

# Mark Scheme (Final)

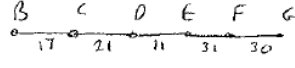
## Summer 2007

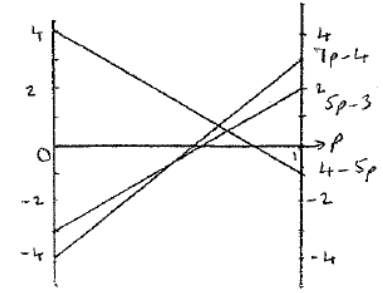
GCE

### GCE Mathematics (6690/01)



June 2007  
6690 Decision Mathematics  
Mark Scheme

Question Number	Scheme	Marks
1) (a)	Adds 32 to $AB + BA$ ( $ACB$ ) 47 to $AE + EA$ ( $ACDE$ ) 32 to $CE + EC$ ( $CDE$ ) 53 to $DG + GD$ ( $DCE$ )	B1 B1 B1 B1 (4)
(b)	$A C B D E F G A$ $15 + 17 + 38 + 11 + 31 + 30 + 23 = 165$ miles	m1 A1 A1 (3)
(c)	eg $BC, CD, DE, EF, FG$  weight of RST = 110 miles Lower bound = $110 + 15 + 23$ = 148 miles	m1 A1 m1 A1 (4) <div style="border: 1px solid black; padding: 2px; display: inline-block;">11</div>

Question Number	Scheme	Marks
2) (a)	$\begin{bmatrix} 2 & -1 & 3 \\ -3 & 4 & -4 \end{bmatrix}$ <p>Row min -1 ← -4</p> <p>col max 2 4 3 ↑</p> <p><math>2 \neq -1</math> <math>\therefore</math> not stable</p>	M1 A1 A1 (3)
(b)	<p>Let Denis play 1 with probability <math>p</math> so he'll play 2 with probability <math>1-p</math></p> <p>If Hilary play 1 Denis wins : <math>2p - 3(1-p) = 5p - 3</math>            If Hilary play 2 Denis wins : <math>-p + 4(1-p) = 4 - 5p</math>            If Hilary play 3 Denis wins : <math>3p - 4(1-p) = 7p - 4</math></p>	M1 A2, 1, 0 (3)
	 <p style="margin-left: 200px;"> <math>5p - 3 = 4 - 5p</math>  <math>10p = 7</math>  <math>p = \frac{7}{10}</math> </p>	M1 A2, 0 (3) M1 A1 ✓ (2)
	<p>Denis should play 1 with probability <math>\frac{7}{10}</math> 2 with probability <math>\frac{3}{10}</math> the value of the game is <math>\frac{1}{2}</math></p>	B1/B1 (2) <span style="border: 1px solid black; padding: 2px;">13</span>

3) (a)

$$\begin{bmatrix} 66 & 101 & 85 & 36 \\ 66 & 98 & 74 & 38 \\ 63 & 97 & 71 & 34 \\ 67 & 102 & 78 & 35 \end{bmatrix}$$

reducing  
rows first

$$\begin{bmatrix} 30 & 65 & 49 & 0 \\ 28 & 60 & 36 & 0 \\ 29 & 63 & 37 & 0 \\ 32 & 67 & 43 & 0 \end{bmatrix}$$

then columns

$$\begin{bmatrix} 2 & 5 & 13 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 4 & 7 & 7 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 12 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 3 & 6 & 6 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & 11 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 1 \\ 2 & 5 & 5 & 0 \end{bmatrix}$$

- A - cutting
- B - stitching
- C - filling
- D - Dressing

(b)  $66 + 98 + 71 + 35 = 270$  seconds

(c)  $20 \times 98 + 66 + 71 + 35 = 2132$  seconds  
 $= 35$  minutes  $32$  seconds

m1 A1  
(2)

m1  
A1 A1 A1  
(3)

m1  
A1 A1 A1

A1  
(4)

B1

m1 A1 A1  
A1 (3)

13

4)

(a)

	A	S	D	seats
1			0	94
2			0	65
3			0	80
	18	200	21	

B2, 1, 0  
(2)

(b) total supply > total demand

B1 (1)

(c)

(d)

	A	S	D
1	18	76	
2		65	
3		59	21

$$\begin{aligned}
 S(1) &= 0 & D(A) &= 5 \\
 S(2) &= -0.7 & D(S) &= 4.5 \\
 S(3) &= -0.5 & D(D) &= 0.5
 \end{aligned}$$

B1

m1 A1 ✓

$$\begin{aligned}
 I_{1D} &= 0 - 0 - 0.5 = -0.5 \quad * \\
 I_{2A} &= 4.2 + 0.7 - 5 = -0.1 \\
 I_{2D} &= 0 + 0.7 - 0.5 = 0.2 \\
 I_{3A} &= 4.6 + 0.5 - 5 = 0.1
 \end{aligned}$$

