# Mark Scheme (Post-Standardisation)
# Summer 2007

GCE

# GCE Mathematics (6674/01)

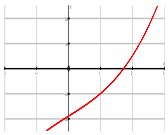
Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

# June 2007
# 6674 Further Pure Mathematics FP1
# Mark Scheme

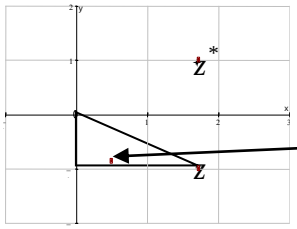
| Question number | Scheme | Marks |
|---|---|---|
| 1. | $1\frac{1}{2}$ and 3 are 'critical values', e.g. used in solution, or both seen as asymptotes | B1 |
| | $(x+1)(x-3) = 2x-3 \Rightarrow \quad x(x-4) = 0$ | |
| | $x = 4, \quad x = 0$ M1: attempt to find at least one other critical value | M1 A1, A1 |
| | $0 < x < 1\frac{1}{2}, \quad 3 < x < 4$ M1: An inequality using $1\frac{1}{2}$ or 3 | M1 A1, A1 (7) |
| | | **7** |
| | First M mark can be implied by the two correct values, but otherwise a method must be seen. (The method may be graphical, but either ($x$ = ) 4 or ($x$ =) 0 needs to be clearly written or used in this case).<br>Ignore 'extra values' which might arise through 'squaring both sides' methods.<br>≤ appearing: maximum one A mark penalty (final mark). | |

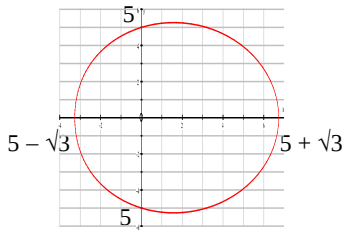
| Question number | Scheme | Marks |
|---|---|---|
| 2. | Integrating factor $e^{\int -\tan x\,dx} = e^{\ln(\cos x)}$ $\left(\text{or } e^{-\ln(\sec x)}\right)$, $= \cos x$ $\left(\text{or } \frac{1}{\sec x}\right)$ | M1, A1 |
| | $\left(\cos x\frac{dy}{dx} - y\sin x = 2\sec^2 x\right)$ | |
| | $y\cos x = \int 2\sec^2 x\,dx$ (or equiv.) $\left(\text{Or: } \frac{d}{dx}(y\cos x) = 2\sec^2 x\right)$ | M1 A1(ft) |
| | $y\cos x = 2\tan x\ (+C)$ (or equiv.) | A1 |
| | $y = 3$ at $x = 0$: $C = 3$ | M1 |
| | $y = \frac{2\tan x + 3}{\cos x}$ (Or equiv. in the form $y = f(x)$) | A1 (7) |
| | | **7** |
| | 1st M: Also scored for $e^{\int \tan x\,dx} = e^{-\ln(\cos x)}$ $\left(\text{or } e^{\ln(\sec x)}\right)$, then A0 for $\sec x$. | |
| | 2nd M: Attempt to use their integrating factor (requires one side of the equation 'correct' for their integrating factor). | |
| | 2nd A: The follow-through is allowed <u>only</u> in the case where the integrating factor used is sec $x$ or $-\sec x$. $\left(y\sec x = \int 2\sec^4 x\,dx\right)$ | |
| | 3rd M: Using $y = 3$ at $x = 0$ to find a value for $C$ (dependent on an integration attempt, however poor, on the RHS). | |
| | <u>Alternative</u><br>1st M: Multiply through the given equation by $\cos x$. | |
| | 1st A: Achieving $\cos x\frac{dy}{dx} - y\sin x = 2\sec^2 x$. (Allowing the possibility of integrating by inspection). | |

