GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Mark Schemes for the Units

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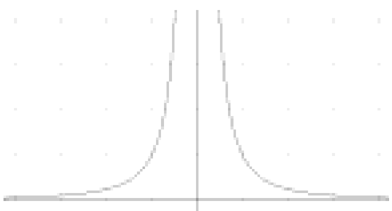
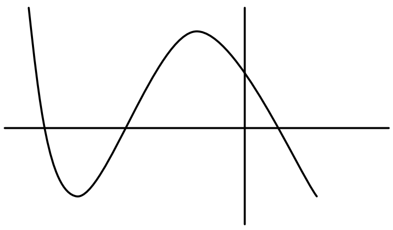
MARK SCHEMES FOR THE UNITS

4721 Core Mathematics 1

1	$3\sqrt{5}+\frac{20\sqrt{5}}{5}$ $=7\sqrt{5}$	B1 M1 A1 3 **3**	$3\sqrt{5}$ soi Attempt to rationalise $\frac{20}{\sqrt{5}}$ cao
2 (i)	x^2	B1 1	cao
(ii)	$\frac{3y^4\times 1000y^3}{2y^5}$ $=1500y^2$	B1 B1 B1 3 **4**	$1000y^3$ soi 1500 y^2
3	Let $y=x^{\frac{1}{3}}$ $3y^2+y-2=0$ $(3y-2)(y+1)=0$ $y=\frac{2}{3}, y=-1$ $x=\left(\frac{2}{3}\right)^3, x=(-1)^3$ $x=\frac{8}{27}, x=-1$	*M1 DM1 A1 DM1 A1 ft 5 **5**	Attempt a substitution to obtain a quadratic or factorise with $\sqrt[3]{x}$ in each bracket Correct method to find roots Both values correct Attempt cube of at least one value Both answers correctly followed through **SR** If M1* not awarded, **B1** x = -1 from T & I
4 (i)		B1 B1 2	Excellent curve in one quadrant or roughly correct curves in correct 2 quadrants Completely correct
(ii)	$y=\frac{1}{(x+3)^2}$	M1 A1 2	$\frac{1}{(x\pm 3)^2}$ $y=\frac{1}{(x+3)^2}$
(iii)	(1, 4)	B1 B1 2 **6**	Correct x coordinate Correct y coordinate

5 (i)	$\frac{dy}{dx} = -50x^{-6}$	M1 A1 2	kx^{-6} Fully correct answer
(ii)	$y = x^{\frac{1}{4}}$ $\frac{dy}{dx} = \frac{1}{4}x^{-\frac{3}{4}}$	B1 B1 B1 3	$\sqrt[4]{x} = x^{\frac{1}{4}}$ soi $\frac{1}{4}x^c$ $kx^{-\frac{3}{4}}$
(iii)	$y = (x^2 + 3x)(1 - 5x)$ $= 3x - 14x^2 - 5x^3$ $\frac{dy}{dx} = 3 - 28x - 15x^2$	M1 A1 M1 A1 4 **9**	Attempt to multiply out fully Correct expression (may have 4 terms) Two terms correctly differentiated from their expanded expression Completely correct (3 terms)
6(i)	$5(x^2 + 4x) - 8$ $= 5[(x+2)^2 - 4] - 8$ $= 5(x+2)^2 - 20 - 8$ $= 5(x+2)^2 - 28$	B1 B1 M1 A1 4	$p = 5$ $(x+2)^2$ seen or $q = 2$ $-8 - 5q^2$ or $-\frac{8}{5} - q^2$ $r = -28$
(ii)	$x = -2$	B1 ft 1	
(iii)	$20^2 - 4 \times 5 \times -8$ $= 560$	M1 A1 2	Uses $b^2 - 4ac$ 560
(iv)	2 real roots	B1 1 **8**	2 real roots
7(i)	$30 + 4k - 10 = 0$ $\therefore k = -5$	M1 A1 2	Attempt to substitute x = 10 into equation of line
(ii)	$\sqrt{(10-2)^2 + (-5-1)^2}$ $= \sqrt{64 + 36}$ $= 10$	M1 A1 2	Correct method to find line length using Pythagoras' theorem cao, dependent on correct value of k in (i)
(iii)	Centre (6, -2) Radius 5	B1 B1 2	
(iv)	Midpoint of AB = (6, -2) Length of AB = 2 x radius Both A and B lie on circumference Centre lies on line 3x + 4y – 10 = 0	B1 B1 2 **8**	One correct statement of verification Complete verification

8 (i)	$x = \dfrac{8 \pm \sqrt{(-8)^2 - (4 \times -1 \times 5)}}{-2}$ $= \dfrac{8 \pm \sqrt{84}}{-2}$ $= -4 - \sqrt{21}$ or $= -4 + \sqrt{21}$	M1 A1 A1 3	Correct method to solve quadratic $x = \dfrac{8 \pm \sqrt{84}}{-2}$ Both roots correct and simplified
(ii)	$x \le -4 - \sqrt{21}$, $x \ge -4 + \sqrt{21}$	M1 A1 2	Identifying $x \le$ their lower root, $x \ge$ their higher root $x \le -4 - \sqrt{21}$, $x \ge -4 + \sqrt{21}$ (not wrapped, no 'and')
(iii)		B1 B1 B1 B1 B1 5 **10**	Roughly correct negative cubic with max and min (-4, 0) (0, 20) Cubic with 3 distinct real roots Completely correct graph
9	$\dfrac{dy}{dx} = 3x^2 + 2px$ When $x = 4$, $\dfrac{dy}{dx} = 0$ $\therefore 3 \times 4^2 + 8p = 0$ $8p = -48$ $p = -6$ $\dfrac{d^2y}{dx^2} = 6x - 12$ When $x = 4$, $6x - 12 > 0$ Minimum point	M1 A1 M1 M1 A1 M1 A1 7 7	Attempt to differentiate Correct expression cao Setting their $\dfrac{dy}{dx} = 0$ Substitution of $x = 4$ into their $\frac{dy}{dx} = 0$ to evaluate p Looks at sign of $\dfrac{d^2y}{dx^2}$, derived correctly from their $\dfrac{dy}{dx}$, or other correct method Minimum point CWO

10(i)	$\frac{dy}{dx} = 2x+1$ $= 5$	M1 A1 2	Attempt to differentiate *y* cao
(ii)	Gradient of normal = $-\frac{1}{5}$ When $x = 2$, $y = 6$ $y-6=-\frac{1}{5}(x-2)$ $x+5y-32=0$	B1 ft B1 M1 A1 4	ft from a non-zero numerical value in (i) May be embedded in equation of line Equation of line, any non-zero gradient, their *y* coordinate Correct equation in correct form
(iii)	$x^2+x=kx-4$ $x^2+(1-k)x+4=0$ One solution => $b^2-4ac=0$ $(1-k)^2-4\times1\times4=0$ $(1-k)^2=16$ $1-k=\pm4$ $k = -3$ or 5	*M1 DM1 DM1 A1 DM1 A1 6 **12**	Equating $y_1 = y_2$ Statement that discriminant = 0 Attempt (involving *k*) to use a, b, c from their equation Correct equation (may be unsimplified) Correct method to find *k*, dep on 1st 3Ms Both values correct

4722 Core Mathematics 2

1 (i)	$\int(x^3+8x-5)\,dx = \frac{1}{4}x^4+4x^2-5x+c$	M1		Attempt integration – increase in power for at least 2 terms
		A1		Obtain at least 2 correct terms
		A1	3	Obtain $\frac{1}{4}x^4+4x^2-5x+c$ (and no integral sign or dx)
(ii)	$\int 12x^{\frac{1}{2}}\,dx = 8x^{\frac{3}{2}}+c$	B1		State or imply $\sqrt{x}=x^{\frac{1}{2}}$
		M1		Obtain $kx^{\frac{3}{2}}$
		A1	3	Obtain $8x^{\frac{3}{2}}+c$ (and no integral sign or dx) (only penalise lack of + c, or integral sign or dx once)
			6	
2 (i)	$140^{\circ} = 140\times\frac{\pi}{180}$	M1		Attempt to convert 140° to radians
	$= \frac{7}{9}\pi$	A1	2	Obtain $\frac{7}{9}\pi$, or exact equiv
(ii)	arc $AB = 7\times\frac{7}{9}\pi$	M1		Attempt arc length using $r\theta$ or equiv method
	= 17.1	A1√		Obtain 17.1, $\frac{49}{9}\pi$ or unsimplified equiv
	chord $AB = 2\times 7\sin\frac{7}{18}\pi$ = 13.2	M1		Attempt chord using trig. or cosine or sine rules
	hence perimeter = 30.3 cm	A1	4	Obtain 30.3, or answer that rounds to this
			6	
3 (i)	$u_1 = 23^1/_3$	B1		State $u_1 = 23^1/_3$
	$u_2 = 22^2/_3$, $u_3 = 22$	B1	2	State $u_2 = 22^2/_3$ and $u_3 = 22$
(ii)	$24 - {}^{2k}/_3 = 0$	M1		Equate u_k to 0
	$k = 36$	A1	2	Obtain 36
(iii)	$S_{20} = \frac{20}{2}\left(2\times 23\frac{1}{3}+19\times\frac{-2}{3}\right)$	M1		Attempt sum of AP with $n = 20$
	= 340	A1		Correct unsimplified S_{20}
		A1	3	Obtain 340
			7	
4	$\int_{-2}^{2}(x^4+3)\,dx = \left[\frac{1}{5}x^5+3x\right]_{-2}^{2}$	M1		Attempt integration – increase of power for at least 1 term
		A1		Obtain correct $\frac{1}{5}x^5+3x$
	$= (\frac{32}{5}+6) - (\frac{-32}{5}-6)$	M1		Use limits (any two of –2, 0, 2), correct order/subtraction
	$= 24\frac{4}{5}$	A1		Obtain $24\frac{4}{5}$
	area of rectangle = 19 x 4	B1		State or imply correct area of rectangle
	hence shaded area = $76 - 24\frac{4}{5}$	M1		Attempt correct method for shaded area
	$= 51\frac{1}{5}$	A1	7	Obtain $51\frac{1}{5}$ aef such as 51.2, $\frac{256}{5}$
OR				
	Area = $19 - (x^4 + 3)$	M1		Attempt subtraction, either order
	$= 16 - x^4$	A1		Obtain $16 - x^4$ (not from $x^4 + 3 = 19$)
	$\int_{-2}^{2}(16-x^4)\,dx = \left[16x-\frac{1}{5}x^5\right]_{-2}^{2}$	M1		Attempt integration
		A1		Obtain $\pm\left(16x-\frac{1}{5}x^5\right)$

	$= (32 - \frac{32}{5}) - (-32 - \frac{-32}{5})$	M1		Use limits – correct order / subtraction
	$= 51\frac{1}{5}$	A1		Obtain $\pm 51\frac{1}{5}$
		A1		Obtain $51\frac{1}{5}$ only, no wrong working
			7	
5 (i)	$\frac{TA}{\sin 107} = \frac{50}{\sin 3}$	M1		Attempt use of correct sine rule to find *TA*, or equiv
	$TA = 914$ m	A1	**2**	Obtain 914, or better
(ii)	$TC = \sqrt{914^2 + 150^2 - 2 \times 914 \times 150 \times \cos 70}$	M1		Attempt use of correct cosine rule, or equiv, to find *TC*
		A1√		Correct unsimplified expression for *TC*, following their (i)
	= 874 m	A1	**3**	Obtain 874, or better
(iii)	dist from *A* = 914 x cos 70 = 313 m beyond *C*, hence 874 m is shortest dist	M1 A1	 **2**	Attempt to locate point of closest approach Convincing argument that the point is beyond *C*, or obtain 859, or better
OR	perp dist = $914 \times \sin 70 = 859$ m			**SR** B1 for 874 stated with no method shown
			7	
6 (i)	$S_\infty = \frac{20}{1-0.9}$	M1		Attempt use of $S_\infty = \frac{a}{1-r}$
	= 200	A1	**2**	Obtain 200
(ii)	$S_{30} = \frac{20(1-0.9^{30})}{1-0.9}$	M1		Attempt use of correct sum formula for a GP, with $n = 30$
	= 192	A1	**2**	Obtain 192, or better
(iii)	$20 \times 0.9^{p-1} < 0.4$ $0.9^{p-1} < 0.02$	B1		Correct $20 \times 0.9^{p-1}$ seen or implied
	$(p-1)\log 0.9 < \log 0.02$ $p - 1 > \frac{\log 0.02}{\log 0.9}$	M1		Link to 0.4, rearrange to $0.9^k = c$ (or >, <), introduce logarithms, and drop power, or equiv correct method
	$p > 38.1$	M1		Correct method for solving their (in)equation
	hence $p = 39$	A1	**4**	State 39 (not inequality), no wrong working seen
			8	
7 (i)	$6k^2a^2 = 24$	M1*		Obtain at least two of 6, k^2, a^2
	$k^2a^2 = 4$	M1dep*		Equate $6k^ma^n$ to 24
	$ak = 2$ **A.G.**	A1	**3**	Show $ak = 2$ convincingly – no errors allowed
(ii)	$4k^3a = 128$	B1		State or imply coeff of x is $4k^3a$
	$4k^3\left(\frac{2}{k}\right) = 128$	M1		Equate to 128 and attempt to eliminate a or k
	$k^2 = 16$	A1		Obtain $k = 4$
	$k = 4$, $a = \frac{1}{2}$	A1	**4**	Obtain $a = ½$
				SR B1 for $k = \pm 4$, $a = \pm\frac{1}{2}$
(iii)	$4 \times 4 \times \left(\frac{1}{2}\right)^3 = 2$	M1		Attempt $4 \times k \times a^3$, following their a and k (allow if still in terms of a, k)
		A1	**2**	Obtain 2 (allow $2x^3$)
			9	

