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Mark Scheme (Final)

Summer 2007

GCE

GCE Mathematics (6680/01)

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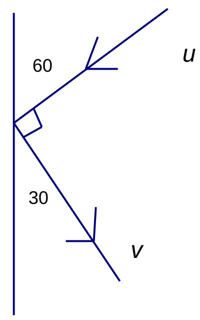
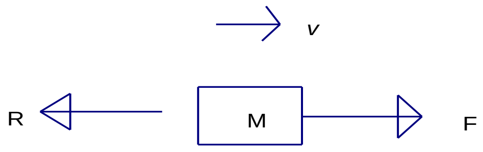
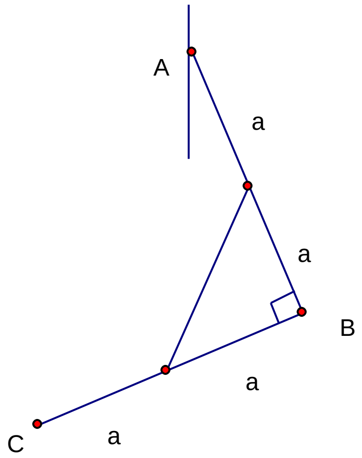
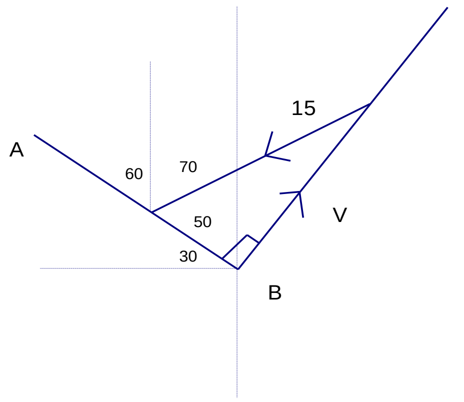
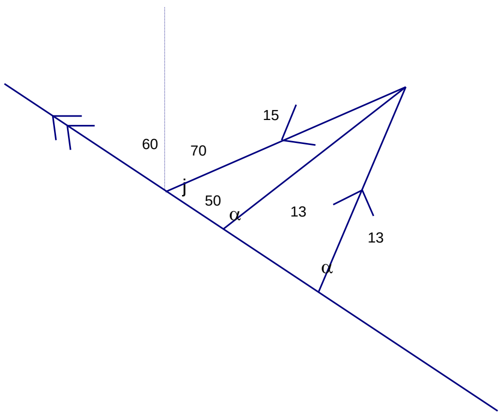
June 2007
6680 Mechanics M4
Mark Scheme

General:
For M marks, correct number of terms, dimensionally correct, all terms that need resolving are resolved.
Omission of g from a resolution is an accuracy error, not a method error.
Omission of mass from a resolution is a method error.
Omission of a length from a moments equation is a method error.
Where there is only one method mark for a question or part of a question, this is for a *complete* method.
Omission of units is not (usually) counted as an error.

Question Number	Scheme	Marks
1(a)	 $u\cos 60^0 = v\cos 30^0$ $u = v\sqrt{3}$ KE lost = $\frac{1}{2}m(u^2 - v^2)$ Fraction of KE lost = $1 - \left(\frac{v}{u}\right)^2$ $= 1 - \frac{1}{3} = \frac{2}{3}$ or at least 3sf ending in 7 or $\frac{3}{4}(1-e^2)$	M1A1 A1 M1 DM1 A1 (6)
(b)	$e = \frac{v\sin 30°}{u\sin 60°}$ $= \frac{v}{u}\cdot\frac{1}{\sqrt{3}}$ $= \frac{1}{3}$	M1A1 DM1 A1 (4)
a)	M1 Resolve parallel to the wall *Alt: reasonable attempt at equation connecting two variables* A1 Correct as above or equivalent *equation correct* A1 u in terms of v or v.v. - not necessarily simplified. *or ration of the two variables correct* M1 expression for KE lost DM1 expression in one variable for fraction of KE lost – could be *u/v* as above A1 cao	*The first three marks can be awarded in (b) if not seen in (a)*
b)	M1 Use NIL perpendicular to the wall and form equation in e A1 Correct unsimplified expression as above or $eu\sin 60^o = v\sin 30^o$ or equivalent DM1 Substitute values for trig functions or use relationship from (a) and rearrange to e = ….. A1 cao accept decimals to at least 3sf	*The first two marks can be awarded in (a)*

