OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4754A**: Applications of Advanced Mathematics: Paper A

Advanced GCE

Mark Scheme for June 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			$\cos\theta - 3\sin\theta = R(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$		
			$\Rightarrow 1 = R\cos\alpha, 3 = R\sin\alpha$	M1 A1	Correct pairs. Condone sign errors for the M mark (so accept $R\sin\alpha = -3$)
			$R^2 = 1^2 + 3^2 = 10 \Rightarrow R = \sqrt{10}$	B1	Or 3.2 or better, not $\pm\sqrt{10}$ unless $+\sqrt{10}$ chosen
			$\tan\alpha = 3 \Rightarrow \alpha = 1.249$	M1 A1	ft their pairs (condone sign errors but division must be the correct way round), A1 for 1.249 or better (accept 1.25), with no errors seen in method for angle
			Maximum value of $\cos\theta - 3\sin\theta$ is $\sqrt{10} < 4$	B1	Or equivalent convincing numerical statement that no solutions exist e.g. $\frac{4}{\sqrt{10}} > 1$. Maybe embedded in an attempt at a solution. Do not accept general statements e.g. 'doesn't work' – must be clear why no solutions exist – **dependent on first B1** **SC**: If candidates state that $\cos\alpha = 1, \sin\alpha = 3 \Rightarrow \tan\alpha = 3$ this could score M0A0B1M1A1B1 (so max 4/6) **Note that those candidates who state $R = \sqrt{10}$ and $\tan\alpha = 3$ with no (wrong) working seen could go on to score full marks**
				[6]	

Question			Answer	Marks	Guidance
2			$\left(1+\frac{x}{p}\right)^q = 1+q\frac{x}{p}+\frac{q(q-1)}{2!}\left(\frac{x}{p}\right)^2+\cdots$	M1*	One of $\frac{q}{p}x$ or $\frac{q(q-1)}{2!}\left(\frac{x}{p}\right)^2$ (soi), for example, $\frac{q}{p}=-1$ scores M1 A1
			$\frac{q}{p}=-1$ $\quad\frac{q(q-1)}{2p^2}=\frac{3}{4}$	A1 A1	Allow x's on both sides of equations (if correct)
			$q=-p \Rightarrow \frac{-p(-p-1)}{2p^2}=\frac{3}{4}$ or $\frac{q(q-1)}{2q^2}=\frac{3}{4}$	M1dep*	Eliminating p (or q) from simultaneous equations (not involving x) involving both variables oe – if M1A1A1 awarded followed by either p or q correct (www) this implies this M mark
			$\Rightarrow p=2$	A1	$p=2$ www (or $q=-2$)
			$\Rightarrow q=-2$	A1ft	$q=-2$ (or $p=2$) for second value, ft their p or q (e.g. the negative of their p or q) provided first 4 marks awarded and only a single computational error in the method – so must be a correct method for solving their equation in p or q (ignore mention of p and/or $q=0$)
			Valid for $\left\|\frac{x}{2}\right\|<1 \Rightarrow \|x\|<2$	A1	or $-2<x<2$ www, allow $-2<\|x\|<2$ but not say, $x<2$ **SC If M0 M0 awarded and no wrong working seen then B1 for p = 2 <u>and</u> q = - 2, B1 for -2 < x < 2 (oe) so max 2 marks** Guidance for solving quadratics on this paper: use of correct quadratic equation formula (if formula is quoted correctly then only one sign slip is permitted, if the formula is quoted incorrectly M0, if not quoted at all substitution must be completely correct to earn the M1) or factorising (giving their x^2 term and one other term when factors multiplied out) or completing the square (must get to the square root stage involving ± and arithmetical errors may be condoned provided that perfect square term was correct)
				[7]	

Question			Answer	Marks	Guidance
3			$V = \pi\int_0^4 x^2 \mathrm{d}y$	M1	M1 for $k(\pi)\int_0^4 x^2(\mathrm{d}y)$ with correct limits and $k = 1$ or $\frac{1}{2}$, allow correct limits seen or implied later, if formula not stated then must substitute for their x^2 correctly to imply this formula – condone lack of π for the M mark and $\mathrm{d}y$ throughout (condone incorrect use of $\mathrm{d}x$ too)
			$V = \pi\int_0^4 y^{\frac{1}{2}} \mathrm{d}y$	A1	Correct (**or** with a 1/2) – limits may be seen or implied through later working
			$\frac{2}{3}y^{\frac{3}{2}}$	B1	$\frac{2}{3}y^{\frac{3}{2}}$ or $\frac{1}{3}y^{\frac{3}{2}}$ (but only if $k = \frac{1}{2}$), condone $\frac{y^{1.5}}{1.5}$ (oe)
			$= \pi\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^4 = \frac{16\pi}{3}$	A1	Exact – mark **final** answer (so no isw if correct answer is subsequently halved) but if exact value seen and is then followed by 16.755… then isw
				[4]	

