{d}x} - y\sin x = \sin x$ $\frac{\mathrm{d}}{\mathrm{d}x}(y\cos x) = \sin x$ $y\cos x = -\cos x + A$ $(y = A\sec x - 1)$	**M1** **A1** **A1** **M1** **M1** **M1** **A1** **A1**	Attempt IF Correct IF Simplified Multiply by IF Recognise derivative Integrate RHS (including constant) LHS	**8**
	$\frac{\mathrm{d}y}{\mathrm{d}x} = (1+y)\tan x$ $\ln(1+y) = \ln\sec x + A$	**M1** **A1** **M1** **A1** **M1** **A1** **M1** **A1**	Rearrange equation Separate variables RHS LHS	**8**
(ii)	$x = 0, y = 0 \Rightarrow 0 = A - 1$ $y = \sec x - 1$ 	**M1** **A1** **B1** **B1**	Use condition Shape and through origin Behaviour at $\pm\frac{1}{2}\pi$	**4**
(b)(i)		**M1** **A1** **M1** **A1**	Attempt one curve Reasonable attempt at one curve Attempt second curve Reasonable attempt at both curves	**4**
(ii)	$y' = (1+y^2)\tan x$ $x = 0, y = 1 \Rightarrow y' = 0$ $y(0.1) = 1 + 0.1\times 0 = 1$ $x = 0.1, y = 1 \Rightarrow y' = 0.201$ $y(0.2) = 1 + 0.1\times 0.201$ $= 1.0201$	**M1** **M1** **A1** **M1** **E1**	Rearrange Use of algorithm Use of algorithm for second step	**5**
(iii)	$\tan\frac{\pi}{2}$ undefined so cannot go past $\frac{\pi}{2}$ So approximation cannot continue to $1.6 > \frac{\pi}{2}$	**M1** **A1**		**2**
(iv)	Reduce step length	**B1**		**1**

3(i)	$\dot{x} = Ae^{-kt}$	**M1**	Any valid method (or no method shown)	
		A1		
	$t=0, \dot{x}=v_1 \Rightarrow A = v_1$	**M1**	Use condition	
	$\dot{x} = v_1 e^{-kt}$	**A1**		
	$x = \int v_1 e^{-kt}\,dt$	**M1**	Integrate	
	$= -\frac{v_1}{k}e^{-kt} + B$	**A1**		
	$t=0, x=0 \Rightarrow B = \frac{v_1}{k}$	**M1**	Use condition	
	$x = \frac{v_1}{k}\left(1-e^{-kt}\right)$	**E1**		
				8
(ii)	$\int \frac{d\dot{y}}{\dot{y}+g/k} = \int -k\,dt$	**M1**	Separate and integrate	
	$\text{In}\left(\dot{y}+\frac{g}{k}\right) = -kt + C$	**A1**	LHS	
		A1	RHS	
	$\dot{y}+\frac{g}{k} = De^{-kt}$	**M1**	Rearrange, dealing properly with constant	
	$t=0, \dot{y}=v_2 \Rightarrow D = v_2 + \frac{g}{k}$	**M1**	Use condition	
	$\dot{y} = \left(v_2+\frac{g}{k}\right)e^{-kt} - \frac{g}{k}$	**A1**		
	$y = \int\left(\left(v_2+\frac{g}{k}\right)e^{-kt} - \frac{g}{k}\right)dt$	**M1**	Integrate	
	$= -\frac{1}{k}\left(v_2+\frac{g}{k}\right)e^{-kt} - \frac{g}{k}t + E$	**A1**		
	$t=0, y=0 \Rightarrow 0 = -\frac{1}{k}\left(v_2+\frac{g}{k}\right) + E$	**M1**	Use condition	
	$y = \frac{1}{k^2}(kv_2+g)\left(1-e^{-kt}\right) - \frac{g}{k}t$	**E1**		
				1 0
(iii)	$1-e^{-kt} = \frac{kx}{v_1}$	**M1**		
	$t = -\frac{1}{k}\ln\left(1-\frac{kx}{v_1}\right)$	**A1**		
	$y = \left(\frac{kv_2+g}{kv_1}\right)x + \frac{g}{k^2}\ln\left(1-\frac{kx}{v_1}\right)$	**M1**	Substitute	
		E1	Convincingly shown	
				4
(iv)	$x = 8 \Rightarrow y = 4.686$	**M1**		
	Hence will not clear wall	**A1**		
				2

4(i)	$4y=-3x+23-\dot{x}$ $4\dot{y}=-3\dot{x}-\ddot{x}$ $\frac{1}{4}(-3\dot{x}-\ddot{x})=2x+\frac{1}{4}(-3x+23-\dot{x})-7$ $-3\dot{x}-\ddot{x}=8x-3x+23-\dot{x}-28$ $\Rightarrow \ddot{x}+2\dot{x}+5x=5$	**M1** **M1** **M1** **M1** **E1**	y or $4y$ in terms of $x,\dot{x}$ Differentiate Substitute for y Substitute for $\dot{y}$	**5**
(ii)	$\alpha^2+2\alpha+5=0$ $\Rightarrow \alpha=-1\pm 2i$ CF $\mathrm{e}^{-t}(A\cos 2t+B\sin 2t)$ PI $x=\frac{5}{5}=1$ GS $x=1+\mathrm{e}^{-t}(A\cos 2t+B\sin 2t)$	**M1** **A1** **M1** **F1** **B1** **B1** **F1**	Auxiliary equation CF for complex roots CF for their roots Constant PI Correct PI PI + CF with two arbitrary constants	**7**
(iii)	$y=\frac{1}{4}(-3x+23-\dot{x})$ $=\frac{1}{4}\Big[-3-3\mathrm{e}^{-t}(A\cos 2t+B\sin 2t)+23$ $+\mathrm{e}^{-t}(A\cos 2t+B\sin 2t)$ $-\mathrm{e}^{-t}(-2A\sin 2t+2B\cos 2t)\Big]$ $y=5-\frac{1}{2}\mathrm{e}^{-t}((A+B)\cos 2t+(B-A)\sin 2t)$	**M1** **M1** **F1** **A1**	 Differentiate and substitute Expression for $\dot{x}$ follows their GS	**4**
(iv)	$t=0, x=8 \Rightarrow 1+A=8 \Rightarrow A=7$ $t=0, y=0 \Rightarrow 5-\frac{1}{2}(A+B)=0 \Rightarrow B=3$ $x=1+\mathrm{e}^{-t}(7\cos 2t+3\sin 2t)$ $y=5-\mathrm{e}^{-t}(5\cos 2t-2\sin 2t)$	**M1** **M1** **A1** **A1**	Use condition Use condition	**4**
(v)	For large t, e^{-t} tends to 0 $y\to 5$ $x\to 1$ $\Rightarrow y>x$	**M1** **B1** **B1** **E1**	 Complete argument	**4**

4761 Mechanics 1

1 (i)	$0 < t < 2$, $v = 2$ $2 < t < 3.5$ $v = -5$	B1 B1	 Condone '5 downwards' and ' – 5 downwards'	 2
(ii)		 B1 B1	Condone intent – e.g. straight lines free-hand and scales not labelled; accept non-vertical sections at $t = 2$ & 3.5. Only horizontal lines used and 1st two parts present. BOD t-axis section. One of 1st 2 sections correct. FT (i) and allow if answer correct with (i) wrong All correct. Accept correct answer with (i) wrong. FT (i) only if 2nd section –ve in (i)	 2
(iii)	(A) upwards; (B) and (C) downwards	E1	All correct. Accept +/- ve but not towards/away from O Accept forwards/backwards. Condone additional wrong statements about position.	 1
				5
2 (i)	$\begin{pmatrix}12\\9\end{pmatrix} = \begin{pmatrix}2\\-3\end{pmatrix} + 4\mathbf{a}$ so $\mathbf{a} = \begin{pmatrix}2.5\\3\end{pmatrix}$	M1 A1	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ If vector $\mathbf{a}$ seen, isw.	 2
(ii)	**either** $\mathbf{r} = \begin{pmatrix}-1\\2\end{pmatrix} + \begin{pmatrix}2\\-3\end{pmatrix} \times 4 + \frac{1}{2}\mathbf{a} \times 4^2$ $\mathbf{r} = \begin{pmatrix}27\\14\end{pmatrix}$ so $\begin{pmatrix}27\\14\end{pmatrix}$ m **or**	 M1 A1 A1 M1 A1 A1	 For use of $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with **their a.** Initial position may be omitted. FT **their a.** Initial position may be omitted. cao. Do not condone magnitude as final answer. Use of $\mathbf{s} = 0.5t(\mathbf{u} + \mathbf{v})$ Initial position may be omitted. Correct substitution. Initial position may be omitted. cao Do not condone mag as final answer. SC2 for $\begin{pmatrix}28\\12\end{pmatrix}$	 3

