



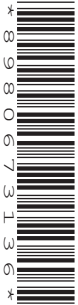
Oxford Cambridge and RSA

Monday 18 October 2021 – Afternoon

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- the Insert
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient Rule } y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{1}{2}\theta^2, \tan \theta \approx \theta \text{ where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi \right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \quad \text{where} \quad S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^n C_r p^r q^{n-r}$ where $q = 1 - p$

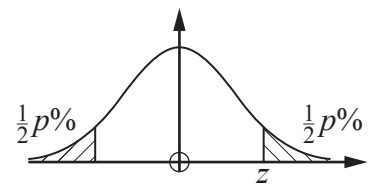
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions.

Section A (60 marks)

- 1 (a) Express $x^2 + 8x + 2$ in the form $(x + a)^2 + b$. [2]
- (b) Write down the coordinates of the turning point of the curve $y = x^2 + 8x + 2$. [1]
- (c) State the transformation(s) which map(s) the curve $y = x^2$ onto the curve $y = x^2 + 8x + 2$. [2]

- 2 Solve the equation $\sin 2x = 0.3$ for $0^\circ \leq x \leq 180^\circ$. Give your answer(s) correct to 1 decimal place. [2]

- 3 (a) Determine, in terms of k , the coordinates of the point where the lines with the following equations intersect.

$$x + y = k$$

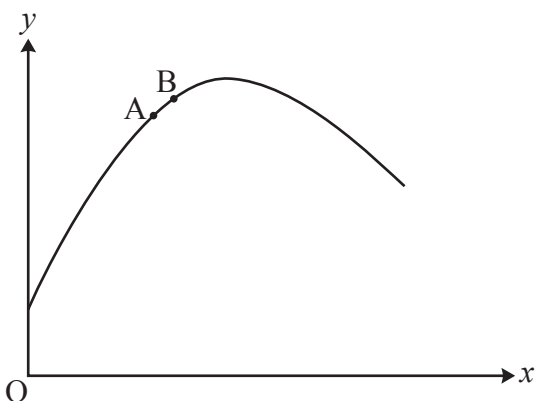
$$2x - y = 1$$

[3]

- (b) Determine, in terms of k , the coordinates of the points where the line $x + y = k$ crosses the curve $y = x^2 + k$. [4]

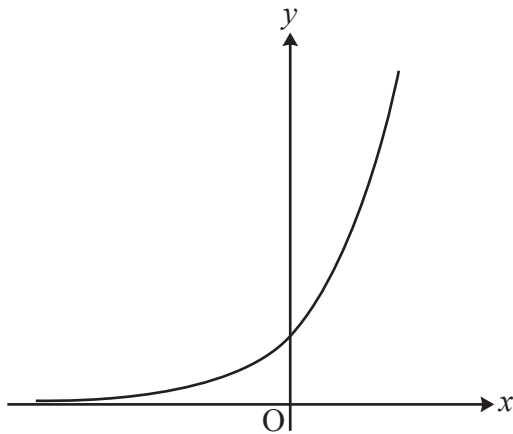
- 4 The diagram shows points A and B on the curve $y = \left(\frac{x}{4}\right)^{-x}$.

The x -coordinate of A is 1 and the x -coordinate of B is 1.1.



- (a) Find the gradient of chord AB. Give your answer correct to 2 decimal places. [2]
- (b) Give the x -coordinate of a point C on the curve such that the gradient of chord AC is a better approximation to the gradient of the tangent to the curve at A. [1]

- 5 (a) The diagram shows the curve $y = e^x$.



On the axes in the Printed Answer Booklet, sketch graphs of

(i) $\frac{dy}{dx}$ against x , [1]

(ii) $\frac{dy}{dx}$ against y . [2]

- (b) Wolves were introduced to Yellowstone National Park in 1995.

The population of wolves, y , is modelled by the equation

$$y = Ae^{kt},$$

where A and k are constants and t is the number of years after 1995.

(i) Give a reason why this model might be suitable for the population of wolves. [1]

(ii) When $t = 0$, $y = 21$ and when $t = 1$, $y = 51$.