8 (a)(i) $\log_a xy = p + q$	B1	**1**	State $p + q$ cwo
(ii) $\log_a\left(\frac{a^2x^3}{y}\right) = 2 + 3p - q$	M1		Use $\log a^b = b\log a$ correctly at least once
	M1		Use $\log \frac{a}{b} = \log a - \log b$ correctly
	A1	**3**	Obtain $2 + 3p - q$
(b)(i) $\log_{10} \frac{x^2-10}{x}$	B1	**1**	State $\log_{10} \frac{x^2-10}{x}$ (with or without base 10)
(ii) $\log_{10} \frac{x^2-10}{x} = \log_{10} 9$	B1		State or imply that $2\log_{10} 3 = \log_{10} 3^2$
$\frac{x^2-10}{x} = 9$	M1		Attempt correct method to remove logs
$x^2 - 9x - 10 = 0$	A1		Obtain correct $x^2 - 9x - 10 = 0$ aef, no fractions
$(x - 10)(x + 1) = 0$	M1		Attempt to solve three term quadratic
$x = 10$	A1	**5**	Obtain $x = 10$ only
		10	
9 (i) $f(1) = 1 - 1 - 3 + 3 = 0$ **A.G.**	B1		Confirm f(1) = 0, or division with no remainder shown, or matching coeffs with $R = 0$
$f(x) = (x - 1)(x^2 - 3)$	M1		Attempt complete division by $(x - 1)$, or equiv
	A1		Obtain $x^2 + k$
	A1		Obtain completely correct quotient (allow $x^2 + 0x - 3$)
$x^2 = 3$	M1		Attempt to solve $x^2 = 3$
$x = \pm\sqrt{3}$	A1	**6**	Obtain $x = \pm\sqrt{3}$ only
(ii) $\tan x = 1, \sqrt{3}, -\sqrt{3}$	B1√		State or imply tan x = 1 or tan x = at least one of their roots from (i)
$\tan x = \sqrt{3} \Rightarrow x = \pi/3, 4\pi/3$	M1		Attempt to solve tan $x = k$ at least once
$\tan x = -\sqrt{3} \Rightarrow x = 2\pi/3, 5\pi/3$	A1		Obtain at least 2 of $\pi/3$, $2\pi/3$, $4\pi/3$, $5\pi/3$ (allow degs/decimals)
$\tan x = 1 \Rightarrow x = \pi/4, 5\pi/4$	A1		Obtain all 4 of $\pi/3$, $2\pi/3$, $4\pi/3$, $5\pi/3$ (exact radians only)
	B1		Obtain $\pi/4$ (allow degs / decimals)
	B1	**6**	Obtain $5\pi/4$ (exact radians only) **SR** answer only is B1 per root, max of B4 if degs / decimals
		12	

4723 Core Mathematics 3

1 (i)	Obtain integral of form ke^{-2x}	M1		any constant k different from 8
	Obtain $-4e^{-2x}$	A1		or (unsimplified) equiv
(ii)	Obtain integral of form $k(4x+5)^7$	M1		any constant k
	Obtain $\frac{1}{28}(4x+5)^7$	A1		in simplified form
	Include … + c at least once	B1		in either part
			5	
2 (i)	Form expression involving attempts at y values and addition	M1		with coeffs 1, 4 and 2 present at least once
	Obtain $k(\ln 4 + 4\ln 6 + 2\ln 8 + 4\ln 10 + \ln 12)$	A1		any constant k
	Use value of k as $\frac{1}{3}\times 2$	A1		or unsimplified equiv
	Obtain 16.27	A1	**4**	or 16.3 or greater accuracy (16.27164…)
(ii)	State 162.7 or 163	B1√	**1**	following their answer to **(i)**, maybe rounded
			5	
3 (i)	Attempt use of identity for $\tan^2\theta$	M1		using $\pm\sec^2\theta \pm 1$; or equiv
	Replace $\dfrac{1}{\cos\theta}$ by $\sec\theta$	B1		
	Obtain $2(\sec^2\theta - 1) - \sec\theta$	A1	**3**	or equiv
(ii)	Attempt soln of quadratic in $\sec\theta$ or $\cos\theta$	M1		as far as factorisation or substitution in correct formula
	Relate $\sec\theta$ to $\cos\theta$ and attempt at least one value of θ	M1		may be implied
	Obtain $60°, 131.8°$	A1		allow 132 or greater accuracy
	Obtain $60°, 131.8°, 228.2°, 300°$	A1	**4**	allow 132, 228 or greater accuracy; and no others between $0°$ and $360°$
			7	
4 (i)	Obtain derivative of form $kx(4x^2+1)^4$	M1		any constant k
	Obtain $40x(4x^2+1)^4$	A1		or (unsimplified) equiv
	State $x = 0$	A1√	**3**	and no other; following their derivative of form $kx(4x^2+1)^4$
(ii)	Attempt use of quotient rule	M1		or equiv
	Obtain $\dfrac{2x\ln x - x^2.\frac{1}{x}}{(\ln x)^2}$	A1		or equiv
	Equate to zero and attempt solution	M1		as far as solution involving e
	Obtain $e^{\frac{1}{2}}$	A1	**4**	or exact equiv; and no other; allow from ± (correct numerator of derivative)
			7	

5 (i)	State 40	B1		
	Attempt value of k using 21 and 80	M1		or equiv
	Obtain $40e^{21k}=80$ and hence 0.033	A1		or equiv such as $\frac{1}{21}\ln 2$
	Attempt value of M for $t = 63$	M1		using established formula or using exponential property
	Obtain 320	A1	**5**	or value rounding to this
(ii)	Differentiate to obtain $ce^{0.033t}$ or $40ke^{kt}$	M1		any constant c different from 40
	Obtain $40\times 0.033e^{0.033t}$	A1√		following their value of k
	Obtain 2.64	A1	**3**	allow 2.6 or 2.64 ± 0.01 or greater accuracy (2.64056…)
			8	
6 (i)	Attempt correct process for finding inverse	M1		maybe in terms of y so far
	Obtain $2x^3-4$	A1		or equiv; in terms of x now
	State $\sqrt[3]{2}$ or 1.26	B1	**3**	
(ii)	State reflection in $y = x$	B1		or clear equiv
	Refer to intersection of $y = x$ and $y = \mathrm{f}(x)$ and hence confirm $x=\sqrt[3]{\frac{1}{2}x+2}$	B1	**2**	AG; or equiv
(iii)	Obtain correct first iterate	B1		
	Show correct process for iteration	M1		with at least one more step
	Obtain at least 3 correct iterates in all	A1		allowing recovery after error
	Obtain 1.39	A1	**4**	following at least 3 steps; answer required to exactly 2 d.p.
	[0 → 1.259921 → 1.380330 → 1.390784 → 1.391684 1 → 1.357209 → 1.388789 → 1.391512 → 1.391747 1.26 → 1.380337 → 1.390784 → 1.391684 → 1.391761 1.5 → 1.401020 → 1.392564 → 1.391837 → 1.391775 2 → 1.442250 → 1.396099 → 1.392141 → 1.391801]			
			9	
7 (i)	Refer to stretch and translation	M1		in either order; allow here informal terms
	State stretch, factor $\frac{1}{k}$, in x direction	A1		or equiv; now with correct terminology
	State translation in negative y direction by a	A1	**3**	or equiv; now with correct terminology
	[SC: If M0 but one transformation completely correct – B1]			
(ii)	Show attempt to reflect negative part in x-axis	M1		ignoring curvature
	Show correct sketch	A1	**2**	with correct curvature, no pronounced 'rounding' at x-axis and no obvious maximum point
(iii)	Attempt method with $x = 0$ to find value of a	M1		... other than (or in addition to) value –12
	Obtain $a = 14$	A1		and nothing else
	Attempt to solve for k	M1		using any numerical a with sound process
	Obtain $k = 3$	A1	**4**	
			9	

8 (i)	Attempt to express x or x^2 in terms of y	M1		
	Obtain $x^2 = \dfrac{1296}{(y+3)^4}$	A1		or (unsimplified) equiv
	Obtain integral of form $k(y+3)^{-3}$	M1		any constant k
	Obtain $-432\pi(y+3)^{-3}$ or $-432(y+3)^{-3}$	A1		or (unsimplified) equiv
	Attempt evaluation using limits 0 and p	M1		for expression of form $k(y+3)^{-n}$ obtained from integration attempt; subtraction correct way round
	Confirm $16\pi(1-\dfrac{27}{(p+3)^3})$	A1	**6**	AG; necessary detail required, including appearance of π prior to final line
(ii)	State or obtain $\dfrac{dV}{dp} = 1296\pi(p+3)^{-4}$	B1		or equiv; perhaps involving y
	Multiply $\dfrac{dp}{dt}$ and attempt at $\dfrac{dV}{dp}$	*M1		algebraic or numerical
	Substitute $p = 9$ and attempt evaluation	M1		dep *M
	Obtain $\frac{1}{4}\pi$ or 0.785	A1	**4**	or greater accuracy
			10	
9 (i)	State $\cos 2\theta \cos\theta - \sin 2\theta \sin\theta$	B1		
	Use at least one of $\cos 2\theta = 2\cos^2\theta - 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$	B1		
	Attempt to express in terms of $\cos\theta$ only	M1		using correct identities for $\cos 2\theta, \sin 2\theta$ and $\sin^2\theta$
	Obtain $4\cos^3\theta - 3\cos\theta$	A1	**4**	AG; necessary detail required
(ii)	Either: State or imply $\cos 6\theta = 2\cos^2 3\theta - 1$	B1		
	Use expression for $\cos 3\theta$ and attempt expansion	M1		for expression of form $\pm 2\cos^2 3\theta \pm 1$
	Obtain $32c^6 - 48c^4 + 18c^2 - 1$	A1	**3**	AG; necessary detail required
	Or: State $\cos 6\theta = 4\cos^3 2\theta - 3\cos 2\theta$	B1		maybe implied
	Express $\cos 2\theta$ in terms of $\cos\theta$ and attempt expansion	M1		for expression of form $\pm 2\cos^2\theta \pm 1$
	Obtain $32c^6 - 48c^4 + 18c^2 - 1$	A1	**(3)**	AG; necessary detail required
(iii)	Substitute for $\cos 6\theta$	*M1		with simplification attempted
	Obtain $32c^6 - 48c^4 = 0$	A1		or equiv
	Attempt solution for c of equation	M1		dep *M
	Obtain $c^2 = \frac{3}{2}$ and observe no solutions	A1		or equiv; correct work only
	Obtain $c = 0$, give at least three specific angles and conclude odd multiples of 90	A1	**5**	AG; or equiv; necessary detail required; correct work only
			12	

4724 Core Mathematics 4

1	Attempt to factorise numerator and denominator	M1	$\frac{A}{f(x)}+\frac{B}{g(x)}$;fg= $6x^2-24x$
	Any (part) factorisation of both num and denom	A1	Corres identity/cover-up
	Final answer = $-\frac{5}{6x}, \frac{-5}{6x}, \frac{5}{-6x}, -\frac{5}{6}x^{-1}$ Not $-\frac{\frac{5}{6}}{x}$	A1	
		3	

2	Use parts with $u = x, \mathrm{d}v = \sec^2 x$	M1	result $f(x) +/- \int g(x)\,\mathrm{d}x$
	Obtain correct result $x\tan x - \int \tan x\,\mathrm{d}x$	A1	
	$\int \tan x\,\mathrm{d}x = k\ln\sec x$ or $k\ln\cos x$, where $k = 1$ or -1	B1	or $k\ln\lvert\sec x\rvert$ or $k\ln\lvert\cos x\rvert$
	Final answer $= x\tan x - \ln\lvert\sec x\rvert + c$ or $x\tan x + \ln\lvert\cos x\rvert + c$	A1	
		4	