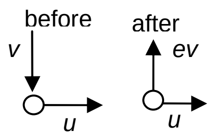
(iii)	Using N2L $\mathbf{F} = 5\mathbf{a} = \begin{pmatrix} 12.5 \\ 15 \end{pmatrix}$ so $\begin{pmatrix} 12.5 \\ 15 \end{pmatrix}$ N	M1 F1	Use of $\mathbf{F} = m\mathbf{a}$ or $\mathbf{F} = mg\mathbf{a}$. FT **their a** only. Do not accept magnitude as final ans.	2
				7
3 (i)	$\lvert\mathbf{F}\rvert = \sqrt{(-1)^2 + 5^2}$ $= \sqrt{26} = 5.0990... = 5.10$ (3 s. f.) Angle with **j** is $\arctan(0.2)$ so 11.309… so 11.3° (3 s. f.)	M1 A1 M1 A1	Accept $\sqrt{-1^2 + 5^2}$ even if taken to be $\sqrt{24}$ accept arctan(p) where $p = \pm 0.2$ or ± 5 o.e. cao	4
(ii)	$\begin{pmatrix} -2 \\ 3b \end{pmatrix} = 4\begin{pmatrix} -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 2a \\ a \end{pmatrix}$ $a = 1$, $b = 7$ so $\mathbf{G} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{H} = \begin{pmatrix} -2 \\ 21 \end{pmatrix}$ or $\mathbf{G} = 2\,\mathbf{i} + \mathbf{j}$ and $\mathbf{H} = -2\,\mathbf{i} + 21\,\mathbf{j}$	M1 M1 A1 A1	$\mathbf{H} = 4\mathbf{F} + \mathbf{G}$ soi Formulating at least 1 scalar equation from **their** vector equation soi a correct **or G** follows from their wrong a **H** cao	4
				8
4(i)	20cos 15 = 19.3185…. so 19.3 N (3 s. f.) in direction BC	B1	Accept no direction. Must be evaluated	1
(ii)	Let the tension be T $T \sin 50 = 19.3185….$ so $T = 25.2185…$ so 25.2 N (3 s. f.)	M1 F1	Accept $\sin \leftrightarrow \cos$ but not (i) × sin 50 FT **their** 19.3… only. cwo	2
(iii)	$R + 20 \sin 15 - 2.5g - 25.2185… \times \cos 50 = 0$ $R = 35.5337…$ so 35.5 N (3 s. f.)	M1 B1 A1 A1	Allow 1 force missing or 1 tension not resolved. FT T. No extra forces. Accept mass used. Accept $\sin \leftrightarrow \cos$. Weight correct All correct except sign errors. FT **their** T cao. Accept 35 or 36 for 2. s.f.	4
(iv)	The horizontal resolved part of the 20 N force is not changed.	E1	Accept no reference to vertical component but do not accept 'no change' to both components. No need to be explicit that value of tension in AB depends only on horizontal component of force at C	
				1
				8

5(i)	$a = 6t - 12$	M1 A1	Differentiating cao	2
(ii)	We need $\int_1^3 (3t^2 - 12t + 14)\,dt$ $= \left[t^3 - 6t^2 + 14t\right]_1^3$ **either** = (27 – 54 + 42) – (1 – 6 + 14) = 15 – 9 = 6 so 6 m **or** $s = t^3 - 6t^2 + 14t + C$ s = 0 when t = 1 gives 0 = 1 – 6 + 14 + C so C = – 9 Put t = 3 to give s = 27 – 54 + 42 – 9 = 6 so 6 m.	M1 A1 M1 A1 M1 A1	Integrating. Neglect limits. At least two terms correct. Neglect limits. Dep on 1st M1. Use of limits with attempt at subtraction seen. cao Dep on 1st M1. An attempt to find C using $s(1) = 0$ and then evaluating $s(3)$. cao	4
(iii)	$v > 0$ so the particle always travels in the same (+ve) direction As the particle never changes direction, the final distance from the starting point is the displacement.	 E1 E1	 Only award if explicit Complete argument	
				2
				8
6 (i)	Component of weight down the plane is $1.5 \times 9.8 \times \frac{2}{7} = 4.2$ N	 M1 E1	 Use of mgk where k involves an attempt at resolution Accept $1.5 \times 9.8 \times \frac{2}{7} = 4.2$ or $14.7 \times \frac{2}{7} = 4.2$ seen	2
(ii)	Down the plane. Take F down the plane. 4.2 – 6.4 + F = 0 so F = 2.2. Friction is 2.2 N down the plane	 M1 A1	 Allow sign errors. All forces present. No extra forces. Must have direction. [Award 1 for 2.2 N seen and 2 for 2.2 N down plane seen]	2
(iii)	F up the plane N2L down the plane 4.2 – F = 1.5 × 1.2 so F = 4.2 – 1.8 = 2.4 Friction is 2.4 N up the plane	 M1 A1 A1 A1	 N2L. $F = ma$. No extra forces. Allow weight term missing or wrong Allow only sign errors ±2.4 cao. Accept no reference to direction if F = 2.4.	4
(iv)	$2^2 = 0.8^2 + 2 \times 1.2 \times s$ s = 1.4 so 1.4 m	M1 A1 A1	Use of $v^2 = u^2 + 2as$ or sequence All correct in 1 or 2-step method	3

(v)	Diagrams	B1 B1	Frictions and coupling force correctly labelled with arrows. All forces present and properly labelled with arrows.	
	either Up the plane $10-3.5\times9.8\times\frac{2}{7}-(2.3+0.7)=3.5a$	M1	N2L. $F = ma$. No extra forces. Condone sign errors. Allow total/part weight **or** total/part friction omitted (but not both). Allow mass instead of weight and mass/weight not or wrongly resolved.	
		B1	Correct overall mass and friction	
	$a = -0.8$ so 0.8 m s^{-2}. down the plane	A1	Clear description or diagram	
	For barge B up the plane $T-2\times9.8\times\frac{2}{7}-0.7=2\times(-0.8)$	M1	N2L on one barge with **their** $\pm a$ ($\neq$ 1.2 or 0). All forces present and weight component attempted. No extra forces. Condone sign errors.	
	$T = 4.7$ so 4.7 N. Tension	A1	cao	
	or (separate equations of motion)		In eom for A **or** B allow weight or friction missing and also allow mass used instead of weight and wt not or wrongly resolved. In other equn weight component attempted and friction term present.	
	Barge A	M1	N2L. Do not allow $F = mga$. No extra forces. Condone sign errors.	
	Barge B	M1	N2L. Do not allow $F = mga$. No extra forces. Condone sign errors.	
		M1	Solving a pair of equns in a and T	
	$a = -0.8$ so 0.8 m s^{-2}. down the plane	A1	Clear description or diagram	
	$T = 4.7$ so 4.7 N. Tension	A1	cao cwo	
				7
				18
7 (i)	$y(0) = 1$	B1		1
(ii)	**Either** $\frac{1}{2}(20+5)-5=7.5$	M1	Use of symmetry e.g. use of $\frac{1}{2}(20+5)$	
		A1	12.5 o.e. seen	
		A1	7.5 cao	
	or	M1	Attempt at y' and to solve $y' = 0$	
		A1	$k(15-2x)$ where $k = 1$ or $\frac{1}{100}$	
		A1	7.5 cao, seen as final answer	
	$y(7.5)=\frac{1}{100}(100+15\times7.5-7.5^2)$	M1	FT **their** 7.5	
	$= \frac{25}{16}$ (1.5625) so 1.5625 m	E1	AG [SC2 only showing 1.5625 leads to $x = 7.5$]	5

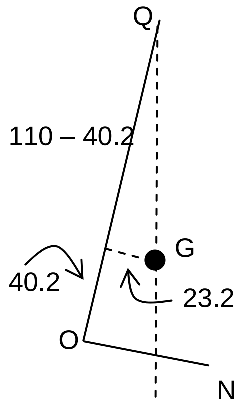
(iii)	$4.9t^2 = \frac{25}{16}$ (1.5625)	M1	Use of $s = ut + 0.5at^2$ with $u = 0$. Condone use of ± 10, ± 9.8, ± 9.81. If sequence of *suvat* used, complete method required.	
		A1	In any method only error accepted is sign error	
	$t^2 = 0.31887\ldots$ so $t = \pm\, 0.56469\ldots$ Hence 0.565 s (3 s. f.)	E1	AG. Condone no reference to –ve value. www. 0.565 must be justified as answer to 3 s. f.	3
(iv)	$\dot{x} = \dfrac{12.5}{0.56469...} = 22.1359...$	M1	or 25 / (2×0.56469..)	
		B1	Use of 12.5 or equivalent	
	so 22.1 m s^{-1} (3 s. f.))	E1	22.1 must be justified as answer to 3 s. f. Don't penalise if penalty already given in (iii).	
	Either Time is $\dfrac{20}{12.5} \times 0.56469...$ s	M1		
	so 0.904 s (3 s. f.)	A1	cao Accept 0.91 (2 s. f.)	
	or Time is $\dfrac{20}{22.1359...}$ s	M1		
	= 0.903507… so 0.904 s (3 s. f.)	A1	cao Accept 0.91 (2 s. f.)	
	or $(\text{iii}) + \dfrac{7.5}{\textbf{their } \dot{x}}$	M1		
	so 0.904 s (3 s. f.)	A1	cao Accept 0.91 (2 s. f.)	5
(v)	$v = \sqrt{\dot{x}^2 + \dot{y}^2}$	M1	Must have attempts at both components	
	$\dot{y}^2 = 0^2 + 2 \times 9.8 \times \frac{25}{16}$ or $\dot{y} = 0 + 9.8 \times 0.5646...$	M1	Or equiv. $u = 0$. Condone use of ± 10, ± 9.8, ± 9.81. Accept wrong s (or t in alternative method)	
	$= \frac{245}{8}$ (30.625) or $\dot{y} = \pm 5.539...$	A1	Or equivalent. May be implied. Could come from (iii) if $v^2 = u^2 + 2as$ used there. Award marks again.	
	so $v = \sqrt{490 + 30.625} = 22.8172\ldots$ m s^{-1} so 22.8 m s^{-1} (3 s. f.)	A1	cao. www	4
				18

