OCR
Oxford Cambridge and RSA

GCE

Mathematics

Unit **4724**: Core Mathematics 4

Advanced GCE

Mark Scheme for June 2016

Annotations and abbreviations

Annotation in scoris	Meaning
✔ and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0 , M1	Method mark awarded 0, 1
A0 , A1	Accuracy mark awarded 0, 1
B0 , B1	Independent mark awarded 0, 1
SC	Special case
∧	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a   Annotations should be used whenever appropriate during your marking.

        The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

        For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

        Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c   The following types of marks are available.

        M
        A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

        A
        Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

        B
        Mark for a correct result or statement independent of Method marks.

        E
        A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g   Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

         Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$2x$ seen in quotient and $4x^3 + 2x$ seen in division	**B1**		if **B0M0, B2** for quotient is $2x + 4$ or for remainder is $-7x + 8$; **B3** for both of these
			$8x^2 + kx \, [+ 12]$ seen in division	**M1**	**NB** $k = -7$	
			$2x + 4$ seen and $-7x + 8$ seen isw	**A1**		ignore wrong labelling
				[3]		
2			$\cos 8x$ seen in integrand	**M1**		
			$F[x] = Ax + B \sin 8x$ oe	**M1***	A and B are non-zero constants	
			$F[x] = 6x - \dfrac{3}{8} \sin 8x$	**A1**		
			$F[\dfrac{1}{8}\pi] - F[\dfrac{1}{16}\pi]$	**M1*dep**		
			$\dfrac{3}{8}\pi + \dfrac{3}{8}$ oe	**A1**		allow eg $0.375\pi + 0.375$ or fractions not in lowest terms
				[5]		

Question			Answer	Marks	Guidance	
3			$2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$	B1	from differentiation of y^2	
			$\sin 2x\dfrac{\mathrm{d}y}{\mathrm{d}x}+2y\cos 2x$	M1	correct use of Product Rule	allow sign error or one incorrect coefficient
			$\sin 2x\dfrac{\mathrm{d}y}{\mathrm{d}x}+2y\cos 2x-\dfrac{1}{x^2}+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$	A1		
			$(\sin 2x+2y)\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{x^2}-2y\cos 2x$ oe	M1	collection of like terms on separate sides, need not be factorised	must be two terms in $\dfrac{\mathrm{d}y}{\mathrm{d}x}$
			$[\dfrac{\mathrm{d}y}{\mathrm{d}x}=]\dfrac{1-2x^2 y\cos 2x}{(\sin 2x+2y)x^2}$ oe isw	A1	eg $[\dfrac{\mathrm{d}y}{\mathrm{d}x}=]\dfrac{x^{-2}-2y\cos 2x}{(\sin 2x+2y)}$	**A0** for eg $y=\ldots$
				[5]		
4			$Ax^{\frac{2}{3}}\ln x-\displaystyle\int Bx^{\frac{2}{3}}\times\dfrac{1}{x}\,\mathrm{d}x$ oe	M1*	A and B are non-zero constants;	
			$\dfrac{3}{2}x^{\frac{2}{3}}\ln x-\displaystyle\int\dfrac{3}{2}x^{\frac{2}{3}}\times\dfrac{1}{x}\,\mathrm{d}x$	A1	ignore $+c$	NB $\dfrac{3}{2}x^{\frac{2}{3}}\ln x-\displaystyle\int\dfrac{3}{2}x^{-\frac{1}{3}}\,\mathrm{d}x$ Allow both marks if dx omitted
			$\mathrm{F}[x]=\dfrac{3}{2}x^{\frac{2}{3}}\ln x-\dfrac{3/2}{2/3}x^{\frac{2}{3}}$	A1	ignore limits for first three marks	
			$\mathrm{F}[8]-\mathrm{F}[1]$	M1*dep	and also dependent on integration of their $\dfrac{3}{2}x^{-\frac{1}{3}}$	
			$18\ln 2-\dfrac{27}{4}$ cao	A1		**NB A0** for $6\ln 8-\dfrac{27}{4}$
				[5]		

