

Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Mechanics 4R (6680/01R)

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# **General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

#### PEARSON EDEXCEL GCE MATHEMATICS

## **General Instructions for Marking**

- 1. The total number of marks for the paper is 75.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:

#### 'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

- (i) should have the correct number of terms
- (ii) be dimensionally correct i.e. all the terms need to be dimensionally correct e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' x.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned. e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

#### 'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. MO A1 is impossible.

#### 'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

### 3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol  $\sqrt{\text{ will be used for correct ft}}$
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- · dp decimal places
- sf significant figures
- \* The answer is printed on the paper
- The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
- 5. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 6. Ignore wrong working or incorrect statements following a correct answer.

# **General Principles for Mechanics Marking**

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.
  - N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations
  - M(A) Taking moments about A.
  - N2L Newton's Second Law (Equation of Motion)
  - NEL Newton's Experimental Law (Newton's Law of Impact)
  - HL Hooke's Law
  - SHM Simple harmonic motion
  - PCLM Principle of conservation of linear momentum
  - RHS, LHS Right hand side, left hand side.

Question Number	Scheme		Marks	3
1.	$v\cos 45^{\circ} = u\sin \alpha$ $v\sin 45^{\circ} = eu\cos \alpha$	parallel perpendicular	M1 A1 M1 A1	
	$e = \tan \alpha$	or square & add	M1 A1	
	$I = m(v\cos 45^{\circ} + u\cos \alpha)$ = $mu(\sin \alpha + \cos \alpha)$	impulse in terms of $u$ , $\alpha$	M1 A1 M1	
	$=\frac{mu(1+e)}{\sqrt{1+e^2}}$	in terms of $u$ , $e$	M1 A1	
				11

Question Number	Scheme	Marks
2. (a)	Vector triangle with perpendicular 20 km	M1
	Minimum speed = $30\sin 60^{\circ}$	M1
	$= 15\sqrt{3}$ oe (26.0)	A1 (3)
	30 km h <sup>-1</sup>	
<b>(b)</b>	$\frac{\sin \theta}{30} = \frac{\sin 60^{\circ}}{32}$	M1 A1
	$\Rightarrow \theta = 54.281^{\circ}$	A1
	$v^{2} = 32^{2} + 30^{2} - 2.32.30\cos(120^{\circ} - \theta)$ $= 33.68 \qquad (15 + \sqrt{349})$	M1 A1
	,	A1
	$t = \frac{20}{33.68} = 0.59379h = 35.62min$ Time is 2.36 pm (14.36)	M1 A1 (8)
	1 mic is 2.50 pm (14.50)	11

Question Number	Scheme		Marks
3.	$-(mg + mkv) = mv \frac{\mathrm{d}v}{\mathrm{d}x}$	Differential equation	M1 A1
	$\int_{0}^{H} \mathrm{d}x = \int_{U}^{0} -\frac{v}{(g+kv)} \mathrm{d}v$	Separate variables	M1
	$\int_{0}^{H} dx = -\int_{U}^{0} \frac{1}{k} - \frac{g}{k(g+kv)} dv$	Split for integration	M1 A1
	$- H = \left[\frac{-v}{k} + \frac{g}{k^2} \ln(g + kv)\right]_U^0$		A1
	$=\frac{U}{k} - \frac{g}{k^2} \ln(\frac{g + kU}{g})$	Use of limits	M1 A1
			8

Question Number	Scheme		Marks
4. (a)	$mv_1 + mv_2 = mu\cos 60^{\circ}$	Momentum	M1 A1
	$-v_1 + v_2 = eu \cos 60^{\circ}$ $v_1 = \frac{u(1-e)}{4}$	Impact law	M1 A1
	Speed of $S = \sqrt{\frac{u^2(1-e)^2}{16} + \frac{3u^2}{4}}$ speed	Solve for $v_1$ and find	M1 A1
	$= \frac{u}{4}\sqrt{e^2 - 2e + 13}$ $\tan \theta = \frac{u\sqrt{3}}{2v_1} = \frac{2\sqrt{3}}{(1-e)}$ dirn $2\sqrt{3}$	Use components to find	M1 A1
	S moves at $\arctan \frac{2\sqrt{3}}{(1-e)}$ to the line of centres $v_2 = \frac{u(1+e)}{4}$	$v_2$ in terms of $u$ , $e$	M1 A1 B1 (12)
(b)	T has speed $\frac{u(1+e)}{4}$ along the line of centres	Conclusion	
	$\theta$ is a max when $e = 1$ then $\theta = 90^{\circ}$ then deflection angle is $90^{\circ} - 60^{\circ} = 30^{\circ}$ $\delta = 30$		M1 A1 A1 (3)
		0-30	15