4762 Mechanics 2

1 (a) (i)	Let vel of Q be v $\rightarrow$ $6\times1=4v+2\times4$ $v=-0.5$ so 0.5 m s^{-1} in opposite direction to R	 M1 A1 A1 A1	 Use of PCLM Any form Direction must be made clear. Accept – 0.5 only if + ve direction clearly shown	4
(ii)	Let velocities after be R: v_R $\rightarrow$; S: v_S $\rightarrow$ PCLM +ve $\rightarrow$ $4\times2-1\times3=2v_R+3v_S$ $2v_R+3v_S=5$ NEL +ve $\rightarrow$ $\frac{v_S-v_R}{-1-4}=-0.1$ so $v_S-v_R=0.5$ Solving gives $v_R = 0.7\rightarrow$ $v_S=1.2$ $\rightarrow$	 M1 A1 M1 A1 A1 A1	 PCLM Any form NEL Any form Direction not required Direction not required Award cao for 1 vel and FT second	6
(iii)	R and S separate at 0.5 m s^{-1} Time to drop T given by $0.5\times9.8T^2=0.4$ so $T=\frac{2}{7}$ $(0.28571\ldots)$ so distance is $\frac{2}{7}\times0.5=\frac{1}{7}$ m $(0.142857\ldots\text{m})$	M1 B1 A1	FT **their** result above. Either from NEL or from difference in final velocities cao	3
(b)	 $u\rightarrow u$ $v\rightarrow(-)ev$ KE loss is $\frac{1}{2}m\left(u^2+v^2\right)-\frac{1}{2}m\left(u^2+e^2v^2\right)$ $=\frac{1}{2}mu^2+\frac{1}{2}mv^2-\frac{1}{2}mu^2-\frac{1}{2}me^2v^2$ $=\frac{1}{2}mv^2\left(1-e^2\right)$	 B1 B1 M1 E1	 Accept $v\rightarrow ev$ Attempt at difference of KEs Clear expansion and simplification of correct expression	
				4
				17

2(i)	GPE is 1200 × 9.8 × 60 = 705 600 Power is (705 600 + 1 800 000) ÷ 120 = 20 880 W = 20 900 W (3 s. f.)	B1 M1 B1 A1	Need not be evaluated power is WD ÷ time 120 s cao	4
(ii)	Using $P = Fv$. Let resistance be R N 13500 = 18F so F = 750 As v const, $a = 0$ so $F - R = 0$ Hence resistance is 750 N We require 750 × 200 = 150 000 J (= 150 kJ)	M1 A1 E1 M1 F1	Use of $P = Fv$. Needs some justification Use of WD = Fd or Pt FT **their** F	5
(iii)	$\frac{1}{2}\times 1200\times\left(9^2-18^2\right)$ $=1200\times 9.8\times x\sin 5-1500x$ Hence 145800 = 475.04846…x so x = 306.91… so 307 m (3 s, f,)	M1 B1 M1 A1 A1 A1	Use of W-E equation with 'x' 2 KE terms present GPE term with resolution GPE term correct All correct cao	6
(iv)	$P = Fv$ and N2L gives $F - R = 1200a$ Substituting gives $P = (R + 1200a)v$ If $a \neq 0$, v is not constant. But P and R are constant so a cannot be constant.	B1 B1 E1 E1	 Shown	
				4
				19
3 (i) (*A*)	Let force be P a.c. moments about C P × 0.125 – 340 × 0.5 = 0 P = 1360 so 1360 N	 M1 A1 A1	 Moments about C. All forces present. No extra forces. Distances correct cao	3
(i) (*B*)	Let force be P c.w. moments about E P × 2.125 – 340 × (2 – 0.5) = 0 P = 240 so 240 N	 M1 A1 A1	 Moments about E. All forces present. No extra forces. Distances correct cao	3

(ii)	$Q\sin\theta\times 2.125+Q\cos\theta\times 0.9$ $=\frac{25.5Q}{13}+\frac{4.5Q}{13}$ $=\frac{30Q}{13}$ so $\frac{30Q}{13}$ N m	M1 B1 E1	Moments expression. Accept $s\leftrightarrow c$. Correct trig ratios **or** lengths Shown	3
(iii)	We need $\frac{30Q}{13}=340\times 1.5$ so $Q=221$ Let friction be F and normal reaction R Resolve $\rightarrow$ $221\cos\theta-F=0$ so $F=85$ Resolve $\uparrow$ $221\sin\theta+R=340$ so $R=136$ $F<\mu R$ as not on point of sliding so $85<136\mu$ so $\mu>\frac{5}{8}$	M1 E1 M1 A1 M1 A1 M1 A1 E1	Moments equn with all relevant forces Shown Accept $\le$ or = Accept $\le$. FT **their** F and R	
				9
				18
4 (i)	$4000\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=4800\begin{pmatrix}30\\40\end{pmatrix}-800\begin{pmatrix}50\\20\end{pmatrix}$ so $\bar{x}=26$ $\bar{y}=44$	M1 A1 E1 A1	Any complete method for c.m. Either one RHS term correct or one component of both RHS terms correct [SC 2 for correct $\bar{y}$ seen if M 0]	4
(ii)	$250\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$ $=110\begin{pmatrix}0\\55\end{pmatrix}+40\begin{pmatrix}20\\0\end{pmatrix}+40\begin{pmatrix}40\\20\end{pmatrix}+20\begin{pmatrix}50\\40\end{pmatrix}+40\begin{pmatrix}60\\60\end{pmatrix}$ $\bar{x}=23.2$ $\bar{y}=40.2$	M1 B1 B1 E1 A1	Any complete method for c.m. Any 2 edges correct mass and c.m. **or** any 4 edges correct with mass and x or y c.m. coordinate correct. At most one consistent error	5

(iii)		B1	Indicating c.m. vertically below Q	
		B1	Clearly identifying correct angle (may be implied) and lengths	
	Angle is $\arctan\left(\frac{23.2}{110-40.2}\right)$	M1	Award for $\arctan\left(\frac{b}{a}\right)$ where b = 23.2 and a = 69.8 or 40.2 or where b = 69.8 or 40.2 and a = 23.2. Allow use of **their** value for y only.	
	= 18.3856…. so 18.4° (3 s. f.)	A1	cao	4
(iv)	$10\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=2\times1.5\times\begin{pmatrix}26\\44\end{pmatrix}+7\begin{pmatrix}23.2\\40.2\end{pmatrix}$	M1	Combining the parts using masses	
		B1	Using both ends	
		A1	All correct	
	$\bar{x}=24.04$ so 24.0 (3 s.f.)	A1	cao	
	$\bar{y}=41.34$ so 41.3 (3 s.f.)	F1	FT **their** y values only.	5
				18

4763 Mechanics 3

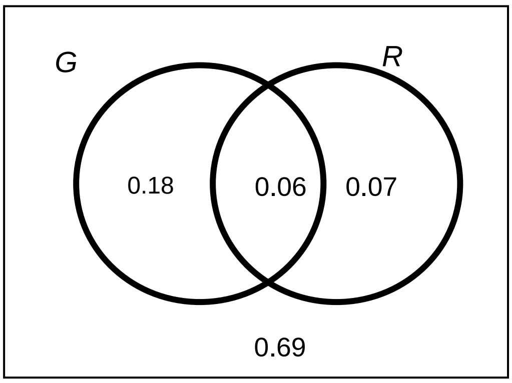
1(a) (i)	[Density] $= \mathrm{M\,L^{-3}}$ [Kinetic Energy] $= \mathrm{M\,L^2\,T^{-2}}$ [Power] $= \mathrm{M\,L^2\,T^{-3}}$	B1 B1 B1	*(Deduct B1 for* $\mathrm{kg\,m^{-3}}$ *etc)*	3
(ii)	$\mathrm{M\,L^2\,T^{-3}} = [\eta]\mathrm{L}(\mathrm{L\,T^{-1}})^2$ $[\eta] = \mathrm{M\,L^{-1}\,T^{-1}}$	B1 M1 A1	For $[v] = \mathrm{L\,T^{-1}}$ *Can be earned in (iii)* Obtaining the dimensions of η	3
(iii)	$\mathrm{M\,L^2\,T^{-3}} = (\mathrm{M\,L^{-3}})^{\alpha}\,\mathrm{L}^{\beta}\,(\mathrm{L\,T^{-1}})^{\gamma}$ $\alpha = 1$ $-3 = -\gamma$ $\gamma = 3$ $2 = -3\alpha + \beta + \gamma$ $\beta = 2$	B1 cao M1 A1 M1 A1 A1	Considering powers of T *(No ft if* $\gamma = 0$ *)* Considering powers of L Correct equation *(ft requires 4 terms)* *(No ft if* $\beta = 0$ *)*	6
(b)	EE at start is $\frac{1}{2}k \times 0.8^2$ EE at end is $\frac{1}{2}k \times 0.3^2$ $\frac{1}{2}k \times 0.8^2 = \frac{1}{2}k \times 0.3^2 + 5.5 \times 9.8 \times 3.5$ Stiffness is $686\ \mathrm{N\,m^{-1}}$	M1 A1 A1 M1 F1 A1	Calculating elastic energy k may be $\frac{\lambda}{l}$ or $\frac{\lambda}{1.2}$ Equation involving EE and PE *(must have three terms)* (A0 for $\lambda = 823.2$)	6
				[18]