Question			Answer	Marks	Guidance	
5	(i)		$3 + 5t = 1 + 2s$ $2 - 3t = 4 - s$ $-5 + t = 5 + 3s$	M1	attempt to solve any two of these simultaneously to obtain a value of s or t	
			$t = -2$ and $s = -4$	A1		
			substitution of their s and t in other equation to obtain eg $-7 = -7$ oe (1st or 3rd equation) or eg $8 = 8$ oe (2nd equation)	B1	may be embedded, eg $-5 + -2 = 5 + 3 \times -4$	**B0** if any subsequent arithmetic errors seen eg $-5 + -2 = 5 + 3 \times -4$ so $7 = -7$
			lines meet at $(-7, 8, -7)$	A1	allow in vector form	
				[4]		
5	(ii)		$\begin{pmatrix} -4 \\ 2 \\ -6 \end{pmatrix} = -2 \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ oe seen	B1	do not allow eg $\begin{pmatrix} -4 \\ 2 \\ -6 \end{pmatrix} \div \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = -2$	or $\begin{pmatrix} 7 \\ 1 \\ 14 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 9 \end{pmatrix}$
			common point identified and justified eg by substitution of correct value of s or u eg $s = 3$ or eg $u = \dfrac{3}{2}$	B1		$= 3 \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$
				[2]		
			Alternatively substitution of eg $s = 3 - 2u$	B1	or eg $u = \dfrac{3}{2} s - \dfrac{1}{2}$	or show one pair of equations consistent
			and completion to $r = \begin{pmatrix} 7 \\ 1 \\ 14 \end{pmatrix} + u \begin{pmatrix} -4 \\ 2 \\ -6 \end{pmatrix}$	B1	and completion to $r = \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} + u \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$	show another pair consistent
				[2]		

Question			Answer	Marks	Guidance	
6			$\dfrac{du}{dx} = 2x$ oe or $\dfrac{dx}{du} = \dfrac{1}{2}\left(u \pm 2\right)^{-\frac{1}{2}}$ oe	**M1**		
			$\dfrac{Ax^2 + B}{2}$ or better from replacing dx NB $\dfrac{6x^3 + 4x}{2x} = \dfrac{6x^2 + 4}{2}$	**M1**		or substitution of $x = (u \pm 2)^{\frac{1}{2}}$ in denominator from $\dfrac{dx}{du}$
			substitution of $x^2 = u \pm 2$ or $x = (u \pm 2)^{\frac{1}{2}}$ in numerator	**M1**	**NB** $3(u+2) + 2$ or $3(u+2)^{\frac{3}{2}} + 2(u+2)^{\frac{1}{2}}$	
			$\int \left(\dfrac{3u + 8}{\sqrt{u}}\right)\left[du\right]$ oe	**A1**	$\dfrac{3(u+2) + 2}{\sqrt{u}}$ or better	
			$\dfrac{3u^{\frac{3}{2}}}{\frac{3}{2}} + \dfrac{8u^{\frac{1}{2}}}{\frac{1}{2}}$ oe	**A1**	or $6u^{\frac{3}{2}} + 16u^{\frac{1}{2}} - 4u^{\frac{3}{2}}$ from integration by parts	
			$2(x^2 - 2)^{\frac{3}{2}} + 16(x^2 - 2)^{\frac{1}{2}} + c$ cao	**A1**	allow $2(x^2 - 2)^{\frac{1}{2}}(x^2 + 6) + c$ for final mark, **A0** if du not seen at some stage in the integral	must see constant of integration here or in previous line and coefficients must be simplified for final **A1**
				[6]		

