

**ADVANCED GCE  
MATHEMATICS (MEI)**

Applications of Advanced Mathematics (C4) Paper A

**4754A**

Candidates answer on the answer booklet.

**OCR supplied materials:**

- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Friday 14 January 2011  
Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

**NOTE**

- This paper will be followed by **Paper B: Comprehension**.

## Section A (36 marks)

- 1 (i) Use the trapezium rule with four strips to estimate  $\int_{-2}^2 \sqrt{1+e^x} dx$ , showing your working. [4]

Fig. 1 shows a sketch of  $y = \sqrt{1+e^x}$ .

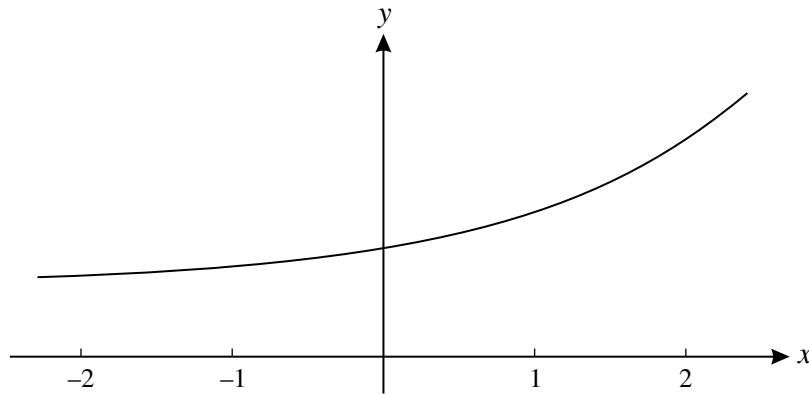


Fig. 1

- (ii) Suppose that the trapezium rule is used with more strips than in part (i) to estimate  $\int_{-2}^2 \sqrt{1+e^x} dx$ . State, with a reason but no further calculation, whether this would give a larger or smaller estimate. [2]

- 2 A curve is defined parametrically by the equations

$$x = \frac{1}{1+t}, \quad y = \frac{1-t}{1+2t}.$$

Find  $t$  in terms of  $x$ . Hence find the cartesian equation of the curve, giving your answer as simply as possible. [5]

- 3 Find the first three terms in the binomial expansion of  $\frac{1}{(3-2x)^3}$  in ascending powers of  $x$ . State the set of values of  $x$  for which the expansion is valid. [7]
- 4 The points A, B and C have coordinates (2, 0, -1), (4, 3, -6) and (9, 3, -4) respectively.
- (i) Show that AB is perpendicular to BC. [4]
- (ii) Find the area of triangle ABC. [3]
- 5 Show that  $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$ . [3]

- 6 (i) Find the point of intersection of the line  $\mathbf{r} = \begin{pmatrix} -8 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$  and the plane  $2x - 3y + z = 11$ . [4]
- (ii) Find the acute angle between the line and the normal to the plane. [4]

**Section B** (36 marks)

- 7 A particle is moving vertically downwards in a liquid. Initially its velocity is zero, and after  $t$  seconds it is  $v \text{ m s}^{-1}$ . Its terminal (long-term) velocity is  $5 \text{ m s}^{-1}$ .

A model of the particle's motion is proposed. In this model,  $v = 5(1 - e^{-2t})$ .

- (i) Show that this equation is consistent with the initial and terminal velocities. Calculate the velocity after 0.5 seconds as given by this model. [3]
- (ii) Verify that  $v$  satisfies the differential equation  $\frac{dv}{dt} = 10 - 2v$ . [3]

In a second model,  $v$  satisfies the differential equation

$$\frac{dv}{dt} = 10 - 0.4v^2.$$

As before, when  $t = 0$ ,  $v = 0$ .

- (iii) Show that this differential equation may be written as

$$\frac{10}{(5-v)(5+v)} \frac{dv}{dt} = 4.$$

Using partial fractions, solve this differential equation to show that

$$t = \frac{1}{4} \ln \left( \frac{5+v}{5-v} \right). \quad [8]$$

This can be re-arranged to give  $v = \frac{5(1 - e^{-4t})}{1 + e^{-4t}}$ . [You are **not** required to show this result.]