2 (a)	$\int \pi x y^2 \, dx = \int_0^a \pi x(a^2 - x^2)\, dx$	M1	*Limits not required*	
	$= \pi\left[\frac{1}{2}a^2x^2 - \frac{1}{4}x^4 \right]_0^a$	A1	For $\frac{1}{2}a^2x^2 - \frac{1}{4}x^4$	
	$= \frac{1}{4}\pi a^4$	A1		
	$\bar{x} = \dfrac{\frac{1}{4}\pi a^4}{\frac{2}{3}\pi a^3}$	M1		
	$= \frac{3}{8}a$	E1		5
(b) (i)	Area is $\int_1^4 (2 - \sqrt{x})\, dx$	M1	*Limits not required*	
	$= \left[2x - \frac{2}{3}x^{3/2} \right]_1^4 \quad (= \frac{4}{3})$	A1	For $2x - \frac{2}{3}x^{\frac{3}{2}}$	
	$\int x y \, dx = \int_1^4 x(2 - \sqrt{x})\, dx$	M1	*Limits not required*	
	$= \left[x^2 - \frac{2}{5}x^{\frac{5}{2}} \right]_1^4 \quad (= \frac{13}{5})$	A1	For $x^2 - \frac{2}{5}x^{\frac{5}{2}}$	
	$\bar{x} = \dfrac{13/5}{4/3} = \dfrac{39}{20} = 1.95$	A1		
	$\int \frac{1}{2}y^2 \, dx = \int_1^4 \frac{1}{2}(2 - \sqrt{x})^2 \, dx$	M1	$\int (2 - \sqrt{x})^2 \, dx$ *or* $\int \left((2-y)^2 - 1\right) y \, dy$	
	$= \left[2x - \frac{4}{3}x^{3/2} + \frac{1}{4}x^2 \right]_1^4 \quad (= \frac{5}{12})$	A2	For $2x - \frac{4}{3}x^{\frac{3}{2}} + \frac{1}{4}x^2$ *or* $\frac{3}{2}y^2 - \frac{4}{3}y^3 + \frac{1}{4}y^4$ Give A1 for two terms correct, or all correct with ½ omitted	
	$\bar{y} = \dfrac{5/12}{4/3} = \dfrac{5}{16} = 0.3125$	A1		9
(ii)	Taking moments about A $T_C \times 3 - W \times 0.95 = 0$	M1 A1	Moments equation (no force omitted) Any correct moments equation (May involve both T_A and T_C) Accept Wg or $W = \frac{4}{3}, \frac{4}{3}g$ here	
	$T_A + T_C = W$	M1	Resolving vertically (or a second moments equation)	
	$T_A = \dfrac{41}{60}W, \quad T_C = \dfrac{19}{60}W$	A1	*Accept* $0.68W, \quad 0.32W$	4
				[18]

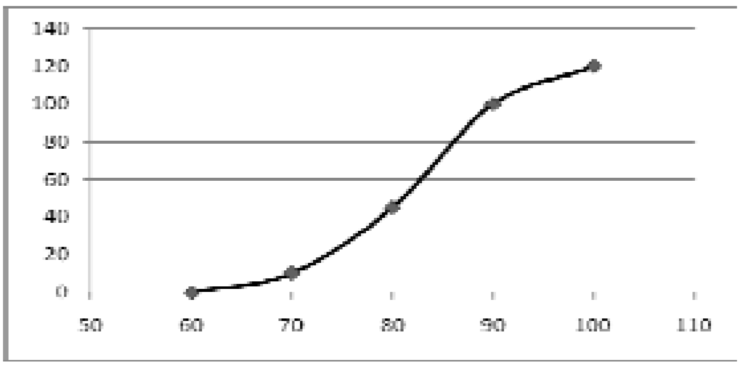
3 (i)	By conservation of energy, $\frac{1}{2}\times 0.6\times 6^2 - \frac{1}{2}\times 0.6v^2 = 0.6\times 9.8(1.25-1.25\cos\theta)$ $36 - v^2 = 24.5 - 24.5\cos\theta$ $v^2 = 11.5 + 24.5\cos\theta$	M1 A1 E1	Equation involving KE and PE	3
(ii)	$T - 0.6\times 9.8\cos\theta = 0.6\times\frac{v^2}{1.25}$ $T - 5.88\cos\theta = 0.48(11.5 + 24.5\cos\theta)$ $T = 5.52 + 17.64\cos\theta$	M1 A1 M1 A1	For acceleration $\frac{v^2}{r}$ Substituting for v^2	4
(iii)	String becomes slack when $T = 0$ $\cos\theta = -\frac{5.52}{17.64}$ ($\theta = 108.2°$ or 1.889 rad) $v^2 = 11.5 - 24.5\times\frac{5.52}{17.64}$ Speed is $1.96\ \mathrm{m\,s^{-1}}$ (3 sf)	M1 A1 M1 A1 cao	 *May be implied* or $0.6\times 9.8\times\frac{5.52}{17.64} = 0.6\times\frac{v^2}{1.25}$ or $-0.6\times 9.8\times\frac{v^2-11.5}{24.5} = 0.6\times\frac{v^2}{1.25}$	4
(iv)	$T_1\cos\theta = mg$ $T_1\times\frac{1.2}{1.25} = 0.6\times 9.8$ (where θ is angle COP) Tension in OP is 6.125 N $T_1\sin\theta + T_2 = \frac{mv^2}{0.35}$ $6.125\times\frac{0.35}{1.25} + T_2 = \frac{0.6\times 1.4^2}{0.35}$ Tension in CP is 1.645 N	M1 A1 A1 M1 F1B1 A1	Resolving vertically Horizontal equation (three terms) For LHS and RHS	7
				[18]

4(i)	$T_{\mathrm{AP}} = \frac{7.35}{1.5} \times 0.05 \quad (= 0.245)$ $T_{\mathrm{BP}} = \frac{7.35}{2.5} \times 0.5 \quad (= 1.47)$ Resultant force up the plane is $T_{\mathrm{BP}} - T_{\mathrm{AP}} - mg\sin 30°$ $= 1.47 - 0.245 - 0.25 \times 9.8 \sin 30°$ $= 1.47 - 0.245 - 1.225$ $= 0$ Hence there is no acceleration	M1 A1 A1 M1 E1	Using Hooke's law or $\frac{7.35}{1.5}(\mathrm{AP} - 1.5)$ or $\frac{7.35}{2.5}(2.05 - \mathrm{AP})$ Correctly shown	5
(ii)	$T_{\mathrm{AP}} = \frac{7.35}{1.5}(0.05 + x) \quad (= 0.245 + 4.9x)$ $T_{\mathrm{BP}} = \frac{7.35}{2.5}(4.55 - 1.55 - x - 2.5)$ $= 2.94(0.5 - x)$ $= 1.47 - 2.94x$	B1 M1 E1		3
(iii)	$T_{\mathrm{BP}} - T_{\mathrm{AP}} - mg\sin 30° = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $(1.47 - 2.94x) - (0.245 + 4.9x) - 1.225 = 0.25\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -31.36x$ Hence the motion is simple harmonic Period is $\frac{2\pi}{\sqrt{31.36}} = \frac{2\pi}{5.6}$ Period is 1.12 s (3 sf)	M1 A2 E1 B1 cao	Equation of motion parallel to plane Give A1 for an equation which is correct apart from sign errors Must state conclusion. Working must be fully correct (cao) *If a is used for accn down plane, then $a = 31.36x$ can earn M1A2; but E1 requires comment about directions* *Accept* $\frac{5\pi}{14}$	5
(iv)	$x = -0.05\cos 5.6t$ $v = 0.28\sin 5.6t$ $-0.2 = 0.28\sin 5.6t$ OR $0.2^2 = 31.36(0.05^2 - x^2)$ $x = (\pm)\ 0.035$ $0.035 = -0.05\cos 5.6t$ M1 $5.6t = \pi + 0.7956$ Time is 0.703 s (3 sf)	M1 A1 M1 M1 A1cao	For $A\sin\omega t$ or $A\cos\omega t$ Allow $\pm 0.05\sin/\cos 5.6t$ *Implied by* $v = \pm 0.28\sin/\cos 5.6t$ Using $v = \pm 0.2$ to obtain an equation for t Fully correct strategy for finding the required time	5
				[18]

4766 Statistics 1

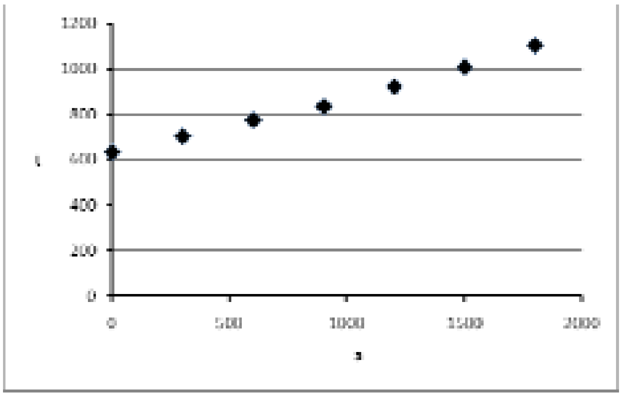
1	(i)	5 | 2 6 | 3 4 7 8 7 | 1 2 2 3 4 5 5 7 9 8 | 1 Key 6 | 3 represents 63 mph	G1 stem G1 leaves CAO G1 sorted G1 key	**[4]**
	(ii)	Median = 72 Midrange = 66.5	B1 FT B1 CAO	**[2]**
	(iii)	*EITHER:* Median since midrange is affected by outlier (52) *OR:* Median since the lack of symmetry renders the midrange less representative	E1 for median E1 for explanation **TOTAL**	**[2]** **[8]**
2	(i)	(A) $P(X = 10) = P(5 \text{ then } 5) = 0.4\times0.25 = 0.1$ (B) $P(X = 30) = P(10 \text{ and } 20) = 0.4\times0.25 + 0.2\times0.5 = 0.2$	B1 **ANSWER GIVEN** M1 for full calculation A1 **ANSWER GIVEN**	**[1]** **[2]**
	(ii)	$E(X) = 10\times0.1+15\times0.4+20\times0.1+25\times0.2+30\times0.2=20$ $E(X^2) =$ $100\times0.1+225\times0.4+400\times0.1+625\times0.2+900\times0.2=445$ $Var(X) = 445 - 20^2 = 45$	M1 for Σrp (at least 3 terms correct) A1 CAO M1 for Σr^2p (at least 3 terms correct) M1 dep for – their $E(X)^2$ A1 FT their E(X) provided Var(X) > 0 **TOTAL**	**[5]** **[8]**
3	(i)		G1 for two labelled intersecting circles G1 for at least 2 correct probabilities G1 for remaining probabilities	**[3]**
	(ii)	$P(G) \times P(R) = 0.24 \times 0.13 = 0.0312 \neq P(G\cap R)$ or $\neq 0.06$ So not independent.	M1 for 0.24×0.13 A1	**[2]**

