OCR
Oxford Cambridge and RSA

Wednesday 14 June 2017 – Morning

A2 GCE MATHEMATICS

4733/01 Probability & Statistics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4733/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **8** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 The governors of a school wish to investigate the opinions of the parents and guardians of the pupils. The secretary of the governors distributes a questionnaire to all parents and guardians who are present at a particular Parents' Evening.

(i) Explain why this method of sampling may not give reliable results. **[1]**

(ii) Suggest a better method of sampling, using random numbers. **[2]**

2 A continuous random variable W has the distribution $N(\mu, \sigma^2)$. It is given that $P(W < 70.0) = 0.8$ and $P(W > 81.0) = 0.1$. Find the value of a if $P(W < a) = 0.1$. **[7]**

3 The number of mistakes made in one session by teams of code transmitters is known to be a random variable with the distribution $Po(\lambda)$. In the past it has been found that $\lambda = 13$. Over a holiday period a new team is used and on one randomly chosen session the new team makes 23 mistakes. Test at the 1% significance level whether the new team makes on average more than 13 mistakes in one session. **[7]**

4 The acidity of paper is measured on the numerical pH scale. It is known that the writing paper generally used by a certain author has a mean pH of 6.3. The pH, X units, of a random sample of 36 pieces of paper thought to have been used by this author was measured, and the results are summarised as follows.

$$n = 36 \qquad \sum x = 222.48 \qquad \sum x^2 = 1380.5264$$

(i) Test at the 5% significance level whether the pH of the paper from which this sample is drawn differs from 6.3. **[11]**

(ii) State where the Central Limit Theorem was used in your test in part **(i)**. **[1]**

5 One game of roulette consists of throwing a ball onto a spinning wheel, which has 37 slots, numbered 0 to 36. On each throw the ball is equally likely to come to rest in any one of the 37 slots on the wheel, independently of all other throws. The number of the slot in which the ball comes to rest is the score of the game.

(i) Use a suitable approximation to find the probability that on 74 throws of the ball a score of 0 is obtained at least four times. Justify your approximation. **[4]**

(ii) 18 of the 37 possible scores are called "impair". 148 games are played. The probability that impair is obtained at least N times is less than 0.0025. Use a suitable approximation to find the smallest possible value of N. **[6]**

6 The editor of a scientific journal receives articles, for possible publication, at random times throughout the year.

(i) State two conditions needed for the number of articles received in one randomly chosen week to be well modelled by a Poisson distribution. **[2]**

Assume now that the number of articles received in one week can be modelled by the distribution Po(2.4).

(ii) Find the probability that in a randomly chosen 2-week period fewer than 5 articles are received. **[2]**

(iii) The number of articles received in one randomly chosen week is denoted by R. Given that $\mathrm{P}(R = r) = 2.5 \times \mathrm{P}(R = r + 1)$, use an algebraic method to find the value of r. **[4]**

(iv) Use an appropriate approximation to find the probability that in a randomly chosen 50-week period at least 140 articles are received. **[5]**

7 A continuous random variable X has probability density function

$$\mathrm{f}(x) = \begin{cases} \frac{1}{64}x(16 - x^2) & 0 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the value of $\mathrm{E}(X)$. **[3]**

(ii) The upper quartile q is defined by the condition $\mathrm{P}(X \leqslant q) = \frac{3}{4}$.

(a) Show that q satisfies the equation

$$q^4 - 32q^2 + 192 = 0.$$ **[3]**

(b) Hence find the exact value of q. **[3]**

8 A random variable X has the distribution $\mathrm{B}(60, p)$. A hypothesis test is to be carried out, at the 5% significance level, of the null hypothesis H_0: $p = 0.95$ against the alternative hypothesis H_1: $p > 0.95$.

(i) Explain why a normal approximation cannot be used. **[1]**

(ii) Verify that the critical region for the test is $X = 60$. **[4]**

(iii) State the value(s) of p for which a Type I error could occur, and give the corresponding probability or probabilities of a Type I error. **[2]**

(iv) Find the range of values of p for which the probability that a Type II error occurs is less than 0.6. **[4]**

END OF QUESTION PAPER