| Question number | Scheme | Marks |
|---|---|---|
| 3. | (a) $(r+1)^3 = r^3 + 3r^2 + 3r + 1$ and $(r-1)^3 = r^3 - 3r^2 + 3r - 1$ | M1 |
| | $(r+1)^3 - (r-1)^3 = 6r^2 + 2$ (*) | A1cso (2) |
| | (b) $r = 1$: $2^3 - 0^3 = 6(1^2) + 2$ | |
| | $r = 2$: $3^3 - 1^3 = 6(2^2) + 2$ | |
| | : : : : : : : : : : : : | |
| | $r = n$: $(n+1)^3 - (n-1)^3 = 6n^2 + 2$ M: Differences: at least first, last and one other. | M1 A1 |
| | Sum: $(n+1)^3 + n^3 - 1 = 6\sum r^2 + 2n$ M: Attempt to sum at least one side. $\left(6\sum r^2 = 2n^3 + 3n^2 + n\right)$ | M1 A1 |
| | $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ (Intermediate steps are not required) (*) | A1cso (5) |
| | (c) $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2$, $= \frac{1}{6}(2n)(2n+1)(4n+1) - \frac{1}{6}(n-1)n(2n-1)$ | M1, A1 |
| | $= \frac{1}{6}n\left(\left(16n^2 + 12n + 2\right) - \left(2n^2 - 3n + 1\right)\right)$ | M1 |
| | $= \frac{1}{6}n(n+1)(14n+1)$ | A1 (4) |
| | | **11** |
| | (b) 1st A: Requires first, last and one other term correct on both LHS and RHS (but condone 'omissions' if following work is convincing). | |
| | (c) 1st M: Allow also for $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n} r^2$. | |
| | 2nd M: Taking out (at some stage) factor $\frac{1}{6}n$, and multiplying out brackets to reach an expression involving $n^2$ terms. | |

| Question number | Scheme | Marks |
|---|---|---|
| 4. | (a) $f'(x) = 3x^2 + 8$ $\quad 3x^2 + 8 = 0$..... or $3x^2 + 8 > 0$..... | M1 |
| | Correct derivative and, e.g., 'no turning points' or 'increasing function'. | A1 |
| |  Simple sketch, (increasing, crossing positive $x$-axis) (or, if the M1 A1 has been scored, a <u>reason</u> such as 'crosses $x$-axis only once'). | B1 (3) |
| | (b) Calculate $f(1)$ and $f(2)$ (<u>Values</u> must be seen) | M1 |
| | $f(1) = -10$, $f(2) = 5$, Sign change, $\therefore$ Root | A1 (2) |
| | (c) $x_1 = 2 - \dfrac{f(2)}{f'(2)}$, $\quad = 2 - \dfrac{5}{20}$ $\quad (= 1.75)$ | M1, A1 |
| | $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$, $\quad \left(= 1.75 - \dfrac{0.359375}{17.1875}\right) = 1.729$ (ONLY) $(\alpha)$ | M1, A1 (4) |
| | (d) Calculate $f(\alpha - 0.0005)$ and $f(\alpha + 0.0005)$ (or a 'tighter' interval that gives a sign change). | M1 |
| | $f(1.7285) = -0.0077...$ and $f(1.7295) = 0.0092...$, $\therefore$ Accurate to 3 d.p. | A1 (2) |
| | | **11** |
| | (a) M: Differentiate and consider sign of $f'(x)$, or equate $f'(x)$ to zero. | |
| | <u>Alternative</u>: M1: Attempt to rearrange as $x^3 - 19 = -8x$ or $x^3 = 19 - 8x$ (condone sign slips), and to sketch a cubic graph and a straight line graph. A1: Correct graphs (shape correct and intercepts 'in the right place'). B1: Comment such as "one intersection, therefore one root"). | |
| | (c) 1st A1 can be implied by an answer of 1.729, provided N.R. has been used. <u>Answer only</u>: No marks. The Newton-Raphson method must be seen. | |
| | (d) For A1, correct <u>values</u> of f(1.7285) and f(1.7295) must be seen, together with a conclusion. If only 1 s.f. is given in the values, allow rounded (e.g. $-0.008$) <u>or</u> truncated (e.g. $-0.007$) values. | |