	(iii)	$P(R\mid G)=\dfrac{P(R\cap G)}{P(G)}=\dfrac{0.06}{0.24}=\dfrac{1}{4}=0.25$	M1 for numerator M1 for denominator A1 CAO	[3]
			TOTAL	**[8]**
4	**(i)**	P(20 correct) = $\binom{30}{20}\times 0.6^{20}\times 0.4^{10}=0.1152$	M1 $0.6^{20}\times 0.4^{10}$ M1 $\binom{30}{20}\times p^{20}q^{10}$ A1 CAO	[3]
	(ii)	Expected number = 100 × 0.1152 = 11.52	M1 A1 FT (Must not round to whole number)	[2]
			TOTAL	**[5]**
5	**(i)**	P(Guess correctly) = 0.1^4 = 0.0001	B1 CAO	[1]
	(ii)	P(Guess correctly) = $\dfrac{1}{4!}=\dfrac{1}{24}$	M1 A1 CAO	[2]
			TOTAL	**[3]**
6	**(i)**	20 × 19 × 18 = 6840	M1 A1	[2]
	(ii)	$20^3 - 20 = 7980$	M1 for figures – 20 A1	[2]
			TOTAL	**[4]**

7	**(i)**	$10 \times 2 = 20$.	M1 for 10×2 A1 CAO	**[2]**
	(ii)	Mean = $\dfrac{10\times65+35\times75+55\times85+20\times95}{120} = \dfrac{9850}{120} = 82.08$ It is an estimate because the data are grouped.	M1 for midpoints M1 for double pairs A1 CAO E1 indep	**[4]**
	(iii)	$10 \times 65^2 + 35 \times 75^2 + 55 \times 85^2 + 20 \times 95^2$ (= 817000) $S_{xx} = 817000 - \dfrac{9850^2}{120}$ (= 8479.17) $s = \sqrt{\dfrac{8479.17}{119}} = 8.44$	M1 for Σfx^2 M1 for valid attempt at S_{xx} A1 CAO	**[3]**
	(iv)	$\bar{x} - 2s = 82.08 - 2\times8.44 = 65.2$ $\bar{x} + 2s = 82.08 + 2\times8.44 = 98.96$ So there are probably some outliers.	M1 FT for $\bar{x} - 2s$ M1 FT for $\bar{x} + 2s$ A1 for both E1 dep on A1	**[4]**
	(v)	Negative.	E1	**[1]**
	(vi)	Upper bound: 60, 70, 80, 90, 100 Cumulative frequency: 0, 10, 45, 100, 120 	C1 for cumulative frequencies S1 for scales L1 for labels 'Length and CF' P1 for points J1 for joining points dep on P1 All dep on attempt at cumulative frequency.	**[5]**
			TOTAL	**[19]**

8	**(i)**	(*A*) P(Low on all 3 days) = 0.5^3 = 0.125 or $^1/_8$	M1 for 0.5^3 A1 CAO	**[2]**
		(*B*) P(Low on at least 1 day) = 1 – 0.5^3 = 1 – 0.125 = 0.875	M1 for 1 – 0.5^3 A1 CAO	**[2]**
		(*C*) P(One low, one medium, one high) = 6 × 0.5 × 0.35 × 0.15 = 0.1575	M1 for product of probabilities 0.5 × 0.35 × 0.15 or $^{21}/_{800}$ M1 × 6 or × 3! or $^3P_3$ A1 CAO	**[3]**
	(ii)	X ~ B(10, 0.15) (*A*) P(No days) = 0.85^{10} = 0.1969 Or from tables P(No days) = 0.1969	M1 A1	**[2]**
		(*B*) *Either* P(1 day) $= \binom{10}{1} \times 0.15^1 \times 0.85^9 = 0.3474$ *or* from tables P(1 day) = $P(X \le 1) - P(X \le 0)$ = 0.5443 – 0.1969 = 0.3474	M1 $0.15^1 \times 0.85^9$ M1 $\binom{10}{1} \times p^1 q^9$ A1 CAO OR: M2 for 0.5443 – 0.1969 A1 CAO	**[3]**
	(iii)	Let X ~ B(20, 0.5) *Either:* $P(X \ge 15)$ = 1 – 0.9793 = 0.0207 < 5% *Or:* Critical region is {15,16,17,18,19,20} 15 lies in the critical region. So there is sufficient evidence to reject H_0 Conclude that there is enough evidence to indicate that the probability of low pollution levels is higher on the new street. H_1 has this form as she believes that the probability of a low pollution level is greater in this street.	*Either:* B1 for correct probability of 0.0207 M1 for comparison *Or:* B1 for CR, M1 for comparison A1 CAO dep on B1M1 E1 for conclusion in context E1 indep	**[5]**
			TOTAL	**[17]**

4767 Statistics 2

1	(i)		G1 For values of a G1 for values of t G1 for axes	[3]
	(ii)	a is independent, t is dependent since the values of a are not subject to random variation, but are determined by the runways which the pilot chooses, whereas the values of t are subject to random variation.	B1 E1dep E1dep	[3]
	(iii)	$\bar{a} = 900$, $\bar{t} = 855.2$ $b = \frac{S_{at}}{S_{aa}} = \frac{6037800 - 5987 \times 6300/7}{8190000 - 6300^2/7} = \frac{649500}{2520000} = 0.258$ OR $b = \frac{6037800/7 - 855.29 \times 900}{8190000/7 - 900^2} = \frac{92785}{360000} = 0.258$ hence least squares regression line is: $t - \bar{t} = b(a - \bar{a})$ $\Rightarrow t - 855.29 = 0.258\,(a - 900)$ $\Rightarrow t = 0.258a + 623$	B1 for $\bar{a}$ and $\bar{t}$ used (SOI) M1 for attempt at gradient (b) A1 for 0.258 **cao** M1 for equation of line A1 FT for complete equation	[5]
	(iv)	(A) For $a = 800$, predicted take–off distance $= 0.258 \times 800 + 623 = 829$ (B) For $a = 2500$, predicted take–off distance $= 0.258 \times 2500 + 623 = 1268$ Valid relevant comments relating to the predictions such as: First prediction is interpolation so should be reasonable Second prediction is extrapolation and may not be reliable	M1 for at least one prediction attempted A1 for both answers (FT their equation if b>0) E1 (first comment) E1 (second comment)	[4]
	(v)	$a = 1200 \Rightarrow$ predicted $t = 0.258 \times 1200 + 623 = 933$ Residual = 923 – 933 = –10 The residual is negative because the observed value is less than the predicted value.	M1 for prediction M1 for subtraction A1 FT E1	[4]
			Total	**[19]**

2	**(i)**	P(1 of 10 is faulty) $= \binom{10}{1} \times 0.02^1 \times 0.98^9 = 0.1667$	M1 for coefficient M1 for probabilities A1	**[3]**
	(ii)	n is large and p is small	B1, B1 Allow appropriate numerical ranges	**[2]**
	(iii)	$\lambda = 150 \times 0.02 = 3$ (A) $\quad$ $\mathrm{P}(X = 0) = \mathrm{e}^{-3}\dfrac{3^0}{0!} = 0.0498$ (3 s.f.) or from tables $= 0.0498$	B1 for mean (soi) M1 for calculation or use of tables A1	**[3]**
		(B) $\quad$ Expected number = 3 Using tables: $\mathrm{P}(X > 3) = 1 - \mathrm{P}(X \leq 3)$ $= 1 - 0.6472 = 0.3528$	B1 expected no = 3 (soi) M1 A1	**[3]**
	(iv)	(A) $\quad$ Binomial(2000,0.02)	B1 for binomial B1 for parameters	**[2]**
		(B) $\quad$ Use Normal approx with $\mu = np = 2000 \times 0.02 = 40$ $\sigma^2 = npq = 2000 \times 0.02 \times 0.98 = 39.2$ $\mathrm{P}(X \leq 50) = \mathrm{P}\left(Z \leq \dfrac{50.5 - 40}{\sqrt{39.2}}\right)$ $= \mathrm{P}(Z \leq 1.677) = \Phi(1.677) = 0.9532$ NB Poisson approximation also acceptable for full marks	B1 B1 B1 for continuity corr. M1 for probability using correct tail A1 CAO	**[5]**
			Total	**[18]**

3	**(i)**	(A) $P(X < 50)$ $= P\left(Z < \frac{50-45.3}{11.5}\right)$ $= P(Z < 0.4087)$ $= \Phi(0.4087)$ $= 0.6585$	M1 for standardising M1 for correct structure of probability calc' A1 CAO inc use of diff tables NB When a candidate's answers suggest that (s)he appears to have neglected to use the difference column of the Normal distribution tables penalise the first occurrence only	**[3]**
		(B) $P(45.3 < X < 50)$ $= 0.6585 - 0.5$ $=0.1585$	M1 A1	**[2]**
	(ii)	From tables $\Phi^{-1}(0.9) = 1.282$ $\frac{k-45.3}{11.5} = 1.282$ $k = 45.3 + 1.282 \times 11.5 = 60.0$	B1 for 1.282 seen M1 for equation in k A1 CAO	**[3]**
	(iii)	P(score = 111) $=P(110.5 < Y < 111.5)$ $= P\left(\frac{110.5-100}{15} < Z < \frac{111.5-100}{15}\right)$ $= P(0.7 < Z < 0.7667)$ $= \Phi(0.7667) - \Phi(0.7)$ $= 0.7784 - 0.7580$ $= 0.0204$	B1 for both continuity corrections M1 for standardising M1 for correct structure of probability calc' A1 CAO	**[4]**
	(iv)	From tables, $\Phi^{-1}(0.3) = -0.5244$, $\Phi^{-1}(0.8) = 0.8416$ $22 = \mu + 0.8416\,\sigma$ $15 = \mu - 0.5244\,\sigma$ $7 = 1.3660\,\sigma$ $\sigma = 5.124$, $\mu = 17.69$	B1 for 0.5244 or 0.8416 seen M1 for at least one equation in z, μ & σ A1 for both correct M1 for attempt to solve two appropriate equations A1 CAO for both	**[5]**
			TOTAL	**[17]**

