## edexcel \#\#

Mark Scheme (Results)

Summer 2013

GCE Further Pure Mathematics 3 (6669/01R)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.


## EDEXCEL GCE MATHEMATI CS

## General Instructions for Marking

1. The total number of marks for the paper is 75 .
2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod - benefit of doubt
- ft - follow through
- the symbol $\sqrt{ }$ will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
-     * The answer is printed on the paper
-     - The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.
8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

## General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

## Method mark for solving 3 term quadratic:

1. Factorisation

$$
\begin{aligned}
& \left(x^{2}+b x+c\right)=(x+p)(x+q), \text { where }|p q|=|c|, \text { leading to } \mathrm{x}= \\
& \left(a x^{2}+b x+c\right)=(m x+p)(n x+q), \text { where }|p q|=|c| \text { and }|m n|=|a|, \text { leading to } \mathrm{x}=
\end{aligned}
$$

2. Formula

Attempt to use correct formula (with values for $a, b$ and $c$ ).
3. Completing the square

Solving $x^{2}+b x+c=0: \quad\left(x \pm \frac{b}{2}\right)^{2} \pm q \pm c, \quad q \neq 0, \quad$ leading to $\mathrm{x}=\ldots$

## Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ( $\left.x^{n} \rightarrow x^{n-1}\right)$

## 2. Integration

Power of at least one term increased by 1 . ( $x^{n} \rightarrow x^{n+1}$ )

## Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

## Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

## Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

| Question | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
|  | Foci ( $\pm 5,0$ ), Directrices $x= \pm \frac{9}{5}$ |  |  |
| 1. | $( \pm) a e=( \pm) 5$ and $( \pm) \frac{a}{e}=( \pm) \frac{9}{5}$ | Correct equations (ignore $\pm$ 's) | B1 |
|  | so $e=\frac{5}{a} \Rightarrow \frac{a^{2}}{5}=\frac{9}{5} \Rightarrow a^{2}=9$ | M1: Solves using an appropriate method to find $a^{2}$ or $a$ | M1A1 |
|  | or $a=\frac{-}{e} \Rightarrow \frac{5}{e^{2}}=\frac{}{5} \Rightarrow e=\frac{5}{3} \Rightarrow a=3$ | A1: $a^{2}=9$ or $a=( \pm) 3$ |  |
|  | $\begin{aligned} & b^{2}=a^{2} e^{2}-a^{2} \Rightarrow b^{2}=25-9 \text { so } \\ & b^{2}=16 \quad(\Rightarrow b=4) \\ & \text { or } b^{2}=a^{2}\left(e^{2}-1\right) \Rightarrow b^{2}=9\left(\frac{25}{9}-1\right) \\ & b^{2}=16 \quad(\Rightarrow b=4) \end{aligned}$ | M1: Use of $b^{2}=a^{2}\left(e^{2}-1\right)$ to obtain a numerical value for $b^{2}$ or $b$ | M1 A1 |
|  |  | A1: : $b^{2}=16$ or $b=( \pm) 4$ |  |
|  | So $\frac{x^{2}}{9}-\frac{y^{2}}{16}=1$ | M1:Use of $\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1$ with their $a^{2}$ and $b^{2}$ | M1 A1 |
|  |  | A1: Correct hyperbola in any form. |  |
|  |  |  | (7) |