3 (i)	$1 + \frac{1}{2}.2x + \frac{\frac{1}{2}.-\frac{1}{2}}{2}\left(4x^2 \text{ or } 2x^2\right) + \frac{\frac{1}{2}.-\frac{1}{2}.-\frac{3}{2}}{6}\left(8x^3 \text{ or } 2x^3\right)$	M1	
	$= 1 + x$	B1	
	$\ldots -\frac{1}{2}x^2 + \frac{1}{2}x^3$ (AE fract coeffs)	A1 (**3**)	For both terms
(ii)	$(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3$	B1	or $(1+x)^3 = 1 + 3x + 3x^2 + x^3$
	Either attempt at their **(i)** multiplied by $(1+x)^{-3}$	M1	or **(i)** long div by $(1+x)^3$
	$1 - 2x \ldots$ √ $1 + (a-3)x$	A1	f.t. **(i)** = $1 + ax + bx^2 + cx^3$
	$\ldots + \frac{5}{2}x^2 \ldots$ √ $(-3a + b + 6)x^2$	A1	
	$\ldots -2x^3$ √ $(6a - 3b + c - 10)x^3$	A1 (**5**)	(AE fract.coeffs)
(iii)	$-\frac{1}{2} < x < \frac{1}{2}$, or $\lvert x\rvert < \frac{1}{2}$	B1 (**1**)	
		9	

4		Attempt to expand $(1+\sin x)^2$ and integrate it	*M1	Minimum of $1+\sin^2 x$
		Attempt to change $\sin^2 x$ into $\mathrm{f}(\cos 2x)$	M1	
		Use $\sin^2 x = \frac{1}{2}(1-\cos 2x)$	A1	dep M1 + M1
		Use $\int \cos 2x \, \mathrm{d}x = \frac{1}{2}\sin 2x$	A1	dep M1 + M1
		Use limits correctly on an attempt at integration	dep* M1	Tolerate g $(\frac{1}{4}\pi)$ − 0
		$\frac{3}{8}\pi - \sqrt{2} + \frac{7}{4}$ AE(3-term)F	A1	WW 1.51… $\rightarrow$ M1 A0
			6	
5	(i)	Attempt to connect du and dx, find $\frac{\mathrm{d}u}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}u}$	M1	But not e.g. du = dx
		Any correct relationship, however used, such as dx = 2u du	A1	or $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}}$
		Subst with clear reduction (≥ 1 inter step) to **AG**	A1 (**3**)	WWW
	(ii)	Attempt partial fractions	M1	
		$\frac{2}{u} - \frac{2}{1+u}$	A1	
		√ $A \ln u + B \ln(1+u)$	√A1	Based on $\frac{A}{u} + \frac{B}{1+u}$
		Attempt integ, change limits & use on f(u)	M1	or re-subst & use 1 & 9
		$\ln \frac{9}{4}$ AEexactF (e.g. 2 ln 3 –2 ln 4 + 2 ln 2)	A1 (**5**)	Not involving ln 1
			8	

6	**(i)**	Solve $0 = t-3$ & subst into $x = t^2 - 6t + 4$	M1	
		Obtain $x = -5$	A1 (**2**)	(–5,0) need not be quoted
		N.B. If **(ii)** completed first, subst $y = 0$ into their cartesian eqn (M1) & find *x* (no f.t.) (A1)		
	(ii)	Attempt to eliminate *t*	M1	
		Simplify to $x = y^2 - 5$ ISW	A1 (**2**)	
	(iii)	Attempt to find $\frac{dy}{dx}$ or $\frac{dx}{dy}$ from cartes or para form	M1	Award anywhere in Que
		Obtain $\frac{dy}{dx} = \frac{1}{2t-6}$ or $\frac{1}{2y}$ or (-)$\frac{1}{2}(x+5)^{-\frac{1}{2}}$	A1	
		If $t = 2$, $x = -4$ and $y = -1$	B1	Awarded anywhere in **(iii)**
		Using their num (x, y) & their num $\frac{dy}{dx}$, find tgt eqn	M1	
		$x + 2y + 6 = 0$ AEF(without fractions) ISW	A1 (**5**)	
			9	
7	**(i)**	Attempt direction vector between the 2 given points	M1	
		State eqn of line using format (**r**) = (either end) + *s*(dir vec)	M1	'*s*' can be '*t*'
		Produce 2/3 eqns containing *t* and *s*	M1	2 different parameters
		Solve giving $t = 3$, $s = -2$ or 2 or -1 or 1	A1	
		Show consistency	B1	
		Point of intersection = $(5, 9, -1)$	A1 (**6**)	
	(ii)	Correct method for scalar product of 'any' 2 vectors	M1	Vectors from this question
		Correct method for magnitude of 'any' vector	M1	Vector from this question
		Use $\cos\theta = \frac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ for the correct 2 vectors $\begin{pmatrix}1\\4\\-2\end{pmatrix}$ & $\begin{pmatrix}2\\-1\\3\end{pmatrix}$	M1	Vects may be mults of dvs
		62.2 (62.188157…) 1.09 (1.0853881)	A1 (**4**)	
			10	

8	**(i)**	$\frac{d}{dx}\left(y^3\right) = 3y^2\frac{dy}{dx}$	B1	
		Consider $\frac{d}{dx}(xy)$ as a product	M1	
		$= \quad x\frac{dy}{dx} + y$	A1	Tolerate omission of '6'
		$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$ ISW AEF	A1 (**4**)	
	(ii)	$x^3 = 2^4$ or 16 and $y^3 = 2^5$ or 32	*B1	
		Satisfactory conclusion	dep* B1	**AG**
		Substitute $\left(2^{\frac{4}{3}}, 2^{\frac{5}{3}}\right)$ into their $\frac{dy}{dx}$	M1	or the numerator of $\frac{dy}{dx}$
		Show or use calc to demo that num = 0, ignore denom **AG**	A1 (**4**)	
	(iii)	Substitute (a, a) into eqn of curve	M1	& attempt to state '$a = \ldots$'
		$a = 3$ only with clear ref to $a \neq 0$	A1	
		Substitute $(3,3)$ or (their a, their a) into their $\frac{dy}{dx}$	M1	
		-1 only WWW	A1 (**4**)	from (their a,their a)
			12	
9	**(i)**	$\frac{d\theta}{dt} = \ldots$	B1	
		$k(160 - \theta)$	B1 (**2**)	The 2 @ 'B1' are indep
	(ii)	Separate variables with $(160 - \theta)$ in denom; or invert	*M1	$\int \frac{1}{160 - \theta}\, d\theta = \int k, \frac{1}{k}, 1\, dt$
		Indication that LHS = $\ln f(\theta)$	A1	If wrong ln, final 3@A = 0
		RHS = kt or $\frac{1}{k}t$ or t $(+\, c)$	A1	
		Subst. $t = 0, \theta = 20$ into equation containing 'c'	dep* M1	
		Subst $t = 5, \theta = 65$ into equation containing 'c' & 'k'	dep* M1	
		$c = -\ln 140$ (-4.94) ISW	A1	
		$k = \frac{1}{5}\ln\frac{140}{95}$ (≈ 0.077 or 0.078) ISW	A1	
		Using their 'c' & 'k', subst $t = 10$ & evaluate θ	dep*M1	
		$\theta = 96(95.535714)$ $\left(95\frac{15}{28}\right)$	A1 (**9**)	
			11	

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1	 $\frac{7}{26}+\frac{17}{26}\text{i}$.	M1 A1 A1 A1	 4 **4**	Multiply by conjugate of denominator Obtain correct numerator Obtain correct denominator
2	**(i)** $\frac{1}{10}\begin{pmatrix}5 & 0\\ -a & 2\end{pmatrix}$ **(ii)** $\begin{pmatrix}3 & -2\\ 2a & 6\end{pmatrix}$	B1 B1 B1 B1	 2 2 **4**	Both diagonals correct Divide by correct determinant Two elements correct Remaining elements correct
3	$n^2(n+1)^2+n(n+1)(2n+1)+n(n+1)$ $n(n+1)^2(n+2)$	M1 A1 A1 M1 A1ft A1	 6 **6**	Express as sum of 3 terms 2 correct unsimplified terms 3rd correct unsimplified term Attempt to factorise Two factors found, ft their quartic Correct final answer a.e.f.
4	 $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$	B1 M1 A1 A1	 4 **4**	State or use correct result Combine matrix and its inverse Obtain **I** or $\mathbf{I}^2$ but not 1 Obtain zero **matrix** but not 0 **S.C. If 0/4, B1 for $\mathbf{AA}^{-1}=\mathbf{I}$**
5	*Either* $4k-4$ $k=1$ *Or*	M1 M1 A1 M1 A1ft M1 A1 M1 A1 A1	 5 **5**	Consider determinant of coefficients of LHS Sensible attempt at evaluating any 3×3 det Obtain correct answer a.e.f. unsimplified Equate det to 0 Obtain k = 1, ft provided all M's awarded Eliminate either x or y Obtain correct equation Eliminate 2nd variable Obtain correct linear equation Deduce that k = 1
6	**(i)** *Either* *Or* **(ii)** **(iii)** $\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$ **(iv)**	B1 DB1 B1 DB1 B1 DB1 B1 B1 B1B1B1	2 2 2 3 **9**	Reflection, in x-axis Stretch parallel to y-axis, s.f. –1 Reflection, in $y=-x$ Each column correct Rotation, 90° ,clockwise about O **S.C. If (iii) incorrect, B1 for identifying their transformation, B1 all details correct**

7	**(i)** $13^n + 6^{n-1} + 13^{n+1} + 6^n$ **(ii)**	B1 M1 A1 B1 B1 B1 B1	 3 4 **7**	Correct expression seen Attempt to factorise both terms in (i) Obtain correct expression Check that result is true for n =1 (or 2) Recognise that (i) is divisible by 7 Deduce that u_{n+1} is divisible by 7 Clear statement of Induction conclusion
8	**(i)** **(ii)** $\alpha + \beta = 6k, \alpha\beta = k^2$ $\alpha - \beta = (4\sqrt{2})k$ **(iii)** $\sum \alpha' = 6k$ $\alpha'\beta' = \alpha\beta - (\alpha - \beta) - 1$ $\alpha'\beta' = k^2 - (4\sqrt{2})k - 1$ $x^2 - 6kx + k^2 - (4\sqrt{2})k - 1 = 0$	M1 A1 B1 B1 M1 A1 B1ft M1 A1ft B1ft	 2 4 4 **10**	Expand at least 1 of the brackets Derive given answer correctly State or use correct values Find value of $\alpha - \beta$ using (i) Obtain given value correctly (allow if –6k used) Sum of new roots stated or used Express new product in terms of old roots Obtain correct value for new product Write down correct quadratic **equation**
9	**(i)** **(ii)** $1 + \frac{1}{3} - \frac{1}{2n-1} - \frac{1}{2n+1}$ **(iii)** $\frac{4}{3}$	M1 A1 M1 M1 A1 A1 M1 A1 B1ft	 2 6 1 **9**	Use correct denominator Obtain given answer correctly Express terms as differences using (i) Do this for at least 1st 3 terms First 3 terms all correct Last 3 terms all correct (in terms or n or r) Show pairs cancelling Obtain correct answer, a.e.f.(in terms of n) Given answer deduced correctly, ft their (ii)

10	**(i)** $x^2 - y^2 = 2, 2xy = \sqrt{5}$	M1 A1		Attempt to equate real and imaginary parts Obtain both results a.e.f.
	$4x^4 - 8x^2 - 5 = 0$	M1		Eliminate to obtain quadratic in x^2 or y^2
		M1		Solve to obtain x (or y) values
	$x = \pm\frac{\sqrt{10}}{2}, y = \pm\frac{\sqrt{2}}{2}$	A1		Correct values for both x & y obtained a.e.f.
	$\pm(\frac{\sqrt{10}}{2} + i\frac{\sqrt{2}}{2})$	A1	6	Correct answers as complex numbers
	(ii) $z^2 = 2 \pm i\sqrt{5}$	M1		Solve quadratic in z^2
	$z = \pm(\frac{\sqrt{10}}{2} \pm i\frac{\sqrt{2}}{2})$	A1		Obtain correct answers
		M1		Use results of (i)
		A1ft	4	Obtain correct answers, ft must include root from conjugate
	(iii)	B1ft	1	Sketch showing roots correctly
	(iv)	B1 B1ft		Sketch of straight line, $\perp$ to α
		B1ft	3	Bisector
			14	