4	**(i)**	H_0: no association between size of business and recycling service used. H_1: some association between size of business and recycling service used.	B1 for both	**[1]**
	(ii)	Expected frequency = 78/285 × 180 = 49.2632 Contribution = $(52 – 49.2632)^2$ / 49.2632 = 0.1520	M1 A1 M1 for valid attempt at $(O-E)^2/E$ A1 ***NB Answer given*** Allow 0.152	**[4]**
	(iii)	Test statistic X^2 = 0.6041 Refer to χ_2^2 Critical value at 5% level = 5.991 Result is not significant There is no evidence to suggest any association between size of business and recycling service used. NB if H_0 H_1 reversed, or 'correlation' mentioned in part (i), do not award B1in part (i) or E1 in part (iii).	B1 B1 for 2 deg of f(seen) B1 CAO for cv B1 for not significant E1	**[5]**
	(iv)	H_0: $\mu = 32.8$; H_1: $\mu < 32.8$ Where μ denotes the population mean weight of rubbish in the bins. Test statistic = $\dfrac{30.9-32.8}{3.4/\sqrt{50}} = -\dfrac{1.9}{0.4808} = -3.951$ 5% level 1 tailed critical value of z = –1.645 –3.951 < –1.645 so significant. There is sufficient evidence to reject H_0 There is evidence to suggest that the weight of rubbish in dustbins has been reduced.	B1 for use of 32.8 B1 for both correct B1 for definition of μ M1 must include √50 A1 B1 for ±1.645 M1 for sensible comparison leading to a conclusion A1 for conclusion in words in context	**[8]**
			TOTAL	**[18]**

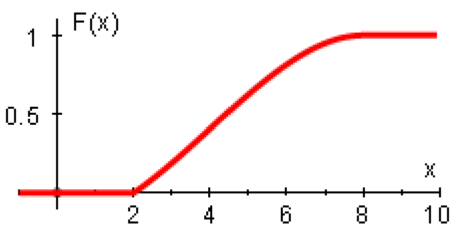
4768 Statistics 3

1 (i)	H_0: The number of eggs hatched can be modelled by B(3, ½) H_1: The number of eggs hatched cannot be modelled by B(3, ½)	B1 B1		

With p = ½

Probability	0.125	0.375	0.375	0.125
Exp'd frequency	10	30	30	10
Obs'd frequency	7	23	29	21

		M1 A1	Probs × 80 for expected frequencies. All correct.	
	X^2 = 0.9 + 1.6333 + 0.0333 + 12.1 = 14.666(7)	M1 A1	Calculation of X^2. c.a.o.	
	Refer to χ_3^2.	M1	Allow correct df (= cells – 1) from wrongly grouped table and ft. Otherwise, no ft if wrong. P(X^2 > 14.667) = 0.00212.	
	Upper 5% point is 7.815. Significant. Suggests it is reasonable to suppose model with p = ½ does not apply.	A1 A1 A1	No ft from here if wrong. ft only c's test statistic. ft only c's test statistic.	**[10]**
(ii)	$\bar{x} = \frac{144}{80} = 1.8$ $\therefore \hat{p} = \frac{1.8}{3} = 0.6$	B1 B1	C.a.o. Use of E(X) = np. ft c's mean, provided $0 < \hat{p} < 1$.	**[2]**
(iii)	Refer to χ_2^2.	M1	Allow df 1 less than in part (i). No ft if wrong.	
	Upper 5% point is 5.991.	A1	No ft if wrong.	
	Suggests it is reasonable to suppose model with estimated p does apply.	A1	ft provided previous A mark awarded.	**[3]**
(iv)	For example: Estimating p leads to an improved fit … … at the expense of the loss of 1 degree of freedom. The model in (i) fails due to a large underestimate for X = 3.	E2	Reward any two sensible points for E1 each.	**[2]**
			Total	**[17]**

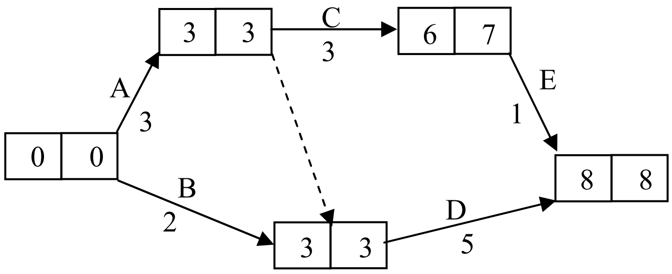
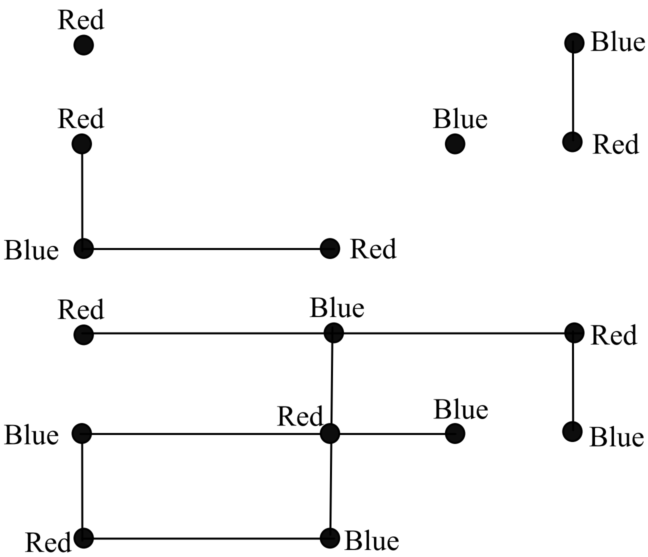
2 (a)	$f(x)=\frac{1}{72}\left(8x-x^2\right)\ ,\ 2\le x\le 8$			
(i)	$F(x)=\int_2^x \frac{1}{72}\left(8t-t^2\right)dt$	M1	Correct integral with limits (which may be implied subsequently).	
	$=\frac{1}{72}\left[4t^2-\frac{t^3}{3}\right]_2^x$	A1	Correctly integrated	
	$=\frac{1}{72}\left(4x^2-\frac{x^3}{3}-16+\frac{8}{3}\right)=\frac{12x^2-x^3-40}{216}$	A1	Limits used. Accept unsimplified form.	**[3]**
(ii)		G1 G1 G1	Correct shape; nothing below $y=0$; non-negative gradient. Labels at (2, 0) and (8, 1). Curve (horizontal lines) shown for $x<2$ and $x>8$.	**[3]**
(iii)	$F(m)=½\quad \therefore \frac{12m^2-m^3-40}{216}=\frac{1}{2}$	M1	Use of definition of median. Allow use of c's F(x).	
	$\therefore 12m^2-m^3-40=108$ $\therefore m^3-12m^2+148=0$	A1	Convincingly rearranged. Beware: answer given.	
	Either $F(4.42)=0.5003(977)\approx 0.5$ Or $4.42^3-12\times 4.42^2+148=-0.0859(12)\approx 0$ $\therefore m\approx 4.42$	E1	Convincingly shown, e.g. 4.418 or better seen.	**[3]**

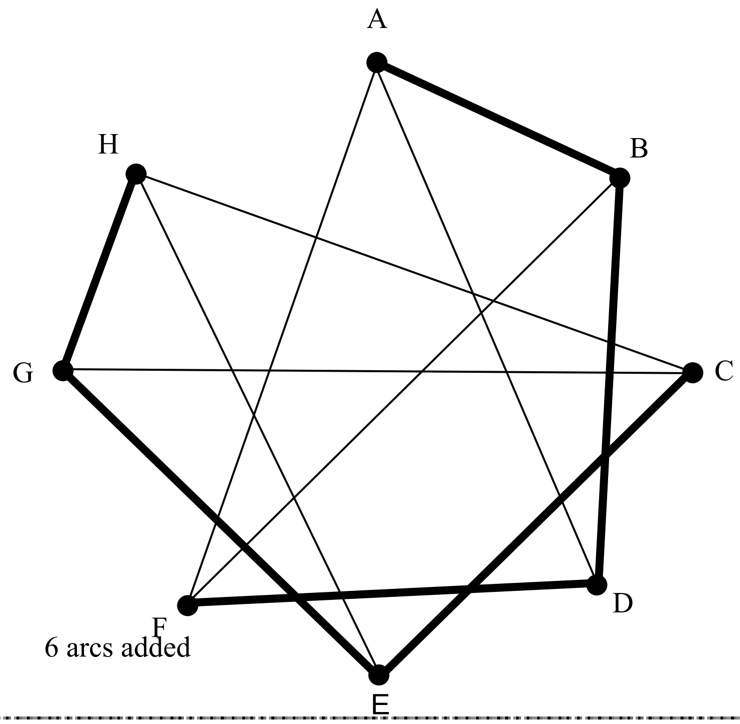
<table>
<tr><td>2 (b)</td><td>H_0: $m = 4.42$ H_1: $m \neq 4.42$
where m is the population median</td><td>B1
B1</td><td>Both. Accept hypotheses in words.
Adequate definition of m to include “population”.</td><td></td></tr>
<tr><td></td><td>
<table>
<tr><th>Weights</th><th>− 4.42</th><th>Rank of |diff|</th></tr>
<tr><td>3.16</td><td>−1.26</td><td>7</td></tr>
<tr><td>3.62</td><td>−0.80</td><td>6</td></tr>
<tr><td>3.80</td><td>−0.62</td><td>4</td></tr>
<tr><td>3.90</td><td>−0.52</td><td>3</td></tr>
<tr><td>4.02</td><td>−0.40</td><td>2</td></tr>
<tr><td>4.72</td><td>0.30</td><td>1</td></tr>
<tr><td>5.14</td><td>0.72</td><td>5</td></tr>
<tr><td>6.36</td><td>1.94</td><td>8</td></tr>
<tr><td>6.50</td><td>2.08</td><td>9</td></tr>
<tr><td>6.58</td><td>2.16</td><td>10</td></tr>
<tr><td>6.68</td><td>2.26</td><td>11</td></tr>
<tr><td>6.78</td><td>2.36</td><td>12</td></tr>
</table>
</td><td>M1
M1
A1</td><td>for subtracting 4.42.
for ranks.
ft if ranks wrong.</td><td></td></tr>
<tr><td></td><td>$W_- = 2 + 3 + 4 + 6 + 7 = 22$</td><td>B1</td><td>($W_+ = 1 + 5 + 8 + 9 + 10 + 11 + 12 = 56$)</td><td></td></tr>
<tr><td></td><td>Refer to Wilcoxon single sample tables for $n = 12$.</td><td>M1</td><td>No ft from here if wrong.</td><td></td></tr>
<tr><td></td><td>Lower 2½% point is 13 (or upper is 65 if 56 used).</td><td>A1</td><td>i.e. a 2-tail test. No ft from here if wrong.</td><td></td></tr>
<tr><td></td><td>Result is not significant.</td><td>A1</td><td>ft only c’s test statistic.</td><td></td></tr>
<tr><td></td><td>Evidence suggests that a median of 4.42 is consistent with these data.</td><td>A1</td><td>ft only c’s test statistic.</td><td>[10]</td></tr>
<tr><td></td><td></td><td></td><td>Total</td><td>[19]</td></tr>
</table>

