OCR
Oxford Cambridge and RSA

Friday 15 June 2018 – Afternoon

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4727/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 **(i)** Find the shortest distance from the point $(3, -1, -2)$ to the plane with equation $x - 2y + 4z = 11$. **[2]**

(ii) Find a cartesian equation of the plane which passes through the point $(3, -1, -2)$ and is parallel to the plane $x - 2y + 4z = 11$. **[2]**

2 A multiplicative group G consists of the elements $\{1, z, z^2, z^3, z^4, z^5\}$.

(i) State the order of the element z^4. **[1]**

(ii) List all the subgroups of G. **[3]**

The group H consists of the set $\{1, 2, 3, 4, 5, 6\}$ with the operation of multiplication modulo 7.

(iii) Determine whether G is isomorphic to H. **[2]**

3 It is given that the differential equation

$$2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 10\mathrm{e}^{-x}$$

has a particular integral of the form $ax\mathrm{e}^{-x}$, where a is a constant. Solve the differential equation subject to the conditions $y = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{9}{2}$ when $x = 0$. **[10]**

4 The operation $*$ is defined by $x * y = xy + k(x + y) + 12$, where x and y are real numbers and k is a real constant. It is given that the operation $*$ is associative.

(i) Show that there are two possible values for k, one of which is 4. **[4]**

(ii) In the case where $k = 4$, determine whether the set of real numbers, under the operation $*$, forms a group. **[4]**

5 The differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2y}{1-x} = 4(1-x^2)\sqrt{y}$$

is to be solved for $x < 1$. Use the substitution $u = \sqrt{y}$ to find the general solution of the differential equation, expressing your answer in the form $y = \mathrm{f}(x)$. **[8]**

6 **(i)** Use de Moivre's theorem to find an expression for $\cot 7\theta$ in terms of $\cot\theta$ and hence find the exact roots of the equation $u^6 - 21u^4 + 35u^2 - 7 = 0$. **[7]**

(ii) State the exact roots of the equation $v^3 - 21v^2 + 35v - 7 = 0$, justifying your answer. Hence find the exact value of

$$\frac{\cot^2\left(\frac{1}{14}\pi\right)\cot^2\left(\frac{3}{14}\pi\right) + \cot^2\left(\frac{3}{14}\pi\right)\cot^2\left(\frac{5}{14}\pi\right) + \cot^2\left(\frac{5}{14}\pi\right)\cot^2\left(\frac{1}{14}\pi\right)}{\cot\left(\frac{1}{14}\pi\right)\cot\left(\frac{3}{14}\pi\right)\cot\left(\frac{5}{14}\pi\right)}. \quad \textbf{[4]}$$

7 The plane Π_1 passes through the points (5, 2, −2), (4, 0, −1) and (2, 1, −3).

(i) Find a cartesian equation of the plane Π_1. **[5]**

The line l_1 has equation $\frac{x}{2} = \frac{y-4}{-1} = \frac{z+3}{3}$.

(ii) Find the acute angle between Π_1 and l_1. **[3]**

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} p \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} q \\ -6 \\ 12 \end{pmatrix}$ and lies in Π_1.

(iii) Find the value of p and show that $q = 12$. **[3]**

The plane Π_2 is perpendicular to Π_1 and l_2 lies in Π_2.

(iv) Find an equation of Π_2, giving your answer in the form $\mathbf{r}.\mathbf{n} = d$. **[3]**

8 **(i)** Show that, if $z \neq \pm 1$ and $z \neq 0$,

$$\sum_{r=1}^{n} z^{2r-1} = \frac{1 - z^{2n}}{z^{-1} - z}. \quad \textbf{[2]}$$

(ii) Hence show that, if $\sin\theta \neq 0$,

$$\sum_{r=1}^{n} \sin(2r-1)\theta = \frac{\sin^2 n\theta}{\sin\theta}. \quad \textbf{[6]}$$

(iii) Hence find the exact value of

$$\int_0^{\frac{1}{6}\pi} \frac{\sin^2 3\theta}{\sin\theta}\,\mathrm{d}\theta. \quad \textbf{[3]}$$

END OF QUESTION PAPER