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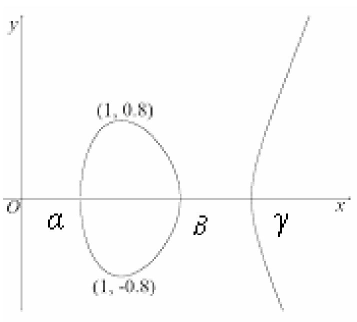
1	**(i)**	Give $1 + 2x + (2x)^2/2$	M1	Reasonable 3 term attempt e.g. allow $2x^2/2$
		Get $1 + 2x + 2x^2$	A1	cao
				SC Reasonable attempt at f′(0) and f″(0) M1 Get $1+2x+2x^2$ cao A1
	(ii)	$\ln((1+2x+2x^2)$ $+ (1-2x+2x^2)) =$	M1	Attempt to sub for e^{2x} and e^{-2x}
		$\ln(2+4x^2) =$	A1√	On their part **(i)**
		$\ln 2 + \ln(1 + 2x^2)$	M1	Use of log law in reasonable expression
		$\ln 2 + 2x^2$	A1	cao
				SC Use of Maclaurin for f ′(x) and f″(x) M1 One correct A1 Attempt f(0), f ′(0) and f″(0) M1 Get cao A1
2	**(i)**	$x_2 = 1.8913115$	B1	x_2 correct; allow answers which round
		$x_3 = 1.8915831$	B1√	For any other from their working
		$x_4 = 1.8915746$	B1	For all three correct
	(ii)	$e_3/e_2 = -0.031(1)$	M1	Subtraction and division on their values; allow ±
		$e_4/e_3 = -0.036(5)$	A1	Or answers which round to −0.031 and −0.037
		State $f'(\alpha) \approx e_3/e_2 \approx e_4/e_3$	B1√	Using their values but only if approx. equal; allow differentiation if correct conclusion; allow gradient for f′
3	**(i)**	Diff. $\sin y = x$	M1	Implicit diff. to $dy/dx = \pm(1/\cos y)$
		Use $\sin^2 + \cos^2 = 1$ to A.G.	A1	Clearly derived; ignore ±
		Justify +	B1	e.g graph/ principal values
	(ii)	Get $2/(\sqrt{(1-4x^2)}$ $+ 1/(\sqrt{(1-y^2)}\, dy/dx = 0$	M1	Attempt implicit diff. and chain rule; allow e.g. $(1-2x^2)$ or $a/\sqrt{(1-4x^2)}$
			A1	
		Find $y = \sqrt{3}/2$	M1	Method leading to y
		Get $-2\sqrt{3}/3$	A1√	AEEF; from their a above
				SC Write $\sin(\frac{1}{2}\pi - \sin^{-1}2x) = \cos(\sin^{-1}2x)$ B1 Attempt to diff. as above M1 Replace x in reasonable dy/dx and attempt to tidy M1 Get result above A1

4	**(i)**	Let $x=\cosh\theta$ such that	M1	
		$dx = \sinh\theta\, d\theta$		
		Clearly use $\cosh^2 - \sinh^2 = 1$	A1	Clearly derive A.G.
	(ii)	Replace $\cosh^2\theta$	M1	Allow $a(\cosh 2\theta \pm 1)$
		Attempt to integrate their expression	M1	Allow $b\sinh 2\theta \pm a\theta$
		Get $\frac{1}{4}\sinh 2\theta + \frac{1}{2}\theta\ (+c)$	A1	
		Clearly replace for x to A.G.	B1	Condone no $+c$
				SC Use expo. def^n; three terms M1
				Attempt to integrate M1
				Get $\frac{1}{8}(e^{2\theta}-e^{-2\theta}) + \frac{1}{2}\theta\ (+c)$ A1
				Clearly replace for x to A.G. B1
5	**(i)**	(**a**) State (x=) α	B1	
		None of roots	B1	No explanation needed
		(**b**) Impossible to say	B1	
		All roots can be derived	B1	Some discussion of values close to 1 or 2 or central leading to correct conclusion
	(ii)		B1	Correct x for y=0; allow 0.591, 1.59, 2.31
			B1	Turning at (1,0.8) and/or (1,-0.8)
			B1	Meets x-axis at 90^0
			B1	Symmetry in x-axis; allow
6	**(i)**	Correct definitions used	B1	
		Attempt at $(e^x-e^{-x})^2/4 + 1$	M1	Allow $(e^x+e^{-x})^2+1$; allow /2
		Clearly derive A.G.	A1	
	(ii)	Form a quadratic in $\sinh x$	M1	
		Attempt to solve	M1	Factors or formula
		Get $\sinh x = -\frac{1}{2}$ or 3	A1	
		Use correct ln expression	M1	On their answer(s) seen once
		Get $\ln(-\frac{1}{2}+\frac{\sqrt{5}}{2})$ and $\ln(3+\sqrt{10})$	A1	
7	**(i)**	$OP= 3 + 2\cos\alpha$		
		$OQ=3 + 2\cos(\frac{1}{2}\pi+\alpha)$	M1	Any other unsimplified value
		$=3 - 2\sin\alpha$		
		Similarly $OR=3 - 2\cos\alpha$	M1	Attempt at simplification of at least two correct expressions
		$OS=3 + 2\sin\alpha$		
		Sum = 12	A1	cao
	(ii)	Correct formula with attempt at r^2	M1	Need not be expanded, but three terms if it is
		Square r correctly	A1	
		Attempt to replace $\cos^2\theta$ with $a(\cos 2\theta \pm 1)$	M1	
		Integrate their expression	A1√	Need three terms
		Get $\frac{11\pi}{4} - 1$	A1	cao

8	**(i)**	Area = $\int 1/(x+1)\,dx$	B1	Include or imply correct limits
		Use limits to $\ln(n+1)$	B1	
		Compare area under curve to areas of rectangles	B1	Justify inequality
		Sum of areas = $1\text{x}(\frac{1}{2} + \frac{1}{3} + \ldots + 1/(n+1))$	M1	Sum seen or implied as 1 x y values
		Clear detail to A.G.	A1	Explanation required e.g. area of last rectangle at $x=n$, area under curve to $x=n$
	(ii)	Show or explain areas of rectangles above curve	M1	
		Areas of rectangles (as above) > area under curve	A1	First and last heights seen or implied; A.G.
	(iii)	Add 1 to both sides in **(i)** to make $\sum(^1/r)$	B1	Must be clear addition
		Add $^1/_{(n+1)}$ to both sides in **(ii)** to make $\sum(^1/r)$	B1	Must be clear addition; A.G.
	(iv)	State divergent	B1	Allow not convergent
		Explain e.g. $\ln(n+1)\to\infty$ as $n\to\infty$	B1	
9	**(i)**	Require denom. = 0	B1	
		Explain why denom. $\neq 0$	B1	Attempt to solve, explain always > 0 etc.
	(ii)	Set up quadratic in x	M1	
		Get $2yx^2-4x+(2a^2y+3a)=0$	A1	
		Use $b^2 \geq 4ac$ for real x	M1	Produce quadratic inequality in y from their quad.; allow use of = or <
		Attempt to solve their inequality	M1	Factors or formula
		Get $y > {^1/_{2a}}$ and $y < {^{-2}/_a}$	A1	Justified from graph SC Attempt diff. by quot./product rule M1 Solve $dy/dx=0$ for two values of x M1 Get $x=2a$ and $x=-a/2$ A1 Attempt to find two y values M1 Get correct inequalities (graph used to justify them) A1
	(iii)	Split into two separate integrals	M1	
		Get $k\ln(x^2+a^2)$	A1	Or $p\ln(2x^2+2a^2)$
		Get $k_1\tan^{-1}(x/a)$	A1	k_1 not involving a
		Use limits and attempt to simplify	M1	
		Get $\ln 2.5 - 1.5\tan^{-1}2 + 3\pi/8$	A1	AEEF SC Sub. $x = a\tan\theta$ and $dx = a\sec^2\theta\,d\theta$ M1 Reduce to $\int p\tan\theta - p_1\,d\theta$ (ignore limits here) A1 Integrate to $p\ln(\sec\theta) - p_1\theta$ A1 Use limits (old or new) and attempt to simplify M1 Get answer above A1

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1 (i) (a)	$(n=)\ 3$	B1	**1**	For correct n
(b)	$(n=)\ 6$	B1	**1**	For correct n
(c)	$(n=)\ 4$	B1	**1**	For correct n
(ii)	$(n=)\ 4,\ 6$	B1		For *either* 4 *or* 6
		B1	**2**	For both 4 and 6 and no extras Ignore all $n \geqslant 8$ **SR** B0 B0 if more than 3 values given, even if they include 4 or 6
		5		
2 (i)	$\frac{\sqrt{3}+\mathrm{i}}{\sqrt{3}-\mathrm{i}} \times \frac{\sqrt{3}+\mathrm{i}}{\sqrt{3}+\mathrm{i}} = \frac{1}{2}+\frac{1}{2}\mathrm{i}\sqrt{3}$	M1		For multiplying top and bottom by complex conjugate
	OR $\frac{\sqrt{3}+\mathrm{i}}{\sqrt{3}-\mathrm{i}} = \frac{2\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}}{2\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}}}$			*OR* for changing top and bottom to polar form
	$=(1)\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$	A1		For $(r=)\ 1$ (may be implied)
		A1	**3**	For $(\theta=)\ \frac{1}{3}\pi$ **SR** Award maximum A1 A0 if $\mathrm{e}^{\mathrm{i}\theta}$ form is not seen
(ii)	$\left(\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}\right)^6 = \mathrm{e}^{2\pi\mathrm{i}} = 1 \Rightarrow (n=)\ 6$	M1		For use of $\mathrm{e}^{2\pi\mathrm{i}}=1$, $\mathrm{e}^{\mathrm{i}\pi}=-1$, $\sin k\pi = 0$ *or* $\cos k\pi = \pm 1$ (may be implied)
		A1	**2**	For $(n=)\ 6$ **SR** For $(n=)\ 3$ only, award M1 A0
		5		
3 (i)	$\mathbf{n} = [2, 1, 3] \times [3, 1, 5]$	M1		For using direction vectors and attempt to find vector product
	$= [2, -1, -1]$	A1	**2**	For correct direction (allow multiples)
(ii)	$d = \frac{\lvert [5, 2, 1] \cdot [2, -1, -1] \rvert}{\sqrt{6}}$	B1		For $(\mathbf{AB}=)\ [5, 2, 1]$ or any vector joining lines
		M1		For attempt at evaluating $\mathbf{AB} \cdot \mathbf{n}$
		M1		For $\lvert \mathbf{n} \rvert$ in denominator
	$= \frac{7}{\sqrt{6}} = \frac{7}{6}\sqrt{6} = 2.8577$	A1	**4**	For correct distance
		6		

4	$m^2+4m+5\,(=0)\Rightarrow m=\dfrac{-4\pm\sqrt{16-20}}{2}$	M1		For attempt to solve correct auxiliary equation
	$=-2\pm\mathrm{i}$	A1		For correct roots
	CF $=\mathrm{e}^{-2x}(C\cos x+D\sin x)$	A1√		For correct CF (here or later). f.t. from m **AEtrig** but not forms including $\mathrm{e}^{\mathrm{i}x}$
	PI $=p\sin 2x+q\cos 2x$	B1		For stating a trial PI of the correct form
	$y'=2p\cos 2x-2q\sin 2x$ $y''=-4p\sin 2x-4q\cos 2x$	M1		For differentiating PI twice and substituting into the DE
	$\cos 2x(-4q+8p+5q)$ $+\sin 2x(-4p-8q+5p)\;=65\sin 2x$	A1		For correct equation
	$\left.\begin{array}{l}8p+q=0\\ p-8q=65\end{array}\right\}\quad p=1,\quad q=-8$	M1		For equating coefficients of $\cos 2x$ and $\sin 2x$ and attempting to solve for p and/or q
	PI $=\sin 2x-8\cos 2x$	A1		For correct p and q
	$\Rightarrow y=$ $\mathrm{e}^{-2x}(C\cos x+D\sin x)+\sin 2x-8\cos 2x$	B1√	9	For using GS = CF + PI, with 2 arbitrary constants in CF and none in PI
			9	
5 (i)	$y=u-\dfrac{1}{x}\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}u}{\mathrm{d}x}+\dfrac{1}{x^2}$	M1 A1		For differentiating substitution For correct expression
	$x^3\left(\dfrac{\mathrm{d}u}{\mathrm{d}x}+\dfrac{1}{x^2}\right)=x\left(u-\dfrac{1}{x}\right)+x+1$	M1		For substituting y and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ into DE
	$\Rightarrow x^2\dfrac{\mathrm{d}u}{\mathrm{d}x}=u$	A1	4	For obtaining correct equation **AG**
(ii)	METHOD 1 $\displaystyle\int\frac{1}{u}\,\mathrm{d}u=\int\frac{1}{x^2}\,\mathrm{d}x\;\Rightarrow\;\ln ku=-\frac{1}{x}$	M1 A1		For separating variables and attempt at integration For correct integration (k not required here)
	$ku=\mathrm{e}^{-1/x}\;\Rightarrow\;k\left(y+\dfrac{1}{x}\right)=\mathrm{e}^{-1/x}$	M1 M1		For any 2 of / For all 3 of { k seen, exponentiating, substituting for u }
	$\Rightarrow\;y=A\mathrm{e}^{-1/x}-\dfrac{1}{x}$	A1	5	For correct solution **AEF** in form $y=\mathrm{f}(x)$
	METHOD 2 $\dfrac{\mathrm{d}u}{\mathrm{d}x}-\dfrac{1}{x^2}u=0\;\Rightarrow\;$ I.F. $\mathrm{e}^{\int -1/x^2\,\mathrm{d}x}=\mathrm{e}^{1/x}$	M1		For attempt to find I.F.
	$\Rightarrow\dfrac{\mathrm{d}}{\mathrm{d}x}\left(u\,\mathrm{e}^{1/x}\right)=0$	A1		For correct result
	$u\,\mathrm{e}^{1/x}=k\;\Rightarrow\;y+\dfrac{1}{x}=k\,\mathrm{e}^{-1/x}$	M1 M1		From $\boxed{u\times\text{I.F.}=}$, { for k seen, for substituting for u } in either order
	$\Rightarrow\;y=k\,\mathrm{e}^{-1/x}-\dfrac{1}{x}$	A1		For correct solution **AEF** in form $y=\mathrm{f}(x)$
			9	