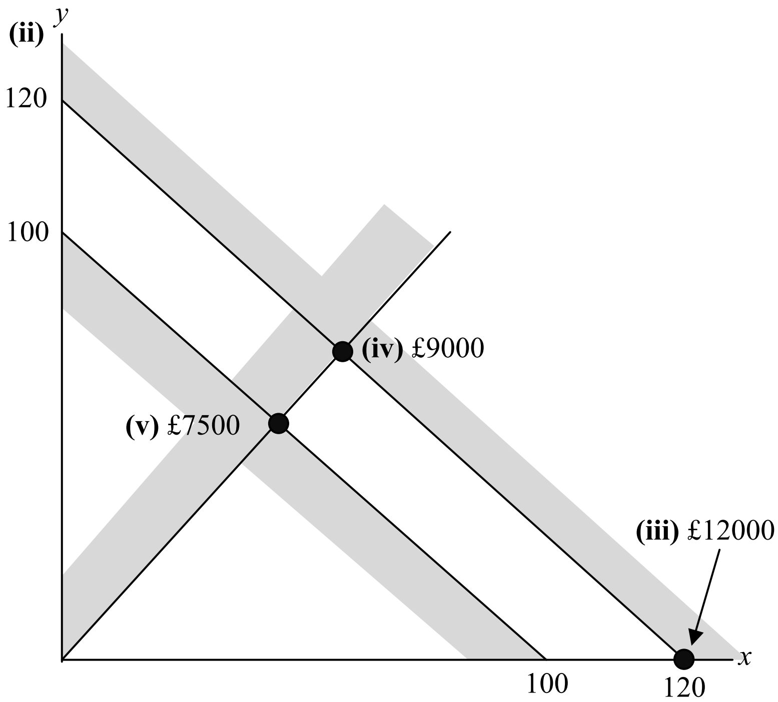
3 (i)	Must assume • Normality of population … • … of <u>differences</u>.	 B1 B1		
	H_0: $\mu_D = 0$ H_1: $\mu_D > 0$	B1	Both. Accept alternatives e.g. $\mu_D <$ 0 for H_1, or $\mu_B - \mu_A$ etc provided adequately defined. Hypotheses in words only must include "population". Do NOT allow "$\overline{X} = ...$" or similar unless $\overline{X}$ is clearly and explicitly stated to be a <u>population</u> mean.	
	Where μ_D is the (population) mean reduction/difference in cholesterol level.	B1	For adequate verbal definition. Allow absence of "population" if correct notation μ is used.	
	<u>MUST</u> be PAIRED COMPARISON t test. Differences (reductions) (before – after) are:		Allow "after – before" if consistent with alternatives above.	
	–0.1 1.7 –1.2 1.1 1.4 0.5 0.9 2.2 –0.1 2.0 0.7 0.3			
	$\bar{x} = 0.7833$ $s_{n-1} = 0.9833(46)$ $(s_{n-1}^2 = 0.966969)$	B1	Do not allow $s_n = 0.9415$ $(s_n^2 = 0.8864)$	
	Test statistic is $\dfrac{0.7833 - 0}{\frac{0.9833}{\sqrt{12}}}$	M1	Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: 0 + (c's 2.718) × $\frac{0.9833}{\sqrt{12}}$ (= 0.7715) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ – (c's 2.718) × $\frac{0.9833}{\sqrt{12}}$ (= 0.0118) for comparison with 0.)	
	= 2.7595.	A1	c.a.o. but ft from here in any case if wrong. Use of $0 - \bar{x}$ scores M1A0, but ft.	
	Refer to t_{11}.	M1	No ft from here if wrong. P(t > 2.7595) = 0.009286.	
	Single-tailed 1% point is 2.718.	A1	No ft from here if wrong.	
	Significant.	A1	ft only c's test statistic.	
	Seems mean cholesterol level has fallen.	A1	ft only c's test statistic.	**[11]**
(ii)	CI is $\bar{x} \pm$	M1	Overall structure, seen or implied.	
	2.201	B1	From t_{11}, seen or implied.	
	$\times \dfrac{s}{\sqrt{12}} = (-0.5380, 1.4046)$	A1	Fully correct pair of equations using the given interval, seen or implied.	
	$\bar{x} = ½(1.4046 - 0.5380) = 0.4333$	B1		
	$s = (1.4046 - 0.4333) \times \dfrac{\sqrt{12}}{2.201}$ = 1.5287	M1 A1	Substitute $\bar{x}$ and rearrange to find s. c.a.o.	
	Using this interval the doctor might conclude that the mean cholesterol level did not seem to have been reduced.	E1	Accept any sensible comment or interpretation of <u>this</u> interval.	**[7]**
			Total	**[18]**

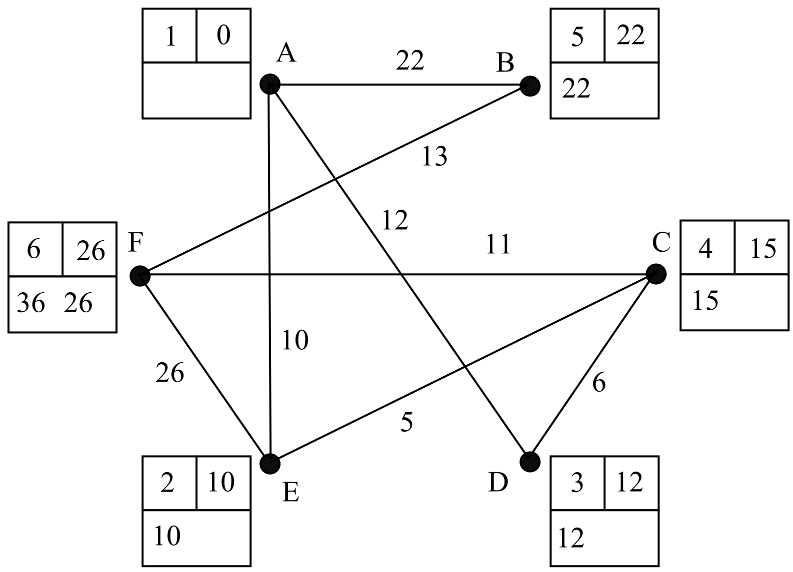
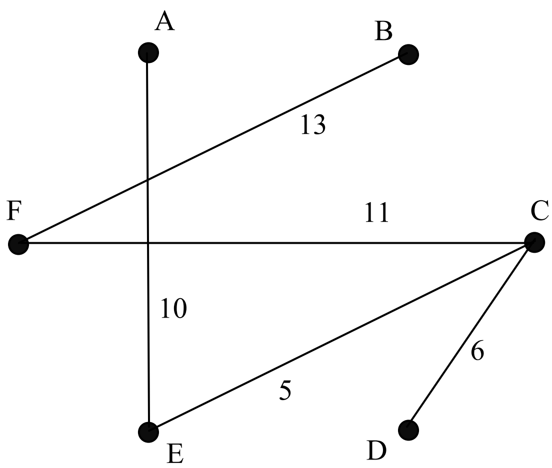
4	$A \sim \mathrm{N}(80,\ \sigma = 11)$ $B \sim \mathrm{N}(70,\ \sigma = v)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.	
(i)	$\mathrm{P}(A < 90) = \mathrm{P}\left(Z < \dfrac{90-80}{11} = 0.9091\right)$ $= 0.8182$	M1 A1 A1	For standardising. Award once, here or elsewhere. c.a.o.	**[3]**
(ii)	$W_B = B_1 + B_2 + \ldots + B_6 + 15 \sim \mathrm{N}(435,$ $\sigma^2 = v^2 + v^2 + \ldots + v^2 = 6v^2)$ $\mathrm{P}(\text{this} < 450) = \mathrm{P}\left(Z < \dfrac{450-435}{v\sqrt{6}}\right) = 0.8463$ $\therefore \dfrac{450-435}{v\sqrt{6}} = \Phi^{-1}(0.8463) = 1.021$ $\therefore v = \dfrac{15}{1.021 \times \sqrt{6}} = 5.9977 = 6$ grams (nearest gram)	B1 B1 M1 B1 A1	Mean. Expression for variance. Formulation of the problem. Inverse Normal. Convincingly shown, beware A.G.	**[5]**
(iii)	$W_A = A_1 + A_2 + \ldots + A_5 + 25 \sim \mathrm{N}(425,$ $\sigma^2 = 11^2 + 11^2 + \ldots + 11^2 = 605\ )$ $D = W_A - W_B \sim \mathrm{N}(-10,$ $605 + 216 = 821)$ Want $\mathrm{P}(W_A > W_B) = \mathrm{P}(W_A - W_B > 0)$ $= \mathrm{P}\left(Z > \dfrac{0-(-10)}{\sqrt{821}} = 0.3490\right) = 1 - 0.6365 = 0.3635$	B1 M1 A1 M1 A1	Mean. Accept "$B - A$". Variance. Accept sd (= 28.65). c.a.o.	**[5]**
(iv)	$\bar{x} = \dfrac{3126.0}{60} = 52.1$, $s = \sqrt{\dfrac{164223.96 - 60 \times 52.1^2}{59}} = 4.8$ CI is given by $52.1 \pm$ 1.96 $\times \dfrac{4.8}{\sqrt{60}}$ $= 52.1 \pm 1.2146 = (50.885(4), 53.314(6))$	B1 M1 B1 M1 A1	Both correct. c.a.o. Must be expressed as an interval.	**[5]**
			Total	**[18]**