Question			Answer	Marks	Guidance
4			$4\sin\theta\cos\theta = 1 + 2\cos^2\theta - 1$	M1*	Use of correct double angle formulae: $\sin 2\theta \equiv 2\sin\theta\cos\theta$ **and** any one of $\cos 2\theta \equiv \cos^2\theta - \sin^2\theta$ or $1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$
			$2\cos\theta(2\sin\theta - \cos\theta) = 0$	A1	Correct equation in solvable form e.g. $2\sin\theta - \cos\theta = 0$ (oe) or $5\sin^4\theta - 6\sin^2\theta + 1 = 0$ or $5\cos^4\theta - 4\cos^2\theta = 0$ but not $4\sin\theta\cos\theta = 2\cos^2\theta$
			$\Rightarrow \tan\theta = \frac{1}{2}$	M1dep*	Use of $\dfrac{\sin\theta}{\cos\theta} \equiv \tan\theta$ on their $\alpha\sin\theta + \beta\cos\theta = 0$ or correct method for solving quadratic in either $\sin^2\theta$ or $\cos^2\theta$ (See guidance in question 2 for solving quadratics)
			$\theta = 26.6^\circ$	A1	www (26.6 or better)
			$\theta = 90^\circ$	B1	Not from incorrect working
					Ignore additional solutions outside the range. If any additional solutions given inside the range $0 \le \theta \le 180^\circ$ **and** full marks would have been awarded then remove last mark (so 4/5)
					Both answers in radians: 0.464 (or better) and $\pi / 2$ scores B1
				[5]	Answers with no working scores B1 B1(so max 2/5)

Question			Answer	Marks	Guidance
5	(i)		$\text{AC} = x\sec\theta$	B1	Accept any equivalent form $\left(\text{e.g. AC}\cos\theta = x\right)$. If AC not seen then there must be a diagram as evidence of correct sides - $x\sec\theta$ with no AC is B0
			$\text{AD} = x\sec^2\theta$ and $\text{AE} = x\sec^3\theta$	B1	Accept $2x = x\sec^3\theta$ (as AE = $2x$) or any equivalent form. Otherwise there must be a corresponding diagram as evidence of correct sides. Accept $\cos^3\theta = x/\text{AC}\times\text{AC}/\text{AD}\times\text{AD}/2x$ for the first two marks
			$\Rightarrow x\sec^3\theta = 2x$ $\Rightarrow \sec^3\theta = 2$*	B1 **[3]**	This line (oe) must be seen before the x's cancelled **NB AG** – dependent on all previous marks
			OR $\text{AD} = 2x\cos\theta$	B1	Same principles as above for each corresponding mark
			$\text{AC} = 2x\cos^2\theta$ and $\text{AB} = 2x\cos^3\theta$	B1	or $x = 2x\cos^3\theta$ (as AB = x)
			$2x\cos^3\theta = x \Rightarrow \sec^3\theta = 2$*	B1	Must see $2x\cos^3\theta = x$ (oe) before given answer
5	(ii)		$\text{ED} = 2x\sin\theta$	B1	oe e.g. $\text{ED} = \sqrt{4x^2 - \text{AD}^2}$ or $\text{ED} = \text{AD}\tan\theta$ with AD correctly expressed in terms of x and θ (or using $\theta = 37.5$ or better) - see (i) for alternatives for AD. Allow ED = 1.22x (or better) but B0 if ED = … missing
			$\text{CB} = x\tan\theta$	B1	oe e.g. $CB = \sqrt{\text{AC}^2 - x^2}$ or $CB = \text{AC}\sin\theta$ with AC correctly expressed in terms of x and θ (or using $\theta = 37.5$ or better) - see (i) for alternatives for AC. Allow CB = 0.77x (or better) but B0 if CB = … missing
			$\dfrac{\text{ED}}{\text{CB}} = \dfrac{2x\sin\theta}{x\tan\theta} = 2\cos\theta$	B1	www must come from exact working (so not using $\theta = 37.46...$ oe) - accept $\dfrac{\text{ED}}{\text{CB}} = \dfrac{2}{\sec\theta}$ or $\dfrac{\text{ED}}{\text{CB}} = \sec^2\theta$ (oe) (as from (i): $\sec^3\theta = 2$)
			$= 2/2^{\frac{1}{3}} = 2^{\frac{2}{3}}$*	B1 **[4]**	**NB AG** – dependent on all previous marks in (ii) – must be one step of intermediate working from $2\cos\theta$ to given answer

