OCR RECOGNISING ACHIEVEMENT

GCE

# Mathematics (MEI)

Advanced GCE

Unit **4764:** Mechanics 4

# Mark Scheme for June 2011

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| | | | | |
|---|---|---|---|---|
| 1(i) | $\frac{dm}{dt} = -\lambda m \Rightarrow m = m_0 e^{-\lambda t}$ | M1 | | |
| | | A1 | | 2 |
| (ii) | $\frac{d}{dt}(mv) = mg - kmv$ | B1 | N2L | |
| | $\frac{dm}{dt}v + m\frac{dv}{dt} = mg - kmv$ | M1 | Expand derivative | |
| | $-\lambda mv + m\frac{dv}{dt} = mg - kmv$ | M1 | Substitute | |
| | $\frac{dv}{dt} = g + (\lambda - k)v$ | A1 | | |
| | $\int \frac{dv}{g+(\lambda-k)v} = \int dt$ | M1 | Separate and integrate | |
| | $\frac{1}{\lambda-k}\ln(g + (\lambda - k)v) = t + c$ | A1√ | | |
| | $g + (\lambda - k)v = Ae^{(\lambda-k)t}$ | | | |
| | $v = 0, t = 0 \Rightarrow A = g$ | M1 | Use condition | |
| | $v = \frac{g}{\lambda-k}\left(e^{(\lambda-k)t} - 1\right)$ AG | E1 | Convincingly shown | 8 |
| (iii) | $m = \frac{1}{2}m_0 \Rightarrow e^{-\lambda t} = \frac{1}{2}$ | M1 | Accept substituted into their expression in part (i) | |
| | $\Rightarrow t = \frac{1}{\lambda}\ln 2$ | | | |
| | $v = \frac{g}{\lambda-k}\left(2^{\frac{\lambda-k}{\lambda}} - 1\right)$ | A1 | Any correct form | 2 |

| | | | | |
|---|---|---|---|---|
| 2(i) | $V = \frac{1}{2}k(2a - x - a)^2 + \frac{1}{2}k\left(\sqrt{a^2+x^2} - a\right)^2$ | M1 | for $E = \frac{1}{2}kx^2$ | |
| | | A1 | | |
| | | A1 | | |
| | $\frac{dV}{dx} = -k(a - x)$ | | | |
| | $+k\left(\sqrt{a^2+x^2} - a\right)\cdot 2x \cdot \frac{1}{2}(a^2 + x^2)^{-1/2}$ | M1 | | |
| | $= -k(a - x) + kx\left(1 - \frac{a}{\sqrt{a^2+x^2}}\right)$ | | | |
| | $= 2kx - ka - \frac{kax}{\sqrt{a^2+x^2}}$ AG | E1 | Convincingly shown | 5 |
| (ii) | $\frac{d^2V}{dx^2} = 2k - \frac{ka\sqrt{a^2+x^2} - kax \cdot x(a^2+x^2)^{-1/2}}{a^2+x^2}$ | M1 | | |
| | $= 2k - \frac{ka^3}{(a^2+x^2)^{3/2}}$ | A1 | | |
| | $(a^2 + x^2)^{3/2} > (a^2)^{3/2} = a^3$ | M1 | | |
| | $\Rightarrow \frac{ka^3}{(a^2+x^2)^{3/2}} < k \Rightarrow V''(x) > 2k - k > 0$ | E1 | Convincingly shown | 4 |
| (iii) | $x = \frac{1}{2}a \Rightarrow V' = ka - ka - \frac{ka \cdot \frac{1}{2}a}{\sqrt{a^2+\left(\frac{1}{2}a\right)^2}} < 0$ | M1 | | |
| | $x = a \Rightarrow V' = 2ka - ka - \frac{ka^2}{\sqrt{a^2+a^2}} = ka - \frac{ka}{\sqrt{2}} > 0$ | | | |
| | Hence (as $V'$ continuous) $V' = 0$ between $\frac{1}{2}a$ and $a$. | E1 | Convincingly shown | |
| | So equilibrium. Stable as $V'' > 0$. | B1 | | 3 |

