

Mark Scheme (Final) Summer 2007

GCE

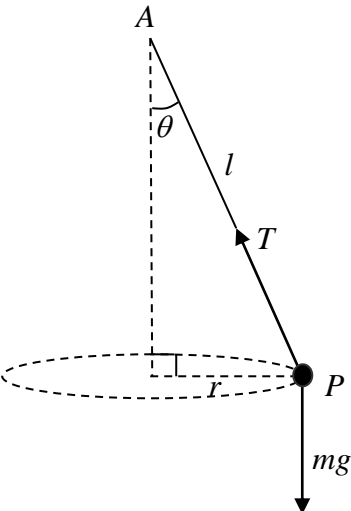
GCE Mathematics (6679/01)

**June 2007
6679 Mechanics M3
Mark Scheme**

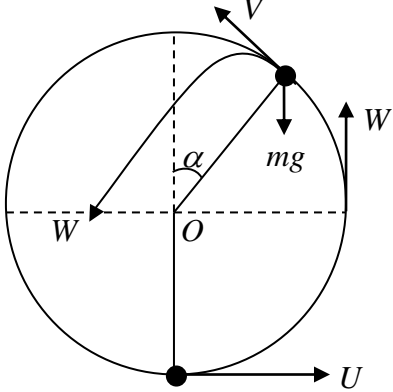
Question Number	Scheme	Marks
1.	<p>(a)</p> $A = \int_0^2 (2x - x^2) dx$ $= \left[x^2 - \frac{x^3}{3} \right]_0^2$ $A = \left[x^2 - \frac{x^3}{3} \right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3} \quad *$ <p style="text-align: right; margin-right: 100px;">cso</p> <p>(b)</p> $\bar{x} = 1 \quad (\text{by symmetry})$ $\frac{4}{3} \bar{y} = \frac{1}{2} \int y^2 dx = \frac{1}{2} \int (2x - x^2)^2 dx$ $= \frac{1}{2} \int (4x^2 - 4x^3 + x^4) dx$ $= \frac{1}{2} \left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5} \right]$ $\frac{4}{3} \bar{y} = \frac{1}{2} \left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5} \right]_0^2 = \frac{8}{15}$ $\bar{y} = \frac{8}{15} \times \frac{3}{4} = \frac{2}{5} \quad \text{accept exact equivalents}$	<p>M1 A1</p> <p>A1</p> <p>A1 (4)</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1 (5)</p> <p>[9]</p>

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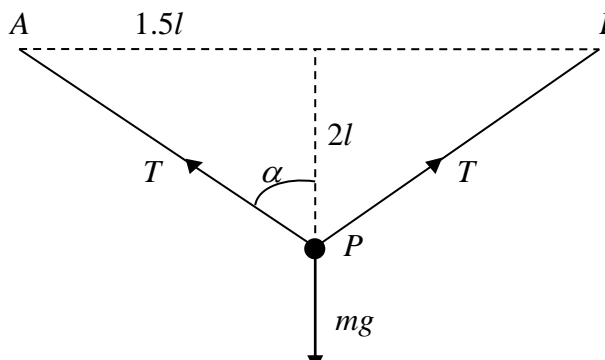
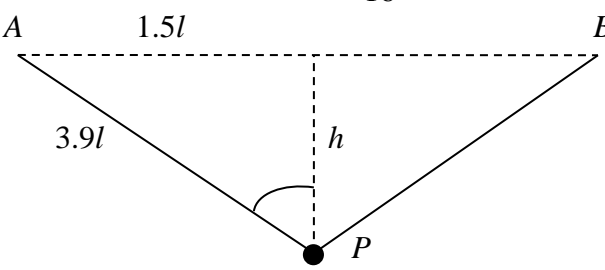
Question Number	Scheme	Marks
3.	<p>(a) At surface</p> $\frac{k}{R^2} = mg \Rightarrow k = mgR^2 \quad *$ <p>(b) N2L</p> $m\ddot{x} = -\frac{mgR^2}{x^2}$ $v \frac{dv}{dx} = -\frac{gR^2}{x^2} \quad \text{or} \quad \frac{d}{dx} \left(\frac{1}{2} v^2 \right) = -\frac{gR^2}{x^2}$ $\int v dv = -gR^2 \int \frac{1}{x^2} dx \quad \text{or} \quad \frac{1}{2} v^2 = -gR^2 \int \frac{1}{x^2} dx$ $\frac{1}{2} v^2 = \frac{gR^2}{x} (+C)$ $x = 2R, v = 0 \Rightarrow C = -\frac{gR}{2}$ $v^2 = \frac{2gR^2}{x} - gR$ <p>At $x = R$,</p> $v^2 = \frac{2gR^2}{R} - gR$ $v = \sqrt{(gR)}$	<p>cso M1 A1 (2)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (7)</p> <p>[9]</p>

Question Number	Scheme	Marks
4.	<div style="text-align: center;">  </div> <div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p>↑ $T \cos \theta = mg$</p> <p>← $T \sin \theta = \frac{mv^2}{r}$</p> <p>$\tan \theta = \frac{r}{\sqrt{(l^2 - r^2)}}$</p> <p>$\tan \theta = \frac{v^2}{rg}$</p> <p>$\frac{r}{\sqrt{(l^2 - r^2)}} = \frac{v^2}{rg}$</p> <p>$gr^2 = v^2 \sqrt{(l^2 - r^2)} *$</p> </div> <div style="width: 45%; vertical-align: top;"> <p>or equivalent</p> <p>Eliminating T</p> <p>Eliminating θ</p> <p>cs0</p> </div> </div>	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1 (9)</p> <p> [9]</p>