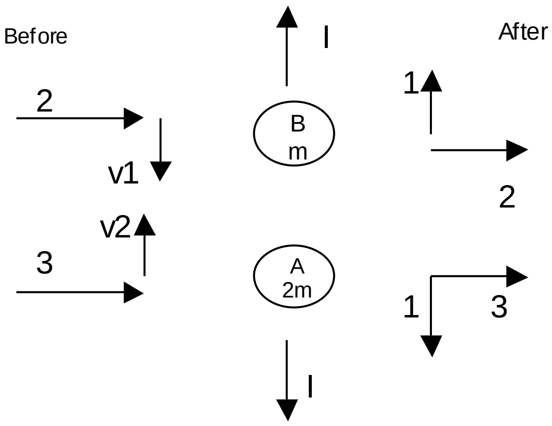
2(a)	 $F=\frac{Ru}{v}$ R($\rightarrow$), $\frac{Ru}{v}-R=M\frac{dv}{dt}$ $R(u-v)=Mv\frac{dv}{dt}$ *	B1 M1 A1 (3)
(b)	$\int_0^T dt=\frac{M}{R}\int_{\frac{1}{4}U}^{\frac{1}{3}U}\frac{vdv}{u-v}$ $\Rightarrow T=\frac{M}{R}\int_{\frac{1}{4}U}^{\frac{1}{3}U}-1+\frac{u}{u-v}dv$ $=\frac{M}{R}\left[-v-u\ln(u-v)\right]_{\frac{1}{4}U}^{\frac{1}{3}U}$ $=\frac{M}{R}\left[-\frac{u}{3}-u\ln\left(\frac{2u}{3}\right)+\frac{u}{4}+u\ln\left(\frac{3u}{4}\right)\right]$ $\quad\left(C=-\frac{Mu}{R}\left(\ln\frac{3u}{4}+\frac{1}{4}\right)\right)$ $=\frac{Mu}{R}\left(-\frac{1}{12}+\ln\frac{9}{8}\right)$ Hence $k=\ln\frac{9}{8}-\frac{1}{12}$	M1A1 DM1 A1 M1 M1 A1 (7)
a) b)	B1 Correct expression involving the driving force. M1 Use of F = ma to form a differential equation. Condone sign errors. a must be expressed as a derivative, but could be any valid form. A1 Rearrange to **<u>given form</u>**. M1 Separate the variables A1 Separation correct (limits not necessarily seen at this stage) DM1 Attempt a complete integration process A1 Integration correct M1 Correct use of both limits – substitute and subtract. Condone wrong order. M1 Simplify to find k from an expression involving a logarithm A1 Answer as given, or exact equivalent. Need to see k = lnA + B	

Question Number	Scheme	Marks
3. (a)	 $V=-mga\cos\theta-mg(2a\cos\theta+a\sin\theta)$ $=-mga(3\cos\theta+\sin\theta)\quad(+const)$ *	M1A1A1 A1 (4)
(b)	$\dfrac{dV}{d\theta}=-mga(-3\sin\theta+\cos\theta)$ $=0\ \Rightarrow\ \tan\theta=\dfrac{1}{3}$ $\Rightarrow\ \theta = 0.32(1)^c$ or 18.4^o accept awrt	M1A1 M1 A1 (4)
(c)	$\dfrac{d^2V}{d\theta^2}=-mga(-3\cos\theta-\sin\theta)$ $=mga(3\cos\theta+\sin\theta)$ Hence, when $\theta = 0.32^c$, $\dfrac{d^2V}{d\theta^2}>0$ i.e. stable	M1A1 M1 A1 (4)
a)	M1 Expression for the potential energy of the two rods. Condone trig errors. Condone sign errors. BC term in two parts A1 correct expression for AB A1 correct expression for BC A1 Answer **<u>as given</u>** .	
b)	M1 Attempt to differentiate V. Condone errors in signs and in constants. A1 Derivative correct M1 Set derivative = 0 and rearrange to a single trig function in θ A1 Solve for θ or M1A1 find the position of the center of mass M1A1 form and solve trig equation for θ	
c)	M1 Differentiate to obtain the second derivative A1 Derivative correct M1 Determine the sign of the second derivative A1 Correct conclusion. cso Or: M1 Find the value of $\dfrac{dV}{d\theta}$ on both sides of the minimum point A1 signs correct M1 Use the results to determine the nature of the turning point A1 Correct conclusion, cso.	*These 4 marks are dependent on the use of derivatives*