| Question number | Scheme | Marks |
|---|---|---|
| 5. | C.F. $m^2+3m+2=0$ $\quad m=-1$ and $m=-2$ | M1 |
| | $y=Ae^{-x}+Be^{-2x}$ | A1 (2) |
| | P.I. $y=cx^2+dx+e$ | B1 |
| | $\frac{dy}{dx}=2cx+d,\ \frac{d^2y}{dx^2}=2c$ $\quad 2c+3(2cx+d)+2(cx^2+dx+e)\equiv 2x^2+6x$ | M1 |
| | $2c=2$ $\quad c=1$ (One correct value) | A1 |
| | $6c+2d=6$ $\quad d=0$ | |
| | $2c+3d+2e=0$ $\quad e=-1$ (Other two correct values) | A1 |
| | General soln: $y=Ae^{-x}+Be^{-2x}+x^2-1$ (Their C.F. + their P.I.) | A1ft (5) |
| | $x=0,\ y=1$: $\ 1=A+B-1$ $\quad (A+B=2)$ | M1 |
| | $\frac{dy}{dx}=-Ae^{-x}-2Be^{-2x}+2x,\quad x=0,\ \frac{dy}{dx}=1$: $\quad 1=-A-2B$ | M1 |
| | Solving simultaneously: $A=5$ and $B=-3$ | M1 A1 |
| | Solution: $y=5e^{-x}-3e^{-2x}+x^2-1$ | A1 (5) |
| | | **12** |
| | 1st M: Attempt to solve auxiliary equation.<br>2nd M: Substitute their $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ into the D.E. to form an identity in $x$ with unknown constants.<br>3rd M: Using $y = 1$ at $x = 0$ in their general solution to find an equation in $A$ and $B$.<br>4th M: Differentiating their general solution (condone 'slips', but the powers of each term must be correct) and using $\frac{dy}{dx}=1$ at $x = 0$ to find an equation in $A$ and $B$.<br>5th M: Solving simultaneous equations to find both a value of $A$ and a value of $B$. | |

| Question number | Scheme | Marks |
|---|---|---|
| 6. | (a) $z^* = \sqrt{3}+\mathrm{i}$ | B1 |
| | $\dfrac{z}{z^*} = \dfrac{(\sqrt{3}-\mathrm{i})(\sqrt{3}-\mathrm{i})}{(\sqrt{3}+\mathrm{i})(\sqrt{3}-\mathrm{i})} = \dfrac{3-2\sqrt{3}\mathrm{i}-1}{3+1}, \quad = \dfrac{1}{2}-\dfrac{\sqrt{3}}{2}\,\mathrm{i}$ (*) | M1, A1cso (3) |
| | (b) $\left\lvert\dfrac{z}{z^*}\right\rvert = \sqrt{\left(\dfrac{1}{2}\right)^2+\left(\dfrac{\pm\sqrt{3}}{2}\right)^2}, \quad =1$ $\left[\text{Or: } \left\lvert\dfrac{z}{z^*}\right\rvert = \dfrac{\lvert z\rvert}{\lvert z^*\rvert} = \dfrac{\sqrt{3+1}}{\sqrt{3+1}}, \quad =1\right]$ | M1, A1 (2) |
| | (c) $\arg(w) = \arctan\left(\pm\dfrac{\text{imag}(w)}{\text{real}(w)}\right)$ or $\arg(w) = \arctan\left(\pm\dfrac{\text{real}(w)}{\text{imag}(w)}\right)$, | M1 |
| | where $w$ is $z$ or $z^*$ or $\dfrac{z}{z^*}$ | |
| | $\arg\left(\dfrac{z}{z^*}\right) = \arctan\left(\dfrac{-\sqrt{3}/2}{1/2}\right) \quad = -\dfrac{\pi}{3}$ | A1 |
| | $\arctan\left(\dfrac{-1}{\sqrt{3}}\right) = -\dfrac{\pi}{6}$ and $\arctan\left(\dfrac{1}{\sqrt{3}}\right) = \dfrac{\pi}{6}$ (Ignore interchanged $z$ and $z^*$) | A1 |
| | $\arg z - \arg z^* = -\dfrac{\pi}{6}-\dfrac{\pi}{6} = -\dfrac{\pi}{3} = \arg\left(\dfrac{z}{z^*}\right)$ | A1 (4) |
| | (d)   $z$ and $z^*$ (Correct quadrants, approx. symmetrical) | B1 |
| | $\dfrac{z}{z^*}$ (Strictly inside the triangle shown here) | B1 (2) |
| | (e) $\left(x-\left(\sqrt{3}-\mathrm{i}\right)\right)\left(x-\left(\sqrt{3}+\mathrm{i}\right)\right)$ | M1 |
| | Or: Use sum of roots $\left(=\dfrac{-b}{a}\right)$ and product of roots $\left(=\dfrac{c}{a}\right)$. | |
| | $x^2-2\sqrt{3}x+4$ | A1 (2) |
| | | **13** |
| | (a) M: Multiplying both numerator and denominator by $\sqrt{3}-\mathrm{i}$, and multiplying out brackets with some use of $\mathrm{i}^2=-1$.<br>(b) Answer 1 with no working scores both marks.<br>(c) Allow work in degrees: –60°, –30° and 30°<br>Allow arg between 0 and $2\pi$: $\dfrac{5\pi}{3}$, $\dfrac{11\pi}{6}$ and $\dfrac{\pi}{6}$ (or 300°, 330° and 30°).<br>Decimals: Allow marks for awrt –1.05 (A1), –0.524 and 0.524 (A1), but then A0 for final mark. (Similarly for 5.24 (A1), 5.76 and 0.524 (A1)).<br>(d) Condone wrong labelling (or lack of labelling), if the intention is clear. | |