A1

(4)

	A	S	D
1	18	76-θ	θ
2		65	
3		59+θ	21-θ

Entering 1D  
Exiting 3D  
θ = 21

	A	S	D
1	18	55	21
2		65	
3		80	

m1 A1 ✓

A1

(3)

(e)

$$\begin{aligned}
 S(1) &= 0 & D(A) &= 4.9 \\
 S(2) &= -0.7 & D(S) &= 4.5 \\
 S(3) &= -0.5 & D(D) &= 0
 \end{aligned}$$

$$\begin{aligned}
 I_{1A} &= 5 - 0 - 4.9 = 0.1 \\
 I_{2D} &= 0 + 0.7 - 0 = 0.7 \\
 I_{3A} &= 4.6 + 0.5 - 4.9 = 0.2 \\
 I_{3D} &= 0 + 0.5 - 0 = 0.5
 \end{aligned}$$

Optimal since all  $II's \geq 0$

cost £ 902.70

m1

A1

A1

A1 (4)

m1 A1 (2)

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5)

Alt 1

Game from R's point of view.

	A1	A2	A3
R1	-6	3	-5
R2	2	-1	-4
R3	3	-2	1

Add 7

	A1	A2	A3
R1	1	10	2
R2	9	6	3
R3	10	5	8

$\beta_1, \beta_1$   
(2)

Let R play 1 with probability  $P_1$   
 2 " " "  $P_2$   
 3 " " "  $P_3$   
 $V$  = value of the game

$\beta_1$   
(1)

maximize  $P = V$

$\beta_1$  (1)

Subject to  $V - P_1 - 9P_2 - 10P_3 \leq 0$   
 $V - 10P_1 - 6P_2 - 5P_3 \leq 0$   
 $V - 2P_1 - 3P_2 - 8P_3 \leq 0$   
 $P_1 + P_2 + P_3 \leq 1$  accept =  
 $V, P_1, P_2, P_3 \geq 0$

$m_1 A1 \checkmark$   
 $A1 \checkmark$   
 $A1$   
 (4)  $\beta$

Alt 2

Add 4 to all entries

	R1	R2	R3
A1	10	2	1
A2	1	5	6
A3	9	8	3

$\beta_1$  (1)

Let R play 1 with probability  $P_1$   
 2 " " "  $P_2$   
 3 " " "  $P_3$  let  $V$  = value of game.

$\beta_1$

let  $x_1 = \frac{P_1}{V}$ ,  $x_2 = \frac{P_2}{V}$ ,  $x_3 = \frac{P_3}{V}$

$\beta_1$  (2)

maximize  $P = x_1 + x_2 + x_3$

$\beta_1$  (1)

Subject to  $10x_1 + 2x_2 + x_3 \leq 1$   
 $x_1 + 5x_2 + 6x_3 \leq 1$   
 $9x_1 + 8x_2 + 3x_3 \leq 1$   
 $x_1, x_2, x_3 \geq 0$  accept  $P_i \geq 0$

$m_1 A1 \checkmark$   
 $A1 \checkmark$   
 $A1$   
 (4)  $\beta$

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(a)

Stage	State	Action	Destination	Value
	J	JY	Y	98 *
1	K	KY	Y	94 *
	L	LY	Y	86 *
	G	GJ	J	$\max(79, 98) = 98 *$
		GK	K	$\max(98, 94) = 98 *$
2	H	HK	K	$\max(95, 94) = 95$
		HL	L	$\max(72, 86) = 86 *$
	I	IL	L	$\max(56, 86) = 86 *$
	C	CG	G	$\max(50, 98) = 98 *$
	D	DG	G	$\max(92, 98) = 98$
3		DH	H	$\max(81, 86) = 86 *$
	E	EH	H	$\max(89, 86) = 89 *$
	F	FH	H	$\max(81, 86) = 86 *$
		FI	I	$\max(72, 86) = 86 *$
	A	AE	E	$\max(95, 98) = 98$
		AD	D	$\max(86, 86) = 86 *$
4		AE	E	$\max(63, 89) = 89$
	B	BE	E	$\max(88, 89) = 89$
		BF	F	$\max(87, 86) = 87 *$
5	X	XA	A	$\max(55, 86) = 86 *$
		XB	B	$\max(85, 87) = 87$

X A D H L Y (minimax = 86)

(b) X B F  $\begin{matrix} \leftarrow H \\ \leftarrow I \end{matrix}$  L Y (minimax = 87) one

B I

m i

A i A i

(4)

m i

A i A i (3)

m i

A i

A i (3)

m i A i (2)

m i A i (2)

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