GCE

## Mathematics

Advanced Subsidiary GCE

## Mark Scheme for January 2011

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| 1 (i) $(1+2 x)^{7}=1+14 x+84 x^{2}$ | B1 |  | Obtain 1+14x | Needs to be simplified, so 1 not $1^{7}$ and $14 x$ not $7 \times 2 x$. <br> B0 if other constant and/or $x$ terms (from terms being sums not products). <br> Must be linked by + sign, so $1,14 x$ is B0, but can still get M1A1 for third term. |
| :---: | :---: | :---: | :---: | :---: |
|  | M1 |  | Attempt third term | Needs to be product of 21 and an attempt at squaring $2 x$ - allow even if brackets never seen, so $42 x^{2}$ gets M1. No need to see powers of 1 explicitly. |
|  | A1 | 3 | Obtain $84 x^{2}$ | Coefficient needs to be simplified. Ignore any further terms, right or wrong. Can isw if they subsequently attempt 'simplification' eg dividing by 14 , but they won't then get the ft mark in part (ii). If manually expanding brackets they need to consider all 7, but may not necessarily show irrelevant terms. If the expansion is attempted in descending powers, only giving the first three will gain no credit in (i), unless they subsequently attempt the relevant terms in (ii) when we will then give appropriate credit for the marks in (i). This only applies if no attempt at the required terms is made in (i). <br> A full expansion with the required terms at the end is marked as per original scheme. |

(ii) $\quad(2-5 x)\left(1+14 x+84 x^{2}\right)$ coeff of $x^{2}=-70+168$

M1

$$
=98
$$

Attempt at least one relevant product

Obtain two correct unsimplified terms (not necessarily summed) - either coefficients or still with powers of $x$ involved

Could be just a single term, or part of a fuller expansion considering terms other than $x^{2}$ as well.
Allow M1 even if second $x^{2}$ term isn't from a relevant product eg $-70+84$ gets M1 A0.

Needs to come from two terms only, and can be awarded for unsimplified terms eg $-5 x \times 14 x$..
If fuller expansion then A0 if other $x^{2}$ terms, but ignore any irrelevant terms.
If expansion is incorrect in (i) and candidate only gives a single final answer in (ii) then examiners need to check and award either M1 A1ft or M0.

Allow $98 x^{2}$.
Allow if part of a fuller expansion and not explicitly picked out. If clearly finding coefficient of $x$, allow as misread.

| 2 (i) | $u_{1}=5, u_{2}=8, u_{3}=11$ | B1 |  | Obtain at least one correct term | ust a list of numbers is fine, no need for labe |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | B1 | 2 | Obtain all three correct terms | Ignore extra terms beyond $u_{3}$. |
| (ii) | arithmetic progression | B1 | 1 | Any mention of arithmetic | Allow AP, but not description eg constant difference. Ignore extra description eg diverging as long as not wrong or contradictory. |
| (iii) | (or $\left.S_{200}-S_{100}=60,700-15,350\right)$ | M1 |  | Attempt relevant $S_{n}$ using correct formula | Must use correct formula to sum an AP - only exception is using $(1 / 2 n-1) d$ rather than $(n-1) d$. <br> Must use $d=3$ (or their $d$ from (i) as long as constant difference). If (i) is incorrect they can still get full marks in (iii) as independent. <br> They need to be finding the sum of $99,100,101$ or 200 terms and make a reasonable attempt at a value of $a$ consistent with their $n-$ if $n=99$ then $a=305$ / if $n=100$ then $a=5$ or $a=305$ / if $n=101$ then $a=5$ / if $n=200$ then $a=5$. Allow slips on $a=305$ as long as clearly intending to find $u_{101}$. <br> If using $1 / 2 n(a+l)$ then there also needs to be a reasonable attempt at $l$. <br> Attempting to sum from $n=101$ to $n=200$ gets both method marks together (assuming that the attempt satisfies above conditions). |
|  |  | M1 |  | Attempt correct method to find required sum | $S_{200}-S_{101}$ is M0. <br> M0 M1 is possible for correct method but with incorrect formula for $S_{N}$ (but must be recognisable as attempt at sum of AP). <br> Need to show subtraction to gain M1, just calculating two relevant sums is not yet enough. <br> Still need $a=5$ and $d=3$. |
|  |  | A1 | 3 | Obtain 45,350 | Answer only gets full marks. |
|  |  |  | 6 |  | SR: if candidates attempt to manually add terms... <br> M1 Attempt to sum all terms from $u_{101}$ to $u_{200}$ <br> A2 Obtain 45,350 |