Question			Answer	Marks	Guidance
6			$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}=\frac{-2/t^2}{2}\left(=-\frac{1}{t^2}\right)$	M1*	M1 for their $(\mathrm{d}y/\mathrm{d}t)$ / their $(\mathrm{d}x/\mathrm{d}t)$ in terms of t with at least one term correct
				A1	A1 cao (oe) – allow unsimplified even if subsequently cancelled incorrectly i.e. can isw
				M1dep*	$y-\frac{2}{t}=\mathrm{f}(t)(x-2t)$ with any non-zero gradient expressed as a function of t - or any equivalent form (e.g. $y = mx + c$) but must have used the correct point $\left(2t,\frac{2}{t}\right)$ - if using $y = mx + c$ must explicitly have $c = \ldots$ before M1 can be awarded
			$y-\frac{2}{t}=-\frac{1}{t^2}(x-2t)$	A1	oe – need not be simplified
			When $x = 0$, $y=\frac{4}{t}$	A1ft	Must be a function of t
			When $y = 0$, $x = 4t$	A1ft	Must be a function of t
			So area of triangle = $\frac{1}{2}\times\frac{4}{t}\times 4t=8$ (which is independent of t)	A1	No ft on this mark – an answer of 8 (www) with no additional comment is sufficient to award this mark
				[7]	
			OR (for the first two marks) $y=\frac{2}{\left(\frac{x}{2}\right)}=\frac{4}{x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{4}{x^2}$	M1*	Attempt to eliminate t and correctly differentiates their Cartesian equation
			$\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=-\frac{4}{(2t)^2}$	A1	

Question			Answer	Marks	Guidance
7	(i)		$\text{AG} = \sqrt{4^2 + 3^2 + 5^2}$	B1	Condone -4^2 etc. if recovered
			$= \sqrt{50}\ \left(= 5\sqrt{2}\right)$	B1	Correct answer implies both marks – accept 7.1 or better
				[2]	
7	(ii)		$\overrightarrow{\text{DP}} = 4\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ or $\overrightarrow{\text{PD}} = -4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$	B1	One correct direction vector in plane DPF (oe e.g. expressed as a column vector)
			$\overrightarrow{\text{DF}} = 4\mathbf{i} + 3\mathbf{j}$ (or $\overrightarrow{\text{PF}} = \mathbf{j} + 5\mathbf{k}$)	B1	Any other correct direction vector in plane DPF
			$\mathbf{n}\cdot\overrightarrow{\text{DP}} = 15\times4 - 20\times2 + 4\times(-5) = 0$	B1	Scalar product with a **direction** vector in the plane (including evaluation and = 0) (**OR** M1 forms vector cross product with at least two correct terms in solution)
			$\mathbf{n}\cdot\overrightarrow{\text{DF}} = 15\times4 - 20\times3 = 0$ (or $\mathbf{n}\cdot\overrightarrow{\text{PF}} = -20\times1 + 4\times5 = 0$)	B1	Scalar product with a second **direction** vector in the plane (including evaluation and = 0) (following **OR** above, A1 all correct ie a multiple of $15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k}$) (NB finding only one direction vector and its scalar product is B1 B0 B1 B0)
				M1	$15x - 20y + 4z = c$ oe (accept any non-Cartesian form for M1 only)
			$\mathbf{r}\cdot\mathbf{n} = \mathbf{a}\cdot\mathbf{n} \Rightarrow 15x - 20y + 4z = 20$	A1	M1A0 for $15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k} = 20$ **or** $15x - 20y + 4z - 20$ **SC1**: if states 'if $15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k}$ is normal then of the form $15x - 20y + 4z = c$' and substitutes one coordinate gets M1A1, then substitutes the other two coordinates A3 (not A1, A1, A1). Then states 'so $15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k}$ is normal' and states the correct equation of the plane this can get B1 provided that there is a clear argument ie M1A1A3B1. Without a clear argument this is B0 **SC2**: if finds two relevant **direction** vectors B1 B1 and then finds equation of the plane from vector form, $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ gets B1. Eliminating parameters B1 cao. If then states 'so $15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k}$ is normal' can get B1, and then a valid reason why $15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k}$ is normal scores the final B mark (each B mark is dependent on the previous one)
				[6]	

