

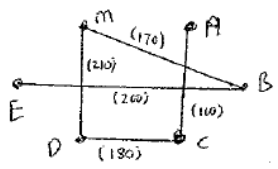
Mark Scheme (Final) Summer 2007

GCE

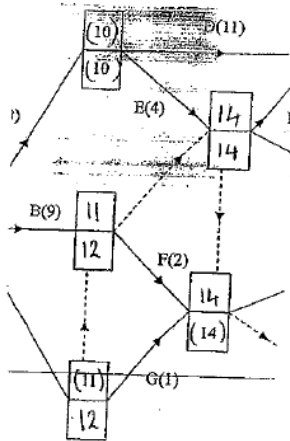
GCE Mathematics (6689/01)

June 2007
6689 Decision Mathematics
Mark Scheme

Question Number	Scheme	Marks																																				
1)	A graph is planar if it <u>can be drawn</u> in a plane in such a way that <u>no two edges meet</u> each other, except at a vertex to which they are both incident	B2,1,0 2																																				
2) (a)	To obtain a complete matching the number of vertices on each side must be equal.	B2,1,0 (2)																																				
(b)	e.g. $L-3=H-5=J-1a=A-4$ c.s. $L=3-H=5-J=1a-A=4$ $A=4 \quad H=5 \quad L=3$ $E=1b \quad J=1a \quad m=2$	M1 A1 A1 (3)																																				
(c)	H and L can now both only do 3. So a complete matching is not possible (other answers possible)	B2,1,0 (2) 1																																				
3) (a)	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>y</th> <th>x even?</th> <th>$x=0?$</th> </tr> </thead> <tbody> <tr><td>0</td><td>54</td><td>Y</td><td></td></tr> <tr><td>126</td><td>27</td><td>N</td><td></td></tr> <tr><td>378</td><td>13</td><td>N</td><td></td></tr> <tr><td>1386</td><td>6</td><td>Y</td><td></td></tr> <tr><td>3402</td><td>3</td><td>N</td><td></td></tr> <tr><td></td><td>2</td><td></td><td>N</td></tr> <tr><td></td><td>1</td><td>N</td><td></td></tr> <tr><td></td><td>0</td><td></td><td>Y</td></tr> </tbody> </table> <p style="text-align: center;">$A = 3402$</p>	x	y	x even?	$x=0?$	0	54	Y		126	27	N		378	13	N		1386	6	Y		3402	3	N			2		N		1	N			0		Y	M1 A1 A1 ✓ M1 A1 ✓ A1 B1 ✓ (7)
x	y	x even?	$x=0?$																																			
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	2		N																																			
	1	N																																				
	0		Y																																			
(b)	The product xy .	B2,1,0 (2)																																				

<p>4) (a)</p>	<p>odd vertices B, D, F, H</p> $BD + FH = 21 + 20 = 41$ $BF + DH = 19 + 20 = 39 \neq$ $BH + DF = 23 + 18 = 41$ <p>[Repeat BE, EF, DG and GH]</p> <p>shortest route = $125 + 39 = 164$ km</p> <p>(b) seek to keep the least pairing - DF/18 Therefore start/finish at B and H.</p>	<p>M A </p> <p>A </p> <p>A </p> <p>A (5)</p> <p>B ✓</p> <p>B ✓ (2)</p> <p>7</p>
<p>5) (a)</p>	<p>MB, BE, MD, DC, CA</p> <p>(b)</p>  <p>(c) $170 + 200 + 210 + 180 + 100 = 860$</p> <p>(d) (A cycle is formed when an arc is used that connects two vertices already connected to each other in the tree) Prim's algorithm always selects arcs that bring a vertex not in the tree into the tree, so cycles can't happen</p>	<p>M A A </p> <p>(3)</p> <p>B ✓ (1)</p> <p>B (1)</p> <p>B 1, 0</p> <p>(2)</p> <p>7</p>

6) (a)



(b) A E H K
A E L

(c) Idea of 'critical'
- zero float, no delay, immediate
- if late, project will finish late
etc.