| Question | Scheme |  | arks |
| :---: | :---: | :---: | :---: |
| 2. | $l_{1}:(\mathbf{i}-\mathbf{j}+\mathbf{k})+\lambda(4 \mathbf{i}+3 \mathbf{j}+2 \mathbf{k})$ | $l_{2}:(3 \mathbf{i}+7 \mathbf{j}+2 \mathbf{k})+\lambda(-4 \mathbf{i}+6 \mathbf{j}+\mathbf{k})$ |  |
|  | $\mathbf{n}=\left\|\begin{array}{rrr} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 2 \\ -4 & 6 & 1 \end{array}\right\|=-9 \mathbf{i}-12 \mathbf{j}+36 \mathbf{k}$ | M1: Correct attempt at a vector product between $4 \mathbf{i}+3 \mathbf{j}+2 \mathbf{k}$ and $-4 \mathbf{i}+6 \mathbf{j}+\mathbf{k}$ (if the method is unclear then 2 components must be correct) allowing for the sign error in the $y$ component. | M1A1 |
|  |  | A1: Any multiple of ( $3 \mathbf{i}+4 \mathbf{j}-12 \mathbf{k}$ ) |  |
|  |  |  | (2) |
| (b) Way 1 | $\mathrm{a}_{1}-\mathrm{a}_{2}= \pm(2 i+8 j+k)$ | M1: Attempt to subtract position vectors <br> A1: Correct vector $\pm \mathbf{( 2 i} \mathbf{+ 8} \mathbf{j} \mathbf{+} \mathbf{k})$ <br> (Allow as coordinates) | M1 A1 |
|  | $\text { So } p=\frac{\left(\begin{array}{l} 2 \\ 8 \\ 1 \end{array}\right) \cdot\left(\begin{array}{c} -9 \\ -12 \\ 36 \end{array}\right)}{\sqrt{9^{2}+12^{2}+36^{2}}}$ | Correct formula for the distance using their vectors: $\frac{" \pm(2 i+8 j+k) " \cdot " n "}{\|" n "\|}$ | M1 |
|  | $p=\frac{ \pm 78}{\sqrt{1521}}=\frac{ \pm 78}{39}=2$ | M1: Correctly forms a scalar product in the numerator and Pythagoras in the denominator. (Dependent on the previous method mark) | dM1 A1 |
|  |  | A1: 2 (not-2) |  |
|  |  |  | (5) |
| (b) Way 2 | $\begin{aligned} & (\mathbf{i}-\mathbf{j}+\mathbf{k}) \bullet(3 \mathbf{i}+4 \mathbf{j}-12 \mathbf{k})=-13\left(d_{1}\right) \\ & (3 \mathbf{i}+7 \mathbf{j}+2 \mathbf{k}) \bullet(3 \mathbf{i}+4 \mathbf{j}-12 \mathbf{k})=13\left(d_{2}\right) \end{aligned}$ | M1: Attempt scalar product between their $\mathbf{n}$ and either position vector <br> A1: Both scalar products correct | M1A1 |
|  | $\frac{ \pm 13}{\sqrt{3^{2}+4^{2}+12^{2}}}(=1)$ | Divides either of their scalar products by the magnitude of their normal vector. $\frac{d_{1} \text { or } d_{2}}{\|" \mathbf{n} "\|}$ | M1 |
|  | $p=\frac{d_{1}}{\|\|\mathbf{n} "\|}-\frac{d_{2}}{\|\|\mathbf{n} "\|} \text { or } 2 \times \frac{d_{1}}{\|" \mathbf{n} "\|}$ | M1: Correct attempt to find the required distance i.e. subtracts their $\frac{d_{1}}{\|" \mathbf{n} "\|}$ and $\frac{d_{2}}{\left\|\mathbf{n}^{n}\right\|}$ or doubles their $\frac{d_{1}}{\|" \mathbf{n} "\|}$ if $\left\|d_{1}\right\|=\left\|d_{2}\right\|$. (Dependent on the previous method mark) A1: 2 (not -2) | dM1 A1 |
|  |  |  | (5) |
|  |  |  | Total 7 |



| Question Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 4. | $\left(\begin{array}{rrr}2 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2\end{array}\right)\left(\begin{array}{c}1+s+t \\ -1+s+2 t \\ 2\end{array}\right)$ | M1: Writes $\Pi_{1}$ as a single vector A1: Correct statement | M1A1 |
|  | $\left(\begin{array}{rrr}2 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2\end{array}\right)\left(\begin{array}{c}1+s+t \\ -1+s+2 t \\ 2\end{array}\right)=$ | $=\left(\begin{array}{l}2+2 s+2 t+6-6 t \\ -2+2 s+4 t-2+2 t \\ -1+s+2 t+4-4 t\end{array}\right)$ | M1A1 |
|  | M1: Correct attempt to multiply A1: Correct vector in any form |  |  |
|  | $=\left(\begin{array}{l}8+2 s-4 t \\ -4+2 s+6 t \\ 3+s-2 t\end{array}\right)$ | Correct simplified vector | B1 |
|  | $\mathbf{r}=\left(\begin{array}{r}8 \\ -4 \\ 3\end{array}\right)+s\left(\begin{array}{l}2 \\ 2 \\ 1\end{array}\right)+t\left(\begin{array}{r}-4 \\ 6 \\ -2\end{array}\right)$ |  |  |
|  | $\mathbf{n}=\left\|\begin{array}{rrc}\mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & 1 \\ -4 & 6 & -2\end{array}\right\|=-10 \mathbf{i}+20 \mathbf{k}$ | M1: Attempts cross product of their direction vectors | M1A1 |
|  |  | A1: Any multiple of $-10 \mathbf{i}+20 \mathbf{k}$ |  |
|  | $\mathbf{( 8 i} \mathbf{- 4} \mathbf{j}+\mathbf{3 k}) . \mathbf{( i}-\mathbf{2 k})=8-6$ | Attempt scalar product of their normal vector with their position vector | M1 |
|  | r. $(\mathbf{i}-\mathbf{2 k})=2$ | Correct equation (accept any correct equivalent $\text { e.g. } \mathbf{r}(-10 \mathbf{i}+20 \mathbf{k})=-20)$ | A1 |
|  |  |  | (9) |