3 (i) $0.5 \times 0.5 \times\{\sqrt{0}+2(\sqrt{0.5}+\sqrt{1}+\sqrt{1.5})+\sqrt{2}\}$ $=1.82$

Use correct $h$ (soi) for their $y$-values must be at equal intervals

Obtain 1.82, or better

Attempt at least 4 correct $y$-coords, and no others

Attempt correct trapezium rule, any $h$, to find area between $x=3$ and $x=5$.
(ii) Underestimate as tops of trapezia are below curve

B1*
B1d* 2 Convincing reason referring to trapezia being below curve

If first term of 0 not explicit then other 4 terms need to be seen. Could be implied by eg $\sqrt{ }(4-3)$, or implied by a table with correct $x$-coords in one column and attempts at $y$-coords in second column.
Allow rounded or truncated decimals.
Allow an error in rearrangement eg $\sqrt{ } x-\sqrt{ } 3$.
Correct structure ie $0.5 \times$ (any $h$ ) $\times$ (first + last $+2 \times$ middles $)-$ no omissions allowed.
The first $y$-coord should correspond to attempt when $x=3$ (though may not be shown explicitly), and last to $x=5$. It could be implied by using $y_{0}$ etc in rule, when these have already been attempted elsewhere and clearly labelled.
It could use other than 4 strips, but these must be at equal widths. Using just one strip is M0.
The 'big brackets' must be seen, or implied by later working (omission of these can lead to 3.41 or 1.91 or $6.21 \ldots$...

If $1 / 2 \mathrm{x} k$ seen at start of rule then assume that $1 / 2$ is part of a correct rule and the $k$ is an incorrect strip width.

Must be in attempt at the trapezium rule, not Simpson's rule. Allow if muddle over placing $y$-values.
Allow if one $y$-value missing (including first or last) or extra.
Allow if $1 / 2$ missing.
Using $h=2$ with only one strip is M0.
More accurate solution is $1.819479 \ldots$

Answer only is $0 / 4$.
Using integration is $0 / 4$.
Using trapezium rule on the result of an integration attempt is $0 / 4$. Using 4 separate trapezia can get full marks. If other than 4 trapezia, mark as above.

Ignore any reasons given.
Referring to gaps between curve and trapezia can get B1.
Could use sketch with brief explanation (but sketch alone is B0) must show more than one trapezium (but not nec 4) or imply this in the text. Trapezia must show clear intention to have top vertices on the curve. Sketching rectangles is B0. Triangle is B0. Explanation that refers to calculated area from integration is B 0 . Only referring to concave / convex is B0.
Can get B 1 for 'rate of change of gradient (or second derivative) is negative', but not for 'gradient is decreasing'.

4 (a) $\log 5^{x-1}=\log 120$
$(x-1) \log 5=\log 120$
$x-1=2.97$
A1
$x=3.97$

## M1* <br> 

M1d*

4 Obtain 3.97, or better

Introduce logarithms throughout (or $\log _{5} 120 / \log _{120} 5$ ) \& drop power

Obtain $(x-1) \log 5=\log 120$, or equiv (eg $x-1=\log _{5} 120$ )

Attempt to solve
(b) $\quad \log _{2} x+\log _{2} 9=\log _{2}(x+5)$ $\log _{2}(9 x)=\log _{2}(x+5)$ $9 x=x+5$
$x=5 / 8$

B1
B1

Use $\log a+\log b=\log a b$, or equiv

Obtain correct equation with single log term on each side (or single $\log =0$ )

A1

Don't need to see base if taking logs on both sides, though if shown it must be the same base.
If taking logs on one side only base must be explicit.
Condone lack of brackets ie $x-1 \log 5=\log 120$, as long as clearly implied by later working.
Attempt at correct process ie ${ }^{\log 120} / \log 5 \pm 1$ or equiv ${ }^{(\log 120+\log 5)} / \log 5$. Allow M1 if ${ }^{\log 120} / \log 5 \pm 1$ subsequently becomes $\log 24 \pm 1$, but M0 if $\log 24$ appears before -1 is dealt with.
Allow M1 if processing slips when evaluating ${ }^{\log 120} / \log 5$ eg 2.23 from incorrect brackets.

Allow more accurate solution, such as 3.975 and then isw if rounded to 3.98 . However, 3.98 without more accurate answer seen is A0.

Answer only is $0 / 3$.
Trial and improvement is $0 / 3$.