6 (i)	METHOD 1 Use 2 of $[-4, 2, 0], [0, 0, 3], [-4, 2, 3], [4, -2, 3]$ or multiples	M1		For finding vector product of 2 appropriate vectors in plane *ACGE*
	$\mathbf{n} = k\,[1, 2, 0]$	A1		For correct **n**
	Use $A[4, 0, 0], C[0, 2, 0], G[0, 2, 3]$ *OR* $E[4, 0, 3]$	M1		For substituting a point in the plane
	$\mathbf{r}.[1, 2, 0] = 4$	A1	**4**	For correct equation. **AEF** in this form
	METHOD 2 $\mathbf{r} = [4, 0, 0] + \lambda[-4, 2, 0] + \mu[0, 0, 3]$	M1		For writing plane in 2-parameter form
	$\Rightarrow x = 4 - 4\lambda,\ y = 2\lambda,\ z = 3\mu$	A1		For 3 correct equations
	$x + 2y = 4$	M1		For eliminating λ (and μ)
	$\Rightarrow \mathbf{r}.[1, 2, 0] = 4$	A1		For correct equation. **AEF** in this form
(ii)	$\theta = \cos^{-1}\dfrac{\lvert[3, 0, -4].[1, 2, 0]\rvert}{\sqrt{3^2+0^2+4^2}\sqrt{1^2+2^2+0^2}}$	B1√ M1 M1		For using correct vectors (allow multiples). f.t. from **n** For using scalar product For multiplying both moduli in denominator
	$\theta = \cos^{-1}\dfrac{3}{5\sqrt{5}} = 74.4°$ $(74.435...°, 1.299...)$	A1	**4**	For correct angle
(iii)	AM: $(\mathbf{r} =)\,[4, 0, 0] + t[-2, 2, 3]$ $(or\ [2, 2, 3] + t[-2, 2, 3])$	M1 A1		For obtaining parametric expression for *AM* For correct expression seen or implied
	$3(4-2t) - 4(3t) = 0$ $(or\ 3(2-2t) - 4(3+3t) = 0)$	M1		For finding intersection of *AM* with *ACGE*
	$t = \frac{2}{3}\ (or\ t = -\frac{1}{3})$ *OR* $\mathbf{w} = \left[\frac{8}{3}, \frac{4}{3}, 2\right]$	A1		For correct t *OR* position vector
	$AW : WM = 2:1$	A1	**5**	For correct ratio
			13	
7 (i) (a)	$x + y - a \in \mathbb{R}$	B1		For stating closure is satisfied
	$(x*y)*z = (x+y-a)*z = x+y+z-2a$	M1		For using 3 distinct elements bracketed both ways
	$x*(y*z) = x*(y+z-a) = x+y+z-2a$	A1		For obtaining the same result twice for associativity **SR** 3 distinct elements bracketed once, expanded, and symmetry noted scores M1 A1
	$x + e - a = x \Rightarrow e = a$	B1		For stating identity $= a$
	$x + x^{-1} - a = a \Rightarrow x^{-1} = 2a - x$	M1 A1	**6**	For attempting to obtain inverse of x For obtaining inverse $= 2a - x$ *OR* for showing that inverses exist, where $x + x^{-1} = 2a$
(b)	$x + y - a = y + x - a \Rightarrow$ commutative	B1	**1**	For stating commutativity is satisfied, with justification
(c)	x order 2 $\Rightarrow x*x = e \Rightarrow 2x - a = e$ $\Rightarrow 2x - a = a \Rightarrow x = a = e$ *OR* $x = x^{-1} \Rightarrow x = 2a - x \Rightarrow x = a = e$ $\Rightarrow$ no elements of order 2	M1 A1	**2**	For obtaining equation for an element of order 2 For solving and showing that the only solution is the identity (which has order 1) *OR* For proving that there are no self-inverse elements (other than the identity)

(ii)	e.g. $2+1-5=-2 \notin \mathbb{R}^+$	M1	For attempting to disprove closure
	$\Rightarrow$ not closed	A1	For stating closure is not necessarily satisfied ($0 < x+y \leqslant 5$ required)
	e.g. $2\times 5-11=-1 \notin \mathbb{R}^+$	M1	For attempting to find an element with no inverse
	$\Rightarrow$ no inverse	A1 **4**	For stating inverse is not necessarily satisfied ($x \geqslant 10$ required)
		13	
8 (i)			z may be used for $e^{i\theta}$ throughout
	$\sin\theta = \frac{1}{2i}\left(e^{i\theta}-e^{-i\theta}\right)$	B1	For expression for $\sin\theta$ seen or implied
		M1	For expanding $\left(e^{i\theta}-e^{-i\theta}\right)^6$
	$\sin^6\theta =$		At least 4 terms and 3 binomial coefficients required.
	$-\frac{1}{64}\left(e^{6i\theta}-6e^{4i\theta}+15e^{2i\theta}-20+15e^{-2i\theta}-6e^{-4i\theta}+e^{-6i\theta}\right)$	A1	For correct expansion. Allow $\frac{\pm(i)}{64}(\ldots\ldots)$
	$=-\frac{1}{64}\left(2\cos 6\theta-12\cos 4\theta+30\cos 2\theta-20\right)$	M1	For grouping terms and using multiple angles
	$\sin^6\theta=-\frac{1}{32}\left(\cos 6\theta-6\cos 4\theta+15\cos 2\theta-10\right)$	A1 **5**	For answer obtained correctly **AG**
(ii)	$\cos^6\theta =$ *OR* $\sin^6\left(\frac{1}{2}\pi-\theta\right)=$	M1	For substituting $\left(\frac{1}{2}\pi-\theta\right)$ for θ throughout
	$-\frac{1}{32}\left(\cos(3\pi-6\theta)-6\cos(2\pi-4\theta)+15\cos(\pi-2\theta)-10\right)$	A1	For correct unsimplified expression
	$\cos^6\theta=\frac{1}{32}\left(\cos 6\theta+6\cos 4\theta+15\cos 2\theta+10\right)$	A1 **3**	For correct expression with $\cos n\theta$ terms **AEF**
(iii)	$\int_0^{\frac{1}{4}\pi}\frac{1}{32}\left(-2\cos 6\theta-30\cos 2\theta\right)\,d\theta$	B1√	For correct integral. f.t. from $\sin^6\theta-\cos^6\theta$
	$=-\frac{1}{16}\left[\frac{1}{6}\sin 6\theta+\frac{15}{2}\sin 2\theta\right]_0^{\frac{1}{4}\pi}$	M1	For integrating $\cos n\theta, \sin n\theta$ *or* $e^{in\theta}$
		A1√	For correct integration. f.t. from integrand
	$=-\frac{11}{24}$	A1 **4**	For correct answer **www**
		12	

4728 Mechanics 1

1 (i)	$0.5\text{x}6 = 0.5\text{x}0.8 + 4m$ $m = 0.65$	M1 A1 A1 [3]	Uses CoLM If g used throughout, possible 3 marks
(ii)	$0.5\text{x}6 = -0.5\text{x}0.8 + 4m$ $m = 0.85$	M1 A1 A1 [3]	After momentums opposite signs If g used throughout, 0 marks
2 (i)	T = 400 N D = 400 + 900 = 1300 N	B1 M1 A1 [3]	Order immaterial Or T + 900; sign correct
(ii)	 500x0.6 = T - 400 T = 700 N 1250x0.6 = D - 900 -700 D = 2350 N *OR* (500 + 1250)x0.6 = D - 400 - 900 D = 2350 N	 M1 A1 A1 M1 A1ft A1 M1 A1 A1 [6]	(Award M marks even if g included in ma terms. M marks require correct number forces) Uses N2L one object only Uses N2L other object ft cv(T from (ii)); allow T instead of its value Uses N2L for both objects
3 (i)	5cos30 or 5 sin 60 or 4.33 5cos 60 or 5sin30 or 2.5	B1 B1 [2]	Order immaterial, accept +/-. May be awarded in (ii) if no attempt in (i)
(ii)	 7-4.33 (= 2.67) and 9 - 2.5 (= 6.5) $R^2 = 2.67^2 + 6.5^2$ R = 7.03 $\tan\theta = 6.5/2.67$ $\theta =$ 67.6, 67.7degrees	M1* A1 D*M 1 A1 D*M 1 A1 [6]	Subtracts either component from either force 3sf or better Valid trig for correct angle 3sf or better
4 (i)	20cos 30 20cos30 = 3a a = 5.77 ms^{-2}	M1 M1 A1 [3]	Resolves 20 (accept 20 sin30) Uses N2L horizontally, accept g in ma term
(ii)	 R = 3x9.8 + 20 sin 30 (= 39.4) F = 20cos 30 (= 17.3) $17.3 = 39.4\mu$ $\mu = 0.44$	M1 A1 B1 M1 A1 [5]	Resolves vertically (accept -, cos if sin in i); correct no. terms Correct (Neither R nor *F* need be evaluated) Uses F = μR

5 (i)	V = ∫ 0.8tdt	M1*	Attempt at integration
	$v = 0.8t^2 /2$ (+c)	A1	Award if c omitted
	t = 0, v =13, (c = 13)	M1	
	$v = 0.4\text{x } 6^2$ (+c)	D*M1	
	$v = 27.4 \text{ ms}^{-1}$	A1	
		[5]	
(ii)	$s = \int 0.4t^2$ (+c)dt	M1*	Attempt at integration of v(t)
	$s = 0.4t^3/3 + 13t$ (+k)	A1ft	ft cv(v(t) in (i))
	t=0, s=0, (k=0)	M1	
	$s = 0.\ 4\text{x}6^3/3 + 13\text{x}6$	D*M1	
	s = 106.8 m	A1	Allow if k=0 assumed. Accept 107 m.
		[5]	
(iii)	Fig. 2	B1	
		[1]	
	Fig.1 has zero initial velocity/gradient	B1	
	Fig. 3 does not have a increasing velocity/gradient	B1	
		[2]	
6 (i) a	$2.5 = 9.8t^2/2$	M1	Uses s = 0 +/- $gt^2/2$
	t = 0. 714 s or better or 5/7	A1	Not awarded if - sign "lost"
		[2]	
b	v^2 = 2x9.8x2.5 *OR* v = 9.8 x 0.714	M1	Uses v^2 = 0 +/-2gs or v = u +/- gt
	$v = 7 \text{ ms}^{-1}$ or 6.99 or art 7.00	A1	Not awarded if - sign "lost"
		[2]	
(ii)	R = 2x9.8sin60 (= 16.97 = 17)	B1	With incorrect angle, e.g
		M1	R = 2x9.8cos60 (=9.8) B0
	F = 0.2x16.97 (=3.395 or 3.4)	A1ft	F = 0.2x9.8 (=1.96) M1A1√
	Cmpt weight = 2x9.8cos60 (= 9.8)	B1	Cmpt wt = 2x9.8sin60 (=16.97) B0
	2a = 9.8 - 3.395	M1	2a = 16.97 - 1.96 M1
	$a = 3.2 \text{ ms}^{-2}$	A1ft	a = 7.5 A1√ ft cv(R and Cmpt weight)
	Distance down ramp = 5 m	B1	
	v^2 = 2x3.2x5	M1	v^2 = 2x7.5x5
	v = 5.66 or 5.7	A1ft	v = 8.66 or 8.7 A1√ ft cv(√ (10a))
		[9]	
7 (i)		M1	Use of v = u - 0.4t
	p = 4 - 2x0.4 (= 3.2)	A1	
	q = 1 - 2x0.4 (= 0.2)	A1	Accept q = -0.2 from -1+2*0.4
		M1	Uses CoLM on reduced velocities
	0.7x3.2 - 0.3x0.2 = (1x)v	A1	
	$v = 2.18 \text{ ms}^{-1}$	A1	
		[6]	

