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Mark Scheme (Final)
Summer 2007

GCE

GCE Mathematics (6691/01)

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6691 Statistics S3
Mark Scheme

Question number	Scheme	Marks
1. (a)		M1A1

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
P Rank	2	6	4	3	1	7	8	5
Q Rank	2	8	1	6	3	5	7	4
d^2	0	4	9	9	4	4	1	1

$\sum d^2 = 32$ M1A1

$r_S = 1 - \frac{6\times 32}{8\times(8^2-1)}$ M1

$= \frac{13}{21}$ or AWRT 0.619 A1 (6)

(b) $H_0 : \rho = 0$ $H_1 : \rho > 0$ (ρ_s is OK) both B1

r_S 1 tail 5% critical value is 0.6429 (Independent of their H_1) B1 (± is OK)

0.619 < 0.6429 or not significant M1

So insufficient evidence of a positive correlation between judges

Or competitor is justified A1f.t. (4)

10

(a) 1st M1 for attempting to rank both *P* and *Q*.
1st A1 for both correct (could be reversed)
2nd M1 for attempting d^2
2nd A1 for $\sum d^2 = 32$.
3rd M1 for correct use of formula for r_S

(b) M1 for a correct comparison or statement about significance (o.e.)
Follow through their r_s provided $0 < r_s < 1$

A1f.t. for a conclusion in context. Must mention judges or marks or competitor.
If they use correlation they must say it is positive.
Follow through their positive r_s with their positive c.v. and ignore hypotheses.
So r_s =0.667 they could say competitor's claim is not justified etc.

S.C. No ranking Typical answer (-3.82) can get mark for use of r_s formula and hypotheses in (b) only
(a) M0A0M0A0M1A0 (b) B1B1M0A0

Question number	Scheme	Marks
2. (a)	H_0 : Maths grades are independent of English grades <u>or</u> No association **…** H_1 : Maths and English grades are dependent <u>or</u> There is an association …	B1
	Expected Frequencies e.g. $\frac{60\times 40}{120}=20$ <table><tr><td>20</td><td>27.5</td><td>12.5</td></tr><tr><td>20</td><td>27.5</td><td>12.5</td></tr></table>	M1 A1
	$\sum\frac{(O-E)^2}{E}=2\times\left(\frac{5^2}{20}+\frac{2.5^2}{27.5}+\frac{2.5^2}{12.5}\right)$, = 3.9545… AWRT <u>3.95</u> or <u>3.955</u>	M1, A1
	$\nu=(3-1)(2-1)=2$; $\chi_2^{\,2}(10\%)$ c.v. = 4.605	B1; B1
	3.95< 4.605 or not significant or do not reject H_0 (allow reject H_1)	M1
	Insufficient evidence of an association between English and maths grades <u>or</u> there is support for the Director's belief <u>or</u> Student's grades in maths and English are independent	A1 (9)
(b)	May have some expected frequencies <5 (and hence need to pool rows/cols)	B1 (1)
		10

(a)	1st B1 for both hypotheses in terms of independence or association and in context. Must mention Maths and English in at least one of the hypotheses. "relationship" or "correlation" or "connection" or "link" is B0 1st M1 for some correct calculation seen 1st A1 for all expected frequencies correct. Accept answers without formula seen. 2nd M1 for some evidence seen of attempt to calculate test statistic. At least one correct term seen. Follow through their expected frequencies. 2nd A1 for AWRT 3.95. Answers only please escalate! 3rd M1 for correct comparison or statement – may be implied by correct conclusion. 3rd A1 for conclusion in context using "association" or "independence" in connection with grades. Don't insist on seeing English or maths mentioned here. Use ISW for comments if a false statement and correct statement are seen.
(b)	B1 If they just say expected frequencies are "small" they must go onto mention need to pool.

Question number	Scheme	Marks
3.	$H_0 : \mu = 18, \quad H_1 : \mu < 18$	B1, B1
	$z = \frac{16.5 - 18}{3/\sqrt{15}} =, -1.9364...$ AWRT – 1.94	M1, A1
	5% one tail c.v. is z = (-) 1.6449 or probability (AWRT 0.026) (±) 1.6449	B1
	- 1.94 < -1.6449 or significant or reject H_0 or in critical region	M1
	There is evidence that the (mean) time to complete the puzzles has reduced Or Robert is getting faster (at doing the puzzles)	A1f.t. **7**

1st & 2nd B1 must see μ and 18

1st M1 for attempting test statistic, allow ±. Or attempt at critical value for $\bar{X}$: $\mu - z \times \frac{3}{\sqrt{15}}$

1st A1 for AWRT – 1.94. Allow use of $|z|$ = +1.94 to score M1A1. Or critical value = AWRT 16.7.

