OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4724/01**

MATHEMATICS

Core Mathematics 4

TUESDAY 22 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 Find the angle between the vectors $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $2\mathbf{i} + \mathbf{j} + \mathbf{k}$. [4]

2 **(i)** Express $\dfrac{x}{(x+1)(x+2)}$ in partial fractions. [3]

(ii) Hence find $\displaystyle\int \frac{x}{(x+1)(x+2)}\,\mathrm{d}x$. [2]

3 When $x^4 - 2x^3 - 7x^2 + 7x + a$ is divided by $x^2 + 2x - 1$, the quotient is $x^2 + bx + 2$ and the remainder is $cx + 7$. Find the values of the constants a, b and c. [5]

4 Find the equation of the normal to the curve

$$x^3 + 4x^2y + y^3 = 6$$

at the point $(1, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

5 The vector equations of two lines are

$$\mathbf{r} = (5\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}) + s(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = (2\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}) + t(2\mathbf{i} - \mathbf{j} - 5\mathbf{k}).$$

Prove that the two lines are

(i) perpendicular, [3]

(ii) skew. [5]

6 **(i)** Expand $(1 + ax)^{-4}$ in ascending powers of x, up to and including the term in x^2. [3]

(ii) The coefficients of x and x^2 in the expansion of $(1 + bx)(1 + ax)^{-4}$ are 1 and -2 respectively. Given that $a > 0$, find the values of a and b. [5]

7 **(i)** Given that

$$A(\sin\theta + \cos\theta) + B(\cos\theta - \sin\theta) \equiv 4\sin\theta,$$

find the values of the constants A and B. [3]

(ii) Hence find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{4\sin\theta}{\sin\theta + \cos\theta}\,\mathrm{d}\theta,$$

giving your answer in the form $a\pi - \ln b$. [5]

8 Water flows out of a tank through a hole in the bottom and, at time t minutes, the depth of water in the tank is x metres. At any instant, the rate at which the depth of water in the tank is decreasing is proportional to the square root of the depth of water in the tank.

(i) Write down a differential equation which models this situation. [2]

(ii) When $t = 0$, $x = 2$; when $t = 5$, $x = 1$. Find t when $x = 0.5$, giving your answer correct to 1 decimal place. [6]

9 The parametric equations of a curve are $x = t^3$, $y = t^2$.

(i) Show that the equation of the tangent at the point P where $t = p$ is

$$3py - 2x = p^3.$$ [4]

(ii) Given that this tangent passes through the point $(-10, 7)$, find the coordinates of each of the three possible positions of P. [5]

10 **(i)** Use the substitution $x = \sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx.$$ [6]

(ii) Find the exact value of

$$\int_1^3 \frac{\ln x}{x^2}\,dx.$$ [5]