edexcel

June 2005
6690 Decision D2
Mark Scheme

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>1) (a)</td><td>

	D	E	F
A	20	4	
B		26	6
C			14

</td><td>M1
A1
(2)</td></tr>
<tr><td>(b)</td><td>$S_A = 0$ $S_B = -1$ $S_C = 7$
$D_D = 21$ $D_E = 24$ $D_F = 18$
$I_{13} = I_{AF} = 16 - 0 - 18 = -2$
$I_{21} = I_{BD} = 18 + 1 - 21 = -2$
$I_{31} = I_{CD} = 15 - 7 - 21 = -13$ *
$I_{32} = I_{CE} = 19 - 7 - 24 = -12$</td><td>M1
A1
M1
A1 √
A1 √ (5)</td></tr>
<tr><td>(c)</td><td>eg CD(+) → AD(-) → AE(+) → BE(-) → BF(+) → CF(-) $\theta = 14$

	D	E	F
A	6	18	
B		12	20
C	14		

Cost £1384</td><td>M1 A1 √
A1 √ A1
(4)
11</td></tr>
</table>

Q1 (a) M1 5 numbers, top LH corner used, a correct solution

A1 c.a.o.

(b) M1 shadow costs stated - all 6

A1 c.a.o.

M1 4 II's stated

A1$\sqrt{}$ at least 2 correct

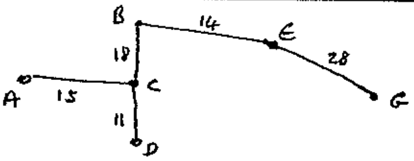
A1$\sqrt{}$ all 4 c.a.o.

(c) M1 Route must $\sqrt{}$ and be clear. Route has 1 entering + 1 exiting square.

A1$\sqrt{}$ route correct + θ correct and clearly given

A1$\sqrt{}$ new improved solution (5 numbers only)

A1 c.a.o

Question Number	Scheme	Marks
2) (a)	Deleting F leaves r.s.t 	M1
	r.s.t. length = 86	A1
	So lower bound = 86 + 16 + 19 = 121	M1 A1 (4)
	∴ best L.B is 129 by deleting C (√ from choice)	B1√ (1)
(b)	Add 33 to BF and FB	B1
	Add 31 to DE and ED	B1 (2)
(c)	Tour, visits each vertex, order correct using table of least distances.	M1 A1
	e.g. F C D A B E G F (actual route F C D C A B E G F)	A1
	upper bound of 138 km	A1 (4)
		11

Q2(a) M1 Finding r.s.t – ie a spanning tree with F removed – needs a "T junction" at C (NN sets M0)

A1 86 c.a.o. maybe implicit

M1 Adding 2 least arcs from F – bod

A1 121 c.a.o – if sensible method 121 gets all 4. No method but 121 gets SC B2

B1√ Chooses greatest of 129 and their lower bound from F.

b) B1 cao

B1 cao

(c) M1 NN each vertex visited once (condone 2 C's if using actual route)

A1 NN starts + finishes at F – stated not drawn

A1 c.a.o

A1 138, if doubled A0 (do not ISW)

3)

Let x_{ij} be number of unit transported from i to j where $i \in \{W, X, Y\}$ (warehouse) and $j \in \{J, K, L\}$ (supermarket) — B1 (1)

objective minimise "C" $= 3x_{WJ} + 6x_{WK} + 3x_{WL} + 5x_{XJ} + 8x_{XK} + 4x_{XL} + 2x_{YJ} + 5x_{YK} + 7x_{YL}$ — B1 B1 (2)

Subject to

$x_{WJ} + x_{WK} + x_{WL} = 34$ — M1 A1

$x_{XJ} + x_{XK} + x_{XL} = 57$

$x_{YJ} + x_{YK} + x_{YL} = 25$

$x_{WJ} + x_{XJ} + x_{YJ} = 20$ — A1 (3)

$x_{WK} + x_{XK} + x_{YK} = 56$

$x_{WL} + x_{XL} + x_{YL} = 40$

$x_{ij} \geq 0 \quad \forall i \in \{W, X, Y\}$ and $j \in \{J, K, L\}$ — B1 (1) $\boxed{7}$

Q3 B1 Introducing decision variables c.a.o (o.e.) need "number" o.e.