| Question Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 5(a) | $I_{n}=\left[x^{n}(2 x-1)^{\frac{1}{2}}\right]_{1}^{5}-\int_{1}^{5} n x^{n-1}(2 x-1)^{\frac{1}{2}} \mathrm{~d} x$ | M1: Parts in the correct direction including a valid attempt to integrate $(2 x-1)^{-\frac{1}{2}}$ | M1 A1 |
|  |  | A1: Fully correct application - may be un-simplified. <br> (Ignore limits) |  |
|  | $I_{n}=\underline{5^{n} \times 3-1}-\int_{1}^{5} n x^{n-1} \underline{(2 x-1)(2 x-1)^{-\frac{1}{2}}} \mathrm{~d} x$ | Obtains a correct (possibly un-simplified) expression using the limits 5 and 1 and writes $(2 x-1)^{\frac{1}{2}} \text { as }(2 x-1)(2 x-1)^{-\frac{1}{2}}$ | B1 |
|  | $I_{n}=5^{n} \times 3-1-2 n I_{n}+n I_{n-1}$ | Replaces $\int x^{n}(2 x-1)^{-\frac{1}{2}} \mathrm{~d} x \text { with } I_{n}$ <br> and $\int x^{n-1}(2 x-1)^{-\frac{1}{2}} \mathrm{~d} x$ with $I_{n-1}$ | dM1 |
|  | $(2 n+1) I_{n}=n I_{n-1}+3 \times 5^{n}-1$ * | Correct completion to printed answer with no errors seen | A1cso |
|  |  |  | (5) |
| (b) | $I_{0}=\int_{1}^{5}(2 x-1)^{-\frac{1}{2}} \mathrm{~d} x=\left[(2 x-1)^{\frac{1}{2}}\right]_{1}^{5}=2$ | $I_{0}=2$ | B1 |
|  | $5 I_{2}=2 I_{1}+74$ and $3 I_{1}=I_{0}+14$ | M1: Correctly applies the given reduction formula twice | M1 A1 |
|  |  | A1: Correct equations for $I_{2}$ and $I_{1}$ (may be implied) |  |
|  | So $I_{1}=\frac{16}{3}$ and $I_{2}=\ldots$ or $5 I_{2}=2 \frac{I_{0}+14}{3}+74$ and $I_{2}=\ldots$ | Completes to obtains a numerical expression for $I_{2}$ | dM1 |
|  | $I_{2}=\frac{254}{15}$ |  | B1 |
|  |  |  | (5) |
|  |  |  | Total 10 |


| Question Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 6. (a) | $\left(\begin{array}{lll}4 & 2 & 3 \\ 2 & b & 0 \\ a & 1 & 8\end{array}\right)\left(\begin{array}{l}1 \\ 2 \\ 0\end{array}\right)=\left(\begin{array}{c}8 \\ \ldots \\ \ldots\end{array}\right),=\lambda\left(\begin{array}{l}1 \\ 2 \\ 0\end{array}\right), \lambda=8$ | M1: Multiplies the given matrix by the given eigenvector | M1, M1, A1 |
|  |  | M1: Equates the $x$ value to $\lambda$ |  |
|  |  | A1: $\lambda=8$ |  |
|  |  |  | (3) |
| (b) | $\left(\begin{array}{c}8 \\ 2+2 b \\ a+2\end{array}\right)=" 8$ " $\left.\begin{array}{l}1 \\ 2 \\ 0\end{array}\right)$ So $a=-2$ and $b=7$ | M1: Their $2+2 b=2 \lambda$ or their $a+2=0$ | M1 A1 A1 |
|  |  | A1: $b=7$ or $a=-2$ |  |
|  |  | A1: $b=7$ and $a=-2$ |  |
|  |  |  | (3) |
| (c) | $\begin{aligned} & \left\|\begin{array}{lcc} 4-\lambda & 2 & 3 \\ 2 & 7-\lambda & 0 \\ -2 & 1 & 8-\lambda \end{array}\right\|=0 \\ & \therefore(4-\lambda)(7-\lambda)(8-\lambda)-2 \times 2(8-\lambda)+3(2+2(7-\lambda))=0 \end{aligned}$ |  | M1 |
|  | Correct attempt to establish the Characteristic Equation. $=0$ is required but may be implied by later work Allow this mark if the equation is in terms of $\mathbf{a}, \mathbf{b}, \mathbf{c}$ |  |  |
|  | Attempts to factorise i.e. $(8-\lambda)\left(30-11 \lambda+\lambda^{2}\right)$ or $(6-\lambda)\left(40-13 \lambda+\lambda^{2}\right)$ or $(5-\lambda)\left(48-14 \lambda+\lambda^{2}\right)\left(\right.$ NB $\left.240-118 \lambda+19 \lambda^{2}-\lambda^{3}=0\right)$ |  | M1 A1 |
|  | M1: Attempt to factorise their cubic - an attempt to identify a linear factor and processes to obtain a simplified quadratic factor <br> A1: Correct factorisation into one linear and one quadratic factor |  |  |
|  | Eigenvalues are 5 and 6 | M1: Solves their equation to obtain the other eigenvalues A1: 5 and 6 | M1 A1 |
|  |  |  | (5) |
|  |  |  | Total 8 |