Could be done at any stage.
Must be correct statement when done, so LHS becoming $\log _{2}(x+9)$ in one step is B0.
Condone lack of base throughout question.
Must be used to combine 2 (or more) terms of $\log x+\log k=\log (x+5$ ), with $k$ most typically (but not exclusively) 6,8 or 9 .
Could move $\log _{2} x$ and/or $\log _{2} 9$ across to RHS and then use $\log a-\log b$, but must still be $\log _{2}(x+5)$ as single term.
$\log _{2}(9 x)=\log _{2}(x+5), \log _{2} x=\log _{2}{ }^{(x+5)} / 9, \log _{2} 9=\log _{2}{ }^{(x+5)} / x$, $\log _{2}{ }^{(x+5)} / 9 x=0$.
Allow A1 for correct equation with logs removed if several steps run together.

| 5 (i) | $4 a=\frac{a}{1-r}$ | M1 |  | Equate $\frac{a}{1-r}$ to $4 a$, or substitute $r=\frac{3}{4}$ into $S_{\infty}$ | $S_{\infty}$ must be quoted correctly. <br> Allow $4 a r^{0}$ for $4 a$. <br> Initially using a numerical value for $a$ is M0. <br> Once equation in $a$ is seen ie $4 a=\frac{a}{1-r}$ assume that $a$ has been cancelled if this subsequently becomes $4=\frac{1}{1-r}$. If initial equation in $a$ is never seen then assume that $a=1$ is being used and mark accordingly. |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $1-r=\frac{1}{4}$ | M1 |  | Attempt to find value for $r$ or evaluate $S_{\infty}$ | Need to get as far as attempting $r$. <br> Need to see at least one extra line of working between initial statement and given answer. <br> Substituting numerical value for $a$ is M0 (so M1 M0 possible depending at what stage the substitution happens). |
|  | $r=\frac{3}{4}$ | A1 | 3 | Obtain $r=\frac{3}{4}$ (or show $\left.S_{\infty}=4 a\right)$ | Allow $r=0.75$. |
| (ii) | $\left(\frac{3}{4}\right)^{2} a=9$ | M1* |  | Attempt use of $a r^{2}$ | Must use $r=3 / 4$ not their incorrect value from (i). <br> Must be clearly intended as $a r^{2}$, so $(3 a / 4)^{2}=9$ is M0, unless correct expression previously seen. <br> Can use equivalent method with ratio of $\frac{4}{3}$ ie $9 \times(4 / 3)^{2}$. |
|  | $a=16$ | M1d* |  | Equate to 9 and attempt to find $a$ | Must get as far as attempting value for $a$. |
|  |  | A1 | 3 | Obtain $a=16$ | Answer only gets full credit. |
| (iii) | $S_{20}=\frac{16\left(1-\frac{3}{4}^{20}\right)}{1-\frac{3}{4}}$ | M1 |  | Attempt use of correct sum formula for a GP | Must be correct formula, with $a=$ their (ii), $r=3 / 4$ and $n=20$. |
|  | $=63.8$ | A1 | 2 8 | Obtain 63.8, or better | More accurate answer is 63.79704... <br> NB using $n-1$ rather than $n$ in the formula gives 63.729 (M0), and using $n+1$ gives 63.848 (M0). <br> Must be decimal, rather than exact answer with power of $3 / 4$. |

## 6(a)

M1 Simplify and attempt integration
$=\frac{1}{3} x^{3}+6 x^{\frac{1}{2}}+c$
Obtain $\frac{1}{3} x^{3}+6 x^{\frac{1}{2}}$
4 Obtain $+c$

Need to attempt to divide both terms by $x$, or multiply entire numerator by $x^{-1}-$ allow if intention is clear even if errors when simplifying, or one term doesn't actually change.
Need to simplify each of the two terms as far as $x^{n}$ before integrating.
For integration attempt, need to increase power by 1 for at least one term.

Allow unsimplified terms.
Coefficients must now be simplified.
Could be $6 \sqrt{ } x$ for second term.
Not dependent on previous marks as long as no longer original function.
B 0 if integral sign or $\mathrm{d} x$ still present in answer. Ignore anything that appears on LHS of an equation eg $y=\ldots, \mathrm{d} x=\ldots$ or even $\int=\ldots$


| 7(i) | $\tan 2 x=1 / 3$ |
| :--- | :--- |
|  | $2 x=18.4^{\circ}, 198.4^{\circ}$ |
|  | $x=9.22^{\circ}, 99.2^{\circ}$ |$\quad$ M1

$x=9.22^{\circ}, 99.2^{\circ}$

Attempt correct solution method
Obtain one of $9.22^{\circ}$ or $99.2^{\circ}$, or better