Question			Answer	Marks	Guidance	
7			$nk = -6$ soi	B1	allow $nkx = -6x$ and /or	
					$\dfrac{n(n-1)k^2}{2!}x^2 = 30x^2$ for first two	
			$\dfrac{n(n-1)k^2}{2!} = 30$ soi	B1	marks	
						NB
						$\dfrac{n(n-1)\times 36}{2\times n^2} = 30$ oe
			substitution of $n = \pm\dfrac{6}{k}$ or $k = \pm\dfrac{6}{n}$ or k=$\pm\sqrt{\dfrac{60}{n(n-1)}}$ oe to eliminate one variable from their equations	M1	allow omission of brackets	$(-\dfrac{6}{k})(\dfrac{-6}{k}-1)k^2 = 60$ oe
			$n = -1.5$ oe	A1	eg allow $-\dfrac{6}{4}$	
			$k = 4$	A1		
			expansion is valid for $\|x\| < \dfrac{1}{4}$ or $-\dfrac{1}{4} < x < \dfrac{1}{4}$ isw	B1FT		
				[6]	**FT** their k	
8	(i)		$3\sin\alpha \times 2\cos\alpha + 2\cos\alpha \times 4\sin\alpha + -1\times 3$	M1		allow one sign error or one coefficient error for **M1**
			$6\sin\alpha\cos\alpha + 8\sin\alpha\cos\alpha - 3 = 0$ soi	A1		
			substitution of sinα cosα = ½ sin 2α oe	M1	**NB** $7\sin 2\alpha = 3$	or squaring both sides and correct substitution from Pythagoras
			$\alpha =$ awrt $12.7°$	A1	awrt 0.221	
			$\alpha =$ awrt $77.3°$	A1	awrt 1.35	if **A0A0**, **SC1** for 13° and 77° or 0.22 and 1.4
				[5]		

Question			Answer	Marks	Guidance	
8	(ii)		their α = 12.7° substituted in $\overrightarrow{OA}$ and $\overrightarrow{OB}$; or in $\left|\overrightarrow{OA}\right|$ and $\left|\overrightarrow{OB}\right|$	M1		**NB** $\begin{pmatrix} 0.6589.. \\ 1.9511.. \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1.9511.. \\ 0.8785.. \\ 3 \end{pmatrix}$
			$\sqrt{(3\sin\alpha)^2 + (2\cos\alpha)^2 + (-1)^2}$ or $\sqrt{(2\cos\alpha)^2 + (4\sin\alpha)^2 + 3^2}$	M1*	allow omission of brackets, one slip in arithmetic and one sign error;	
			$\frac{1}{2}\sqrt{9\sin^2\alpha + 4\cos^2\alpha + 1}\sqrt{4\cos^2\alpha + 16\sin^2\alpha + 9}$	M1*dep	may be implied by numerical value for lengths; allow one sign or coefficient error α may be unspecified or any acute angle for these method marks	**NB** √5.241..= 2.289… and √13.579.. = 3.685… **NB** hypotenuse is 4.34 and other angles in triangle are 58.2° and 31.8°
			awrt 4.22	A1 [4]		
9	(i)		sin*t*sin2*t* = 0 oe seen	M1		**NB** $t = 0$, ½π, π
			(0, 0) (1, 0) and (2, 0) or $x = 0$, $x = 1$, $x = 2$ cao	A2 [3]	**A1** for two of three correct	deduct 1 mark if all three correct plus extra values if **A0**, allow **SC1** for $t = 0$, ½π, π if unsupported, full marks for all three values correct

Question			Answer	Marks	Guidance	
9	**(ii)**		$\left[\dfrac{\mathrm{d}y}{\mathrm{d}t}\right] = 2\sin t\cos 2t + \cos t\sin 2t$	**B1**	or $4\sin t\cos^2 t - 2\sin^3 t$	
			$\dfrac{(2\sin t\cos 2t + \cos t\sin 2t)}{\sin t}$ or $\dfrac{(4\sin t\cos^2 t - 2\sin^3 t)}{\sin t}$	**M1**	allow sign errors and/or one incorrect coefficient	
			substitution of $\sin 2t = 2\sin t\cos t$ in their	**M1**	may be seen before differentiation	
			$\dfrac{(2\sin t\cos 2t + \cos t\sin 2t)}{\sin t}$ and completion to			
			$2\cos 2t + 2\cos^2 t$ www **NB AG**	**A1**	at least one correct intermediate step needed	
			eg $2(2\cos^2 t - 1) + 2\cos^2 t = 0$ or $2\cos 2t + 2\times\frac{1}{2}(1 + \cos 2t) = 0$	**M1**	use of double angle formula to obtain quadratic equation in eg $\cos t$ or linear equation in $\cos 2t$; may be seen before differentiation	mark intent: allow sign error, bracket error, omission of one coefficient
			$(1 + \dfrac{1}{\sqrt{3}}, \dfrac{-4}{3\sqrt{3}})$ oe isw	**A1**		eg $(\dfrac{\sqrt{3}+3}{3}, -\dfrac{4\sqrt{3}}{9})$
			$(1 - \dfrac{1}{\sqrt{3}}, \dfrac{4}{3\sqrt{3}})$ oe isw	**A1** [7]	if **A0**, **A0**, allow **A1** for both x values correct	

