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Mark Scheme (Final)
Summer 2007

GCE

GCE Mathematics (6678/01)

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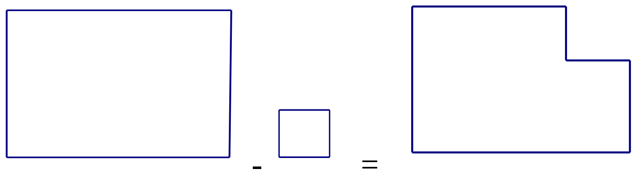
June 2007
6678 Mechanics M2
Mark Scheme

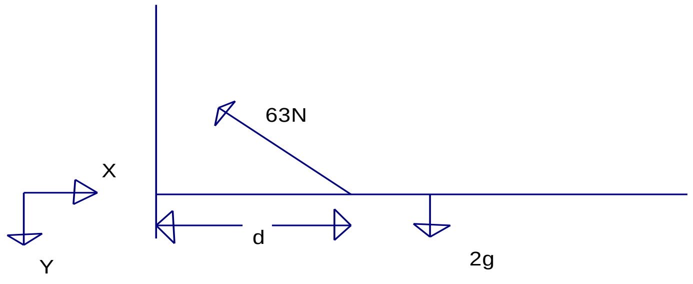
General:
For M marks, correct number of terms, dimensionally correct, all terms that need resolving are resolved.
Omission of *g* from a resolution is an accuracy error, not a method error.
Omission of mass from a resolution is a method error.
Omission of a length from a moments equation is a method error.
Where there is only one method mark for a question or part of a question, this is for a *complete* method.
Omission of units is not (usually) counted as an error.
When resolving, condone sin/cos confusion for M1, but M0 for tan or dividing by sin/cos.

Question Number	Scheme	Marks
1	Force exerted = 444/6 (= 74 N) $R + 90g \sin \alpha = 44$ $\Rightarrow R = \underline{3}$	B1 M1 A1 A1 (4)
	B1 444/6 seen or implied M1 Resolve parallel to the slope for a 3 term equation – condone sign errors and sin/cos confusion A1 All three terms correct – expression as on scheme or exact equivalent A1 32(N) only	
2 .(a) (b)	$\mathbf{a} = \mathrm{d}\mathbf{v}/\mathrm{d}t = 6t\mathbf{i} - 4\mathbf{j}$ Using $\mathbf{F} = ½\mathbf{a}$, sub $t = 2$, finding modulus e.g. at $t = 2$, $\mathbf{a} = 12\mathbf{i} - 4\mathbf{j}$ $\mathbf{F} = 6\mathbf{i}$ - $\lvert\mathbf{F}\rvert = \sqrt{(6^2 + 2^2)} \approx \underline{6.3}$	M1 A1 M1, M1, M1 A1(CSO)
	M1 Clear attempt to differentiate. Condone **i** or **j** missing. A1 both terms correct (column vectors are OK) The 3 method marks can be tackled in any order, but for consistency on epen grid please enter as: M1 **F**=m**a** (their **a**, (correct **a** or following from (a)), not **v**. **F**=$\frac{1}{2}$ **a**). Condone **a** not a vector for this mark. M1 subst t = 2 into candidate's vector **F** or **a** (**a** correct or following from (a), not **v**) M1 Modulus of candidate's **F** or **a** (not **v**) A1 CSO All correct (beware fortuitous answers e.g. from 6t**i**+4**j**)) Accept 6.3, awrt	

	6.32, any exact equivalent e.g. $2\sqrt{10}$, $\sqrt{40}$, $\frac{\sqrt{160}}{2}$	

