OCR

Wednesday 18 June 2014 – Afternoon

A2 GCE MATHEMATICS

4724/01 Core Mathematics 4

QUESTION PAPER

3170841111

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4724/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Express $x+\frac{1}{1-x}+\frac{2}{1+x}$ as a single fraction, simplifying your answer. **[3]**

2 The points $O(0, 0, 0)$, $A(2, 8, 2)$, $B(5, 5, 8)$ and $C(3, -3, 6)$ form a parallelogram $OABC$. Use a scalar product to find the acute angle between the diagonals of this parallelogram. **[5]**

3 **(i)** Find the first three terms in the expansion of $(1-2x)^{-\frac{1}{2}}$ in ascending powers of x, where $|x|<\frac{1}{2}$. **[3]**

(ii) Hence find the coefficient of x^2 in the expansion of $\frac{x+3}{\sqrt{1-2x}}$. **[2]**

4 Show that $\int_0^{\frac{1}{4}\pi}\frac{1-2\sin^2 x}{1+2\sin x\cos x}\,\mathrm{d}x=\frac{1}{2}\ln 2$. **[5]**

5 The equations of three lines are as follows.

Line A: $\mathbf{r}=\mathbf{i}+4\mathbf{j}+\mathbf{k}+s(-\mathbf{i}+2\mathbf{j}+2\mathbf{k})$

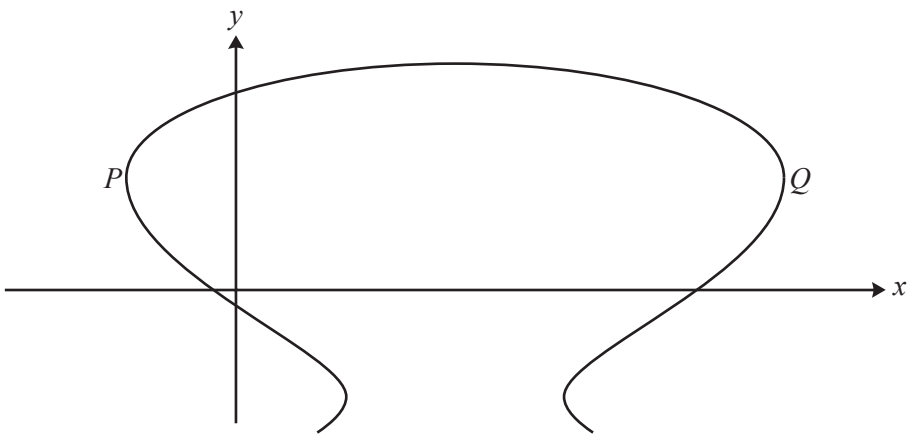
Line B: $\mathbf{r}=2\mathbf{i}+8\mathbf{j}+2\mathbf{k}+t(\mathbf{i}+3\mathbf{j}+5\mathbf{k})$

Line C: $\mathbf{r}=-\mathbf{i}+19\mathbf{j}+15\mathbf{k}+u(2\mathbf{i}-4\mathbf{j}-4\mathbf{k})$

(i) Show that lines A and B are skew. **[4]**

(ii) Determine, giving reasons, the geometrical relationship between lines A and C. **[2]**

6



The diagram shows the curve with equation $x^2+y^3-8x-12y=4$. At each of the points P and Q the tangent to the curve is parallel to the y-axis. Find the coordinates of P and Q. **[8]**

7 A curve has parametric equations

$$x = 2\sin t, \quad y = \cos 2t + 2\sin t$$

for $-\frac{1}{2}\pi \leqslant t \leqslant \frac{1}{2}\pi$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - 2\sin t$ and hence find the coordinates of the stationary point. **[5]**

(ii) Find the cartesian equation of the curve. **[3]**

(iii) State the set of values that x can take and hence sketch the curve. **[3]**

8 **(i)** Use division to show that $\frac{t^3}{t+2} \equiv t^2 - 2t + 4 - \frac{8}{t+2}$. **[3]**

(ii) Find $\int_1^2 6t^2 \ln(t+2)\,\mathrm{d}t$. Give your answer in the form $A + B\ln 3 + C\ln 4$. **[6]**

9 Express $\frac{2+x^2}{(1+2x)(1-x)^2}$ in partial fractions and hence show that $\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2}\mathrm{d}x = \frac{1}{2}\ln\frac{3}{2} + \frac{1}{3}$. **[9]**

10 A container in the shape of an inverted cone of radius 3 metres and vertical height 4.5 metres is initially filled with liquid fertiliser. This fertiliser is released through a hole in the bottom of the container at a rate of $0.01\,\mathrm{m}^3$ per second. At time t seconds the fertiliser remaining in the container forms an inverted cone of height h metres.

[The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.]

(i) Show that $h^2\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{9}{400\pi}$. **[5]**

(ii) Express h in terms of t. **[4]**

(iii) Find the time it takes to empty the container, giving your answer to the nearest minute. **[2]**

END OF QUESTION PAPER