Question		Answer	Marks	Guidance	
9	(iii)	$y = 2(1 - \cos^2 t)\cos t$ oe may be implicit equation, may be implied by partial substitution for $\cos t$ eg $(1 - x)^2 + \dfrac{y}{2\cos t} = 1$	M1	or $\dfrac{dy}{dx} = 6\cos^2 t - 2$	use of double angle formula (and Pythagoras) to obtain expression for y or $\dfrac{dy}{dx}$ in terms of $\cos t$ only;
		$y = 2(1 - (1-x)^2)(1-x)$	M1	or $\dfrac{dy}{dx} = 6(1-x)^2 - 2$	substitution of $\cos t = \pm 1 \pm x$ to obtain expression in terms of x only allow sign errors, bracket errors or minor slips in arithmetic eg omission of 2 for these method marks
		$y = 2x^3 - 6x^2 + 4x$ or $y = 2x(x^2 - 3x + 2)$ or $y = 2x(x-1)(x-2)$ oe cao	A1	integration and substitution of eg (0, 0) to obtain correct answer must see $y =$ at some stage for **A1**	
			[3]		
9	(iv)	cubic with two turning points and of correct orientation through (0, 0)	M1		
		x-intercepts correct and only for $0 \le x \le 2$	A1		
			[2]		

Question			Answer	Marks	Guidance	
10	**(i)**		$\dfrac{A}{(x+4)} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+1)^2}$	**B1**		may be awarded later
			$[16 + 5x - 2x^2] = A(x+1)^2 + B(x+1)(x+4) + C(x+4)$	**M1**		allow sign errors only
			$A = -4$	**A1**	**NB** $36 = -9A$	if **B0M0,** allow **SC3** for $\dfrac{2x+5}{(x+1)^2} - \dfrac{4}{x+4}$
			$C = 3$	**A1**	$9 = 3C$	
			$B = 2$ isw	**A1**	$-2 = A + B,\ 5 = 2A + 5B + C$ $16 = A + 4B + 4C$	
				[5]	**NB** $\dfrac{-4}{(x+4)} + \dfrac{2}{(x+1)} + \dfrac{3}{(x+1)^2}$	

Question		Answer	Marks	Guidance	
10	**(ii)**	$\int \dfrac{dy}{y} = \int \dfrac{16 + 5x - 2x^2}{(x+1)^2(x+4)}\,dx$	**B1**	separation of variables	allow omission of integral signs; allow omission of dy or dx but not both
		$\dfrac{3}{(x+1)^2} + \dfrac{2}{(x+1)} - \dfrac{4}{(x+4)}$ seen in RHS, may be embedded	**M1***	**FT** their partial fractions if two or three terms; ignore LHS	may be implied by correct integration of two of their terms
		$\dfrac{-3}{x+1} + 2\ln(x+1) - 4\ln(x+4) + c$	**A1FT**	**FT** their non-zero 3, 2 and 4; allow recovery from $x + 1^2$ in denominator; if brackets in log terms omitted, allow **A1** if recovery seen in substitution	allow omission of $+ c$ here
		$\ln(\dfrac{1}{256}) = -3 + 2\ln 1 - 4\ln 4 + c$	**M1*dep**	substitution of $x = 0$ and $y = \dfrac{1}{256}$; allow if error in manipulation following integration;	$+ c$ must be included and LHS must be correctly obtained
		$c = 3$ cao	**A1**	or $A = e^{-3}$ from $y = Ae^{\frac{-3}{x+1}} \dfrac{(x+1)^2}{(x+4)^4}$	
		$\ln y = \dfrac{-3}{2+1} + 2\ln(2+1) - 4\ln(2+4) + 3$	**M1*dep**	substitution of $x = 2$; dependent on award of previous **M1M1** and numerical value found for c	allow **M1** if substitution follows incorrect manipulation eg to find expression for y
		$y = \dfrac{e^2}{144}$ oe	**A1**		
			[7]		

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