| Question Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 7(a) | Put $6 \cosh x=9-2 \sinh x$ |  | M1 |
|  | $6 \times \frac{1}{2}\left(e^{x}+e^{-x}\right)=9-2 \times \frac{1}{2}\left(e^{x}-e^{-x}\right)$ | Replaces $\cosh x$ and $\sinh x$ by the correct exponential forms | M1 |
|  | $4 e^{x}+2 e^{-x}-9=0 \Rightarrow 4 e^{2 x}-9 e^{x}+2=0$ | M1: Multiplies by $\mathrm{e}^{x}$ | M1 A1 |
|  |  | A1: Correct quadratic in $\mathrm{e}^{x}$ in any form with terms collected |  |
|  | So $e^{x}=\frac{1}{4}$ or 2 and $x=\ln 2$ or $\ln \frac{1}{4}$ | M1: Solves their quadratic in $\mathrm{e}^{x}$ | M1 A1 |
|  |  | A1: Correct values of $x$ (Any correct equivalent form) |  |
|  |  |  | (6) |
| (b) | Area is $\int(9-2 \sinh x-6 \cosh x) \mathrm{d} x$ | $\begin{aligned} & \int(9-2 \sinh x-6 \cosh x) \mathrm{d} x \text { or } \\ & \int(6 \cosh x-(9-2 \sinh x)) \mathrm{d} x \\ & \text { or the equivalent in exponential } \\ & \text { form } \end{aligned}$ | M1 |
|  | $\begin{gathered} \pm(9 x-2 \cosh x-6 \sinh x) \text { or } \\ \pm\left(9 x-4 \mathrm{e}^{x}+2 \mathrm{e}^{-x}\right) \end{gathered}$ | M1: Attempt to integrate | M1 A1 |
|  |  | A1: Correct integration |  |
|  | $\pm\left([9 \ln 2-2 \cosh \ln 2-6 \sinh \ln 2]-\left[9 \ln \frac{1}{4}-2 \cosh \ln \frac{1}{4}-6 \sinh \ln \frac{1}{4}\right]\right)$ |  | dM1 |
|  | Complete substitution of their limits from part (a). Depends on both previous M's |  |  |
|  | $= \pm\left(9 \ln \left(2 \div \frac{1}{4}\right)-\left(e^{\ln 2}+\mathrm{e}^{-\ln 2}\right)-3\left(\mathrm{e}^{\ln 2}-\mathrm{e}^{-\ln 2}\right)+\left(\mathrm{e}^{\ln \frac{1}{4}}+\mathrm{e}^{-\ln \frac{1}{4}}\right)+3\left(\mathrm{e}^{\ln \frac{1}{4}}-\mathrm{e}^{-\ln \frac{1}{4}}\right)\right)$ |  | M1 |
|  | Combines logs correctly and uses cosh and sinh of In correctly at least once |  |  |
|  | $\left(9 \ln 8-\frac{5}{2}-\frac{18}{4}+4.25-11.25\right)=9 \ln 8-14 \text { or } 27 \ln 2-14$ <br> Any correct equivalent |  | A1cao |
|  | Subtracting the wrong way round could score 5/6 max |  |  |
|  |  |  | (6) |
|  |  |  | Total 12 |
|  | Note <br> If they use $4 e^{2 x}-9 e^{x}+2$ in (b) to find the area - no marks |  |  |



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