OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754A

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet
  (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Friday 14 January 2011
Afternoon

Duration: 1 hour 30 minutes

4754A

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Turn over

Section A (36 marks)

1 **(i)** Use the trapezium rule with four strips to estimate $\int_{-2}^{2} \sqrt{1 + e^x}\, dx$, showing your working. **[4]**

Fig. 1 shows a sketch of $y = \sqrt{1 + e^x}$.

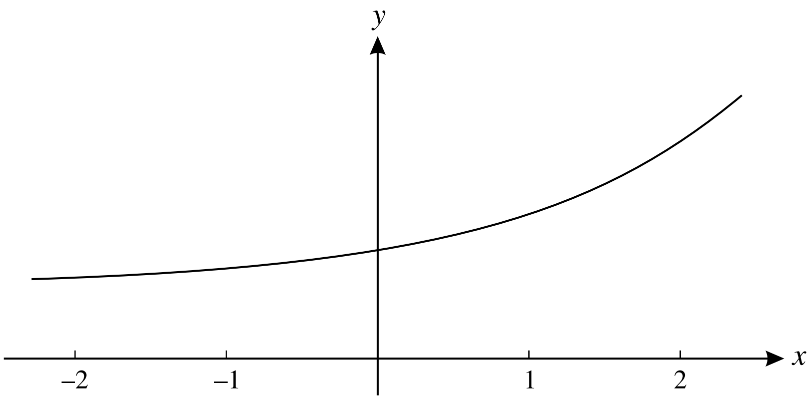

Fig. 1

(ii) Suppose that the trapezium rule is used with more strips than in part **(i)** to estimate $\int_{-2}^{2} \sqrt{1 + e^x}\, dx$. State, with a reason but no further calculation, whether this would give a larger or smaller estimate. **[2]**

2 A curve is defined parametrically by the equations

$$x = \frac{1}{1+t}, \qquad y = \frac{1-t}{1+2t}.$$

Find t in terms of x. Hence find the cartesian equation of the curve, giving your answer as simply as possible. **[5]**

3 Find the first three terms in the binomial expansion of $\frac{1}{(3-2x)^3}$ in ascending powers of x. State the set of values of x for which the expansion is valid. **[7]**

4 The points A, B and C have coordinates (2, 0, −1), (4, 3, −6) and (9, 3, −4) respectively.

(i) Show that AB is perpendicular to BC. **[4]**

(ii) Find the area of triangle ABC. **[3]**

5 Show that $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$. **[3]**

6 **(i)** Find the point of intersection of the line $\mathbf{r} = \begin{pmatrix} -8 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$ and the plane $2x - 3y + z = 11$. **[4]**

(ii) Find the acute angle between the line and the normal to the plane. **[4]**

Section B (36 marks)

7 A particle is moving vertically downwards in a liquid. Initially its velocity is zero, and after t seconds it is $v\,\mathrm{m\,s^{-1}}$. Its terminal (long-term) velocity is $5\,\mathrm{m\,s^{-1}}$.

A model of the particle's motion is proposed. In this model, $v = 5\left(1 - \mathrm{e}^{-2t}\right)$.

(i) Show that this equation is consistent with the initial and terminal velocities. Calculate the velocity after 0.5 seconds as given by this model. **[3]**

(ii) Verify that v satisfies the differential equation $\frac{\mathrm{d}v}{\mathrm{d}t} = 10 - 2v$. **[3]**

In a second model, v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 10 - 0.4v^2.$$

As before, when $t = 0$, $v = 0$.

(iii) Show that this differential equation may be written as

$$\frac{10}{(5-v)(5+v)}\frac{\mathrm{d}v}{\mathrm{d}t} = 4.$$

Using partial fractions, solve this differential equation to show that

$$t = \tfrac{1}{4}\ln\left(\frac{5+v}{5-v}\right).$$ **[8]**

This can be re-arranged to give $v = \frac{5\left(1 - \mathrm{e}^{-4t}\right)}{1 + \mathrm{e}^{-4t}}$. [You are **not** required to show this result.]