- (iv) Verify that this model also gives a terminal velocity of  $5 \text{ m s}^{-1}$ .

Calculate the velocity after 0.5 seconds as given by this model. [3]

The velocity of the particle after 0.5 seconds is measured as  $3 \text{ m s}^{-1}$ .

- (v) Which of the two models fits the data better? [1]

- 8 Fig. 8 shows a searchlight, mounted at a point A, 5 metres above level ground. Its beam is in the shape of a cone with axis AC, where C is on the ground. AC is angled at  $\alpha$  to the vertical. The beam produces an oval-shaped area of light on the ground, of length DE. The width of the oval at C is GF. Angles DAC, EAC, FAC and GAC are all  $\beta$ .

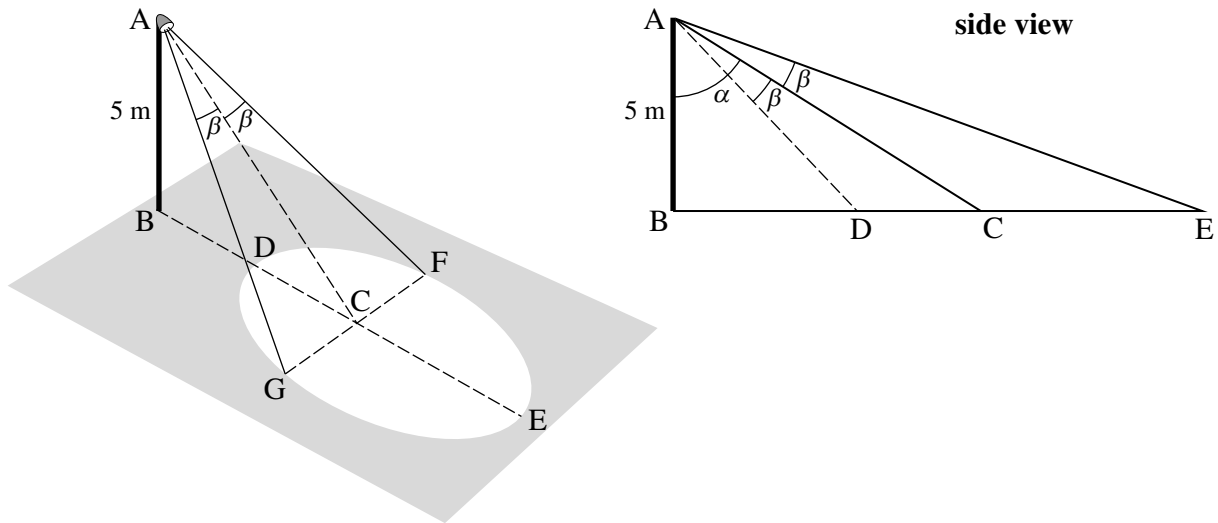


Fig. 8

In the following, all lengths are in metres.

- (i) Find AC in terms of  $\alpha$ , and hence show that  $GF = 10 \sec \alpha \tan \beta$ . [3]

- (ii) Show that  $CE = 5(\tan(\alpha + \beta) - \tan \alpha)$ .

Hence show that  $CE = \frac{5 \tan \beta \sec^2 \alpha}{1 - \tan \alpha \tan \beta}$ . [5]

Similarly, it can be shown that  $CD = \frac{5 \tan \beta \sec^2 \alpha}{1 + \tan \alpha \tan \beta}$ . [You are **not** required to derive this result.]

You are now given that  $\alpha = 45^\circ$  and that  $\tan \beta = t$ .

- (iii) Find CE and CD in terms of  $t$ . Hence show that  $DE = \frac{20t}{1 - t^2}$ . [5]

- (iv) Show that  $GF = 10\sqrt{2}t$ . [2]

For a certain value of  $\beta$ ,  $DE = 2GF$ .

- (v) Show that  $t^2 = 1 - \frac{1}{\sqrt{2}}$ .

Hence find this value of  $\beta$ . [3]

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