

Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Statistics 3 (6691/01)

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- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 75
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.
- 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- d... or dep dependent
- indep independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper or ag- answer given
- _ or d... The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1(a)	(This is a sample where) every (possible) sample (of size <i>n</i>) has an equal chance of being chosen.	B1
		(1)
(b)	'When it is impossible to provide a sampling frame ' or a correct example with an indication	B1
	of sampling frame being impossible.	
		(1)
(c)(i)	A list/register of all the students.	B1
(ii)	Number the students (from 0 to 74, 1 to 75 etc.)	B1
	Using the random no. table read off the nos. and identify or select the students allocated	B1
	those nos.	
		(3)
		Total 5
	Notes	
(a)	Require all / each / every etc sample and same/equal etc chance / probability etc for B1	
(b)	Require impossible / no / doesn't exist etc and sampling frame for B1	
(c)(i)	Require list/register etc and all/every/75 etc students for B1	
	List of 8 students is B0	
(ii)	First B1 accept 'in the corresponding position' o.e. if numbering omitted	
	Second B1 require both for mark.	

Question Number	Scheme	Marks
2a(i)	Only contains known data / function of data only / no population parameters	B1
	therefore it is a statistic	B1d
(ii)(iii)	(ii) and (iii) contain unknown parameters / population parameters / μ and / or σ	B1
	therefore it is not a statistic .	B1d
		(4)
(b)	$(E(\frac{3X_1-X_{20}}{2}) = \frac{3\mu-\mu}{2} =) \mu$	B1
	$(E(\frac{3X_1 - X_{20}}{2}) = \frac{3\mu - \mu}{2} =) \mu$ $Var(\frac{3X_1 - X_{20}}{2}) = \frac{9\sigma^2 + \sigma^2}{2^2}$ $= \frac{5\sigma^2}{2}$	M1
	$=\frac{5\sigma^2}{2}$	A1
	2	(3)
		Total 7
	Notes	I
(a)(i)	First B1 for known / no unknowns o.e. in (i)	
	Second B1 dependent on first B1 for 'Yes' / is a statistic o.e. in (i)	
	Third B1 for unknowns o.e. in both (ii) and (iii)	
	Fourth B1 dependent on third B1 for 'No' / not a statistic o.e. in both (ii) and (ii)	
(b)	B1 for μ	
	M1 for some squaring on numerator or denominator and must add on numerator	
	A1 for $\frac{5\sigma^2}{2}$ o.e.	

Question Number	Scheme									
3	Happiness									
			Not happy	Fairly happy	Very happy					
		Female	13.51	41.77	30.71		M1			
	Gender	Male	8.49	26.23	19.29		A1			
	H ₀ : Happiness and gender are independent/ not associated									
	H_1 : Happ			endent/ associated			B1			
	0	1	F	(O-E)) ²	0^{2}	dM1			
			2.51	E 1.500	5.000	E				
	9 43		3.51 1.77	1.508	5.996					
	34		80.71	0.0361	44.26					
	13		3.49	2.402	19.91					
	25		26.23	0.0575	23.82					
	16		9.29	0.560	13.27		A1			
							A1			
	$\sum \frac{(O-I)}{r}$	= 4.91	or $\sum \frac{O^2}{E} - N =$	= 144.91 - 140 = 4	.91					
		(2-1) = 2	-E				B1			
		, , ,								
	$\sum \frac{(O-I)}{2}$	$\frac{E^{2}}{2} < 5.991$					B1ft			
	L									
	4.91 < 5.9	M1								
	No associa	A1								
		(1								
		Total 1								
	Notes									
	1^{st} M1 for some use of $\frac{Row Total \times Column Total}{Grand Total}$. May be implied by at least 1 correct <i>Ei</i>									
	1 st A1 awrt 13.5, 41.8, 30.7, 8.5, 26.2 and 19.3 Allow M1A0 for <i>Ei</i> rounded to integers									
	1 st B1 for both hypotheses. Must mention "happiness" and "gender" at least once.									
	Use of "relationship" or "correlation" or "connection" is B0									
	2^{nd} dM1 for at least 2 correct terms (in 3^{rd} or 4^{th} columns) or correct expressions with their <i>Ei</i>									
	Dependent on 1st M1. Accept 2sf accuracy for the M mark.									
	2 nd A1 for all correct terms (2sf or better). May be implied by a correct ans									
	Allow truncation e.g. 44.2									
	3 rd A1 awr									
	3 rd A1 awrt 4.91 . Condone 4.915 2 rd B1 for correct degrees of freedom (may be implied by a cv of 5.991)									
	3^{rd} B1ft for cv that follows from their degrees of freedom									
	3 rd M1 for									
	Contradictory statements score M0 e.g. "significant, do not reject H0"									
	Condone '									
	4th A1 for	appiness"								
	Condone '									
	e.g. "Ther	e is no eviden	ce of a relationsl	nip between gende	r and happines	S"				
						5"				
	No follow	through. If e.	g hypotheses are	nip between gende the wrong way ar nay get M0A0B1N	ound A0 here.					