3rd B1 for AWRT 0.026 (i.e. correct probability only) or ± 1.6449. (May be seen in cv formula)

2nd M1 for correct comparison or statement relating their test statistic and 1.6449 or their probability and 0.05. Ignore their hypotheses if any or assume they were correct.

2nd A1f.t. for conclusion in context which refers to "speed" or "time". Depends only on previous M

Question number	Scheme	Marks
4. (a)	$\frac{0\times17+1\times31+...}{17+31+...}=\left(\frac{200}{100}=2\right)$, $\hat{p}=\frac{2}{20}=0.1$ (Accept $\frac{2}{20}$ or 2 per 20)	M1, A1 (2)
(b)	e.g. $r=100\times\binom{20}{2}(0.1)^2(0.9)^{18}$	M1
	r = 28.5, s = AWRT 9	A1, A1 (3)
(c)	x: 0 | 1 | 2 | 3 | ≥ 4 O_i: 17 | 31 | 19 | 14 | 19 E_i: 12.2 | 27.0 | 28.5 | 19.0 | 13.3 $\frac{(O-E)^2}{E}$: 1.89 | 0.59 | 3.17 | 1.32 | 2.44	
	Pooling	M1
	$\sum\frac{(O-E)^2}{E}=$ AWRT 9.4	M1A1c.a.o.
	$v=5-2=3$, $\chi_3^2(5\%)=7.815$	B1ft, B1ft
	H_0 : Binomial distribution is a good/suitable model/fit [Condone: B(20, 0.1) is…]	
	H_1 : Binomial distribution is not a suitable model — both	B1
	(Significant result) Binomial distribution is not a suitable model	A1cao (7)
(d)	defective items do not occur independently **or** not with constant probability	B1ft (1)
		13

(a) M1 for attempt to find mean or $\hat{p}$ (as printed or better). The 0.1 must be seen in part (a).

(b) M1 for correct expression for r or s using the binomial distribution. Follow through their $\hat{p}$.

(c) 1st M1 for some pooling (accept $x \geq 5$, obs.freq. …14, 9, 10 and exp.freq. 19.0, s, 4.3)

2nd M1 for calculation of test statistic (N.B. $x \geq 5$ gives 14.5). One correct term seen.

1st B1ft for number of classes – 2 (N.B. $x \geq 5$ will have 6 – 2 = 4)

2nd B1ft for the appropriate tables value, ft their degrees of freedom. (NB $\chi_4^2(5\%)=9.488$)

3rd B1 (for hypotheses) allow just "X ~ B(20, 0.1)" for null etc.

2nd A1 for correctly rejecting Binomial model. No ft and depends on 2nd M1.

(d) B1ft for independence or constant probability – must mention defective items or defectives
Follow through their conclusion in (c). So if they do not reject they may say "defectives occur with probability 0.1". Stating the value implies constant probability.

Question number	Scheme	Marks
5. (a)	$\hat{\mu} = \bar{x} = \frac{361.6}{80}, = \underline{4.52}$	M1, A1
	$\hat{\sigma}^2 = s^2 = \frac{1753.95 - 80 \times \bar{x}^2}{79} = (1.51288...)$	M1A1ft
	AWRT $\underline{1.51}$	A1 (5)
(b)	$H_0 : \mu_A = \mu_B$ $\quad H_1 : \mu_A > \mu_B$	B1 B1
	Denominator	M1
	$z = \frac{4.52 - 4.06}{\sqrt{\frac{1.51...}{80} + \frac{2.50}{60}}} = \left(\frac{0.46}{\sqrt{0.0605...}} \right)$ $\quad z$	dM1
	$= (\underline{+})\ 1.8689\ldots$ AWRT $(\underline{+})\ \underline{1.87}$	A1
	One tail c.v. is $z = 1.6449$ (AWRT 1.645 or probability AWRT 0.0307 or 0.0308)	B1
	(significant) there is evidence that diet *A* is better than diet *B* or	
	evidence that (mean) weight lost in first week using diet *A* is more than with *B*	A1ft (7)
(c)	CLT enables you to assume that $\bar{A}$ and $\bar{B}$ are normally distributed	B1 (1)
(d)	Assumed ${\sigma_A}^2 = {s_A}^2$ and ${\sigma_B}^2 = {s_B}^2$ (either)	B1 (1)
		14

(a) 2nd M1 for a correct attempt at s or s^2, A1ft for correct expression for s^2, ft their mean.