Idea of 'path'
- from start to end event + continuous
- the event forming end of one activity
forms the start of the next
- sequence or series or link or run

S.c. "longest path" gets 100% only.

(a) B 3, 2, 1, 0
(3)

(b) B 2, 1, 0
(2)

(c) B 1

B 1 (2)

m 1

A 1 (2)

B 2, 1, 0 (2)

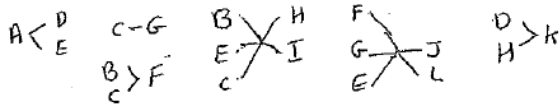
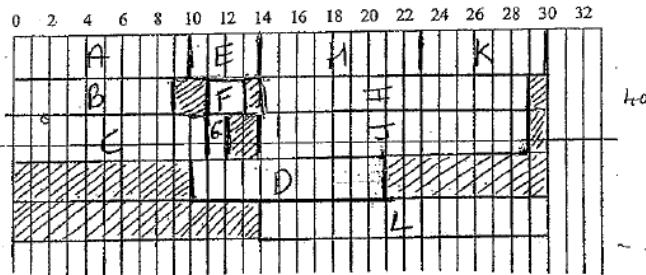
B 1 (1)

(d) $\frac{110}{30} (= 3.7/36) \therefore 4$ workers

(e) D, H, I, J, L

(f) It will not be possible to find a solution with 4 workers to complete the project in the minimum time. 5 workers will be needed.
Accept "an extra worker is required"

(g) e.g.



m 1

A 1

A 1

(3)

15

7) (a) $P - 2x - 4y - 3z = 0$ (o.e.)

(b) $12x + 4y + 5z \leq 246$
 $9x + 6y + 3z \leq 153$
 $5x + 2y - 2z \leq 171$

(c)

basic variable	x	y	z	r	s	t	Value
r	12	4	5	1	0	0	246
s	9	6	3	0	1	0	153
t	5	2	-2	0	0	1	171
P	-2	-4	-3	0	0	0	0

b.v.	x	y	z	r	s	t	Value	Row operations
r	6	0	3	1	-2/3	0	144	$R_1 - 4R_2$
y	3/2	1	1/2	0	1/6	0	25.5	$R_2 \div 6$
t	2	0	-3	0	-1/3	1	120	$R_3 - 2R_2$
P	4	0	-1	0	2/3	0	102	$R_4 + 4R_2$

b.v.	x	y	z	r	s	t	Value	Row operations
z	2	0	1	1/3	-2/9	0	48	$R_1 \div 3$
y	1/2	1	0	-1/6	5/18	0	1.5	$R_2 - \frac{1}{2}R_1$
t	8	0	0	1	-1	1	264	$R_3 + 3R_1$
P	6	0	0	1/3	4/9	0	150	$R_4 + R_1$

(d) $P = 150$ $x = 0$ $y = 1.5$ $z = 48$
 $r = 0$ $s = 0$ $t = 264$

(e) (The third constraint) $t \neq 0$

B2, 0 (2)

B1

B1

B1 (3)

m1 A1

m1 A1 ✓

B1 ✓

m1 A1

m1 A1

(9)

m1 A1 ✓

A1 ✓

(3)

B1 ✓ (1)

18

<p>8) (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p>85</p> <p>$C_1 = 140$, $C_2 = 104$</p> <p>e.g.</p> <p>S B D F H J T - 4</p> <p>S B D F G T - 1</p> <p>S B D F C H I T - 2</p> <p>S B D F C H J T - 2</p> <p>S B D E G T - 10</p> <p>max flow - min cut theorem, flow is 104, min cut is C_2</p>	<p>B1</p> <p>B1, B1 (3)</p> <p>M1, A1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>(5)</p> <p>M1, A1 (2)</p> <p>10</p>