4771 Decision Mathematics 1

1	**(i) & (ii)**	 Critical activities: A and D	M1 A1 A1 M1 A1 M1 A1 B1	activity-on-arc C and E OK D OK forward pass backward pass
2	**(i)**	 Subgraph Swap colours on connected vertices and complete	M1 A1 M1 A1 A1 A1 B1	subgraph Changing colours top right bottom left not singletons
	(ii)	The rule does not specify a well-defined and terminating set of actions.	B1	

3	**(i)**	No repeated arcs. No loops	B1 B1
	(ii)	Two disconnected sets, {A,B,D,F} and {C,E,G,H}	M1 A1
	(iii)	 6 arcs added	M1 A1 B1
	(iv)	$4\times4 = 16$ or $\binom{8}{2} - 12 = 28 - 12 = 16$	B1

4	**(i)**	e.g. Let x be the number of adult seats sold. Let y be the number of child seats sold. $x + y \le 120$ $x + y \ge 100$ $x \ge y$	 M1 A1 B1 B1 B1
	(ii)		B3 lines (scale must be clear) B1 shading (axes must be clear) B1 point + amount M1 point A1 amount M1 point A1 amount
	(vi)	6000+60c > 10000 => c $\ge$ 67	M1 A1

5	**(i)** **&** **(ii)**	 shortest route: A E C F distance: 26 miles	M1 network A1 arcs A1 lengths M1 Dijkstra A1 working values B1 order of labelling B1 labels B1 B1
	(iii)	CE CD AE CF ~~AD~~ BF ~~AB~~ ~~EF~~  total length of connector = 45	M1 5 arc connector A1 AD not included A1 all OK, inc order B1 B1
	(iv)	A 3 miles (or length = 9) B 2 miles (or length = 10)	B1 B1

6	**(i)**	e.g. 0, 1, 2 → fall 3, 4, 5, 6, 7, 8 → not fall 9 → redraw	M1 A1 A1	ignore at least 1 proportions correct efficient
	(ii)	apple r n fall? 1 1 yes 2 3 no 3 8 no 4 0 yes 5 2 yes 6 7 no Three apples fall in this simulation.	M1 A2 B1√	 –1 each error
	(iii)	apple r n fall? 2 0 yes 3 1 yes 6 4 no apple r n fall? 6 4 no apple r n fall? 6 8 no apple r n fall? 6 0 yes 5 days before all have fallen	M1 A2 A1√	 –1 each error
	(iv)	apple r n fall? 1 picked 2 1 yes 3 3 no 4 8 no 5 0 yes 6 2 yes apple r n fall? 3 picked 4 7 no apple r n fall? 4 picked 3 days before none left	M1 A2 B1√	 –1 each error
	(v)	more simulations	B1	

4776 Numerical Methods

1

x	LHS		mpe		
1.3	2.868415	< 3			
1.5	3.181981	> 3			**[M1A1]**
			mpe	*(may be implied)*	
1.4	3.017945		0.1		**[M1]**
1.35	2.941413		0.05		**[A1]**
1.375	2.979232		0.025		**[A1]**
1.3875			0.0125	*finishing at this point:*	**[A1]**

mpe: 0.00625 0.003125 0.001563 0.000781 < 0.001 so 4 more iterations **[M1A1]**

[TOTAL 8]

2

h	M	T	S		
1	2.579768	2.447490	**2.535675**	T	**[M1A1]**
0.5	2.547350	**2.513629**	**2.536110**	S	**[M1A1A1]**

2.536 secure by comparison of S values. **[E1A1]**

[TOTAL 7]

3(i) $f'(x) = 3x^2 - 2x$ so $f'(0.5) = -0.25$ **[B1B1]**

$f(0.5) = 0.875$ hence given result **[B1]**

(ii) Require $-0.0005 < 0.25h < 0.0005$ **[M1A1]**

Hence $-0.002 < h < 0.002$ **[A1]**

And so $0.498 < x < 0.502$ **[B1]**

[TOTAL 7]

4(i) Convincing algebra to given result **[M1A1]**

(ii) Eg $k = 1000$ correct evaluation to 2 **[B1]**

$k = 1000000$ incorrect evaluation to zero (NB some will need larger k) **[B1]**

Mathematically equivalent expressions do not always evaluate equally **[E1]**
(because calculators do not store (large) numbers exactly)

Subtraction of nearly equal quantities often causes problems **[E1]**

[TOTAL 6]

5(i)

x	f(x)	Δf(x)	Δ^2f(x)	Δ^3f(x)		
0	1.883				*1st diff:*	**[M1A1]**
1	2.342	0.459			*2nd, 3rd*	**[F1]**
2	2.874	0.532	0.073			
3	3.491	0.617	0.085	0.012		
4	4.206	0.715	0.098	0.013		

3rd diffs almost constant **[E1]**

(ii) f(1.5) = 1.883 + 0.459 × 1.5 + 0.073 × 1.5 × 0.5 / 2! + 0.012 × 1.5 × 0.5 × (–0.5) / 3! **[M1A1A1]**
f(1.5) = 2.598125 or 2.598 to 3 dp **[A1]**
[TOTAL 8]

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6 (i) Sketch of smooth curve and its tangent. **[G1]**
Forward and central difference chords. **[G1G1]**
Clear statement or implication that the central difference chord has gradient closer to that of the tangent **[E1]**

[subtotal 4]

(ii)

h	tan 60º	tan (60 + h)º	derivative	
2	1.732051	1.880726	0.074338	**[M1A1]**
1	1.732051	1.804048	0.071997	**[A1]**
0.5	1.732051	1.767494	0.070886	**[A1]**

[subtotal 4]

(iii)

h	tan (60 + h)º	tan (60 – h)º	derivative	
2	1.880726	1.600335	0.070098	**[M1A1]**
1	1.804048	1.664279	0.069884	**[A1]**
0.5	1.767494	1.697663	0.069831	**[A1]**

[subtotal 4]

(iv)

	derivative	diffs	ratio of diffs		
forward difference:	0.074338				
	0.071997	–0.00234			
	0.070886	–0.00111	0.474407	(about 0.5, may be implied)	**[M1A1A1]**

	derivative	diffs	ratio of diffs		
central difference:	0.070098				
	0.069884	–0.00021			
	0.069831	–5.3E–05	0.24896	(about 0.25, less than forward difference, hence faster)	**[M1A1E1]**

[subtotal 6]
[TOTAL 18]

7 (i) Sketch showing $y = 3\sin x$ and $y = x$ with intersection in $(\frac{1}{2}\pi, \pi)$ **[G1G1]**
State or show that there is only one other non-zero root **[E1]**
[subtotal 3]

(ii)

r	0	1	2	3	4	5
x_r	2	2.727892	1.206001	2.80259	0.997639	2.52058

[M1A1A1]

clearly not converging **[B1]**
Cobweb diagram to illustrate process **[G3]**
[subtotal 7]

(iii) Convincing algebra to given result. **[M1]**

r	0	1	2	3	4	5
x_r	2	2.242631	2.277768	2.278844	2.278862	2.278863

[M1A1A1]

Root appears to be 2.27886 to 5 dp **[A1]**

x		$\sin x + \frac{2}{3}x$
2.278855	$<$	2.2788625
2.278865	$>$	2.2788627

hence result is correct to 5 dp **[M1A1E1]**
[subtotal 8]

[TOTAL 18]

Grade Thresholds

Advanced GCE Mathematics 3895 7895

January 2010 Examination Series

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	52	46	40	34	28	0
4752	Raw	72	59	52	45	38	32	0
4753/01	Raw	72	57	50	43	36	29	0
4753/02	Raw	18	15	13	11	9	8	0
4754	Raw	90	74	65	56	48	40	0
4755	Raw	72	55	47	39	31	24	0
4756	Raw	72	54	46	39	32	25	0
4758	Raw	72	61	53	45	37	29	0
4758/02	Raw	18	15	13	11	9	8	0
4761	Raw	72	58	49	41	33	25	0
4762	Raw	72	62	54	46	38	31	0
4763	Raw	72	64	56	48	41	34	0
4766/G241	Raw	72	58	50	42	35	28	0
4767	Raw	72	62	54	46	39	32	0
4768	Raw	72	55	48	41	34	27	0
4771	Raw	72	60	53	46	39	33	0
4776/01	Raw	72	60	53	46	40	33	0
4776/02	Raw	18	14	12	10	8	7	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895-7898	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	27.9	61.3	84.3	95.7	98.7	100	395
7896	54.3	62.9	88.6	100	100	100	35
7897							0
7898							0
3895	27.1	54.1	74.2	88.2	97.3	100	947
3896	41.3	67.5	86.3	95	100	100	80
3897	100	100	100	100	100	100	1
3898	50	50	100	100	100	100	2

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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