N.B. $\sigma^2_n = 1.49...$ so $\frac{80}{79} \times 1.49...$ is M1A1ft

(b) 1st B1 can be given for $\mu_1 = \mu_2$, but 2nd B1 must specify which is *A* or *B*.
1st M1 for the denominator, follow through their 1.51.

Must have square root can condone 2.50^2 but $\sqrt{\frac{1.51^2}{80} + \frac{2.50^2}{60}}$ is M0.

Allow $\sqrt{\frac{1.51}{79} + \frac{2.50}{59}}$ leading to AWRT 1.85 to score M1M1A0 in (b) and can score in (d).

2nd dM1 for attempting the correct test statistic, dependent on denominator mark
1st A1 for AWRT $\pm$ 1.87, may be implied by a correct probability.
2nd A1ft ft their test statistic vs their cv **only if** H_1 is correct and both Ms are scored

(c) B1 for stating either $\bar{A}$ or $\bar{B}$ (but not *A* or *B*) are normally distributed
(d) B1 for either, can be stated in words in terms of variances or standard deviations.

Question number	Scheme	Marks
6.	$\bar{x} = \frac{1}{2}(123.5+154.7) = \underline{139.1}$	B1
	2.5758	B1
	"their 2.5758" $\frac{\sigma}{\sqrt{n}} = 154.7 - 139.1 = 15.6$	M1
	AWRT 1.96	B1
	"their 1.96" $\frac{\sigma}{\sqrt{n}} = \frac{15.6 \times 1.96}{2.5758} = (11.87...)$	M1
	So 95% C.I. = 139.1 $\pm$ 11.87…= (127.22…, 150.97…) AWRT <u>(127, 151)</u>	A1
		6

1st B1 for mean = 139.1 only

1st M1 for UL – mean or mean – LL set equal to *z* value times standard error or some equivalent expression for standard error. Follow through their 2.5758 provided a *z* value.

May be implied by $\frac{\sigma}{\sqrt{n}}$ = 6.056… [N.B. $\frac{15.6}{2.3263} = 6.705...$]

Condone poor notation for standard error if it is being used correctly to find CI.

2nd M1 for full method for semi-width (or width) of 95% interval

Follow through their *z* values for both M marks

N.B. Use of 2.60 instead of 2.5758 should just lose 2nd B1 since it leads to AWRT (127, 151)

Question number	Scheme	Marks
7. (a)	Let $X = L - 4S$ then $E(X) = 19.7 - 4\times4.9$,$= 0.1$	M1, A1
	$Var(X) = Var(L) + 4^2 Var(S) = 0.5^2 + 16\times0.2^2$	M1, M1
	$= 0.89$	A1
	$P(X > 0) = [P(Z > -0.10599\ldots)]$	M1
	$=$ AWRT (0.542 – 0.544)	A1 (7)
(b)	$T = S_1 + S_2 + S_3 + S_4$ (May be implied by 0.16)	M1
	$T \neg N(19.6, 0.16)$ $E(T) = 19.6$	B1
	$Var(T) = 0.16$ or 0.4^2	A1 (3)
(c)	Let $Y = L - T$ $E(Y) = E(L) - E(T) = [0.1]$	M1
	$Var(Y) = Var(L) + Var(T) = [0.41]$	M1
	Require $P(-0.1 < Y < 0.1)$	M1
	$= P(Z < 0) - P(Z < -0.31..)$ or $0.5 - P(Z < -0.31..)$ or $P(Z < 0.31..) - P(Z < 0)$	M1
	$= 0.1217$ (tables) or 0.1226.. (calc) AWRT (0.122 – 0.123)	A1 (5)
		15

(a) 1st M1 for defining X and attempting $E(X)$

1st A1 for 0.1. Answer only will score both marks.

2nd M1 for $Var(L) + \ldots..$

3rd M1 for $\ldots. 4^2 Var(S)$. For those who don't attempt $L - 4S$ this will be their only mark in (a).

2nd A1 for 0.89

4th M1 for attempting a correct probability, correct expression and attempt to find, which should involve some standardisation: ft their $\sqrt{0.89}$ and their 0.1.

If 0.1 is used for $E(X)$ answer should be > 0.5, otherwise M0.

(c) 1st M1 for a correct method for $E(Y)$, ft their $E(T)$.

2nd M1 for a correct method for $Var(Y)$, ft their $Var(T)$. Must have +.

3rd M1 for dealing with the modulus and a correct probability statement. Must be modulus free.

May be implied by e.g. $P(Z < \frac{0.2}{\sqrt{\text{their } 0.41}}) - 0.5$, or seeing both 0.378… (or 0.622…) and 0.5

4th M1 for correct expression for the correct probability, as printed or better. E.g. 0.5 + 0.378.. is M0

A1 for AWRT in range.