(ii) **a**		B1 B1 B1 [3]	Straight line with larger y intercept slopes towards t axis, but does not reach it. Straight line with negative y intercept slopes towards t axis, and gets to t axis before other line ends. **SR** if t=2 in ii give B1 if line stops before axis
b	0 = 1 - 0.4t t = 2.5 s P = 4x3 - 0.5x0.4x3^2 Q = 1x2.5 - 0.5x0.4x2.5^2 PQ = 10.2 + 1.25 = 11.45 m	M1 A1 M1 A1 A1 A1 [6]	Finds when Q comes to rest (any method) Uses s = ut - 0.4t^2/2 (nb $0^{(2)} = 1^{(2)} - 0.4Q^2/2$ B1; convincing evidence (graph to scale, or calculation that Q comes to rest and remains at rest at t less than 3, M1A1;graph A1 needs –ve v intercept) **SR** if t=2 in iib, allow M1 for s= ut - 0.4t^2/2 And A1 for PQ=8.4

Alternative for Q3 where 7 N and 9N forces combined initially

3 (i)	5cos30 or 5 sin 60 or 4.33 5cos 60 or 5sin30 or 2.5	B1 B1 [2]	Order immaterial, accept +/-. May be awarded in (ii) if no attempt in (i)
(ii)	Z^2 = 7^2 + 9^2 (= 130, Z = 11.4017...) cos(angle of Z with y axis) = 9/11.4017.. angle of Z with y axis = 37.8746... Angle opposite R in triangle of forces = 180 -(37.8746+90+30) = 22.125 (Accept 22) R^2 = 5^2 + 11.4017^2 - 2x5x11.4017cos22.125 R (= 7.0269) = 7.03 N 11.4017^2 = 5^2 + 7.0269^2 - 2x5x7.0269cosA (A = 142.33) Angle between R and y axis = 142.33-30-90 (=22.33) θ (= 90-22.33) =67.7 degrees	 M1* A1 D*M1 A1 D*M1 A1 [6]	Z is resultant of 7N and 9N forces only R is resultant of all 3 forces Complete method Cosine rule to find R Or Sine Rule. A is angle between R and 5N forces Complete method θ is angle between R and x axis

4729 Mechanics 2

1	$(20 \sin \theta)^2 = 2 \times 9.8 \times 17$	M1	or B2 for max ht = $v^2\sin^2\theta/2g$
		A1	
	$\sin \theta = \sqrt{(2 \times 9.8 \times 17)} \div 20$	M1	subst. values in above
	$\theta = 65.9°$	A1 **4**	**4**

2	$\bar{x} = 8$	B1	
	T sin30° x 12 = 8 x 2 x 9.8	M1	ok if g omitted
		A1 ft	ft their $\bar{x}$
	T = 26.1	A1 **4**	**4**

3 (i)	140 x X = 40 x 70	M1	
	X = 20 N	A1	
	at *F* 20 N to the right	B1	inspect diagram
	at *G* 20 N to the left	B1 **4**	**SR** B1 for correct directions only
(ii)	đ = (2x40sinΠ/2)÷3Π/2	M1	must be radians
		A1	
	đ = 17.0	A1	16.98 160/3Π (8/15Π m)
	$70\bar{y} = 100 \times 60 + 217 \times 10$	M1	
		A1 ft	ft 200 + their đ or 2 + their đ (m)
	$\bar{y} = 117$	A1 **6**	116.7 **10**

4 (i)	$P/10 - 800 \times 9.8\sin12° - 100k = 800 \times 0.25$	M1	$P/10 = D_1$ ok
		A1	D_1 ok
	$P/20 - 400k = 800 \times 0.75$	M1	$P/20 = D_2$ ok
		A1	$D_1 = 2D_2$ needed for this A1
	solving above	M1	
	$k = 0.900$	A1	**AG** 0.9000395
	$P = 19\,200$	A1 **7**	or 19.2 kW (maybe in part (ii))
(ii)	$0.9 v^2 = 28\,800/v$	M1	ok if $19200/v$
	solving above	M1 *	($v^3 = 32\,000$)
	$v = 31.7$ m s^{-1}	A1 **3**	**10**

5 (i)	0.8 *S*	B1	vert comp of *S*
	0.6 *T*	B1	vert comp of *T*
	$S \cos\alpha = T \cos\beta + 0.2 \times 9.8$	M1	
	$0.8 S = 0.6 T + 1.96$ aef	A1 **4**	**AG** $4S = 3T + 9.8$
(ii)	0.6 *S*	B1	
	0.8 *T*	B1	
	$0.2 \times 0.24 \times 8^2$	B1	3.072 384/125
	$S \sin\alpha + T \sin\beta = 0.2 \times 0.24 \times 8^2$	M1	must be $mr\omega^2$
	$6S + 8T = 30.72$	A1	aef
	eliminate *S* or *T*	M1	
	S = 3.4 N	A1	3.411
	T = 1.3 N	A1 **8**	1.282 **12**

6 (i)	x = vcosθ t	B1	
	y = vsinθ t – ½ x 9.8 t^2	B1	or g
	substitute t = x/vcosθ	M1	
	y = xtanθ – $4.9x^2/v^2\cos^2\theta$	A1 **4**	**AG**
(ii)	Sub y = -*h* , x = *h* , v = 14 , θ =30	M1	signs must be correct
	$-h = h/\sqrt{3} - h^2/30$	A1	aef
	solving above	M1	
	h = 47.3	A1 **4**	
(iii)	$v_v^2 = (14\sin30°)^2 - 2x9.8x(-47.3)$	M1	14cos30° t=47.3 ft & v_v =14sin30°-9.8t
	(double negative needed) ft their -47.3	A1 ft	t = 3.90 (or dy/dx=1/√3 – x/15 etc ft)
	v_v = ±31.2	A1	v_v = ±31.2 (tanα = 1/√3 – 47.3/15)
	$\tan^{-1}(31.2/14\cos30°)$	M1	$\tan^{-1}(31.2/14\cos30°)$
	α= 68.8° below horiz/21.2° to d'vert.	A1 **5**	68.8°/…..
(iv)	$½mx14^2 + mx9.8x47.3 = ½mv^2$	M1	ft $(12.1^2 + 31.2^2)$
	v = 33.5	A1 **2**	33.5 **15**

7 (i)	p = 4 m s^{-1}	B1	P's first speed
	$0.8 = 0.2p_1 + 0.3q_1$	M1	
		A1	
	$0.5 = (q_1 - p_1)/4$	M1	
		A1	
	solving above	M1	
	q_1 = 2.4 12/5	A1	Q's first speed
	p_1 = 0.4 2/5	A1 **8**	may be in (ii). **SR** 1 for both negative
(ii)	$0.8 = 0.2p_2 + 0.3q_2$	M1	
		A1	
	$0.5 = (p_2 - q_2)/2$	M1	
		A1	
	solving above	M1	
	p_2 = 2.2 11/5	A1	
	q_2 = 1.2 6/5	A1 **7**	
(iii)	R = 0.3 x 1.2^2 /0.4	M1	
	R = 1.08 N	A1 **2**	**17**

4730 Mechanics 3

1 (i)	For triangle sketched with sides (0.5)2.5 and (0.5)6.3 and angle θ correctly marked OR Changes of velocity in i and j directions $2.5\cos\theta - 6.3$ and $2.5\sin\theta$, respectively.	B1	May be implied in subsequent working.
	For sides 0.5x2.5, 0.5x6.3 and 2.6 (or 2.5, 6.3 and 5.2) OR $-2.6\cos\alpha = 0.5(2.5\cos\theta - 6.3)$ and $2.6\sin\alpha = 0.5(2.5\sin\theta)$	B1ft	May be implied in subsequent working.
	$[5.2^2 = 2.5^2 + 6.3^2 - 2\text{x}2.5\text{x}6.3\cos\theta$ OR $2.6^2 = 0.5^2\{(2.5\cos\theta - 6.3)^2 + (2.5\sin\theta)^2\}]$	M1	For using cosine rule in triangle or eliminating α.
	$\cos\theta = 0.6$	A1 [4]	AG
(ii)		M1	For appropriate use of the sine rule or substituting for θ in one of the above equations in θ and α
	$\sin\alpha = 2.5\text{x}0.8/5.2$ OR $-2.6\cos\alpha = 0.5(2.5\text{x}0.6 - 6.3)$	A1	
		M1	For evaluating $(180 - \alpha)^{o}$ or $(\pi - \alpha)^{c}$
	Impulse makes angle of 157^{o} or 2.75^{c} with original direction of motion of P.	A1 [4]	SR (relating to previous 2 marks; max 1 mark out of 2) $\alpha = 23^{o}$ or 0.395^{C} B1

2 (i)	[70x2 = 4X – 4Y]	M1	For taking moments about A for AB (3 terms needed)
	X – Y = 35	A1 [2]	
(ii)	[110x3 = -4X + 6Y]	M1	For taking moments about C for BC (3 terms needed)
	2X – 3Y + 165 = 0	A1 [2]	AG
(iii)		M1	For attempting to solve for X and Y
	X = 270, Y = 235	A1ft	ft any (X, Y) satisfying the equation given in (ii)
		M1	For using magnitude = $\sqrt{X^2 + Y^2}$
	Magnitude is 358N	A1ft [4]	ft depends on all 4 Ms

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3 (i)	$[T_A = (24x0.45)/0.6,\ T_B = (24x0.15)/0.6]$ $T_A - T_B = 18 - 6 = 12 = W$ ➔ P in equil'm.	M1 A1 [2]	For using T = λ x/L for PA or PB
(ii)	Extensions are 0.45 + x and 0.15 – x Tensions are 18 + 40x and 6 – 40x	B1 B1 [2]	AG From T = λ x/L for PA and PB
(iii)	$[12 + (6 - 40x) - (18 + 40x) = 12\ddot{x}/g]$ $\ddot{x} = -80gx/12$ ➔ SHM Period is 0.777s	M1 A1 A1 [3]	For using Newton's second law (4 terms required) AG From Period = $2\pi\sqrt{12/(80g)}$
(iv)	$[v_{max} = 0.15\sqrt{80g/12}$ or $v_{max} = 2\pi x0.15/0.777$ or $½(12/g)v_{max}^2 + mg(0.15)$ $+24\{0.45^2 + 0.15^2 - 0.6^2\}/(2x0.6) = 0]$ Speed is $1.21ms^{-1}$	M1 A1 [2]	For using v_{max} = An or $v_{max} = 2\pi A/T$ or conservation of energy (5 terms needed)

4 (i)	Loss in PE = $mg(0.5\sin\theta)$ $[½mv^2 - ½m3^2 = mg(0.5\sin\theta)]$ $v^2 = 9 + 9.8\sin\theta$	B1 M1 A1 [3]	For using KE gain = PE loss (3 terms required) AG
(ii)	$a_r = 18 + 19.6\sin\theta$ $[ma_t = mg\cos\theta]$ $a_t = 9.8\cos\theta$	B1 M1 A1 [3]	Using $a_r = v^2/0.5$ For using Newton's second law tangentially
(iii)	$[T - mg\sin\theta = ma_r]$ $T - 1.96\sin\theta = 0.2(18 + 19.6\sin\theta)$ $T = 3.6 + 5.88\sin\theta$ $\theta = 3.8$	M1 A1 A1 B1 [4]	For using Newton's second law radially (3 terms required) AG

5	Initial **i** components of velocity for A and B are $4ms^{-1}$ and $3ms^{-1}$ respectively.	B1	May be implied.
		M1	For using p.c.mmtm. parallel to l.o.c.
	3x4 + 4x3 = 3a + 4b	A1	
		M1	For using NEL
	0.75(4 – 3) = b – a	A1	
		M1	For attempting to find a
	a = 3	A1	Depends on all three M marks
	Final **j** component of velocity for A is $3ms^{-1}$	B1	May be implied
		M1	For using $\tan^{-1}(v_j/v_i)$ for A
	Angle with l.o.c. is 45^o or 135^O	A1ft	ft incorrect value of a ($\neq 0$) only
		[10]	
			SR for consistent sin/cos mix (max 8/10) 3x3 + 4x4 = 3a + 4b and b – a = 0.75(3 – 4) M1 M1 as scheme and A1 for *both* equ's a = 4 M1 as scheme A1 j component for A is $4ms^{-1}$ B1 Angle $\tan^{-1}(4/4) = 45^o$ M1 as scheme A1