3		
(a)	M (AF) $\quad 4a^2.a \;-\; a^2.3a/2 \quad = \quad 3a^2.\bar{x}$ $\bar{x} = \underline{5a/6}$	M1 A2,1,0 A1 (4)
(b)	Symmetry $\Rightarrow$ $\bar{y} = 5a/6$, or work from the top to get 7a/6 $\tan q = \dfrac{5a/6}{2a - 5a/6} \qquad \left(\dfrac{\bar{x}}{2a-\bar{y}}\right)$ $q \approx \underline{35.5°}$	B1√ M1 A1√ A1 (4)
	M1 Taking moments about AF or a parallel axis, with mass proportional to area. Could be using a difference of two square pieces, as above, but will often use the sum of a rectangle and a square to make the L shape. Need correct number of terms but condone sign errors for M1. A1 A1 All correct A1 A0 At most one error A1 5a/6, (accept 0.83a or better) *Condone consistent lack of a's for the first three marks.* *NB: Treating it as rods rather than as a lamina is M0* B1ft $\bar{x} = \bar{y} =$ their 5a/6, or $\bar{y}$ =distance from AB = 2a - their 5a/6. Could be implied by the working. Can be awarded for a clear statement of value in (a). M1 Correct triangle identified and use of tan. $\dfrac{2a - 5a/6}{5a/6}$ is OK for M1. Several candidates appear to be getting 45□ without identifying a correct angle. This is M0 unless it clearly follows correctly from a previous error. A1ft Tan□ expression correct for their 5a/6 and their $\bar{y}$ A1 35.5 (Q asks for 1d.p.) NB: Must suspend from point A. Any other point is not a misread.	

4. (a) (b)	PE lost = $2mgh - mgh\sin\alpha$ (= $7mgh/5$) Normal reaction $R = mg\cos\alpha$ (= $4mg/5$) Work-energy: $\frac{1}{2}mv^2 + \frac{1}{2}.2mv^2 = \frac{7mgh}{5} - \frac{5}{8}.\frac{4mg}{5}.h$ $\Rightarrow \frac{3}{2}mv^2 = \frac{9mgh}{10} \Rightarrow v^2 = \frac{3}{5}gh$	M1 A1 (2) B1 M1 A2,1,0 A1 (5)
	M1 Two term expression for PE lost. Condone sign errors and sin/cos confusion, but must be vertical distance moved for A A1 Both terms correct, sin□ correct, but need not be simplified. Allow 13.72*mh*. Unambiguous statement. B1 Normal reaction between A and the plane. Allow when seen in (b) provided it is clearly the normal reaction. Must use cos□□but need not be substituted. M1(NB QUESTION SPECIFIES WORK & ENERGY) substitute into equation of the form PE lost = Work done against friction plus KE gained. Condone sign errors. They ***must include KE of both particles.*** A1A1 All three elements correct (including signs) A1A0 Two elements correct, but follow their GPE and □ x their R x *h*. A1 V^2 correct (NB *kgh* specified in the Q)	

5.(a)		
	M(A) 63 sin 30 . 14 = 2g . d Solve: d = 0.225m Hence AB = <u>45 cm</u>	M1 A1 A1 A1 (4)
(b)	R(→) X = 63 cos 30 (≈ 54.56) R(↑) Y = 63 sin 30 – 2g (≈ 11.9) $R = \sqrt{(X^2 + Y^2)}$ ≈ <u>55.8, 55.9 or 56 N</u>	B1 M1 A1 M1 A1 (5)
	M1 Take moments about A. 2 recognisable force x distance terms involving 63 and 2(g). A1 63 N term correct A1 2g term correct. A1 AB = 0.45(m) or 45(cm). No more than 2sf due to use of g. B1 Horizontal component (Correct expression – no need to evaluate) M1 Resolve vertically – 3 terms needed. Condone sign errors. Could have cos for sin. Alternatively, take moments about B : $0.225 \times 2g = 0.31 \times 63 \sin 30 + 0.45Y$ or C : $0.14Y = 0.085 \times 2g$ A1 Correct expression (not necessarily evaluated) - direction of Y does not matter. M1 Correct use of Pythagoras A1 55.8(N), 55.9(N) or 56 (N) OR For X and Y expressed as Fcos□□ and Fsin□ . M1 Square and add the two equations, or find a value for tan □ , and substitute for sin □□or cos □ A1 As above . N.B. Part (b) can be done before part (a). In this case, with the extra information about the resultant force at A, part (a) can be solved by taking moments about any one of several points. M1 in (a) is for a complete method - they must be able to substitute values for all their forces and distances apart from the value they are trying to find..	