Question Number	Scheme	Marks
4	E(A) = E(B) + 4E(C) - 3E(D)	M1
	= 22	A1
	$\operatorname{Var}(A) = \operatorname{Var}(B) + 16\operatorname{Var}(C) + 9\operatorname{Var}(D)$	M1
	= 168.25	A1
	P (A < 45) = P $\left(Z < \frac{45 - 22}{\sqrt{168.25}} \right)$ = P (Z < 1.773)	M1
	= 0.9616 awrt 0.962	A1
		(6)
		Total 6
	Notes	
	$1^{\text{st}} \text{ M1 for E}(4C) = 4\text{E}(C) \text{ and } - \text{E}(3D) = -3\text{E}(D)$	
	1 st A1 for 22 cao	
	2^{nd} M1 for use of Var $(aX) = a^2$ Var X and + their '9Var (D) '	
	2 nd A1 for 168.25 cao	
	3 rd M1 for standardising using their mean and their sd	
	3 rd A1 for awrt 0.962. NB Calculator gives 0.961899	

Question Number	Scheme	Mark	S						
5(a)	The seeds are independent / There are a fixed number of seeds in a row / There are only								
	two outcomes to the seed germinating – either it germinates or it does not / The probability								
	of a seed germinating is constant	B1 B1	(2)						
(b)	$\frac{(0\times2) + (1\times6) + (2\times11) + (3\times19) + (4\times25) + (5\times32) + (6\times16) + (7\times9)}{504} = \frac{504}{100}$	M1	(2)						
	120×7 840 = 0.6 **	Alcso	(2)						
(c)	p = 0.6 $q = 0.4$		(2)						
	$s = 120 \times 21q^5 p^2 = 120 \times 21 \ge 0.4^5 \ge 0.6^2 = 9.29$	B1							
	$t = 120 \times 35q^3p^4 = 120 \times 35 \ge 0.4^3 \ge 0.6^4 = 34.84$	B1							
(d)	H_0 : A binomial distribution is a suitable model. H_1 : A binomial distribution is not a suitable model.	B1	(2)						
		M1							
	Observed number of rows 19 19 25 32 25 Expected number of rows 11.55 23.22 34.84 31.35 19.04								
	$\begin{array}{c c c c c c c c c c c c c c c c c c c $								
	$\mathfrak{v}=5-2=3$	B1ft							
	Critical value for $\chi^2 = 11.345$ $\sum \frac{(O-E)^2}{E} = 10.23$ or $\sum \frac{O^2}{E} - N = 130.23 - 120 = 10.23$	B1ft M1A1							
	E $E10.23 < 11.345 therefore do not reject H0$								
	A binomial is a suitable model.	A1	(7)						
		Total 13	(7)						
(a)	Notes Any two and at least one must have context. 2 correct, no context B1B0. Do not award B0B1.	T							
(a) (b)	M1 require at least two correct terms in numerator and $/(120x7)$ or $/120$ then $/7$ A1 cso as given answer								
(c)	Cao for each B1								
(d)	1 st B1 for both hypotheses. B0 if they include 0.6 Condone $X \sim B(n,p)$ etc								
	1 st M1 for using some combined columns (<8) 2 nd B1ft follows from 'their no of columns' -2 3 rd B1ft follows from the degrees of freedom								
	2^{nd} M1 for attempting $\frac{(O-E)^2}{E}$ or $\frac{O^2}{E}$ with at least 2^{nd} (3 seeds) and 4^{th} (5 seeds) accurate								
	to 2sf Contradictory statements score M0 e.g. "significant" do not reject H0								
	1 st A1 for awrt 10.2								
	2 nd A1 dependent on 2 nd M for a correct comment suggesting that binomial model is suitable. No follow through . Condone mention of 0.6 here. Hypotheses wrong way round scores A0								