6(i)	Initial speed in medium is $\sqrt{2g \times 10}$ (= 14)	B1	
	[0.125dv/dt = 0.125g – 0.025v]	M1	For using Newton's second law with a = dv/dt (3 terms required)
	$\int \frac{5dv}{5g - v} = \int dt$	M1	For separating variables and attempt to integrate
	-5 ln(5g – v) = t (+A)	A1	
	[-5 ln35 = A]	M1	For using v(0) = 14
	t = 5 ln{35/(49 – v)}	A1	
		M1	For method of transposition
	$v = 49 - 35e^{-0.2t}$	A1	AG
		[8]	
(ii)		M1	For integrating to find x(t)
	$x = 49t + 175e^{-0.2t}$ (+B)	A1	
	$[x(3) = (49x3 + 175e^{-0.6}) - (0 + 175)]$	M1	For using limits 0 to 3 or for using x(0) = 0 and evaluating x(3)
	Distance is 68.0m	A1	
		[4]	

7(i)	Gain in EE = $20x^2/(2x2)$ Loss in GPE = $0.8g(2 + x)$ [$\frac{1}{2}$ $0.8v^2 = (15.68 + 7.84x) - 5x^2$] $v^2 = 39.2 + 19.6x - 12.5x^2$	B1 B1 M1 A1 [4]	Accept 0.8gx if gain in KE is $\frac{1}{2}$ $0.8(v^2 - 19.6)$ For using the p.c.energy AG
(ii)	**(a)** Maximum extension is 2.72m	M1 A1 [2]	For attempting to solve $v^2 = 0$
	(b) [$19.6 - 25x = 0$, $v^2 = 46.8832 - 12.5(x - 0.784)^2$] x = 0.784 or c = 46.9 [$v_{max}{}^2 = 39.2 + 15.3664 - 7.6832$] Maximum speed is 6.85ms^{-1}	 M1 A1 M1 A1 [4]	For solving 20x/2 = 0.8g or for differentiating and attempting to solve $d(v^2)/dx = 0$ or $dv/dx = 0$ or for expressing v^2 in the form $c - a(x - b)^2$. For substituting x = 0.784 in the expression for v^2 or for evaluating $\sqrt{c}$
	(c) $\pm(0.8g - 20x/2) = 0.8a$ or $2v\,dv/dx = 19.6 - 25x$ $a = \pm(9.8 - 12.5x)$ or $\ddot{y} = -12.5y$ where $y = x - 0.784$ [$\lvert a\rvert_{max} = \lvert 9.8 - 12.5x2.72\rvert$ or $\lvert \ddot{y}_{max}\rvert = \lvert -12.5(2.72 - 0.784\rvert$] Maximum magnitude is 24.2ms^{-2}	 M1 A1 A1 M1 A1 [5]	For using Newton's second law (3 terms required) or a = v dv/dx For substituting x = ans(ii)(a) into a(x) or y = ans(ii)(a) – 0.784 into $\ddot{y}$ (y)

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Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to ≥ 3sfs, ISW for later rounding. Penalise over-rounding only once in paper.

1 (i)	$0.2^2 + 0.7 \times 0.1 \times 2$ $= 0.18$ **AG**	M2 A1 3	0.2^2 or 0.7×0.1: M1 no errors seen NB $2 \times 0.9 \times 0.1 = 0.18$ M0A0
(ii)	$0.28 + 2\times0.18 + 3\times0.04 + 4\times0.01$ $= 0.8$ oe $0.28 + 2^2\times0.18 + 3^2\times0.04 + 4^2\times0.01$ $-$ "0.8"^{2} $= 0.88$ oe	M1 A1 M1 M1 A1 5	≥ 2 terms correct (excl 0×0.49) $\div$ 5 (or 4 or 10 etc): M0 ≥ 2 terms correct (excl $0^2\times0.49$) dep +ve result cao $\Sigma(x-\mu)^2$: 2 terms: M1; 5 terms M2 $0.8^2\times0.49+0.2^2\times0.28+1.2^2\times0.18+2.2^2\times0.04+3.2^2\times0.01$ SC Use original table, 0.4:B1 0.44: B1
Total		**8**	
2(i)(a)	$\dfrac{8736.9 - \frac{202\times245.3}{7}}{7300 - \frac{202^2}{7}}$ or $\dfrac{1658.24}{1470.86}$ $= 1.127\ldots$ (= 1.13 **AG**)	M1 A1 2	correct sub in any correct formula for b eg $\dfrac{236.8921}{210.1249}$ must see 1.127… ; 1.127.. alone: M1A1
(b)	$y - {}^{245.3}/_7 = 1.13(x - {}^{202}/_7)$ $y = 1.1x + 2.5$ (or 2.4) or $y = 1.13x + 2.43$	M1 A1 2	or $a = {}^{245.3}/_7 - 1.13 \times {}^{202}/_7$ 2 sfs suff. (exact: $y = 1.127399..x + 2.50934…$)
(ii)(a)	$(1.1(..) \times 30 + 2.5(..)) = 35.5$ to 36.5	B1f 1	
(b)	$(1.1(..) \times 100 + 2.5(..)) = 112.4$ to 115.6	B1f 1	
(iii)	(a) Reliable (b) Unreliable because extrapolated	B1 B1 2	Both reliable: B1 | (a) more reliable than (b) B1 because (a) within data or (b) outside data B1 Ignore extras
Total		**8**	
3(i)(a)	Geo stated $({}^7/_8)^2({}^1/_8)$ ${}^{49}/_{512}$ or 0.0957 (3 sfs)	M1 M1 A1 3	or impl. by $({}^7/_8)^n({}^1/_8)$ or $({}^1/_8)^n({}^7/_8)$ alone
(b)	$({}^7/_8)^3$ alone ${}^{343}/_{512}$ or 0.670 (3 sfs) allow 0.67	M2 A1 3	or $1-({}^1/_8+{}^7/_8\times{}^1/_8+({}^7/_8)^2\times{}^1/_8)$: M2 one term incorrect, omit or extra: M1 $1 - ({}^7/_8)^3$ or $({}^7/_8)^2$ alone: M1
(ii)	8	B1 1	
(iii)	Binomial stated or implied ${}^{15}C_2({}^7/_8)^{13}({}^1/_8)^2$ $= 0.289$ (3 sfs)	M1 M1 A1 3	eg by $({}^7/_8)^a({}^1/_8)^b$ ($a+b = 15$, $a,b \neq 1$), not just ${}^nC_r$
Total		**10**	
4 (i)	1 2 3 4 5 or 5 4 3 2 1 3 5 4 1 2 3 1 2 5 3 Σd^2 (= 32) $1 - {}^{6\times\text{"32"}}/_{5(25-1)}$ $= -0.6$	M1 A1 M1dep M1dep A1 5	attempt ranks correct ranks S_{xx} or $S_{yy}=55-15^2/_5(=10)$ or $S_{xy}=39-15^2/_5(= -6)$ ${}^{-6}/\sqrt{(10\times10)}$

(ii)	1 & 3 Largest neg r_s or large neg r_s or strong neg corr'n or close(st) to -1 or lowest r_s	B1ind B1dep 2	ft if -1 < (i) < -0.9, ans 1 & 2 NOT: furthest from 0 or closest to ±1 little corr'n most disagreement
Total		**7**	

5 (i)	68 75 – 59 = 16	B1 M1 A1 3	 attempt 6^{th} & 18^{th} or 58-60, 74-76 & subtr must be from 75 – 59
(ii)	Unaffected by outliers or extremes (allow less affected by outliers) sd can be skewed by one value	B1 1	NOT: … by anomalies or freaks easier to calculate
(iii)	Shows each data item, retains orig data can see how many data items can find (or easier to read) mode or modal class can find (or easier to read) frequs can find mean Harder to read med (or Qs or IQR) Doesn't show med (or Qs or IQR) B&W shows med (or Qs or IQR) B&W easier to compare meds	 B1 B1 2	NOT: shows freqs shows results more clearly B&W does not show freqs NOT: B&W easier to compare B&W shows spread or variance or skew B&W shows highest & lowest Assume in order: Adv, Disadv, unless told Allow disadv of B&W for adv of S&L & vice versa Ignore extras
(iv)	m = 68.1 NOT by restart sd = 9.7 (or same) NOT by restart	B1 B1 2	 Restart mean or mean & sd: 68.1 or 68.087 & 9.7 or 9.73 B1 only
Total		**8**	

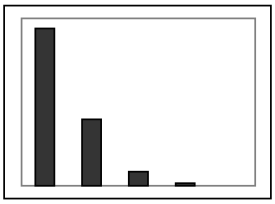
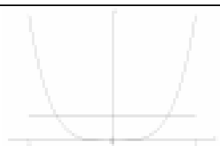
6 (i) (a)	8! = 40320	M1 A1 2	Allow $^4P_4$ & $^3P_3$ instead of 3! & 4! thro'out Q6
(b)	$^4/_8 \times ^4/_7 \times ^3/_6 \times ^3/_5 \times ^2/_4 \times ^2/_3 \times ^1/_2$ $\times 2$ $= ^1/_{35}$ or 0.0286 (3 sfs)	M1 M1dep A1 3	$4! \times 4! \div 8!$ $\times 2$ | $4! \times 4! + 4! \times 4!$ $\div 8!$ allow 1 – above for M1 only oe, eg $^{1152}/_{40320}$
(ii)(a)	$4! \times 4!$ = 576	M1 A1 2	allow $4! \times 4! \times 2$: M1
(b)	$^1/_{16}$ or 0.0625	B1 1	
(c)	Separated by 5 or 6 qus stated or illus $^1/_4 \times ^1/_4 \times 3$ or $^1/_{16} \times 3$ ($^1/_4 \times ^1/_4$ or $^1/_{16}$ alone or ×(2 or 6): M1) $^3/_{16}$ or 0.1875 or 0.188	M1 M2 A1 4	allow 5 only or 6 only or (4, 5 or 6) can be impl by next M2 or M1 $3! \times 3! \times 3$ ($3! \times 3!$ alone or ×(2 or 6); or $(3! + 3!) \times 3$: M1) (÷ 576) correct ans, but clearly B, J sep by 4: M0M2A0 1- P(sep by 0, 1, 2, 3, (4)) M1 $1-(^1/_4+^1/_4+^1/_4\times^3/_4+^1/_4\times^1/_2)$ or $1-(^1/_4\times^1/_4+^1/_2\times^1/_4+^3/_4\times^1/_4+1\times^1/_4+^3/_4\times^1/_4)$ M2 (one omit: M1)
Total		**12**	

7 (i)	Binomial $n = 12, p = 0.1$ Plates (or seconds) independent oe Prob of fault same for each plate oe	B1 B1 B1 B1 4	 B(12, 0.1) : B2 NOT: batches indep Comments must be in context Ignore incorrect or irrelevant
(ii)(a)	$0.9744 - 0.8891$ or $^{12}C_3\times0.9^9\times0.1^3$ = 0.0852 or 0.0853 (3 sfs)	M1 A1 2	
(b)	$1 - 0.2824$ or $1- 0.9^{12}$ =0.718 (3 sfs)	M1 A1 2	allow $1 - 0.6590$ or $1 - 0.9^{11}$
(iii)	"0.718" and 1 – "0.718" used $(1\text{-}0.718)^4 + 4(1\text{-}0.718)^3\times0.718$ $+ ^4C_2(1\text{-}0.718)^2\times0.718^2$ = 0.317 (3 sfs)	B1 M2 A1 4	ft (b) for B1M1M1 M1 for any one term correct (eg opp tail or no coeffs) 1 – P(3 or 4) follow similar scheme M2 or M1 1 – correct wking (= 0.623) B1M2 cao
Total		**12**	

8 (i)	$^1/_6 + 3 \times (^1/_6)^2$ $= ^1/_4$	M2 A1 3	or $3 \times (^1/_6)^2$ or $^1/_6 + (^1/_6)^2$ or $^1/_6 + 2(^1/_6)^2$ or $^1/_6 + 4(^1/_6)^2$ M1
(ii)	$^1/_3$	B1 1	
(iii)	3 routes clearly implied out of 18 possible (equiprobable) routes $^1/_6$	M1 M1 A1 3	or $^1/_3 \times ^1/_6 \times 3$ M2 or $^1/_3 \times ^1/_6$ or $^1/_6 \times ^1/_6 \times 3$ or $^1/_3 \times ^1/_3 \times 3$ or $^1/_4 - ^1/_6$ M1 but $^1/_6 \times ^1/_6 \times 2$ M0 $\frac{(\frac{1}{6})^2 \times 3}{\frac{1}{2}}$ or $\frac{\frac{1}{4}-\frac{1}{6}}{\frac{1}{2}}$ or $\frac{\frac{1}{2}\times\frac{1}{6}}{\frac{1}{2}}$ oe M2 or $\frac{\text{P(4\&twice)}}{\text{P(twice)}}$ stated or $\frac{\text{prob}}{\frac{1}{2}}$ M1 Whatever 1st, only one possibility on 2nd M2 $^1/_6$, no wking M1M1A1 $^1/_{12}$, no wking M0
Total		**7**	