4 (a)		
	Fix A $v_{\min} = 15\sin 50°$ $= 11.5$ km h^{-1} (3 s.f.)	M1A1 A1 (3)
	or: triangle without the right angle identified and $\dfrac{15}{\sin\theta} = \dfrac{v_B}{\sin 50}$ $\Rightarrow v_B = \dfrac{15\sin 50}{\sin\theta}$ minimum value $\Rightarrow \theta = 90$ for M1 As above for A1A1	
(b)	Ambiguous Sine Rule: 2 possible solutions for α	B1B1 (2)
		
(c)	$\dfrac{\sin\alpha}{15} = \dfrac{\sin 50}{13}$	M1A1
	<u>α = 62,1º</u> (or 118º) (smaller value gives larger relative velocity)	A1
	$\Rightarrow$ either $v = 13\cos 62.1 + 15\cos 50 = 15.72\,kmh^{-1}$	M1A1
	Or $v^2 = 15^2 + 13^2 - 390\cos 67.9 = 247.27$ $v = 15.7\,kmh^{-1}$	M1 A1
	Time = $\dfrac{20}{their 15.72.....}$ = 1.272…… hrs	M1 A1
	Earliest time is 13.16hrs or 13.17 hrs accept 1.16 (pm) or 1.17 (pm)	A1 (8)

a)	M1 Velocity of B relative to A is in the direction of the line joining AB. Minimum V requires a right angled triangle. Convincing attempt to find the correct side. A1 15 x sin(their 50^0) A1 Q specifies 3sf, so 11.5 only	
b)	B1B1 Convincing argument B1B0 Argument with some merit	
c)	M1 Use of Sine Rule A1 Correct expression A1 (2 possible values,) pick the correct value. M1 Use trig. to form an equation in v A1 correct equation M1 $time = \frac{distance}{speed}$ A1ft correct expression with their v (not necessarily evaluated) A1 correct time in hours & minutes Or: M1 Use of cosine rule A1 $13^2 = 15^2 + v^2 - 2 \times 15 \times v \times \cos 50$ A1 (Award after the next two marks) 15.72 or awrt 15.72 M1 Attempt to solve the equation for v A1 $\frac{30\cos 50 \pm \sqrt{(30\cos 50)^2 - 4 \times 56}}{2}$ (15.72 or 3.562) Finish as above	

5. (a)



CLM: $2v_2 - v_1 = 1 - 2 = -1$ M1A1

NIL: $1 + 1 = \frac{1}{2}(v_1 + v_2)$ M1A1

$\therefore$ $v_2 = 1$, $v_1 = 3$ *Dependent on both M's above* DM1

Horizontal components unchanged (i.e. 2 & 3) *Independent of all other marks* A1

$\mathbf{v}_A = 3\mathbf{i} + \mathbf{j}$; $\mathbf{v}_B = 2\mathbf{i} - 3\mathbf{j}$ A1

(7)

(b)

For B: I = m(1-(-3)) = 4m M1A1

(Or For A: -I = 2m(-1 – 1) $\therefore$ I = 4m)

(2)

(c)

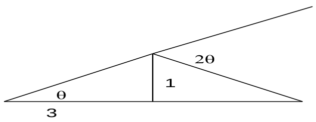
$$\begin{pmatrix}3\\1\end{pmatrix}.\begin{pmatrix}3\\-1\end{pmatrix} = \sqrt{3^2+1^2}.\sqrt{3^2+(-1)^2}\cos\theta$$ M1A1

$\Rightarrow 8 = 10\cos\theta$ M1

$\theta = 37^o$ A1

Alternative:

(4)