Question Number	Scheme	Marks
6(a)	$\overline{X} = \frac{1}{n} \left(X_1 + \ldots + X_n \right)$	
	$E\left(\overline{X}\right) = \frac{1}{n}E(X_1 + \ldots + X_n)$	
	$= \frac{1}{n} (E(X_1) + \dots + E(X_n))$	
	$= \frac{1}{n} (\mu + + \mu)$	
	$=\frac{n\mu}{n}=\mu$	Blcso
		(1)
(b)	$\bar{x} = \frac{1}{5}(197 + 203 + 205 + 201 + 195)$	
	= 200.2(g)	B1
		M1
	$s^{2} = \frac{1}{n-1} (\sum x^{2} - n\bar{x}^{2})$ or $\frac{n}{n-1} V \text{ ar } x$	
	$=\frac{1}{5-1}(200469 - 5(200.2^2))$	
	= 17.2	A1
		(3)
(C)	We require $2 \times 1.25 \ge$ Width of confidence interval	
	$2.5 \ge \frac{2 \times 1.96 \times 4.8}{\sqrt{n}}$ or $1.25 \ge \frac{1.96 \times 4.8}{\sqrt{n}}$ or $\frac{1.25}{\frac{4.8}{\sqrt{n}}} \ge 1.96$	M1B1
	$\sqrt{n} \ge \frac{2 \times 1.96 \times 4.8}{2.5} = 7.5264$	
	$n \ge 56.6(5)$	A1
	Minimum sample size is 57	A1
		(4)
		Total 8
	Notes	
(a)	B1 cso: require $E(\overline{X}) = \mu$ with at least 1 correct intermediate step and no incorrect working.	
(b)		
	B1 for 200.2 or $\frac{1001}{5}$	
	M1 for use of correct formula. Accept $\frac{1}{4}S_{xx} = \frac{1}{4} \times 68.8$	
	A1 for awrt 17.2	
(c)	M1 for use of any equivalent expression. Accept equality. Accept their s instead of 4.8	
	B1 for 1.96 seen with s.e.	
	1 st A1 for 56.6(5)	
	2 nd A1 for 57. Must follow from correct working e.g. $\sqrt{n} \le 7.5264$ resulting in $n = 57$ award A0	

Question Number	Scheme	Marl	<s< th=""></s<>				
7(a)	$z = \pm 3.2905$	B1					
	$\sigma = \frac{30}{3.2905}$	M1					
	3.2905						
	$\sigma = 9.117 **$	Alcso					
			(3)				
(b)	H ₀ : $\mu = 1000$ H ₁ : $\mu < 1000$	B1					
	mean weight = 999.54	B1					
	$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{(999.54 - 1000)}{\frac{9.117}{\sqrt{10}}} = -0.160 \text{or} \frac{c - 1000}{\sqrt{83.12/10}} = -2.3263 \therefore \text{CR } c < 993.29$	M1A1					
	1% critical value = -2.3263	B1					
	-2.3263 < -0.160						
	Accept H ₀ / not in critical region	dM1					
	There is no evidence that the machine is delivering packets of mean weight less than 1 kg	A1ft	(7)				
		Tot	al 10				
	Notes						
(a)	M1 for 30/'their $ z $ '>1						
	A1 cso as given answer						
(b)	1 st B1 both hypotheses correct.						
	Accept 1kg in hypotheses if consistent units used in working usually either kg or g.						
	2 nd B1 999.54 (g) or 0.99954 (kg)						
	1 st M1 for standardising using their mean allow \pm , 1000 and $\frac{9.117}{\sqrt{10}}$ o.e. in kg						
	1 st A1 awrt -0.160 unless clearly using $ z $ (stated) then accept 0.160 or CR awrt 993						
	Condone -0.16 if fully correct expression seen.						
	3^{rd} B1 ± 2.3263 sign consistent with test statistic or $p = 0.4364 > 0.01$ NB $p = 0.5636 < 0.99$						
	2 nd dM1 dependent upon 1 st M for a correct statement linking their test statistic and their cv						
	Contradictory statements score M0 e.g. "significant, do not reject H ₀ "						
	2 nd A1 for correct conclusion in context. Must mention 'machine' and 'packets'.						

