Mark Scheme (Results)

Summer 2012

GCE Statistics S4
(6686) Paper 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

June 2012
6686 Statistics S4
Mark Scheme

Question Number	Scheme	Marks
1 (a)	$H_0: \mu_d = 0$, $H_1: \mu_d > 0$ (or $H_1: \mu_d < 0$)	B1
	where μ_d is the (population) mean difference :- BP sitting down – BP standing. (BP standing – BP sitting down)	
	Assume the differences are normally distributed	B1
		(2)
(b)	d: 4, -1, 6, 6, 3, -2, 9, -1, 4, 7, -11, 7	M1
	$(\Sigma d = 31, \Sigma d^2 = 419)$ $\bar{d} = \pm 2.5833$; sd = 5.55073. (or Var = 30.8106)	A1; A1
	$t = \dfrac{\pm 2.5833\sqrt{12}}{5.55073} = \pm 1.612\ldots$ Formula and substitution, 1.61	M1, A1
	Critical value $t_{11}(1\%) = 2.718$(1 tail)	B1
	Not significant. Insufficient evidence to support that the blood pressure of a person sitting down is more than the blood pressure of a person after standing up.	A1 ft
		(7) **Total 9 marks**

	Notes
1a	B1 both hypotheses. B1 must be differences
1b	M1 at least 2 correct or may be implied by correct Σd or Σd^2 or $\bar{d}$ or sd or var or implied by correct t value A1 correct $\bar{d}$ awrt ± 2.58- may be implied by correct t value A1 correct sd awrt 5.55 or var awrt 30.8 - may be implied by correct t value M1 $\dfrac{\pm \text{their } \bar{d}\sqrt{12}}{\text{their sd}}$ A1 awrt 1.61 B1 CV A1ft follow through their t value – need context of **blood pressure** and **sitting** and **standing**

Question Number	Scheme	Marks
2 (a)	$S_F^2 = \frac{1}{5}\{2308.01 - 6\times19.6^2\} = 0.61$	B1
	$S_M^2 = \frac{1}{11}\{2262.57 - 12\times13.7^2\} = 0.93545..$	B1
	H_0: $\mu_F = \mu_M + 5$; H_1: $\mu_F \neq \mu_M + 5$ both	B1
	CR: $t_{16}(0.025) > 2.120$ 2.12	B1
	$S_p^2 = \frac{5\times0.61+11\times0.93545...}{16} = 0.83375$	M1 A1
	$t = \frac{19.6-13.7-5}{\sqrt{0.83375\left(\frac{1}{6}+\frac{1}{12}\right)}} = 1.971$	M1 A1ftA1
	Since 1.971 is not in the critical region we accept H_0 and conclude that the mean shell length of female turtles does exceed the shell length of male turtles by 5cm.(or Biologists claim is correct)	A1 ft
		(10)
(b)(i)	$-1.96 < \frac{\overline{X}_F - \overline{X}_M - 5}{\sqrt{\left(\frac{0.9}{6}+\frac{0.9}{12}\right)}} < 1.96$	B1 M1
	$4.07 < \overline{X}_F - \overline{X}_M < 5.93$	A1cso
(ii)	P(Type II error) = P($4.07 < \overline{X}_F - \overline{X}_M < 5.93$ | N(6, 0.225))	M1
	$= P(\frac{4.07-6}{\sqrt{0.225}} < z < \frac{5.93-6}{\sqrt{0.225}})$	M1
	$= 0.44$ awrt 0.44	A1
		(6)
		Total 16 marks
2(a)	B1 – awrt 0.61 B1 – awrt 0.935 Both may be implied by correct t value or S_p B1 allow rearrangements eg $\mu_F - \mu_M = 5$. If M and F not used then they must make clear what each letter is. B1 CV (if using one tail test allow 1.746) M1 $\frac{5\times\text{their }0.61+11\times\text{their }0.93545...}{16}$ A1 awrt 0.834 M1 $\pm\left(\frac{19.6-13.7-5}{\sqrt{p\left(\frac{1}{6}+\frac{1}{12}\right)}}\right)$ where p is either their 0.61 or 0.94 or their S_p^2 (awrt 0.834) (Allow 13.7 - 19.6 - 5) A1 ft their S_p^2 A1 awrt 1.97	
(b)	B1 1.96 M1 must use z value	
(c)	M1 writing or using N(6, 0.225) M1 finding correct area and standardising (must use 6 but allow use of 0.9 and (0.9/18) for var)	