6. (a)	$0 \;=\; (35 \sin \alpha)^2 - 2gh$ $h = \underline{40 \text{ m}}$	M1 A1 A1 (3)
(b)	$x = 168 \;\Rightarrow\; 168 = 35 \cos$ □ $. t \quad (\Rightarrow t = 8\text{s})$	M1 A1
	At $t = 8$, $\quad y = 35\sin\alpha \times t - \frac{1}{2}gt^2 \quad (= 28.8 - ½.g.8^2 = -89.6 \text{ m})$	M1 A1
	Hence height of A = $\underline{89.6 \text{ m}}$ or 90 m	DM1 A1 (6)
(c)	$½mv^2 = 1/2.m.35^2 + mg.89.6$ $\Rightarrow v = \underline{54.6 \text{ or } 55 \text{ m s}^{-1}}$	M1 A1 A1 (3)
	M1 Use of $v^2 = u^2 + 2as$, or possibly a 2 stage method using $v = u + at$ and $s = ut + \frac{1}{2}at^2$ A1 Correct expression. Alternatives need a complete method leading to an equation in h only. A1 40(m) No more than 2sf due to use of g. M1 Use of $x = u\cos$ □ $. t$ to find t. A1 $168 = 35 \times \mathit{their} \cos\alpha \times t$ M1 Use of $s = ut + \frac{1}{2}at^2$ to find vertical distance for their t. (AB or top to B) A1 $y = 35\sin\alpha \times t - \frac{1}{2}gt^2$ (u,t consistent) DM1 This mark dependent of the previous 2 M marks. Complete method for AB. Eliminate t and solve for s. A1 cso. (NB some candidates will make heavy weather of this, working from A to max height (40m) and then down again to B (129.6m)) OR : Using $y = x\tan\alpha - \frac{gx^2 \sec^2\alpha}{2u^2}$ M1 formula used (condone sign error) A1 x,u substituted correctly M1 □□terms substituted correctly. A1 fully correct formula M1, A1 as above M1 Conservation of energy: change in KE = change in GPE. All terms present. One side correct (follow their h). (will probably work A to B, but could work top to B). A1 Correct expression (follow their h) A1 54.6 or 55 (m/s) OR: M1 horizontal and vertical components found and combined using Pythagoras	

	$v_x = 21$ $v_y = 28 - 9.8 \times 8$ (-50.4) A1 v_x and v_y expressions correct (as above). Follow their *h*,*t*. A1 54.6 or 55 *NB Penalty for inappropriate rounding after use of g only applies once per question.*	

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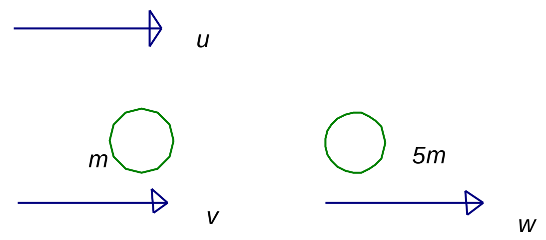
7.		
(a)	CLM: $mv + 5mw = mu$ NLI: $w - v = eu$ Solve v: $v = \frac{1}{6}(1 - 5e)u$, so speed = $\frac{1}{6}(5e - 1)u$ (**NB – answer given on paper**) Solve w: $w = \frac{1}{6}(1 + e)u$ * The M’s are dependent on having equations (not necessarily correct) for CLM and NLI	B1 B1 M1* A1 M1* A1 (6)
(b)	After B hits C, velocity of B = “v” = $\frac{1}{6}(1 - 5.\frac{4}{5})u = -\frac{1}{2}u$ velocity $< 0 \Rightarrow$ change of direction $\Rightarrow$ B hits A	M1 A1 A1 CSO (3)
(c)	velocity of C after = $\frac{3}{10}u$ When B hits A, “u” = $\frac{1}{2}u$, so velocity of B after = $-\frac{1}{2}(-\frac{1}{2}u) = \frac{1}{4}u$ Travelling in the same direction but $\frac{1}{4} < \frac{3}{10}$ $\Rightarrow$ <u>no second collision</u>	B1 B1 M1 A1 CSO (4)
	B1 Conservation of momentum – signs consistent with their diagram/between the two equations B1 Impact equation M1 Attempt to eliminate w A1 correct expression for v. Q asks for speed so final answer must be verified positive with reference to e>1/5. **Answer given so watch out for fudges**. M1 Attempt to eliminate v A1 correct expression for w M1 Substitute for e in speed or velocity of P to obtain $\boldsymbol{v}$ in terms of $\boldsymbol{u}$. Alternatively, can obtain v in terms of w A1 (+/-) u/2 ($v = -\frac{5w}{3}$) A1 CSO <u>Justify direction</u> (and correct conclusion) B1 speed of C = value of w = $(\pm)\frac{3u}{10}$ (Must be referred to in (c) to score the B1.)	