B1 minimise

B1 Function accept any letter, but need equation

M1 At least 3 equalities listed, with 3 variables in each (accept $\leq$ or $\geq$ here)

A1 3 correct

A1 6 correct – penalise $\leq$ here

B1 non-negativity constraints – all x values dealt with.

penalise bad notation only once per question, by losing first A or B mark earned

4) (a) The route from start to finish in which the arc of minimum length is as large as possible. — B2, 1, 0

e.g. must be practical, involve choice of route, have arc 'cost'. — B1 (3)

(b)

Stage	State	Action	Value
1	H	HK	18 *
	I	IK	19 *
	J	JK	21 *
2	F	FH	min(16, 18) = 16
		FI	min(23, 19) = 19 *
		FJ	min(17, 21) = 17
	G	GH	min(20, 18) = 18
		GI	min(15, 19) = 15
		GJ	min(28, 21) = 21 *
3	B	BG	min(18, 21) = 18 *
	C	CF	min(25, 19) = 19 *
		CG	min(16, 21) = 16
	D	DF	min(22, 19) = 19 *
		DG	min(19, 21) = 19 *
	E	EF	min(14, 19) = 14 *
4	A	AB	min(24, 18) = 18
		AC	min(25, 19) = 19 *
		AD	min(27, 19) = 19 *
		AE	min(23, 14) = 14

Marks: M1 A1 (2); M1 A1 A1 (3); M1 A1√; A1√ (3); A1√

(c) Routes A C F I K, A D F I K, A D G J K — A1√ A1 (3)

[14]

(a) B2 Good definition : route, arc of min length maximised. Clear

B1 pretty good – all there but confused. 'bod' gets B1

B1 Practical, choice of route, arc 'cost'.

(b) M1 1st stage completed

A1 c.a.o. (+ *) — only penalise silly slips etc here and on last A marks.

minimum + maximum stick to scheme – so M1A1, M1A0A0, M1A0A0, A1√, A1√, A0 (max)

M1 2nd stage completed bod

A1 F state correct finding mins + then maximum c a o

A1 G state correct .. .. .. c a o

Penalise * with 1st A earned once only (but maybe 2nd later)

M1 3rd + 4th stage completed bod

A1√ 3rd stage c.a.o

A1√ 4th stage c.a.o

penalise * once only [but maybe scored]

A1√ 1 correct route (must be able to √ on these 3 A marks condone if only these 1 silly)

(c) A1√ a second correct route

A1 a third correct route – and no more! ie, c.a.o

5) (a) To maximise, subtract all entries from $n \geq 30$ — m1

e.g.

$$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 5 & 3 & 6 \\ 0 & 3 & 5 & 9 \end{bmatrix}$$

A2, 1, 0 (3)

minimum uncovered element is 1 : so

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 6 & 3 & 0 \\ 0 & 4 & 2 & 5 \\ 0 & 2 & 4 & 8 \end{bmatrix}$$

m1 A2√1√0 (3)

or — m1

min. el. = 2 | min. el. = 2

$$\begin{bmatrix} 7 & 0 & 0 & 2 \\ 0 & 4 & 1 & 0 \\ 0 & 2 & 0 & 5 \\ 0 & 0 & 2 & 8 \end{bmatrix} \qquad \begin{bmatrix} 7 & 0 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 2 & 6 \end{bmatrix}$$

A2√1√0 (3)

A – 2 B – 4 C – 3 D – 1 — m1 A1√ (2)
A – 3 B – 4 C – 1 D – 2

(b) £1160 000 — B2, 1, 0 (2)

(c) Gives other solution — m1 A1 (2)

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Q5(a) m1 Attempt to subtract all terms from n, $n \geq 30$ + Rows + cols if necess.

A2, 1, 0 – 1 e.e. for my table

m1 (Drawing 2 lines) + dble covered + e, uncovered – e, once covered unchanged at least once

A2√1√ c.a.o. – 1 e.e ~~for my table~~

m1 (Drawing 3 lines, 3 elements 1 dble covered, 1 once covered + 1 uncovered treated correctly.