Find values of A and k consistent with the data. [3]

(iii) Give a reason why the model will not be a good predictor of wolf populations many years after 1995. [1]

6 In this question you must show detailed reasoning.

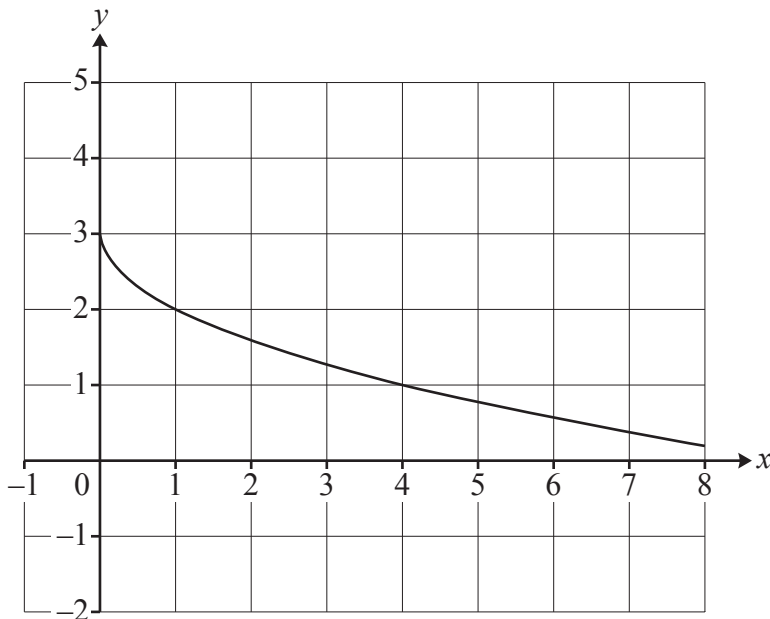
Show that $\sum_{r=1}^3 \frac{1}{\sqrt{r+1} + \sqrt{r}} = 1$. [4]

7 Determine $\int x \cos 2x \, dx$. [3]

8 For a particular value of a , the curve $y = \frac{a}{x^2}$ passes through the point $(3, 1)$.

Find the coordinates of all the other points on the curve where both the x -coordinate and the y -coordinate are integers. [3]

9 The diagram shows the curve $y = 3 - \sqrt{x}$.



(a) Draw the line $y = 5x - 1$ on the copy of the diagram in the Printed Answer Booklet. [1]

(b) In this question you must show detailed reasoning.

Determine the exact area of the region bounded by the curve $y = 3 - \sqrt{x}$, the lines $y = 5x - 1$ and $x = 4$ and the x -axis. [10]

10 (a) Express $\frac{1}{(4x+1)(x+1)}$ in partial fractions. [3]

(b) A curve passes through the point $(0, 2)$ and satisfies the differential equation

$$\frac{dy}{dx} = \frac{y}{(4x+1)(x+1)},$$

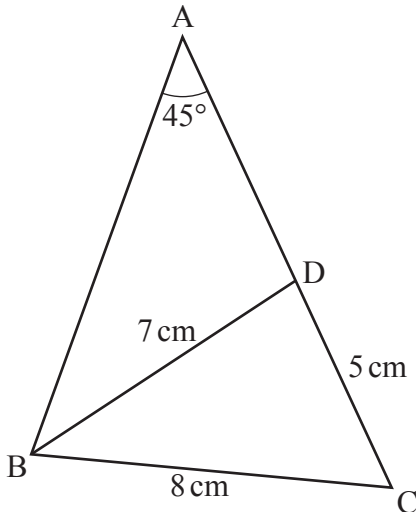
for $x > -\frac{1}{4}$.

Show by integration that $y = A\left(\frac{4x+1}{x+1}\right)^B$ where A and B are constants to be determined. [6]

11 In this question you must show detailed reasoning.

The diagram shows triangle ABC, with $BC = 8$ cm and angle $BAC = 45^\circ$.

The point D on AC is such that $DC = 5$ cm and $BD = 7$ cm.



Determine the exact length of AB. [5]

Answer **all** the questions.

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

12 Show that $\beta = \arctan\left(\frac{1}{3}\right)$, as given in line 15. [3]

13 (a) Use triangle ABE in **Fig. C2** to show that $\arctan x + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$, as given in line 29. [1]

(b) Sketch the graph of $y = \arctan x$. [1]

(c) What property of the arctan function ensures that $y > \frac{1}{x} \Rightarrow \arctan y > \arctan\left(\frac{1}{x}\right)$, as given in line 30? [1]

14 (a) Show that

$$\arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) = \arctan\left(\frac{1}{n}\right) \Rightarrow \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan 1. \quad [1]$$

(b) Use the arctan addition formula in line 23 to show that

$$\arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) = \arctan\left(\frac{1}{n}\right), \text{ as given in line 39.} \quad [4]$$

15 Prove that $\arctan 1 + \arctan 2 + \arctan 3 = \pi$, as given in line 41. [4]

END OF QUESTION PAPER

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Insert

Time allowed: 2 hours



INSTRUCTIONS

- Do **not** send this Insert for marking. Keep it in the centre or recycle it.

INFORMATION

- This Insert contains the article for Section B.
- This document has **4** pages.

Adding arctangents

Where does the name ‘arctangent’ come from?