| | | | | |
|---|---|---|---|---|
| 3(i) | $800v\frac{dv}{dx} = \frac{8v^4}{v} - 8v^2$ | M1 | N2L with $P/v$ | |
| | | A1 | | |
| | $\int \frac{100dv}{v^2-v} = \int dx$ | M1 | Separate | |
| | $\int 100\left(\frac{1}{v-1} - \frac{1}{v}\right)dx = \int dx$ | M1 | Partial fractions | |
| | $100(\ln(v-1) - \ln v) = x + c$ | A1 | | |
| | $x = 0, v = 2 \Rightarrow c = -100\ln 2$ | M1 | Use condition | |
| | $100\ln\left(\frac{2(v-1)}{v}\right) = x$ | A1 | AEF, condone *m* | |
| | $v = 20 \Rightarrow x = 100\ln\left(2 \times \frac{19}{20}\right) = 100\ln 1.9$ | E1 | | |
| | $\frac{2(v-1)}{v} = e^{0.01x}$ | | | |
| | $2v - 2 = ve^{0.01x}$ | M1 | Rearrange | |
| | $v = \frac{2}{2-e^{0.01x}}$ | A1 | Cao without *m* | |
| | | | | 10 |
| (ii) | $\frac{dx}{dt} = \frac{2}{2-e^{0.01x}}$ | M1 | | |
| | $\int(2 - e^{0.01x})dx = \int 2dt$ | M1 | Separate and integrate | |
| | $2x - 100e^{0.01x} = 2t + c_2$ | A1 | | |
| | $x = 0, t = 0 \Rightarrow c_2 = -100$ | M1 | Use condition | |
| | $2x - 100e^{0.01x} = 2t - 100$ | A1 | Any correct form | |
| | $x = 100\ln 1.9 \Rightarrow t \approx 19.2$ AG | E1 | | |
| | | | | 6 |
| (iii) | $800\frac{dv}{dt} - -8v^2$ | M1 | N2L | |
| | | A1 | | |
| | $\int 100v^{-2}dv = \int -1dt$ | M1 | Separate and integrate | |
| | $-100v^{-1} = -t + c_3$ | A1 | | |
| | $t = 19.2, v = 20 \Rightarrow -5 = -19.2 + c_3$ | M1 | Use condition | |
| | $c_3 = 14.2$ | | | |
| | $v = \frac{100}{t-14.2}$ | M1 | Rearrange | |
| | | A1 | CAO | |
| | $2 = \frac{100}{t-14.2} \Rightarrow t = 64.2$ | B1 | Accept $t = 45$ (time for this part of motion) | |
| | | | | 8 |

| | | | | |
|---|---|---|---|---|
| 4(i) | $I_x = \frac{1}{2}my^2$ | B1 | | |
| | $2I_{\text{diameter}} = I_x$ | M1 | Use perpendicular axes theorem | |
| | $I_{\text{diameter}} = \frac{1}{4}my^2$ | B1 | | |
| | $I = \frac{1}{4}my^2 + mx^2$ | M1 | Use parallel axes theorem | |
| | $= m\left(\frac{1}{4}\left(\frac{1}{2}x\right)^2 + x^2\right)$ | M1 | Use $y = \frac{1}{2}x$ | |
| | $= \frac{17}{16}mx^2$ AG | E1 | Complete argument | |
| | | | | 6 |
| (ii) | Mass of slice $\approx M\left(\frac{\pi y^2 \delta x}{\frac{1}{3}\pi a^2 \cdot 2a}\right)$ | M1 | | |
| | $= \frac{3M}{2a^3}y^2\delta x$ | B1 | Deal correctly with mass/density | |
| | $I_{\text{slice}} \approx \frac{17}{16}\left(\frac{3M}{2a^3}y^2\delta x\right)x^2$ | M1 | | |
| | | A1 | | |
| | $= \frac{51M}{128a^3}x^4\delta x$ | M1 | Substitute for $y$ | |
| | $I = \int_0^{2a}\frac{51M}{128a^3}x^4\,dx$ | M1 | | |
| | $= \frac{51M}{128a^3}\left[\frac{1}{5}x^5\right]_0^{2a}$ | A1 | | |
| | $= \frac{51}{20}Ma^2$ AG | E1 | Complete argument | |
| | | | | 8 |
| (iii) | $\frac{1}{2}I\dot{\theta}^2 - Mg\cdot\frac{3}{2}a\cos\theta = -Mg\cdot\frac{3}{2}a\cos\alpha$ | M1 | Energy equation | |
| | | B1 | Position of centre of mass | |
| | | A1 | KE term | |
| | | F1 | GPE terms ft their CoM only | |
| | $\dot{\theta}^2 = \frac{3Mga}{I}(\cos\theta - \cos\alpha)$ | | | |
| | $= \frac{20g}{17a}(\cos\theta - \cos\alpha)$ | E1 | Complete argument | |
| | | | | 5 |
| (iv) | $2\dot{\theta}\ddot{\theta} = -\frac{20g}{17a}\sin\theta\,\dot{\theta}$ | M1 | Differentiate or use $I\ddot{\theta}$ = torque | |
| | $\ddot{\theta} = -\frac{10g}{17a}\sin\theta$ | A1 | | |
| | $\approx -\frac{10g}{17a}\theta$ for small $\theta$ | M1 | Use $\sin\theta \approx \theta$ | |
| | Hence SHM | E1 | | |
| | Period $2\pi\sqrt{\frac{17a}{10g}}$ | B1 | | |
| | | | | 5 |

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