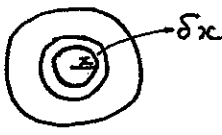



January 2006
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
1.	$\underline{AB} = k(2\underline{i} - \underline{j} + 2\underline{k}); AB = 6 \Rightarrow k\sqrt{(2^2+1^2+2^2)} = 6$ $\Rightarrow k = 2 \Rightarrow \underline{AB} = 4\underline{i} - 2\underline{j} + 4\underline{k}$ $\text{Work done} = (7\underline{i} + 4\underline{j} - 2\underline{k}) \cdot (4\underline{i} - 2\underline{j} + 4\underline{k})$ $= \underline{12J}$	M1 A1 M1 A1 (4)
2.	 $\delta m = \frac{m}{\pi a^2} \cdot 2\pi x \delta x$ $\delta I = \frac{m}{\pi a^2} \cdot 2\pi x^3 \delta x$ $I = \frac{2m}{a^2} \int_0^a x^3 dx = \underline{\frac{1}{2} ma^2}$	M1 A1 M1 M1 A1 (5)
3.	<p>Either CF: $\underline{\dot{\Gamma}} + 2\underline{\Gamma} = \underline{0}$ OR IF = e^{2t}</p> <p>$\Rightarrow \underline{\Gamma} = \underline{A} e^{-2t}$ M1 A1</p> <p>PI $\underline{\Gamma} = 2\underline{i}$ B1</p> <p>GS $\underline{\Gamma} = \underline{A} e^{-2t} + 2\underline{i}$ A1</p> <hr style="border-left: 1px dashed black;"/> <p>$\frac{d}{dt}(\underline{\Gamma} e^{2t}) = 4\underline{i} e^{2t}$</p> <p>$\underline{\Gamma} e^{2t} = 2\underline{i} e^{2t} + \underline{A}$</p> <p>$\underline{\Gamma} = 2\underline{i} + \underline{A} e^{-2t}$</p> <p>$t = 0 \quad \underline{\Gamma} = 2\underline{j} \Rightarrow \underline{A} = 2\underline{j} - 2\underline{i}$</p> <p>$\underline{\Gamma} = (2\underline{j} - 2\underline{i})e^{-2t} + 2\underline{i}$ or $2\underline{i}(1 - e^{-2t}) + 2\underline{j}e^{-2t}$</p>	B1 M1 A1 A1 M1 A1 (6)
4.	 <p>(a) $m(A): \frac{4}{3} ma^2 \ddot{\theta} = -mga \sin \theta$</p> <p>Small $\theta \Rightarrow \sin \theta \approx \theta \rightarrow \ddot{\theta} = -\frac{3g}{4a} \theta$</p> <p>$\therefore$ Approx. SHM</p> <p>(b) Time = $\frac{1}{4}$ period = $\frac{1}{4} \cdot 2\pi \sqrt{\frac{4a}{3g}} = \underline{\pi \sqrt{\frac{a}{3g}}}$</p>	M1 A1 M1 A1 (4) M1 A1 (2) (6)

Question Number	Scheme	Marks
5.	<p>MI of disc abt diameter = $\frac{1}{4} ma^2$ \therefore MI of disc about axis = $\frac{1}{4} ma^2 + ma^2 = \frac{5}{4} ma^2$</p> <p>CAM: $m \cdot a \cdot \sqrt{kg} a = \left(\frac{5}{4} ma^2 + ma^2\right) \omega$ $\Rightarrow \omega = \frac{4}{9} \sqrt{\frac{kg}{a}}$</p> <p>Energy: $\frac{1}{2} \cdot \frac{9ma^2}{4} \cdot \frac{16kg}{81a} = 2mga$ $\Rightarrow \underline{k = 9}$</p>	<p>MI MI AI MI AI AI MI AI AI AI (10)</p>
6.	<p>(a) $\underline{SQ} = \underline{q} - \underline{s} = 4\underline{j} - 3\underline{k} \Rightarrow \underline{F}_1 = 16\underline{j} - 12\underline{k}$ $\underline{SR} = -3\underline{i} - 2\underline{j} \Rightarrow \underline{F}_2 = -6\underline{i} - 4\underline{j}$</p> <p>Net couple alone $\Rightarrow \Sigma \underline{F}_i = \underline{0}$ $\Rightarrow \underline{F}_3 = \underline{6i} - 12\underline{j} + 12\underline{k}$</p> <p>(b) $M(s) \quad \underline{G} = \underline{SP} \times \underline{F}_3$ $= (-7\underline{i} + 4\underline{j} - 2\underline{k}) \times (6\underline{i} - 12\underline{j} + 12\underline{k})$ (or complete expressⁿ if about another pt.)</p> <p>$\underline{G} = 24\underline{i} + 72\underline{j} + 60\underline{k}$ $= 12(2\underline{i} + 6\underline{j} + 5\underline{k})$ $\underline{G} = \underline{12\sqrt{65}}$</p>	<p>MI AI MI AI MI AI (6) MI AI AI MI AI MI AI (6)</p>