Question Number	Scheme	Marks
3.	H_0: $\sigma_A^2 = \sigma_B^2$; H_1: $\sigma_A^2 \neq \sigma_B^2$	B1
	$S_A^2 / S_B^2 = \frac{225}{36} = 6.25 \quad \left(\frac{36}{225} = 0.16\right)$	M1A1
	CR: $F_{10,8} > 3.35 \quad \left(\frac{1}{F_{10.8}} = 0.299\right)$	B1
	Since 6.25 is in the critical region we can assume that the lengths of paving slabs sold by the builders merchant differ in variability.	A1ft
		(5)
		Total 5 marks
	B1 both correct. Must use σ .May use different notation to *A* and *B*	
	M1 $\frac{225}{36}$ or $\frac{36}{225}$ allow $\frac{15}{6}$ or $\frac{6}{15}$	
	A1 either 6.25 or 0.16	
	B1 CR must match their method	
	A1 context must include “lengths of slabs”	

Question Number	Scheme	Marks
4 (a)	$\bar{x} = 4.9$	B1
	$s = \sqrt{0.191..}$ (0.437...)	B1
	(NB: $\Sigma x = 49$; $\Sigma x^2 = 241.82$)	
(i)	95% confidence interval is given by	
	$4.9 \pm 2.262 \times \sqrt{\dfrac{0.191..}{10}}$	M1A1ft B1
	i.e: (4.587…, 5.212 …)	A1 A1
(ii)	95% confidence interval is given by	
	$\dfrac{9\times 0.437...^2}{19.023} < \sigma^2 < \dfrac{9\times 0.437...^2}{2.7}$ use of $\dfrac{(n-1)s^2}{\chi^2_{n-1}}$	M1B1B1A1
	i.e; (0.0904, 0.63704)	A1 A1
(b)		**(13)**
	5 lies inside the confidence interval	B1ft
	0.49(0.7^2) lies inside the confidence interval	B1ft
	Yes it does meet the time requirement	B1 ft
		(3)
		Total 16 marks

Question Number	Scheme	Marks
(a)	B1 B1 may be implied by correct a correct answer to (i) or (ii)	
(i)	M1 - "their 4.9" ± t value × $\sqrt{\frac{\text{their } 0.191..}{10}}$ A1ft - "their 4.9" ± 2.262 × $\sqrt{\frac{\text{their } 0.191..}{10}}$ B1 2.262 A1 either correct to 3sf or better or both correct to 2sf or better A1 both correct to 3sf or better	
(ii)	M1 – writing and attempting to use $\frac{(n-1)s^2}{\chi^2_{n-1}}$ or may be implied by correct formula used with their 0.437 B1 19.023 B1 2.7 A1ft follow through their 0.437 and two chi squared values A1 either correct to 2sf or better A1 awrt (0.09, 0.637)	
(b)	For the second B1. If both 0.7 and 0.49 lie in interval they must state variance = 0.49 or the interval for standard deviation. For the third B1 their must not be two conflicting conclusions unless they give just one overall as well.	

Question Number	Scheme	Marks
5.(a)	H_0: $\sigma^2 = 36$; H_1: $\sigma^2 > 36$	B1
	$\upsilon = 24$, X^2_{24} (0.05) = 36.415	B1
	$\frac{(n-1)S^2}{\sigma^2} = \frac{24\times 55}{36} = 36.67$	M1 A1
	Since 36.67 > 36.415 there is sufficient evidence to reject H_0.	A1 ft
	There is evidence to suggest that the variance is greater than 36.	A1 ft
		(6)
(b)		
	$H_0 : \mu = 450$ $H_1 : \mu > 450$	B1
	$t_{24} = 1.711$	B1
	$t = \pm \frac{455-450}{\sqrt{\frac{55}{25}}} = \pm 3.37\ldots$	M1 A1
	Significant; The **<u>mean weight</u>** of chocolates is **<u>greater than 450,</u>** Or **<u>μ is more than 450</u>**	A1ft; A1ft **(6)**
(c)	The **<u>weights</u>** are normally distributed	B1 **(1)**
		Total 13 marks
	Notes	
(a)	B1 both correct. Also allow H_0: $\sigma = 6$; H_1: $\sigma > 6$ B1 36.415 M1 use of $\frac{(n-1)S^2}{\sigma^2}$ A1 awrt 36.7	
(b)	M1 $\pm \frac{455-450}{\sqrt{\frac{55}{25}}}$ A1 awrt 3.4	
	A1ft any statement – no conflicting A1ft contextual statement must include " weight of chocolate" and is "greater than 50"	