A2√, 1√ cao – 1ee

m1 All 4 people allocated to all 4 companies – a list

A1 cao

(b) B2 £1160 000

B1 116 or £116 etc (multiple of 10 out)

(c) m1 All 4 people allocated to all 4 companies – a list

A1 cao

6) (a) A zero-sum game is one in which the sum of the gains for all players is zero. (o.e.) — B1 (1)

(b)

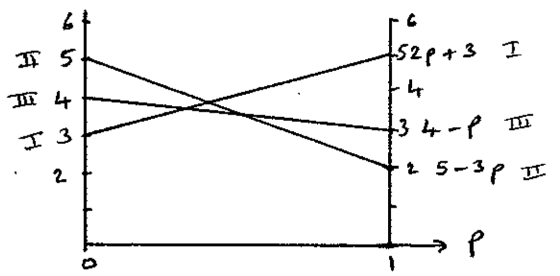
	I	II	III	
I	5	2	3	min 2
II	3	5	4	min 3 ← max
	max 5	5	4 ↑ min	

M1 A1

Since $3 \neq 4$ not stable — A1 (3)

(c) Let A play I with probability p
" " play II " " $(1-p)$

If B plays I A's gains are $5p + 3(1-p) = 2p + 3$
" " " II " " " $2p + 5(1-p) = 5 - 3p$
" " " III " " " $3p + 4(1-p) = 4 - p$ — M1 A1 (2)



A2, 1, 0 (2)

Intersection of $2p+3$ and $4-p$ $\Rightarrow p = \frac{1}{3}$ — M1 A1√ (2)

∴ A should play I $\frac{1}{3}$ of time and II $\frac{2}{3}$ of time; value (to A) $= 3\frac{2}{3}$ — A1√ A1√ (2)

(d) Let B play I with probability q_1, II with probability q_2 and III with probability q_3 — B1

e.g. Let $x_1 = \frac{q_1}{V}$ $x_2 = \frac{q_2}{V}$ $x_3 = \frac{q_3}{V}$ (If reduced x_2 vanishes) don't √ if wrong row deleted — M1

Maximise $P = x_1 + x_2 + x_3$ — A1

Subject to $5x_1 + 2x_2 + 3x_3 \le 1$
$3x_1 + 5x_2 + 4x_3 \le 1$
$x_1, x_2, x_3 \geq 0$ — A2, 1, 0 (5)

Alt1 eg. $\begin{bmatrix} -5 & -3 \\ -2 & -5 \\ -3 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 3 & 2 \end{bmatrix}$ (If reduced $\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$ + q_2 vanishes.) don't √ if wrong row deleted

maximise $P = V$

subject to $V - q_1 - 4q_2 - 3q_3 \le 0$ $q_1 + q_2 + q_3 \le 1$
$V - 3q_1 - q_2 - 2q_3 \le 0$
$V, q_1, q_2, q_3 \geq 0$

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Q6(a) B1 cao (o.e) Condone assumption its a 2 player game

(b) M1 Finding row maximin and column minimax. All 5 values listed enough

A1 row maximin = 3, col minimax = 4. Identified in some way

A1 row maximin $\neq$ col minimax ($3 \neq 4$) stated + statement (not stable) a clear link

(c) M1 Set up 3 probability equations (implicit definition of p)

A1 all correct - may be unsimplified.

A2,1,0 3 lines correctly drawn + domain $0 \le p \le 1$ + scale clear + lines labelled -1 e.e.

M1 Using correct eqns to find max - but √ from their eqns.

A1√ $p = \frac{1}{3}$ cao but √ their eqns.

A1√ strategy clear - both row + value √ their eqns

A1√ value clear must √ their eqns.

(d) B1 Set up of B probabilities - all 3 (or reducing)

M1 Setting up to formulate an LP. Defining x's, adapting matrix etc. Still dealing with 3 (or 2 if reduced)

A1 objective fn + maximise, 3 variables. (or 2 if reduced)

A2,1,0 constraints (incl non negativity) -1 each constraint error (non-neg count a 1 error)
-1 for equations

Alt2 maximise $P = V$ — If reduced

subject to $V + 2q_1 - q_2 \le 3$

$V - q_1 + q_2 \le 2$ $\quad q_1, q_2, V \ge 0$

Alt3 If adapt matrix main scheme becomes. — If reduced "x_2" vanishes

minimise $P = x_1 + x_2 + x_3$

subject to $x_1 + 4x_2 + 3x_3 \le 1$

$3x_1 + x_2 + 2x_3 \le 1$ $\quad x_1, x_2, x_3 \ge 0$