	B1 speed of B after second collision $(\pm)\frac{1}{4}u$ or $(\pm)\frac{5}{6}w$ M1 Comparing their speed of *B* after 2nd collision with their speed of *C* after first collision. A1 CSO. Correct conclusion .	
8. (a)	$0 \le t \le 4$: $a = 8 - 3t$ $a = 0 \Rightarrow t = 8/3$ s $\rightarrow v = 8.\frac{8}{3} - \frac{3}{2}.\left(\frac{8}{3}\right)^2 = \frac{32}{3}$ (m/s) second M1 dependent on the first, and third dependent on the second.	M1 DM1 DM1 A1 (4)
(b)	$s = 4t^2 - t^3/2$ $t = 4$: $s = 64 - 64/2 = \underline{32 \text{ m}}$	M1 M1 A1 (3)
(c)	$t > 4$: $v = 0 \Rightarrow t = \underline{8 \text{ s}}$	B1 (1)
(d)	*Either* $t > 4$ $s = 16t - t^2$ $(+ C)$ $t = 4, s = 32 \rightarrow C = -16 \Rightarrow s = 16t - t^2 - 16$ $t = 10 \rightarrow s = 44$ m But direction changed, so: $t = 8, s = 48$ Hence total dist travelled = 48 + 4 = <u>52 m</u> *Or (probably accompanied by a sketch?)* t=4 v=8, t=8 v=0, so area under line = $\frac{1}{2}\times(8-4)\times 8$ t=8 v=0, t=10 v=-4, so area above line = $\frac{1}{2}\times(10-8)\times 4$ total distance = 32(from b) + 16 + 4 = <u>52 m.</u>	M1 M1 A1 M1 A1 M1 DM1 A1 (8) M1A1A1 M1A1A1 M1A1 (8)

	Or M1, A1 for t > 4 $\frac{dv}{dt}=-2$, =constant t=4, v=8; t=8, v=0; t=10, v=-4 M1, A1 $s=\frac{u+v}{2}t=\frac{32}{2}t$, =16 working for t = 4 to t = 8 M1, A1 $s=\frac{u+v}{2}t=\frac{-4}{2}t$, =-4 working for t = 8 to t = 10 M1, A1 total = 32+14+4, =52	

M1 Differentiate to obtain acceleration
DM1 set acceleration. = 0 and solve for t
DM1 use their t to find the value of v
A1 32/3, 10.7oro better

OR using trial an improvement:
M1 Iterative method that goes beyond integer values
M1 Establish maximum occurs for t in an interval no bigger than $2.5<t<3.5$
M1 Establish maximum occurs for t in an interval no bigger than $2.6<t<2.8$
A1

Or M1 Find/state the coordinates of both points where the curve cuts the x axis.
DM1 Find the midpoint of these two values.
M1A1 as above.

Or M1 Convincing attempt to complete the square:
DM1 substantially correct $8t-\frac{3t^2}{2}=-\frac{3}{2}(t-\frac{8}{3})^2+\frac{3}{2}\times\frac{64}{9}$
DM1 Max value = constant term
A1 CSO

M1 Integrate the correct expression
DM1 Substitute t = 4 to find distance (s=0 when t=0 - condone omission / ignoring of constant of integration)
A1 32(m) only

B1 t = 8 (s) only

M1 Integrate 16-2t
M1 Use t=4, s= their value from (b) to find the value of the constant of integration.
or 32 + integral with a lower limit of 4 (in which case you probably see these two marks
occurring with the next two. First A1 will be for 4 correctly substituted.)
A1 $s = 16t - t^2 - 16$ or equivalent
M1 substitute t = 10
A1 44
M1 Substitute t = 8 (their value from (c))
DM1 Calculate total distance (M mark dependent on the previous M mark.)
A1 52 (m)

OR the candidate who recognizes v = 16 – 2t as a straight line can divide the shape into two triangles:
M1 distance for t = 4 to t = candidates's 8 = ½ x change in time x change in speed.
A1 8-4
A1 8-0
M1 distance for t = their 8 to t = 10 =½ x change in time x change in speed.
A1 10-8
A1 0-(-4)

	M1 Total distance = their (b) plus the two triangles (=32 + 16 + 4). A1 52(m) NB: This order on epen grid (the A's and M's will not match up.)	