| Question number | Scheme | Marks |
|---|---|---|
| 7. | (a)   Shape (closed curve, approx. symmetrical about the initial line, in all 'quadrants' and 'centred' to the right of the pole/origin). B1<br>Scale (at least one correct 'intercept' $r$ value… shown on sketch or perhaps seen in a table). B1 (Also allow awrt 3.27 or awrt 6.73). | (2) |
| | (b) $y = r\sin\theta = 5\sin\theta + \sqrt{3}\sin\theta\cos\theta$ | M1 |
| | $\frac{dy}{d\theta} = 5\cos\theta - \sqrt{3}\sin^2\theta + \sqrt{3}\cos^2\theta \quad \left(= 5\cos\theta + \sqrt{3}\cos 2\theta\right)$ | A1 |
| | $5\cos\theta - \sqrt{3}\left(1-\cos^2\theta\right) + \sqrt{3}\cos^2\theta = 0$ | M1 |
| | $2\sqrt{3}\cos^2\theta + 5\cos\theta - \sqrt{3} = 0$ | |
| | $\left(2\sqrt{3}\cos\theta - 1\right)\left(\cos\theta + \sqrt{3}\right) = 0 \qquad \cos\theta = \ldots \ (0.288\ldots)$ | M1 |
| | $\theta = 1.28$ and $5.01$ (awrt) (Allow $\pm 1.28$ awrt) $\left(\text{Also allow} \pm \arccos\frac{1}{2\sqrt{3}}\right)$ | A1 |
| | $r = 5 + \sqrt{3}\left(\frac{1}{2\sqrt{3}}\right) = \frac{11}{2}$ (Allow awrt 5.50) | A1 (6) |
| | (c) $r^2 = 25 + 10\sqrt{3}\cos\theta + 3\cos^2\theta$ | B1 |
| | $\int 25 + 10\sqrt{3}\cos\theta + 3\cos^2\theta\, d\theta = \underline{\frac{53\theta}{2}} + 10\sqrt{3}\sin\theta + \underline{\underline{3\left(\frac{\sin 2\theta}{4}\right)}}$ | M1 A1ft A1ft |
| | (ft for integration of $(a + b\cos\theta)$ and $c\cos 2\theta$ respectively) | |
| | $\frac{1}{2}\left[25\theta + 10\sqrt{3}\sin\theta + \frac{3\sin 2\theta}{4} + \frac{3\theta}{2}\right]_0^{2\pi} = \ldots\ldots$ | M1 |
| | $= \frac{1}{2}(50\pi + 3\pi) = \frac{53\pi}{2}$ or equiv. in terms of $\pi$. | A1 (6) |
| | | **14** |
| | (b) 2nd M: Forming a quadratic in $\cos\theta$.<br>3rd M: Solving a 3 term quadratic to find a value of $\cos\theta$ (even if called $\theta$).<br>Special case: Working with $r\cos\theta$ instead of $r\sin\theta$:<br>1st M1 for $r\cos\theta = 5\cos\theta + \sqrt{3}\cos^2\theta$<br>1st A1 for derivative $-5\sin\theta - 2\sqrt{3}\sin\theta\cos\theta$, then no further marks.<br>(c) 1st M: Attempt to integrate at least one term.<br>2nd M: Requires use of the $\frac{1}{2}$, correct limits (which could be 0 to $2\pi$, or $-\pi$ to $\pi$, or 'double' 0 to $\pi$), and subtraction (which could be implied). | |