Question Number	Scheme	Marks
5.(a)	$-12mgl\sin\theta \qquad \text{rod}$ $-mg(L-4l\sin\frac{1}{2}\theta) \qquad \text{particle}$ $4mgl(\sin\frac{1}{2}\theta - 3\sin\theta) + \text{constant} \qquad *\text{given answer*}$	B1 M1 A1 A1 (4)
(b)	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 4mgl(\frac{1}{2}\cos\frac{1}{2}\theta - 3\cos\theta) \qquad \text{Differentiate}$ $4mgl(\frac{1}{2}\cos\frac{1}{2}\theta - 3\cos\theta) = 0 \qquad \text{Derivative} = 0$ $\frac{1}{2}\cos\frac{1}{2}\theta - 3(2\cos^2\frac{1}{2}\theta - 1) = 0 \qquad \text{In terms of } \cos\frac{1}{2}\theta$ $\cos\frac{1}{2}\theta = \frac{3}{4} \text{ or } -\frac{2}{3}$ $\cos\theta = \frac{1}{8} \text{ or } -\frac{1}{9} \qquad \text{Solve for } \theta$ $\theta = 1.45 \text{ as } \theta < \frac{1}{2}\pi \qquad (83.0^\circ)$ $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 4mgl(-\frac{1}{4}\sin\frac{1}{2}\theta + 3\sin\theta) \qquad \text{Second derivative}$ When $\theta = 1.45$ , $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 11.25mgl > 0$ , hence stable	M1 A1 M1 A1 M1 A1 M1 A1 A1 M1 A1 A1 A1 A1

Question	Scheme	Marks	
Number	Scheme		
6.(a)	$-Mkv - T = M\ddot{x}$	M1 A1	
	$-Mk\dot{x} - \frac{Mn^2ax}{a} = M\ddot{x}$ In terms of x	M1	
	$\ddot{x} + k\dot{x} + n^2x = 0 \text{ *Answer Given*}$		
(b)	Aux equn: $p^2 + \frac{5n}{2}p + n^2 = 0$ $p = -\frac{n}{2} \text{ or } -2n$	M1	
	$x = Ae^{\frac{-n}{2}t} + Be^{-2nt}$ General solution		
	$t = 0, x = 0 \Rightarrow 0 = A + B$	A1	
	$\dot{x} = -\frac{nA}{2}e^{-\frac{n}{2}t} - 2nBe^{-2nt}$ Differentiate	B1	
	$t = 0, x = U \implies U = -\frac{nA}{2} - 2nB$	M1	
	$\Rightarrow -\frac{2U}{n} = A + 4B$	A1	
	$A = \frac{2U}{3n}; B = -\frac{2U}{3n}$ Solve for A, B		
	$x = \frac{2U}{3n} e^{-\frac{n}{2}t} - \frac{2U}{3n} e^{-2nt}$	M1	
	3n $3n$	A1 (7)	
(c)	$U_{2,1,2nt}$ $\frac{n}{2}$		
	$\dot{x} = \frac{U}{3} (4e^{-2nt} - e^{-\frac{n}{2}t})$	M1	
	$\dot{x} = 0 \Rightarrow 4e^{-2nt} - e^{-\frac{n}{2}t} = 0$ $e^{\frac{3n}{2}t} = 4$	M1	
	$e^{\frac{-1}{2}} = 4$	A1	
	$x = \frac{2U}{3n} (2^{-\frac{2}{3}} - 2^{-\frac{8}{3}}) = \frac{U}{n} (2^{-\frac{5}{3}}) \qquad \left(0.31 \frac{U}{n}\right)$		
	$x = \frac{1}{3n}(2^3 - 2^3) = \frac{1}{n}(2^3) \qquad \left(0.31 - \frac{1}{n}\right)$	M1 A1 (5)	
		16	