The two commonly used ways to denote the angle which has a tangent x are $\tan^{-1}x$ and $\arctan x$. The first of these is related to inverse function notation, $f^{-1}(x)$. Arctangent comes from radian measure, where an angle is represented by an arc on a unit circle; $\arctan x$ is the arc whose tangent is x .

5

An interesting result

It can be shown that $\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan 1$.

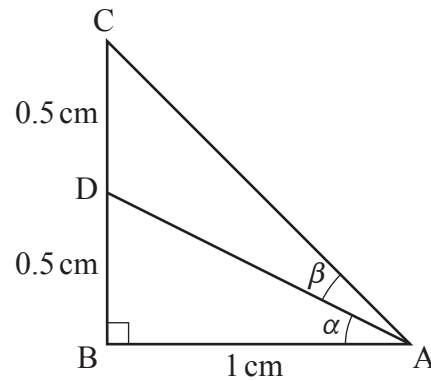
Consider the diagram in **Fig. C1**.

Triangle ABC is right-angled at B.
 $AB = BC = 1$ cm.
 D is the midpoint of BC.

Using triangle ABD, $\tan \alpha = \frac{DB}{BA} = \frac{1}{2}$ so $\alpha = \arctan\left(\frac{1}{2}\right)$.

Using triangle ABC, $\tan(\alpha + \beta) = 1$ so $\alpha + \beta = \arctan 1$.

Hence $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = 1$.



10

Fig. C1

Using $\tan \alpha = \frac{1}{2}$ and finding $\tan \beta$, it follows that $\beta = \arctan\left(\frac{1}{3}\right)$,

15

which gives the required result that $\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan 1$.

Generalising the result

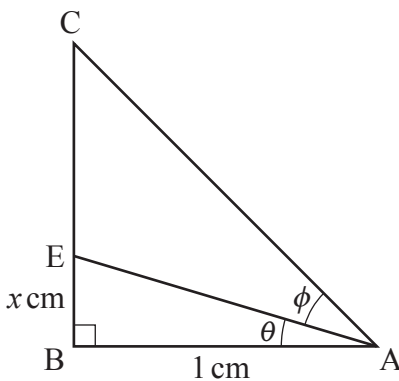


Fig. C2

Triangle ABC in **Fig. C2** is the same as triangle ABC in **Fig. C1** but E is a point on BC such that $EB = x$ cm and $\theta = \arctan x$.

Following the same method as above, $\arctan x + \arctan\left(\frac{1-x}{1+x}\right) = \arctan 1$.

20

The arctan addition formula

The arctangent addition formula is a further generalization:

$$\arctan x + \arctan y = \arctan\left(\frac{x+y}{1-xy}\right), \text{ as long as } xy < 1.$$

This result is equivalent to the addition formula $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$ where $\alpha = \arctan x$ and $\beta = \arctan y$. 25

To see why the restriction $xy < 1$ is necessary, consider what happens if $xy \geq 1$.

Clearly, $\frac{x+y}{1-xy}$ is undefined when $xy = 1$, so the formula does not apply in this case.

Suppose next that $xy > 1$, and that x and y are both positive; in this case $y > \frac{1}{x}$.

For any positive x , $\arctan x + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$.

$y > \frac{1}{x} \Rightarrow \arctan y > \arctan\left(\frac{1}{x}\right)$ so it follows that $\arctan x + \arctan y > \frac{\pi}{2}$. 30

However, $\arctan\left(\frac{x+y}{1-xy}\right)$ cannot be greater than $\frac{\pi}{2}$ as the range of the arctan function is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

The formula $\arctan x + \arctan y = \arctan\left(\frac{x+y}{1-xy}\right)$ therefore cannot be valid in this case.

A similar argument can be used to show that the formula cannot be valid when $xy > 1$ and x and y are both negative.

If $xy > 1$, the arctangent addition formula needs to be adapted, as shown below. 35

$$\arctan x + \arctan y = \arctan\left(\frac{x+y}{1-xy}\right) - \pi, \text{ when } xy > 1 \text{ and } x, y < 0$$

$$\arctan x + \arctan y = \arctan\left(\frac{x+y}{1-xy}\right) + \pi, \text{ when } xy > 1 \text{ and } x, y > 0$$

Some additional results

- For n a positive integer, $\arctan\left(\frac{1}{n+1}\right) + \arctan\left(\frac{1}{n^2+n+1}\right) = \arctan\left(\frac{1}{n}\right)$; this follows directly from the arctan addition formula in line 23. 40
- $\arctan 1 + \arctan 2 + \arctan 3 = \pi$. This can be proved by using $\arctan x + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$ together with $\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan 1$.

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