Question Number				Schei	me						Mar	ks
8(a)	r =										M1	
	$r = \frac{1}{\sqrt{0.0632 \times 1957.5556}} = 0.840$										A1	
	0.040										211	(2)
(b)	$H_0: \rho = 0 \ H_1: \rho > 0$										B1	(-)
(-)	Critical value = 0.5822										B1	
	0.840 > 0.5822 There is e	evidence	to reject	t H₀.							M1	
	There is evidence of a po				en a m	an's h	eight a	nd his	weigh	t.	A1ft	
	1						U		U			(4)
(c)	Man	Α	В	С	D	Е	F	G	Н	Ι		
	Actual weight	1	2	7	3	4	5	8	6	9	B1	
	Peter's order	1	4	2	6	3	8	5	9	7	B1	
	d^2	0	4	25	9	1	9	9	9	4		
	$\sum d^2 = 70$										M1A1	
	$\sum_{1} 6\sum d^2$										dM1	
	$r_{s} = 1 - \frac{6\sum d^{2}}{n(n^{2} - 1)}$ $= 1 - \frac{6\times70}{9(81 - 1)}$										unin	
	$= 1 - \frac{6 \times 70}{9(81-1)}$											
	= 0.417										A1	
	,											(6)
(d)	$H_0: \rho = 0 \ H_1: \rho > 0$										B1	(0)
(4)	$\begin{array}{c} n_0 \cdot p = 0 n_1 \cdot p \neq 0 \\ \text{Critical value } 0.600 \end{array}$									B1		
	(0.417 < 0.600) There is insufficient evidence to reject H ₀ .									M1		
	Peter does not have the al						ght fra	om the	ir phot	ograph	A1	
				01401	, .	/j	D,	,	n phot	081 0 911		(4)
											Та	tal 16

	Notes					
(a)	M1 Clear use of $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$					
	A1 0.840 cao					
(b)	1^{st} B1 for both hypotheses in terms of ρ , one tail H1 must be compatible with their r					
	Hypotheses just in words e.g. "no correlation" score B0					
	2 nd B1 for 0.5822 cao					
	M1 for a statement comparing 'their r' with 'their cv'					
	A1 for a correct contextualised comment. Must mention positive correlation, be					
	carrying out a 1-tailed test and mention height and weight.					
(c)	Follow through their <i>r</i> and their cv (provided their $ cv < 1$ and their $ r < 1$) 1 st B1 for attempt to rank actual weight / Peter's order with at least 4 correct					
(0)	2^{nd} B1 for correct rankings for both (one or both may be reversed)					
	1 st M1 for use of $\sum d^2$ with at least 4 values correct and attempt to add					
	1 st A1 for 70 or 170 with reversed rankings					
	2^{nd} dM1 for use of the correct formula, follow through their $\sum d^2$. Dependent on 1^{st} M1					
	If answer is not correct, a correct expression is required.					
	2^{nd} A1 for awrt 0.417 or $\frac{5}{12}$					
(d)	1 st B1 for both hypotheses in terms of ρ or ρ_s One tail H1 must be compatible with their					
	ranking					
	Hypotheses just in words e.g. "no correlation" score B0					
	2^{nd} B1 for cv of 0.6(00) cao					
	Their cv must be compatible with their H1 which may be in words					
	M1 for statement comparing 'their r' with 'their cv'					
	A1 for a correct contextualised comment. Must mention Peter and Men.					
	Follow through their <i>r</i> and their cv (provided their $ cv < 1$ and their $ r_s < 1$)					

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