Question Number	Scheme	Marks	
6(a)(i)	$\text{E}(\hat{p}_1) = \text{E}\left(\frac{X}{n}\right)$		
	$= \frac{1}{n}\text{E}(X)$		
	$= \frac{1}{n} \times np$	M1	
	$= p$ unbiased	A1cso	
(ii)	$\text{Var}(\hat{p}_1) = \text{Var}\left(\frac{X}{n}\right)$		
	$= \frac{1}{n^2}\text{Var}(X)$	M1	
	$= \frac{1}{n^2} \times np(1- p)$		
	$= \frac{p(1-p)}{n}$	A1	
			(4)
b(i)	$\text{E}(\hat{p}_3) = 3a\,\text{E}(\hat{p}_1) + 2a\,\text{E}(\hat{p}_2)$	M1	
	$= 3ap + 2ap$		
	$= 5ap$		
	$5ap = p$	M1	
	$a = \frac{1}{5}$	A1	
(ii)	$\text{Var}(\hat{p}_3) = \frac{9}{25}\,\text{Var}(\hat{p}_1) + \frac{4}{25}\,\text{Var}(\hat{p}_2)$	M1	
	$= \frac{9p(1-p)}{25n} + \frac{4p(1-p)}{25m}$	M1d	
	$= \frac{p(1-p)}{25}\left(\frac{9}{n} + \frac{4}{m}\right)$	A1	
			(6)
(c)	$\frac{p(1-p)}{25}\left(\frac{9}{n} + \frac{4}{m}\right) < \frac{p(1-p)}{n}$	M1	
	$9m + 4n < 25m$		
	$4n < 16m$		
	$\frac{n}{m} < 4$		
	$\frac{p(1-p)}{25}\left(\frac{9}{n} + \frac{4}{m}\right) < \frac{p(1-p)}{m}$	M1	
	$9m + 4n < 25n.$		

Question Number	Scheme	Marks
	$9m < 21n$ $\frac{9}{21} < \frac{n}{m}$ or $\frac{3}{7} < \frac{n}{m}$	
	$\frac{3}{7} < \frac{n}{m} < 4$	A1 **(3)**
(d)	Var($\hat{p}_1$) = 0.05 $p(1 - p)$ Var($\hat{p}_2$) = 0.0167 $p(1 - p)$ Var($\hat{p}_3$) = 0.0207 $p(1 - p)$	M1
	Or since $\frac{1}{3}$ is not in the range $\frac{9}{21} < \frac{n}{m} < 4$ Var($\hat{p}_3$) is not the smallest variance. Var($\hat{p}_1$) = 0.05 $p(1 - p)$ Var($\hat{p}_2$) = 0.0167 $p(1 - p)$	
	Therefore $\hat{p}_2$; is the best estimator as it has the smallest variance	A1ft; A1ft **(3)** **Total 16 marks**
	Notes	
(a) (i)	M1 either $\frac{1}{n}$ E(X) or $\frac{1}{n} \times np$	
	A1 cso	
(ii)	M1 either $\frac{1}{n^2}$ Var(X) or $\frac{1}{n^2} \times np(1- p)$ A1 cso	
(b) (i)	M1 For either $3a$ E($\hat{p}_1$) + $2a$ E($\hat{p}_2$) or $3ap + 2ap$ M1 Putting their E($\hat{p}_3$) = p	
(ii)	M1 for $\frac{9}{25}$ Var($\hat{p}_1$) + $\frac{4}{25}$ Var($\hat{p}_2$) M1d for substituting (aii) for Var($\hat{p}_1$) and (aii) with m instead of n for Var($\hat{p}_2$) A1 cso	
(c)	M1 Putting Var($\hat{p}_3$) < their Var($\hat{p}_1$) leading to an inequality of the form $\frac{n}{m} < a$ or $\frac{n}{m} > a$ where a is a constant.	
	M1 Putting Var($\hat{p}_3$) < their Var($\hat{p}_2$) leading to an inequality of the form $\frac{n}{m} > a$ or	

	$\frac{n}{m} < a$ where a is a constant.		
(d)	1/3 is not in their range in part(c) M1 attempt to find all 3 variances or eliminating Var($\hat{p}_3$) with reason and finding the other 2 variances. A1ft correct estimator chosen. A1ft correct supporting reason from correct working for their var formulae SC if 1/3 is in their range in part(c) they may get B1 for stating $\hat{p}_3$ B1dependent on the previous B being awarded- stating smallest variance award first two marks on epen.		

PMT

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