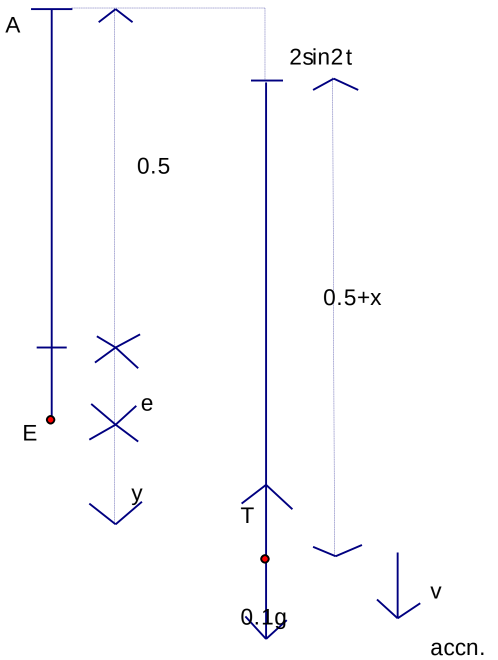


M1

where $\tan\theta = \frac{1}{3}$ A1

required angle is 2θ M1A1

a)	M1 Conservation of momentum along the line of centres. Condone sign errors A1 equation correct M1 Impact law along the line of centres. *e* must be used correctly, but condone sign errors. A1 equation correct. The signs need to be consistent between the two equations M1 Solve the simultaneous equations for their v_1 and v_2. A1 **i** components correct – independent mark A1 $\mathbf{v}_A$ & $\mathbf{v}_B$ correct	
b)	M1 Impulse = change in momentum for one sphere. Condone order of subtraction. A1 Magnitude correct.	
c)	M1 Any complete method to find the trig ratio of a relevant angle. A1 $\cos\theta = \frac{4}{5}$, $\tan\frac{\theta}{2} = \frac{1}{3}$, … Or M1 find angle of approach to the line of centres and angle after collision. A1 values correct. (both 71.56 …..) M1 solve for θ A1 37^0 (Q specifies nearest degree) Special case: candidates who act as if the line of centres is in the direction of **i**: CLM u+2v = 8 NIL v-u = 2 u=4/3, v=10/3 4/3i + j ; 10/3i – j Impulse 2m-4/3m = 2/3m $\frac{10+1}{\sqrt{10}\sqrt{\frac{109}{9}}} = \cos\theta$ θ = 1.70^0 Work is equivalent, so treat as a MR: M1A0M1A0M1A1A1 M1A1 M1A1M1A1	

6 (a)	 At E, $\frac{2.45e}{0.5} = 0.1g$ $\Rightarrow$e=0.2 $\Rightarrow$ $0.5(orl) + 0.2 + y = 2\sin 2t + 0.5(orl) + x$ $\Rightarrow 0.2 + y = 2\sin 2t + x$ *	M1 A1 B1 (3)
(b)	$0.1g - T = 0.1\ddot{y}$ R(↓) $0.1g - \frac{2.45x}{0.5} = 0.1\ddot{y}$ $0.98 - 4.9(0.2 + y - 2\sin 2t) = 0.1\ddot{y}$ $(-4.9y + 9.8\sin 2t = 0.1\ddot{y})$ $\Rightarrow \frac{d^2y}{dt^2} + 49y = 98\sin 2t$ *	M*1 M1 DM*1A1 A1 cso (5)
(c)	CF is $y = A\cos 7t + B\sin 7t$ Hence GS is $y = A\cos 7t + B\sin 7t + \frac{98}{45}\sin 2t$ t = 0, y = o: 0 = A so, $y = B\sin 7t + \frac{98}{45}\sin 2t$ $\dot{y} = 7B\cos 7t + \frac{196}{45}\cos 2t$ t = 0, $\dot{y} = 0 : 0 = 7B + \frac{196}{45}$ $\Rightarrow B = -\frac{28}{45}$ $\Rightarrow y = \frac{14}{45}(7\sin 2t - 2\sin 7t)$	M1 A1 B1 M1 A1 (5)
(d)	$\dot{y} = \frac{14}{45}(14\cos 2t - 14\cos 7t)$ $\dot{y} = 0 \Rightarrow \cos 2t = \cos 7t$ $\Rightarrow 7t = 2k\pi \pm 2t$ k=1 $\Rightarrow$ 9t = 2π (or 5t=2π) $t = \frac{2\pi}{9}$, accept 0.698s, 0.70s.	B1 M1 M1 A1 (4)

a)	M1 Hooke's law to find extension at equilibrium A1 cao B1 Q specifies reference to a diagram. Correct reasoning leading to **given answer**.	
b)	M1 Use of F=ma. Weight, tension and acceleration. Condone sign errors. M1 Substitute for tension in terms of x M1 Use given result to substitute for x in terms of y A1 Correct unsimplified equation A1 Rearrange to **given form** cso.	
c)	M1 Correct form for CF A1 GS for y correct B1 Deduce coefficient of cos θ = 0 M1 Differentiate their y and substitue t=0, $\dot{y} = 0$ A1 y in terms of t. Any exact equivalent.	
d)	B1 $\dot{y}$ correct M1 set $\dot{y} = 0$ M1 solve for general solution for t: $7t = 2k\pi \pm 2t$ or: $\sin\frac{9t}{2} \times \sin\frac{5t}{2} = 0 \Rightarrow \sin\frac{9t}{2} = 0 or \sin\frac{5t}{2} = 0$ A1 Select smallest value	