Question			Answer	Marks	Guidance
7	(iii)		$\mathbf{r} = 4\mathbf{i} + \ldots$	B1	Need $\mathbf{r}$ (or another single letter) =… **or** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \ldots$ for first B1
			$\ldots + \lambda(-4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$	B1	**NB answer is not unique** e.g. $\mathbf{r} = 3\mathbf{j} + 5\mathbf{k} + \mu(4\mathbf{i} - 3\mathbf{j} - 5\mathbf{k})$ - accept column vector form and condone row vectors (non-vector form scores B1 only)
			$15(4 - 4\lambda) - 20(3\lambda) + 4(5\lambda) = 20$	M1	Substituting their line in their plane equation from (ii) (condone a slip if intention clear) – their line and plane must be of the correct form (e.g. the line must be of the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$)
			$\Rightarrow 40 = 100\lambda$, $\lambda = 0.4$	A1	www cao **NB λ is not unique as depends on choice of line seen in this part**
			Q is (2.4, 1.2, 2)	A1	www - condone answer given as a position vector
			AQ : QG = 2:3	A1 **[6]**	oe www
7	(iv)			M1*	Selecting $15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k}$ and **their** direction vector from (iii)
			Angle between $(-4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$ and $(15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k})$ is θ where $\cos\theta = \dfrac{(-4\times15)+(3\times-20)+(5\times4)}{\sqrt{50}\sqrt{641}}$	M1dep*	Correct formula (including cosine), with correct substitution, using their direction vector from (iii) and the correct normal vector - condone either a single numerical slip **or** a single sign slip (but not one of each) if intention is clear. So it must be clear where the slip comes from e.g. if the magnitude of one vector is stated incorrectly **with no working** then this is M0
			$\theta = 56.0$ or 124.0	A1	www cao (accept 56 or better, 124 or better, 2.16 radians or better, 0.978 radians or better)
			angle between line and plane = 34.0°	A1 **[4]**	www cao (accept 34 or better, 0.593 radians or better)

Question			Answer	Marks	Guidance
8	(i)		$\frac{1}{2+x}+\frac{1}{2-x}=\frac{2-x+2+x}{(2+x)(2-x)}=\frac{4}{(2+x)(2-x)}$ *	B1 **[1]**	**NB AG** – must be at least one intermediate step before given answer – correct application of partial fractions is fine
8	(ii)		$\ln\left(\frac{2+x}{2-x}\right)=0\Rightarrow\frac{2+x}{2-x}=1$ $2+x=2-x\Rightarrow x=0$	B1 B1 **[2]**	or $=e^0$ or $\ln(2+x)=\ln(2-x)$ If only this line seen then award B0B1 **SC: Allow B1 only for verifying that when x = 0, t = 0**
8	(iii)		$t=\ln\left(\frac{2+x}{2-x}\right)=\ln(2+x)-\ln(2-x)$ $\left(\frac{dt}{dx}=\right)\frac{1}{2+x}+\frac{1}{2-x}$ $\frac{dt}{dx}=\frac{4}{(2+x)(2-x)}\Rightarrow\frac{dx}{dt}=\frac{1}{4}(2+x)(2-x)$ $k=\frac{1}{4}$ **See next page for an alternative solution**	B1 M1 A1 A1 **[4]**	 **Correct** differentiation of their t **OR for first two marks -** If no subtraction law of logs seen e.g. $\frac{dt}{dx}=\left(\frac{1}{\left(\frac{2+x}{2-x}\right)}\right)\left(\frac{(2-x)(1)-(2+x)(-1)}{(2-x)^2}\right)$ award B1 for correct first bracket (reciprocal expression) and B1 for second correct bracket (quotient/chain rule)(oe) – if additional constant(s) added (e.g. $t = k\ln(\ldots)$) then award B1 only for a constant times a fully correct derivative $\frac{dt}{dx}$ and $\frac{dx}{dt}$ must be correctly attributed to the correct expression for this mark Explicitly stating (that the constant of proportionality is) $\frac{1}{4}$ - therefore it is possible to score A0A1in this part

