# Edexcel GCE Statistics S4 Advanced/Advanced Subsidiary Wednesday 16 June 2004 - Afternoon Time: 1 hour 30 minutes 

Materials required for examination<br>Items included with question papers<br>Answer Book (AB16)<br>Graph Paper (ASG2)<br>Mathematical Formulae (Lilac)

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Statistics S4), the paper reference (6686), your surname, other name and signature.
Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. The random variable $X$ has an $F$-distribution with 8 and 12 degrees of freedom.

Find $\mathrm{P}\left(\frac{1}{5.67}<X<2.85\right)$.
2. A mechanic is required to change car tyres. An inspector timed a random sample of 20 tyre changes and calculated the unbiased estimate of the population variance to be 6.25 minutes $^{2}$. Test, at the $5 \%$ significance level, whether or not the standard deviation of the population of times taken by the mechanic is greater than 2 minutes. State your hypotheses clearly.
3. It is suggested that a Poisson distribution with parameter $\lambda$ can model the number of currants in a currant bun. A random bun is selected in order to test the hypotheses $\mathrm{H}_{0}: \lambda=8$ against $\mathrm{H}_{1}: \lambda \neq 8$, using a $10 \%$ level of significance.
(a) Find the critical region for this test, such that the probability in each tail is as close as possible to $5 \%$.
(b) Given that $\lambda=10$, find
(i) the probability of a type II error,
(ii) the power of the test.
4. A doctor believes that the span of a person's dominant hand is greater than that of the weaker hand. To test this theory, the doctor measures the spans of the dominant and weaker hands of a random sample of 8 people. He subtracts the span of the weaker hand from that of the dominant hand. The spans, in mm, are summarised in the table below.

|  | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ | $H$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dominant hand | 202 | 251 | 215 | 235 | 210 | 195 | 191 | 230 |
| Weaker hand | 195 | 249 | 218 | 234 | 211 | 197 | 181 | 225 |

Test, at the 5\% significance level, the doctor's belief.
5. (a) Explain briefly what you understand by
(i) an unbiased estimator,
(ii) a consistent estimator.
of an unknown population parameter $\theta$.

From a binomial population, in which the proportion of successes is $p, 3$ samples of size $n$ are taken. The number of successes $X_{1}, X_{2}$, and $X_{3}$ are recorded and used to estimate $p$.
(b) Determine the bias, if any, of each of the following estimators of $p$.

$$
\begin{align*}
& \hat{p}_{1}=\frac{X_{1}+X_{2}+X_{3}}{3 n} \\
& \hat{p}_{2}=\frac{X_{1}+3 X_{2}+X_{3}}{6 n}, \\
& \hat{p}_{3}=\frac{2 X_{1}+3 X_{2}+X_{3}}{6 n} . \tag{4}
\end{align*}
$$

(c) Find the variance of each of these estimators.
(d) State, giving a reason, which of the three estimators for $p$ is
(i) the best estimator,
(ii) the worst estimator.
6. A supervisor wishes to cheek the typing speed of a new typist. On 10 randomly selected occasions, the supervisor records the time taken for the new typist to type 100 words. The results, in seconds, are given below.

$$
110,125,130,126,128,127,118,120,122,125
$$

The supervisor assumes that the time taken to type 100 words is normally distributed.
(a) Calculate a 95\% confidence interval for
(i) the mean,
(ii) the variance
of the population of times taken by this typist to type 100 words.

The supervisor requires the average time needed to type 100 words to be no more than 130 seconds and the standard deviation to be no more than 4 seconds.
(b) Comment on whether or not the supervisor should be concerned about the speed of the new typist.
7. A grocer receives deliveries of cauliflowers from two different growers, $A$ and $B$. The grocer takes random samples of cauliflowers from those supplied by each grower. He measures the weight $x$, in grams, of each cauliflower. The results are summarised in the table below.

|  | Sample size | $\Sigma x$ | $\Sigma x^{2}$ |
| :---: | :---: | :---: | :---: |
| $A$ | 11 | 6600 | 3960540 |
| $B$ | 13 | 9815 | 7410579 |

(a) Show, at the $10 \%$ significance level, that the variances of the populations from which the samples are drawn can be assumed to be equal by testing the hypothesis $\mathrm{H}_{0}: \sigma_{A}^{2}=\sigma_{B}^{2}$ against hypothesis $\mathrm{H}_{1}: \sigma_{A}^{2} \neq \sigma_{B}^{2}$.
(You may assume that the two samples come from normal populations.)

The grocer believes that the mean weight of cauliflowers provided by $B$ is at least 150 g more than the mean weight of cauliflowers provided by $A$.
(b) Use a 5\% significance level to test the grocer's belief.
(c) Justify your choice of test.

