OCR
RECOGNISING ACHIEVEMENT

Monday 10 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4764/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 marks)

1 An empty railway truck of mass m_0 is moving along a straight horizontal track at speed v_0. The point P is at the front of the truck. The horizontal forces on the truck are negligible. As P passes a fixed point O, sand starts to fall vertically into the truck at a constant mass rate k. At time t after P passes O the speed of the truck is v and OP $= x$.

(i) Find an expression for v in terms of m_0, v_0, k and t, and show that $x = \frac{m_0 v_0}{k} \ln\left(1 + \frac{kt}{m_0}\right)$. **[9]**

(ii) Find the speed of the truck and the distance OP when the mass of sand in the truck is $2m_0$. **[2]**

2 A uniform rod AB of length 0.5 m and mass 0.5 kg is freely hinged at A so that it can rotate in a vertical plane. Attached at B are two identical light elastic strings BC and BD each of natural length 0.5 m and stiffness $2\,\mathrm{N\,m^{-1}}$. The ends C and D are fixed at the same horizontal level as A and with AC = CD = 0.5 m. The system is shown in Fig. 2.1 with the angle BAC $= \theta$. You may assume that $\frac{1}{3}\pi \leqslant \theta \leqslant \frac{5}{3}\pi$ so that both strings are taut.

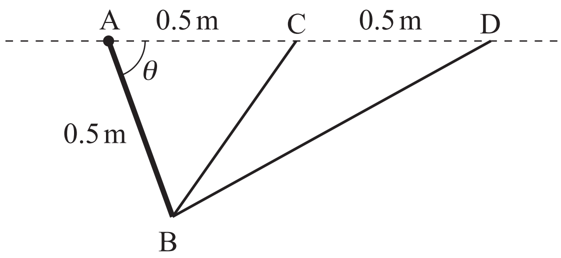

Fig. 2.1

(i) Show that the length of BC in metres is $\sin\frac{1}{2}\theta$. **[1]**

(ii) Find the potential energy, V J, of the system relative to AD in terms of θ. Hence show that

$$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 1.5\sin\theta - 1.225\cos\theta - \frac{0.5\sin\theta}{\sqrt{1.25 - \cos\theta}} - 0.5\cos\frac{1}{2}\theta.$$

[8]

(iii) Fig. 2.2 shows a graph of the function $\mathrm{f}(\theta) = 1.5\sin\theta - 1.225\cos\theta - \frac{0.5\sin\theta}{\sqrt{1.25 - \cos\theta}} - 0.5\cos\frac{1}{2}\theta$ for $0 \leqslant \theta \leqslant 2\pi$.

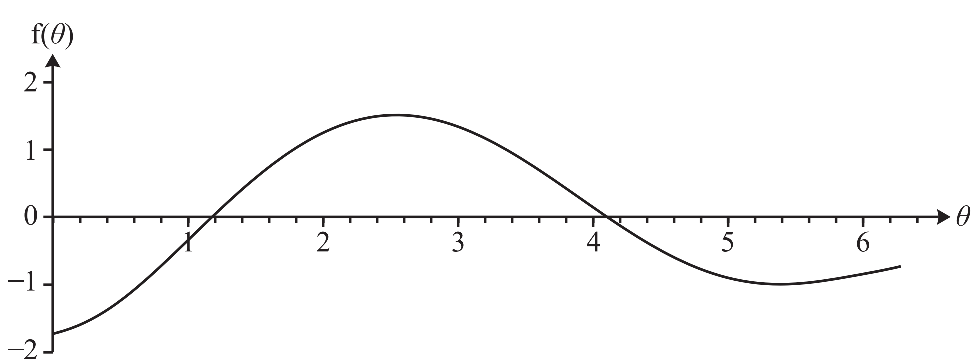

Fig. 2.2

Use the graph both to estimate, correct to 1 decimal place, the values of θ for which the system is in equilibrium and also to determine their stability. **[4]**

Section B (48 marks)

3 A model car of mass 2 kg moves from rest along a horizontal straight path. After time t s, the velocity of the car is $v\,\text{m}\,\text{s}^{-1}$. The power, P W, developed by the engine is initially modelled by $P = 2v^3 + 4v$. The car is subject to a resistance force of magnitude $6v$ N.

(i) Show that $\frac{\mathrm{d}v}{\mathrm{d}t} = (1 - v)(2 - v)$ and hence show that $t = \ln\frac{2 - v}{2(1 - v)}$. **[10]**

(ii) Hence express v in terms of t. **[2]**

Once the power reaches 4.224 W it remains at this constant value with the resistance force still acting.

(iii) Verify that the power of 4.224 W is reached when $v = 0.8$ and calculate the value of t at this instant. **[2]**

(iv) Find v in terms of t for the motion at constant power. Deduce the limiting value of v as $t \to \infty$. **[10]**

4 A uniform lamina of mass m is in the shape of a sector of a circle of radius a and angle $\frac{1}{3}\pi$. It can rotate freely in a vertical plane about a horizontal axis perpendicular to the lamina through its vertex O.

(i) Show by integration that the moment of inertia of the lamina about the axis is $\frac{1}{2}ma^2$. **[6]**

(ii) State the distance of the centre of mass of the lamina from the axis. **[1]**

The lamina is released from rest when one of the straight edges is horizontal as shown in Fig. 4.1. After time t, the line of symmetry of the lamina makes an angle θ with the downward vertical.

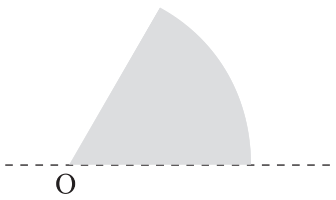

Fig. 4.1

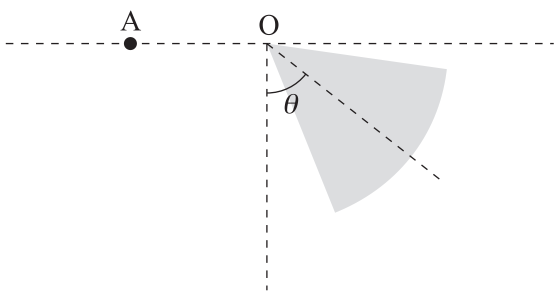

Fig. 4.2

(iii) Show that $\dot{\theta}^2 = \frac{4g}{\pi a}(2\cos\theta + 1)$. **[4]**

(iv) Find the greatest speed attained by any point on the lamina. **[4]**

(v) Find an expression for $\ddot{\theta}$ in terms of θ, a and g. **[2]**

The lamina strikes a fixed peg at A where $\text{AO} = \frac{3}{4}a$ and is horizontal, as shown in Fig. 4.2. The collision reverses the direction of motion of the lamina and halves its angular speed.

(vi) Find the magnitude of the impulse that the peg gives to the lamina. **[4]**

(vii) Determine the maximum value of θ in the subsequent motion. **[3]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.