7.

 $0 \uparrow \bigcirc \delta m$
 $\uparrow v + \delta v$
 $v \uparrow \bigcirc m$
 $\bigcirc m + \delta m$

(a)

$$(m + \delta m)(v + \delta v) - mv = -mg \delta t$$

$$mv + m\delta v + v\delta m + \delta m\delta v - mv = -mg \delta t$$

$$\underline{m \frac{dv}{dt} + v \frac{dm}{dt} = -mg} \quad (*)$$

(b)

$$m = Me^{kt} \Rightarrow \frac{dm}{dt} = kMe^{kt}$$

$$\text{Hence } Me^{kt} \frac{dv}{dt} + v \cdot kMe^{kt} = -Me^{kt}g$$

$$\Rightarrow \underline{\frac{d}{dt}(ve^{kt}) = -ge^{kt}} \quad (*)$$

(c)

$$ve^{kt} = -g \int e^{kt} dt$$

$$= -\frac{g}{k} e^{kt} + C$$

$$t = 0, v = \frac{g}{2k} \Rightarrow C = \frac{3g}{2k}$$

$$\text{At highest pt } v = 0: -\frac{g}{k} e^{kt} + \frac{3g}{2k} = 0$$

$$\Rightarrow e^{kt} = \frac{3}{2}$$

$$\text{Hence } m = Me^{kt} = \underline{\underline{\frac{3M}{2}}}$$

m1 A2,1,0

m1 A1
(5)

B1

m1

A1
(3)

m1

A1

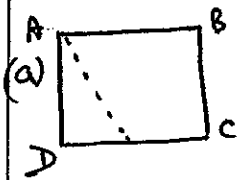
B1

m1

A1

m1 A1
(7)

8.



$$I_{AB} = 2 \times \frac{4}{3} ma^2 + m(2a)^2 = \frac{20}{3} ma^2$$

(BC, AD) (CD)

By \perp axis: $I_A = 2 \times \frac{20}{3} ma^2 = \frac{40}{3} ma^2$ (*)

[or $I_A = 2 \times \frac{4}{3} ma^2 + 2 \left(\frac{1}{3} ma^2 + 5ma^2 \right)$
 (AB, AD) (BC, CD)
 $= \frac{40}{3} ma^2$]

m | A | A

m | A | (5)

[m |, m | A | A |
 A |]

(b) M(A) $\frac{40ma^2}{3} \ddot{\theta} = 4mg \cdot a\sqrt{2}$
 $\ddot{\theta} = \frac{3g\sqrt{2}}{10a}$

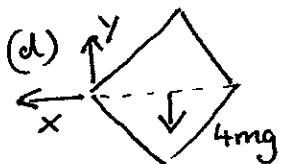
m | A |

A | (3)

(c) $\frac{1}{2} \cdot \frac{40ma^2}{3} \dot{\theta}^2 = 4mg \cdot a\sqrt{2}$
 $\dot{\theta} = \sqrt{\frac{3g\sqrt{2}}{5a}}$

m | A |

A | (3)



R(\leftarrow) $X = 4ma\sqrt{2} \dot{\theta}^2$
 $= 4ma\sqrt{2} \cdot \frac{3g\sqrt{2}}{5a} = \frac{24mg}{5}$

R(\uparrow) $4mg - Y = 4ma\sqrt{2} \ddot{\theta}$
 $Y = 4mg - 4ma\sqrt{2} \cdot \frac{3g\sqrt{2}}{10a}$

$Y = \frac{8mg}{5}$

$R = \sqrt{X^2 + Y^2} = \frac{8mg}{5} \sqrt{1^2 + 3^2}$
 $= \frac{8\sqrt{10}}{5} mg$

m |
 A |

m |

A |

m |

A |