

ADVANCED GCE
MATHEMATICS (MEI)
Mechanics 4

4764

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

- Scientific or graphical calculator

Tuesday 15 June 2010
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (24 marks)

- 1 At time t a rocket has mass m and is moving vertically upwards with velocity v . The propulsion system ejects matter at a constant speed u relative to the rocket. The only additional force acting on the rocket is its weight.

(i) Derive the differential equation $m \frac{dv}{dt} + u \frac{dm}{dt} = -mg$. [4]

The rocket has initial mass m_0 of which 75% is fuel. It is launched from rest. Matter is ejected at a constant mass rate k .

(ii) Assuming that the acceleration due to gravity is constant, find the speed of the rocket at the instant when all the fuel is burnt. [8]

- 2 A particle of mass m kg moves horizontally in a straight line with speed v m s⁻¹ at time t s. The total resistance force on the particle is of magnitude $mkv^{\frac{3}{2}}$ N where k is a positive constant. There are no other horizontal forces present. Initially $v = 25$ and the particle is at a point O.

(i) Show that $v = 4\left(kt + \frac{2}{5}\right)^{-2}$. [7]

(ii) Find the displacement from O of the particle at time t . [3]

(iii) Describe the motion of the particle as t increases. [2]

Section B (48 marks)

- 3 A uniform rod AB of mass m and length $4a$ is hinged at a fixed point C, where $AC = a$, and can rotate freely in a vertical plane. A light elastic string of natural length $2a$ and modulus λ is attached at one end to B and at the other end to a small light ring which slides on a fixed smooth horizontal rail which is in the same vertical plane as the rod. The rail is a vertical distance $2a$ above C. The string is always vertical. This system is shown in Fig. 3 with the rod inclined at θ to the horizontal.

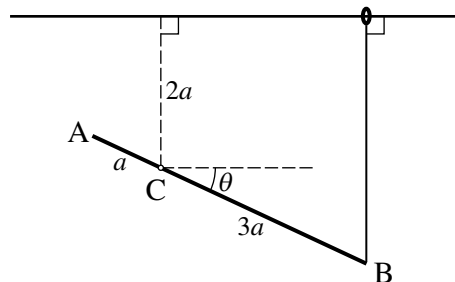


Fig. 3

(i) Find an expression for V , the potential energy of the system relative to C, and show that $\frac{dV}{d\theta} = a \cos \theta \left(\frac{9}{2} \lambda \sin \theta - mg \right)$. [6]

- (ii) Determine the positions of equilibrium and the nature of their stability in the cases

(A) $\lambda > \frac{2}{9}mg$, [10]

(B) $\lambda < \frac{2}{9}mg$, [4]

(C) $\lambda = \frac{2}{9}mg$. [4]

- 4 Fig. 4.1 shows a uniform cone of mass M , base radius a and height $2a$.

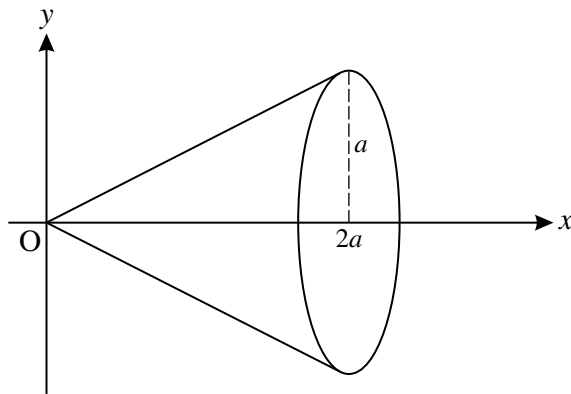


Fig. 4.1

- (i) Show, by integration, that the moment of inertia of the cone about its axis of symmetry is $\frac{3}{10}Ma^2$.
 [You may assume the standard formula for the moment of inertia of a uniform circular disc about its axis of symmetry and the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.] [8]

A frustum is made by taking a uniform cone of base radius 0.1 m and height 0.2 m and removing a cone of height 0.1 m and base radius 0.05 m as shown in Fig. 4.2. The mass of the frustum is 2.8 kg.

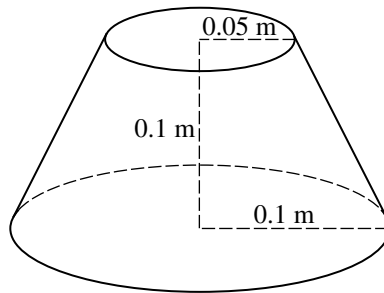


Fig. 4.2

The frustum can rotate freely about its axis of symmetry which is fixed and vertical.

- (ii) Show that the moment of inertia of the frustum about its axis of symmetry is 0.0093 kg m^2 . [4]

The frustum is accelerated from rest for t seconds by a couple of magnitude 0.05 N m about its axis of symmetry, until it is rotating at 10 rad s^{-1} .

- (iii) Calculate t . [4]

- (iv) Find the position of G, the centre of mass of the frustum. [3]

The frustum (rotating at 10 rad s^{-1}) then receives an impulse tangential to the circumference of the larger circular face. This reduces its angular speed to 5 rad s^{-1} .

- (v) To reduce its angular speed further, a parallel impulse of the same magnitude is now applied tangentially in the horizontal plane through G at the curved surface of the frustum. Calculate the resulting angular speed. [5]

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.



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