Question Number	Scheme	Marks
5.	<p>(a) $\ddot{x} = -\omega^2 x \Rightarrow 1 = \omega^2 \times 0.04 \quad (\Rightarrow \omega = 5)$</p> $T = \frac{2\pi}{5}$ <p style="text-align: right;">awrt 1.3</p> <p>(b) $v^2 = \omega^2 (a^2 - x^2) \Rightarrow 0.2^2 = 5^2 (a^2 - 0.04^2)$</p> $a = \frac{\sqrt{2}}{25}$ <p style="text-align: right;">accept exact equivalents or awrt 0.057</p> <p>(c) Using $x = a \cos \omega t$</p> $\frac{1}{2} a = a \cos \omega t$ $5t = \frac{\pi}{3}$ $t = \frac{\pi}{15}$ $T' = 4t = \frac{4\pi}{15}$ <p style="text-align: right;">ft their ω</p> <p style="text-align: right;">awrt 0.84</p>	<p>M1 A1</p> <p>A1 (3)</p> <p>M1 A1ft</p> <p>A1 (3)</p> <p>M1 A1ft</p> <p>A1</p> <p>M1 A1 (5)</p> <p>[11]</p>
	<p><i>Alternative to (c)</i></p> <p>Using $x = a \sin \omega t$</p> $\frac{1}{2} a = a \sin \omega t$ $5t = \frac{\pi}{6}$ $t = \frac{\pi}{30}$ $T' = T - 4t = \frac{4\pi}{15}$ <p style="text-align: right;">ft their ω</p> <p style="text-align: right;">awrt 0.84</p>	<p>M1 A1ft</p> <p>A1</p> <p>M1 A1 (5)</p>

Question Number	Scheme	Marks
6.	<div style="text-align: center;">  </div> <p>(a) Energy $\frac{1}{2}m(U^2 - v^2) = mga(1 + \cos \alpha)$</p> <p style="margin-left: 40px;">□ $(T +) mg \cos \alpha = \frac{mv^2}{a}$</p> <p>Leaves circle when $T = 0$</p> $g \cos \alpha = \frac{U^2 - 2ga - 2ga \cos \alpha}{a}$ <p>Leading to $U^2 = ag(2 + 3 \cos \alpha) *$</p> <p>(b) Using conservation of energy from the lowest point of the surface</p> $\frac{1}{2}m(U^2 - W^2) = mga$ $W^2 = U^2 - 2ag$ <p>Using $\cos \alpha = \frac{1}{\sqrt{3}}$,</p> $W^2 = ag \left(2 + \frac{3}{\sqrt{3}} \right) - 2ag$ $= ag \sqrt{3} *$ <p><i>Alternatives for (b) are given on the next page.</i></p>	<p>M1 A1=A1</p> <p>M1 A1</p> <p>M1</p> <p>Eliminating v</p> <p>cs0</p> <p>A1 (7)</p> <p>M1 A1=A1</p> <p>M1</p> <p>cs0</p> <p>A1 (5)</p> <p>[12]</p>

Question Number	Scheme	Marks
6.	<p><i>Alternative to part (b) using conservation of energy from the point where P loses contact with surface.</i></p> $\left(V^2 = ag \cos \alpha = \frac{ga}{\sqrt{3}} \right)$ <p>Energy $\frac{1}{2}m(W^2 - V^2) = mga \cos \alpha$</p> $\frac{1}{2}m\left(W^2 - \frac{1}{\sqrt{3}}ag\right) = mga \times \frac{1}{\sqrt{3}}$ <p>Leading to $W^2 = ag \sqrt{3} *$</p> <p><i>Alternative to part (b) using projectile motion from the point where P loses contact with surface.</i></p> $V^2 = ag \cos \alpha = \frac{ga}{\sqrt{3}}$ <p>↓ $W_y^2 = V^2 \sin^2 \alpha + 2ga \cos \alpha$</p> $= \frac{1}{\sqrt{3}}ag \left(1 - \frac{1}{3}\right) + 2ga \times \frac{1}{\sqrt{3}} = \frac{8\sqrt{3}}{9}ag$ <p>← $V_x = V \cos \alpha$</p> $W^2 = W_y^2 + V_x^2 = \frac{8\sqrt{3}}{9}ag + \frac{1}{3}ag \sqrt{3} \times \frac{1}{3} = ag \sqrt{3} *$	<p>M1 A1</p> <p>A1</p> <p>M1 A1 (5)</p> <p>cs0</p> <p>M1 A1</p> <p>A1</p> <p>M1 A1 (5)</p> <p>cs0</p>

Question Number	Scheme	Marks
7.	<p>(a)</p>  $AP = \sqrt{(1.5l)^2 + (2l)^2} = 2.5l$ $\cos \alpha = \frac{4}{5}$ <p>Hooke's Law $T = \frac{\lambda(2.5l - 1.5l)}{1.5l} \left(= \frac{2\lambda}{3} \right)$</p> <p>↑ $2T \cos \alpha = mg \quad \left(T = \frac{5mg}{8} \right)$</p> $2 \times \frac{2\lambda}{3} \times \frac{4}{5} = mg \quad \left(\frac{2\lambda}{3} = \frac{5mg}{8} \right)$ $\lambda = \frac{15mg}{16} *$ <p>(b)</p>  $h = \sqrt{(3.9l)^2 - (1.5l)^2} = 3.6l$ <p>Energy $\frac{1}{2}mv^2 + mg \times h = 2 \times \frac{15mg}{16} \times \frac{(2.4l)^2}{2 \times 1.5l}$</p> <p>Leading to $v = 0 *$</p>	<p>M1 A1</p> <p>B1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>cs0 A1 (9)</p> <p>M1 A1</p> <p>ft their h M1 A1ft = A1</p> <p>cs0 A1 (6)</p> <p>[15]</p>