(iv) Verify that this model also gives a terminal velocity of $5\,\mathrm{m\,s^{-1}}$.

Calculate the velocity after 0.5 seconds as given by this model. **[3]**

The velocity of the particle after 0.5 seconds is measured as $3\,\mathrm{m\,s^{-1}}$.

(v) Which of the two models fits the data better? **[1]**

8 Fig. 8 shows a searchlight, mounted at a point A, 5 metres above level ground. Its beam is in the shape of a cone with axis AC, where C is on the ground. AC is angled at α to the vertical. The beam produces an oval-shaped area of light on the ground, of length DE. The width of the oval at C is GF. Angles DAC, EAC, FAC and GAC are all β.

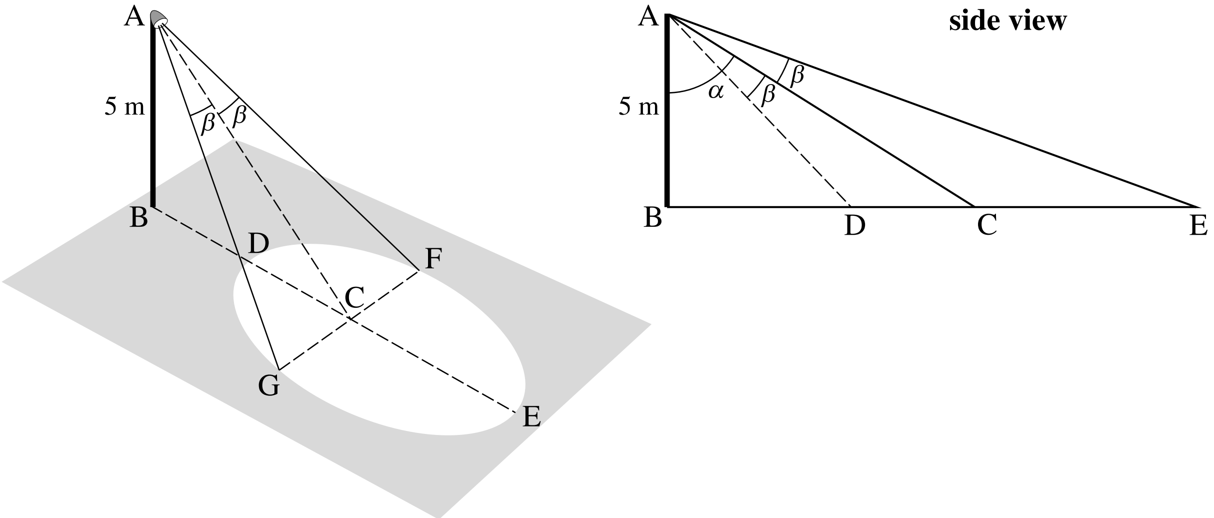

Fig. 8

In the following, all lengths are in metres.

(i) Find AC in terms of α, and hence show that $GF = 10\sec\alpha\tan\beta$. **[3]**

(ii) Show that $CE = 5\big(\tan(\alpha+\beta) - \tan\alpha\big)$.

Hence show that $CE = \dfrac{5\tan\beta\sec^2\alpha}{1-\tan\alpha\tan\beta}$. **[5]**

Similarly, it can be shown that $CD = \dfrac{5\tan\beta\sec^2\alpha}{1+\tan\alpha\tan\beta}$. [You are **not** required to derive this result.]

You are now given that $\alpha = 45^\circ$ and that $\tan\beta = t$.

(iii) Find CE and CD in terms of t. Hence show that $DE = \dfrac{20t}{1-t^2}$. **[5]**

(iv) Show that $GF = 10\sqrt{2}t$. **[2]**

For a certain value of β, DE = 2GF.

(v) Show that $t^2 = 1 - \dfrac{1}{\sqrt{2}}$.

Hence find this value of β. **[3]**