A1ft 3 Obtain second correct angle
(ii) $3\left(1-\sin ^{2} x\right)+2 \sin x-3=0$
$3 \sin ^{2} x-2 \sin x=0$
$\sin x(3 \sin x-2)=0$
$\sin x=0, \sin x=2 / 3$
$x=0^{\circ}, 180^{\circ} \quad x=41.8^{\circ}, 138^{\circ}$

M1

Use $\cos ^{2} x=1-\sin ^{2} x$, aef

Obtain $3 \sin ^{2} x-2 \sin x=0$
Attempt to solve equation to find solutions for $x$

Attempt $\tan ^{-1}(1 / 3)$ and then halve answer.
Allow radian equiv (0.161 or 1.73).
Maximum of 2 marks if angles not in degrees.
A0 if extra solutions in given range, but ignore extra outside range If M1 A0 given, award A1ft for adding $90^{\circ}$ or $\pi / 2$ to their angle.

SR: if no working shown then allow B1 for each correct solution. Maximum of B1 if in radians, or extra solutions in given range.

SR: if using $\tan 2 x$ identity then...
M1 Attempt to find $x$ from solving quadratic equation in $\tan 2 x$, derived from correct $\tan 2 x$ identity.
A1 Obtain at least one of $9.22^{\circ}$ or $99.2^{\circ}$, or better (or radian equiv) A1 Obtain second correct angle

Must be used not just stated.
Must be used correctly, so $1-3 \sin ^{2} x$ is M0.
Allow aef, but must be simplified (ie no constant term; allow 0 ).
Not dependent on first M1 so could get M0 M1 if $\cos ^{2} x=\sin ^{2} x-1$ previously used.
Must be quadratic in $\sin x$ (must have $\sin x$ term), but can still get M1 if constant term in their quadratic as well.
Candidates need to be solving for $x$, so need to $\sin ^{-1}$ at least one of the solutions to their quadratic.
Must be acceptable method - if factorising then it must give correct lead term and one other on expansion (inc $c=0$ ), if using formula then allow sign slips but no other errors.
SR If solving the quadratic involves cancelling by $\sin x$ rather than factorising then M 0 , but give B1 if both $41.8^{\circ}$ and $138^{\circ}$ found (or radian equivs)

Must come from correct factorisation of correct quadratic equation ie $\sin x(3 \sin x+2)=0$ leading to $\sin x=0$ and hence $x=0^{\circ}, 180^{\circ}$ is A 0 .
Allow radian equivs $-0, \pi$ (or 3.14), $0.73,2.41$.
Must now all be in degrees, with no extra in given range (ignore any outside range).
SR If no working out seen, then allow B1 for each of $41.8^{\circ}$ and $138^{\circ}$, and B1 for both $0^{\circ}$ and $180^{\circ}$. Maximum of B2 if in radians or extra solutions in given range.


9(i) $\mathrm{f}(3)=-108+81+30-3=0$
hence $(x-3)$ is a factor

2 State $(x-3)$ as factor (allow $(3-x)$ as the factor)

Substitute $x=3$ and confirm $\mathrm{f}(3)=0$ - must show detail of
substitution rather than just state $f(3)=0$.
Allow $f(3)=-4 \times 3^{3}+9 \times 3^{2}+10 \times 3-3=0$ for B1.
Not dependent on first B1.
Must be seen in (i) so no back credit from (ii).
Allow if not explicitly stated as factor (and allow $\mathrm{f}(x)=x-3$ ).
Ignore other factors if also given at this stage.
(ii) $\mathrm{f}(x)=(x-3)\left(-4 x^{2}-3 x+1\right)$
or
$\mathrm{f}(x)=(3-x)\left(4 x^{2}+3 x-1\right)$
or
$\mathrm{f}(x)=(x+1)\left(-4 x^{2}+13 x-3\right)$
$\mathrm{f}(x)=(-x-1)\left(4 x^{2}-13 x+3\right)$
or
$\mathrm{f}(x)=(1-4 x)\left(x^{2}-2 x-3\right)$
$\mathrm{f}(x)=(4 x-1)\left(-x^{2}+2 x+3\right)$

M1
Attempt complete division by $(x-3)$, or
equiv
(allow division by $(3-x)$ )

Must be a full attempt to find three term quadratic. Can use inspection, but must be a reasonable attempt at middle term, with first and last correct.
Can use coefficient matching, but must be full method with reasonable attempts at all 3 coefficients. Allow M1 if actually factorising $-\mathrm{f}(x)$.

