OCR
Oxford Cambridge and RSA

Monday 25 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4764/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 marks)

1 A rocket is launched vertically upwards from rest. The initial total mass of the rocket and its fuel is 1500 kg. The propulsion system of the rocket burns fuel at a constant rate of $20\,\mathrm{kg\,s^{-1}}$ and the fuel is ejected vertically downwards with a speed of $1800\,\mathrm{m\,s^{-1}}$ relative to the rocket. The only other force acting on the rocket is its weight. The acceleration due to gravity is constant throughout the motion. At time t s after launch, where $t \leqslant 60$, the speed of the rocket is $v\,\mathrm{m\,s^{-1}}$. The rocket stops burning fuel 60 seconds after the launch.

(i) Show that, while fuel is being burnt,

$$\frac{\mathrm{d}v}{\mathrm{d}t} - \frac{1800}{75-t} = -g.$$

[6]

(ii) Solve this differential equation to find an expression for v in terms of t. Calculate, correct to 3 significant figures, the speed of the rocket when $t = 30$. **[6]**

2 Fig. 2 shows a uniform rigid rod AB of mass m and length a. The rod is freely hinged at A so that it can rotate in a vertical plane. The end B of the rod is attached to one end of a light elastic string BC of modulus λ and natural length a. The other end of the string, C, is fixed at a point vertically above A, where the distance AC is a. The rod makes an angle 2θ with the downward vertical, where $0 < \theta \leqslant \frac{\pi}{4}$.

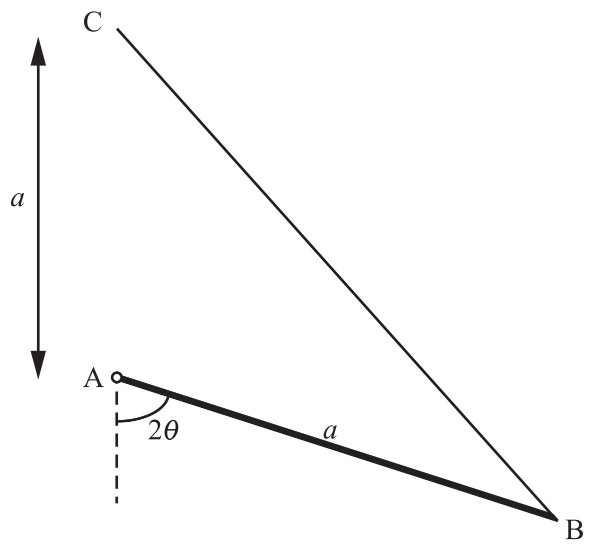

Fig. 2

(i) Find the potential energy, V, of the system relative to a situation in which the rod AB is horizontal, and hence show that

$$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 2a\sin\theta(\lambda + mg\cos\theta - 2\lambda\cos\theta).$$

[5]

(ii) Show that if there is a position of equilibrium then $mg < \lambda \leqslant mg\left(1 + \frac{\sqrt{2}}{2}\right)$. Deduce that any such position of equilibrium is stable. **[7]**

Section B (48 marks)

3 A particle P of mass m kg is held at rest at a point O on a fixed plane inclined at an angle of 30° to the horizontal. P is released and moves down a line of greatest slope. The total resistance acting on P is kv^2 N, where k is a positive constant and where $v\,\mathrm{m\,s^{-1}}$ is the velocity of P when P has travelled a distance x m from O.

(i) Write down an equation of motion for P and show that

$$v^2 = \frac{mg}{2k}\left(1 - \mathrm{e}^{-\frac{2kx}{m}}\right).$$

[7]

It is given that $k = 0.2$, $m = 3$ and P travels a distance of 1.5 m before reaching the foot of the plane.

(ii) Show, by integration, that the work done against the resistance in the first 1.5 m of the motion is

$$\frac{9}{4}g\left(5\mathrm{e}^{-0.2} - 4\right)\mathrm{J},$$

and verify that this is equal to the loss in mechanical energy of P. **[6]**

At the bottom of the slope the particle P moves onto a smooth horizontal plane without loss of speed; a force then acts on P. This force, which acts in the direction of motion of P, has a magnitude of $\ln(2t + 1)$ N where t s is the time from the moment that P begins to move horizontally. When travelling horizontally there are no resistances to motion acting on P.

(iii) Given that the impulse of the force over the first T seconds is 20 N s show that T satisfies

$$T = \frac{40 + 2T - \ln(2T + 1)}{2\ln(2T + 1)}.$$

[7]

(iv) Use an iterative process based on the equation in part **(iii)**, with a suitable starting value, to find T correct to 3 decimal places. **[2]**

(v) Find the velocity of P after P has travelled horizontally for T seconds. **[2]**

Question 4 begins on page 4.

4 **(i)** Show, by integration, that the moment of inertia of a thin uniform rigid rod of length $3a$ and mass $2m$ about an axis through one end and perpendicular to the rod is $6ma^2$. **[4]**

A pendulum consists of a thin uniform rigid rod AB of length $3a$ and mass $2m$ and a uniform circular disc of radius a, mass m and centre C. The end B of the rod is rigidly attached to a point on the circumference of the disc in such a way that ABC is a straight line. The pendulum is initially at rest with B vertically below A. The pendulum is free to rotate in a vertical plane about a smooth fixed horizontal axis passing through A where the axis is perpendicular to the plane of the disc (see Fig. 4). At time $t = 0$ the pendulum is set in motion with initial angular velocity ω.

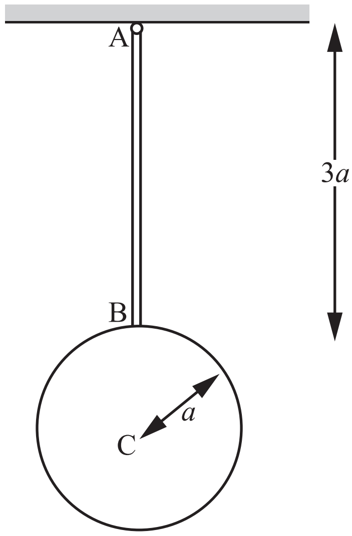

Fig. 4

(ii) Show that the angular velocity $\dot{\theta}$ when the pendulum makes an angle θ with the downward vertical is given by

$$\dot{\theta}^2 = \omega^2 + k(\cos\theta - 1),$$

where k is a constant to be determined in terms of a and g. **[8]**

(iii) Find, in terms a, g and θ, the angular acceleration of the pendulum. **[2]**

The pendulum is making small oscillations about the equilibrium position.

(iv) Show that the motion is approximately simple harmonic, and find the approximate period of oscillations in terms of a and g. **[2]**

(v) Now suppose θ is such that θ^3 and higher powers can be neglected. Show that

$$\frac{dt}{d\theta} \approx \left(\omega^2 - \frac{1}{2}k\theta^2\right)^{-\frac{1}{2}},$$

and hence, by integration, express θ in terms of k, ω and t. **[8]**

END OF QUESTION PAPER