

ADVANCED GCE MATHEMATICS

4723

Core Mathematics 3

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:

- Printed answer book 4723
- List of Formulae (MF1)

Other materials required:

· Scientific or graphical calculator

Monday 13 June 2011 Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the printed answer book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer all the questions.
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [] at the end of each question or part question on the question paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

• Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 Find

(i)
$$\int 6e^{2x+1} dx,$$

(ii)
$$\int 10(2x+1)^{-1} dx$$
. [5]

2 The curve $y = \ln x$ is transformed by:

a reflection in the x-axis,

followed by a stretch with scale factor 3 parallel to the y-axis,

followed by a translation in the positive y-direction by ln 4.

Find the equation of the resulting curve, giving your answer in the form $y = \ln(f(x))$. [4]

- 3 (a) Given that $7 \sin 2\alpha = 3 \sin \alpha$, where $0^{\circ} < \alpha < 90^{\circ}$, find the exact value of $\cos \alpha$. [3]
 - (b) Given that $3\cos 2\beta + 19\cos \beta + 13 = 0$, where $90^{\circ} < \beta < 180^{\circ}$, find the exact value of $\sec \beta$. [5]
- 4 (i) Show by means of suitable sketch graphs that the equation

$$(x-2)^4 = x + 16$$

has exactly 2 real roots.

(ii) State the value of the smaller root. [1]

[3]

[4]

(iii) Use the iterative formula

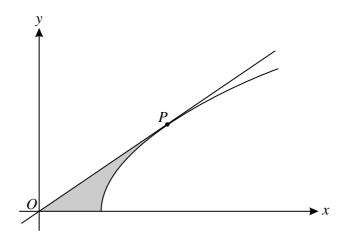
$$x_{n+1} = 2 + \sqrt[4]{x_n + 16},$$

with a suitable starting value, to find the larger root correct to 3 decimal places.

5 The equation of a curve is $y = x^2 \ln(4x - 3)$. Find the exact value of $\frac{d^2y}{dx^2}$ at the point on the curve for which x = 2.

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The diagram shows the curve with equation $y = \sqrt{3x - 5}$. The tangent to the curve at the point *P* passes through the origin. The shaded region is bounded by the curve, the *x*-axis and the line *OP*. Show that the *x*-coordinate of *P* is $\frac{10}{3}$ and hence find the exact area of the shaded region. [9]

7 The functions f, g and h are defined for all real values of x by

$$f(x) = |x|,$$
 $g(x) = 3x + 5$ and $h(x) = gg(x).$

(i) Solve the equation
$$g(x+2) = f(-12)$$
. [3]

(ii) Find
$$h^{-1}(x)$$
. [3]

(iii) Determine the values of x for which

$$x + f(x) = 0. ag{2}$$

[2]

8 An experiment involves two substances, Substance 1 and Substance 2, whose masses are changing. The mass, M_1 grams, of Substance 1 at time t hours is given by

$$M_1 = 400e^{-0.014t}.$$

The mass, M_2 grams, of Substance 2 is increasing exponentially and the mass at certain times is shown in the following table.

t (hours)	0	10	20
M_2 (grams)	75	120	192

A critical stage in the experiment is reached at time T hours when the masses of the two substances are equal.

- (i) Find the rate at which the mass of Substance 1 is decreasing when t = 10, giving your answer in grams per hour correct to 2 significant figures. [3]
- (ii) Show that T is the root of an equation of the form $e^{kt} = c$, where the values of the constants k and c are to be stated. [5]
- (iii) Hence find the value of T correct to 3 significant figures.

[Question 9 is printed overleaf.]

9 (i) Prove that
$$\frac{\sin(\theta - \alpha) + 3\sin\theta + \sin(\theta + \alpha)}{\cos(\theta - \alpha) + 3\cos\theta + \cos(\theta + \alpha)} = \tan\theta \text{ for all values of } \alpha.$$
 [5]

(ii) Find the exact value of
$$\frac{4\sin 149^\circ + 12\sin 150^\circ + 4\sin 151^\circ}{3\cos 149^\circ + 9\cos 150^\circ + 3\cos 151^\circ}.$$
 [3]

(iii) It is given that k is a positive constant. Solve, for $0^{\circ} < \theta < 60^{\circ}$ and in terms of k, the equation

$$\frac{\sin(6\theta - 15^{\circ}) + 3\sin 6\theta + \sin(6\theta + 15^{\circ})}{\cos(6\theta - 15^{\circ}) + 3\cos 6\theta + \cos(6\theta + 15^{\circ})} = k.$$
 [4]



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