Obtain $-4 x^{2}-3 x+c$ or $-4 x^{2}+b x+1$
(or the negative of these if dividing by $(3-x)$ )

Obtain $(x-3)\left(-4 x^{2}-3 x+1\right)$
(or $\left.(3-x)\left(4 x^{2}+3 x-1\right)\right)$

| (iii)$-4 x^{2}-3 x+1=0$ <br> $(1-4 x)(x+1)=0$ <br> $x=1 / 4, x=-1$ | M1 | Attempt to solve quadratic | If factorising, needs to give two correct terms when brackets <br> expanded. If using formula allow sign slips only - need to <br> substitute and attempt one further step. If completing the square <br> must get to $(x+p)= \pm \sqrt{ } q$, with reasonable attempts at $p$ and $q$. |
| :--- | :--- | :--- | :--- |
|  | A1 | $\mathbf{2}$ | Obtain $(1 / 4,0),(-1,0)$ | | Condone only $x$ values given rather than coordinates. |
| :--- |
| Allow if $x=3$ is still present as well. |

(iv) $\int \mathrm{f}(\mathrm{x}) \mathrm{d} x=-x^{4}+3 x^{3}+5 x^{2}-3 x$
$\mathrm{F}(3)-\mathrm{F}(1 / 4)=(36)-\left({ }^{-101 / 256}\right)=36^{101 / 256}$ $F(1 / 4)-F(-1)=\left({ }^{-101 / 256}\right)-(4)=-4^{101} / 256$

Hence area $=36^{101} / 256+4^{101} / 256=40^{101} / 128$

Obtain $-x^{4}+3 x^{3}+5 x^{2}-3 x$

Attempt F(3) - F (1/4) or $F(1 / 4)-F(-1)$

Obtain at least one correct area, including decimal equivs

Attempt full method to find total area including dealing correctly with negative area

5 Obtain $40^{101} / 128$ or $5221 / 128$ or 40.8

Allow unsimplified coefficients.
Condone $+c$.

Allow use of incorrect limits from their (iii).
Limits need to be in correct order, and subtraction.
Allow slips when evaluating but clear subtraction attempt must be seen or implied at least once.
If minimal method shown then it must appear to be a plausible attempt eg $F(3)=198$ or even $F(3)-F(1 / 4)=198.4$.

Obtain $36^{101} / 256$ or ${ }_{1125}^{9317} / 256$ or 36.4
or $-4^{101} / 256$ or $-1125 / 256$ or -4.4
Can get A1 if both areas attempted and one is correct but the other isn't.

Need to see modulus of negative integral from attempt at
$F(1 / 4)-F(-1)$ (just changing sign from $-v e$ to + ve is sufficient). If values incorrect in (iii) then can only get this mark if their integral gives negative value.
Need to have positive integral from $F(3)-F(1 / 4)$.
Allow exact fraction (including unsimplified ie ${ }^{10442} / 256$ ), or decimal answer to 3 dp or better (rounding to 40.8 with no errors seen)

SR: If candidate attempts $F(3)-F(1 / 4)$ and $F(-1)-F(1 / 4)$ as an alternative method for dealing with negative area then mark as B1 correct integral
M2 complete method
A1 obtain one correct area
A1 obtain correct total area
Any attempts using this method must be fully supported by evidence of intention, especially -1 as top limit and $1 / 4$ as bottom limit used consistently throughout integration attempt. It should not be awarded if candidate appears to have simply confused their order of subtraction.

## Guidance for marking C2

## Accuracy

Allow answers to 3sf or better, unless an integer is specified or clearly required.
3sf is sometimes explicitly specified in a question - this is telling candidates that a decimal is required rather than an exact answer eg in logs, and more than 3sf should not be penalised unless stated in mark scheme.
If more than 3sf is given, allow the marks for an answer that rounds to the one in the mark scheme, with no obvious errors.

## Extra solutions

Candidates will usually be penalised if an extra, incorrect, solution is given. However, in trigonometry questions only look at solutions in the given range and ignore any others, correct or incorrect.

## Solving equations

With simultaneous equations, the method mark is given for eliminating one variable allowing sign errors, addition / subtraction confusion or incorrect order of operations. Any valid method is allowed ie balancing or substitution for two linear equations, substitution only if at least one is non-linear.

## Solving quadratic equations

Factorising - candidates must get as far as factorising into two brackets which, on expansion, would give the correct coefficient of $x^{2}$ and at least one of the other two coefficients.
Completing the square - candidates must get as far as $(x+p)= \pm \sqrt{ } q$, with reasonable attempts at $p$ and $q$.
Using the formula - candidates need to substitute values into the formula and do at least one further step. Sign slips are allowed on $b$ and $4 a c$, but all other aspects of the formula must be seen correct, either algebraic or numerical. If the algebraic formula is quoted then candidates are allowed to make one slip when substituting their values. Condone not dividing by $2 a$ as long as it has been seen earlier.

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