Question			Answer	Marks	Guidance
			OR		
			$\frac{\mathrm{d}x}{\mathrm{d}t}=k(2+x)(2-x)\Rightarrow\int\frac{\mathrm{d}x}{(2+x)(2-x)}=k\int\mathrm{d}t$	B1	Allow omission of dx and/or dt
			$\lambda\ln(2+x)+\mu\ln(2-x)=kt(+c)$	M1	Any non-zero constant λ,μ - further guidance in (v) for this and the next mark
			$\frac{1}{4}\left[\ln(2+x)-\ln(2-x)\right]=kt(+c)$	A1	www oe (condone absence of c)
			$x=0,t=0\Rightarrow c=0$		
			$\ln\left(\frac{2+x}{2-x}\right)=4kt\Rightarrow k=\frac{1}{4}$	A1	www – must include c and show that c = 0. Must explicitly state that $k=\frac{1}{4}$
8	**(iv)**		$\mathrm{e}^t=\frac{2+x}{2-x}$	B1	
			$(2-x)\mathrm{e}^t=2+x$		
			$\Rightarrow 2\mathrm{e}^t-x\mathrm{e}^t=2+x$ $\Rightarrow x(1+\mathrm{e}^t)=2\mathrm{e}^t-2$	M1	Multiplying out, collecting x terms (condone sign slips and numerical errors (eg loss of a 2) only but M0 if e^t incorrectly replaced with e^{-t}) **and** factorising their x terms correctly
			$x=\frac{2(\mathrm{e}^t-1)}{1+\mathrm{e}^t}=\frac{2(\mathrm{e}^t-1)\mathrm{e}^{-t}}{(1+\mathrm{e}^t)\mathrm{e}^{-t}}=\frac{2(1-\mathrm{e}^{-t})}{1+\mathrm{e}^{-t}}$ *	A1	www **NB AG** – as AG must be an indication of how previous line leads to the required result (eg stating or showing multiplying by e^{-t})
			$(x\to)2$ (as $t\to\infty$)	B1 **[4]**	
			OR (for first three marks)		
			$\mathrm{e}^{-t}=\frac{2-x}{2+x}$	B1	
			$(2+x)\mathrm{e}^{-t}=2-x$		
			$\Rightarrow 2\mathrm{e}^{-t}+x\mathrm{e}^{-t}=2-x$ $\Rightarrow x(1+\mathrm{e}^{-t})=2-2\mathrm{e}^{-t}$	M1	Multiplying out, collecting x terms (condone sign slips as above) and factorising their x terms correctly
			$x=\frac{2(1-\mathrm{e}^{-t})}{1+\mathrm{e}^{-t}}$ *	A1	www **NB AG**

Question			Answer	Marks	Guidance
8	(v)		$\int \frac{1}{(2-x)(2+x)}\,\mathrm{d}x = k\int \mathrm{e}^{-t}\,\mathrm{d}t$	M1*	Separating variables - condone sign slips and issues with placement of *k* but M0 for $\int (2-x)(2+x)\mathrm{d}x = \ldots$ or equivalent algebraic error in separating variables unless recovered. If no subsequent work integral signs needed, but allow omission of d*x* and/or d*t* but must be correctly placed if present
			$\alpha \ln(2+x) + \beta \ln(2-x) = \gamma \mathrm{e}^{-t}(+c)$	A1	Any non-zero constants α, β, γ **- this line must be seen and cannot be implied by later working (as this is an AG) –** condone absence of *c* or if a constant present condone the use of *k* for their constant. Do not condone invisible brackets e.g. ln 2 + *x* unless recovered before subtraction law of logs applied – all of these points apply to the next A mark too
			$\ln(2+x) - \ln(2-x) = -4k\mathrm{e}^{-t}(+c)$	A1	www oe
			When $t = 0, x = 0 \Rightarrow c = 4k$	M1dep*	Substituting $x = 0, t = 0$ into each term in an attempt to find their *c* (must get *c* = …) – if they integrate and use *k* as their constant they must use $x = 0, t = 0$ to find this single *k* term only
			$\Rightarrow \ln\left(\frac{2+x}{2-x}\right) = 4k(1-\mathrm{e}^{-t})$ *	A1 **[5]**	www **NB AG** must have obtained all previous marks in this part
			OR (for first 3 marks) – final M1A1 as above $\int \frac{1}{(2-x)(2+x)}\,\mathrm{d}x = k\int \mathrm{e}^{-t}\,\mathrm{d}t$	M1*	Separating variables. If no subsequent work integral signs needed, but allow omission of d*x* or d*t*, but must be correctly placed if present
			$\int \frac{1}{4-x^2}\,\mathrm{d}x = k\int \mathrm{e}^{-t}\,\mathrm{d}t \Rightarrow \frac{1}{4}\ln\left(\frac{2+x}{2-x}\right) = -k\mathrm{e}^{-t}(+c)$	A2	**Must see** $1/\left(4-x^2\right)$ on lhs – please note that one A mark cannot be awarded
8	(vi)		as $t \to \infty$, $x \to 1.85 \Rightarrow \ln 3.85/0.15 = 4k$	M1	Sets e^{-t} to 0 and substitutes *x* = 1.85 – condone substitution of a ‘large’ value of *t* only if it leads to the correct value of *k*
			$\Rightarrow k = 0.811$	A1 **[2]**	$k = 0.25\ln\left(77/3\right)$ or 0.81 or better

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