Total 72 marks

4733 Probability & Statistics 2

Question		Answer	Marks		Guidance
1		$U \sim B(800, 0.005) \approx Po(4)$ $P(U \le 6)$ $= \mathbf{0.8893}$ $n > 50$/large, $np < 5/p$ small	B1 M1 A1 B1	4	Po(np) stated or implied Tables or formula ± 1 term, e.g. 0.7851, 0 .9489, 0.1107, *not* 1– Answer 0.889 or a.r.t. 0.8893 Both conditions
2		$\frac{23.625-23}{5/\sqrt{n}} = 2$ $\sqrt{n} = 16$ $n = \mathbf{256}$	M1 A1 M1 A1	4	Standardise with √n, allow √/^{2} errors Equate to 2 or a.r.t. 2.00, signs correct Solve for √n, needs Φ^{-1}, *not* from /n 256 only, allow from wrong signs
3	(i)	(a) $e^{-0.42}$ $= \mathbf{0.657}$ (b) $0.42\, e^{-0.42} = \mathbf{0.276}$	M1 A1 A1	3	Correct formula for R = 0 or 1 P(0), a.r.t. 0.657 P(1), a.r.t. 0.276
	(ii)	Po(2.1): $1 - P(\le 3) = 1 - 0.8386$ $= \mathbf{0.1614}$	M1 M1 A1	3	Po(2.1) stated or implied Tables or formula, e.g. 0.8386 or 0.6496 or 0.9379 or complement; Answer, in range [0.161, 0.162]
	(iii)		B2	2	At least 3 separate bars, all decreasing *Allow histogram. Allow convex* P(0) < P(1) but otherwise OK: B1 Curve: B1 *[no hint of normal allowed]*
4	(i)	$H_0 : p = 0.14$ $H_1 : p < 0.14$ B(22, 0.14) $P(\le 2) = .86^{22} + (22 \times .86^{21} \times .14) + (231 \times .86^{20} \times .14^2) = \mathbf{0.3877}$ > 0.1 Do not reject H_0. Insufficient evidence that company overestimates viewing proportion	B2 M1 A1 A1 B1 M1 A1	8	Both correct. 1 error, B1, but x or r or $\bar{x}$ etc: 0 B(22, 0.14) stated or implied, e.g. N(3.08, 2.6488) or Po(3.08) Correct formula for 2 or 3 terms, *or* P(≤ 0) = 0.036 and CR Correct answer, a.r.t. 0.388, *or* CR is = 0 Explicitly compare 0.1 or CR with 2, OK from Po but *not* from N Correct comparison type and conclusion, needs binomial, at least 2 terms, *not* from P(< 2) Contextualised, some acknowledgement of uncertainty [SR: Normal: B2 M1 A0 B0 M0] [SR: 2-tailed, or p > 0.14, P(≥ 2): B1M1A2B0M1A1]
	(ii)	Selected independently Each adult equally likely to be chosen	B1 B1	2	Independent *selection* Choice of sample elements equally likely (no credit if not focussed on selection) [Only "All samples of size n equally likely": B1 only unless related to Binomial conditions]
5	(i)		B1 B1 B1	3	Horizontal straight line Symmetrical U-shaped curve Both correct, including relationship between the two and not extending beyond [–2, 2], curve through (0,0)
	(ii)	S is equally likely to take any value T is more likely at extremities	B2	2	Correct statement about both distributions, √ on their graph [Correct for one only, or partial description: B1] *Not* "probability of S is constant", etc.
	(iii)	$\frac{5}{64}\int_{-2}^{2} x^6 dx = \frac{5}{64}\left[\frac{x^7}{7}\right]_{-2}^{2} \quad \left[= \frac{20}{7}\right]$ $- 0^2$ $= \frac{20}{7}$	M1 A1 B1 A1	4	Integrate $x^2 g(x)$, limits –2, 2 Correct indefinite integral [$= 5x^7/448$] 0 or 0^2 subtracted or E(X) = 0 seen, *not* $\int x^2 f(x)dx - \int x f(x)dx$ Answer $\frac{20}{7}$ or $2\frac{6}{7}$ or a.r.t. 2.86, don't need 0

6	(i)	$50.0 \pm 1.96\sqrt{\frac{20.25}{81}}$ = 50.0 ± 0.98 = 49.02, 50.98 $\overline{W}$ < 49.02 and $\overline{W}$ > 50.98	M1 B1 A1A1 A1√ **5**	50.0 ± z√(1.96/81), allow one sign only, allow √ errors z = 1.96 in equation (*not* just stated) Both critical values, min 4 SF at some stage (if both 3SF, A1) CR, allow ≤ / ≥, don't need $\overline{W}$, √ on their CVs, can't recover [Ans 50 ± 0.98: A1 only] [SR: 1 tail, M1B0A0; 50.8225 or 49. 1775: A1]
	(ii)	$\frac{50.98-50.2}{0.5}$ = 1.56 $\frac{49.02-50.2}{0.5}$ = –2.36 Φ(1.56) – Φ(–2.36) = **0.9315**	M1 A1 A1 M1 A1 **5**	Standardise one limit with same SD as in (i) A.r.t. 1.56, allow – } Can allow √ here A.r.t. –2.36, allow + } if very unfair Correct handling of tails for Type II error Answer in range [0.931, 0.932] [SR 1-tail M1; –1.245 or 2.045 A1; 0.893 or 0.9795 A1]
	(iii)	It would get smaller	B1 **1**	No reason needed, but withhold if definitely wrong reason seen. Allow from 1-tail
7	(i)	$\hat{\mu} = \bar{t} = 13.7$ $\frac{12657.28}{64} - 13.7^2$ [= 10.08] ; $\times \frac{64}{63}$ = **10.24** $H_0 : \mu$ = 13.1, $H_1 : \mu$ > 13.1 $\frac{13.7-13.1}{\sqrt{10.24/64}}$ = 1.5 or p = 0.0668 1.5 < 1.645 or 0.0668 > 0.05 Do not reject H_0. Insufficient evidence that time taken on average is greater than 13.1 min	B1 M1 M1 A1 B2 M1 A1 B1 M1 A1 **11**	13.7 stated Correct formula for biased estimate $\times \frac{64}{63}$ used, or equivalent, can come in later Variance or SD 10.24 or 10.2 Both correct. [SR: One error, B1, but x or t or $\bar{x}$ or $\bar{t}$, 0] Standardise, or find CV, with √64 or 64 z = a.r.t. 1.50, or p = 0.0668, or CV 13.758 [√ on z] Compare z & 1.645, or p & 0.05 (must be correct tail), or z = 1.645 & 13 with CV Correct comparison & conclusion, needs 64, *not* μ = 13.7 Contextualised, some acknowledgement of uncertainty [13.1 – 13.7: (6), M1 A0 B1 M0]
	(ii)	Yes, not told that dist is normal	B1 **1**	Equivalent statement, *not* "n is large", don't need "yes"
8	(i)	N(14.7, 4.41) Valid because np = 14.7 > 5; nq = 6.3 > 5 $1-\Phi\left(\frac{15.5-14.7}{\sqrt{4.41}}\right)$ = 1 – Φ(0.381) = 1 – 0.6484 = **0.3516**	M1 A1 B1 B1 M1 A1 A1 **7**	Normal, attempt at np Both parameters correct Check np > 5; } If both asserted but not both nq or npq > 5 } 14.7 and 6.3 seen: B1 only [Allow "n large, p close to ½"] Standardise, answer < 0.5, no √n z, a.r.t. 0.381 Answer in range [0.351, 0.352] [Exact: M0]
	(ii)	$\bar{K}$ ~ N(14.7, 4.41/36) [= N(14.7, 0.35^2)] Valid by Central Limit Theorem as 36 is large $\Phi\left(\frac{14.0+\frac{1}{72}-14.7}{\sqrt{4.41/36}}\right)$ = Φ(–1.96) = **0.025**	M1 A1√ B1 M1 A1 A1 A1 **7**	Normal, their np from (i) Their variance/36 Refer to CLT or large n (= 36, *not* 21), or "K ~ N so $\bar{K}$ ~ N", *not* same as (i), *not* np > 5, nq > 5 for $\bar{K}$ Standardise 14.0 with 36 or √36 cc included, allow 0.5 here, e.g. 14.5 – 14.7 z = –1.96 or –2.00 or –2.04, allow + if answer < 0.5 0.025 or 0.0228 [0.284 loses last 2] [Po(25.2) etc: probably 0]
	OR:	B(756, 0.7) ≈ N(529.2, 158.76) $\Phi\left(\frac{504.5-529.2}{\sqrt{158.76}}\right)$ = Φ(–1.96) = **0.025**	M1M1A1 B1 M1 A1 A1	×36; N(529.6,…); 158.76 CLT as above, or np > 5, nq > 5, can be asserted here Standardise 14×36 cc correct and √npq 0.025 or 0.0228

4734 Probability & Statistics 3

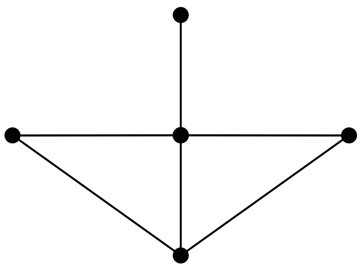
1	T has a Poisson distribution E(T)=28×0.75+4×6.4 = 46.6 Var(T)=46.6	B1 M1 A1 B1√ **4**	 From sum of Poissons Ft E(T) only if Poisson
2 (i)	Use F(Q_3)=0.75 or $\int_{Q_3}^{\infty} \frac{1}{5} e^{-\frac{1}{4}u} \, du = 0.25$ Solve to obtain Q_3 = 4.65 AEF eg 4ln(16/5)	M1 M1A1 **3**	 M1 for solving similar eqn A0 for ≥4.65
(ii)	$f(u) = \begin{cases} \frac{1}{5} e^{u} & u < 0, \\ \frac{1}{5} e^{-\frac{1}{4}u} & u \geq 0. \end{cases}$	B1 B1 **2**	u < 0 unless evidence of ∫ u ≥ 0
3 (i)	Use 28± zs z=2.326 s^2 = 28×72/1200 (25.0, 31.0)	M1 B1 B1 A1 **4**	Accept s=c/√n for M1 Accept 0.28 with corresponding s Or 1199 Accept (25, 31)
(ii)	2×2.326 √(0.28×0.72/n) ≤0.05 AEF Solve to obtain n Smallest n = 1745 e.g. Variance is an approximation	M1 M1 A1 B1 **4**	Or = or ≥ Solving similar equn Accept 1746 ,1750 Or normal is approx or Or p only an estimate
4 (i)	c = 1/20	B1 **1**	
(ii)	$\int_{25}^{45} \frac{400\sqrt{x} - 240}{20} \, dx$ $= \left[\frac{40}{3}x^{3/2} - 12x\right]$ = 2118(£)	M1 A1 A`1 **3**	 Correct indefinite integral 2120 or better than 2118
(iii)	400√X –240 > 2000 , X > 31.36 P(X > 31.36) = (45 – 31.36)/20 = 0.682	M1 M1 A1 **3**	Or 31.4 cao

5 (i)	$H_o: \mu_2 = \mu_1$, $H_1: \mu_2 > \mu_1$, where μ_1 and μ_2 are the mean concentrations in the lake before and after the spillage respectively	B1 B1 **2**	For both hypotheses Allow in words if population mean used**.**
(ii)	$\bar{X}_2 - \bar{X}_1 \geq zs$ z=1.645 s=0.24√(1/5+1/6) ≥ 0.2391	M1 A1 B1 A1 **4**	Accept >, =, <. ≤, *ts* Or **>;** 0.239
(iii)	$P(\bar{X}_2 - \bar{X}_1 < 0.2391)$ *z* =[0.2391 – 0.3]/*s* *p*=0.3376 This is a large probability for this error	M1 M1 A1 B1 **4**	May be implied ART 0.337 or 0.338 Relevant comment
6 (i)	Use *B* ~ B(29, 0.3) , *G* ~ B(26, 0.2) E(*F*)=29×0.3+26×0.2=13.9 Var(*F*) = 29×0.3×0.7+26×0.2×0.8=10.25	M1 M1A1 M1A1 **5**	
(ii)	*B: np* = 8.7, *nq*=20.3 *G*: *np* = 5.2, *nq*= 20.8 All exceed 5, so normal approximation valid for each *F* ~ N(13.9, 10.25) (approximately) (Requires P(*F* ≤*n*) = 0.99) [*n* + 0.5 – 13.9]/√(10.25) ; = 2.326, their 10.25 *n* = 20.85 Need to have 21 spares available SR Using B(55, 0.2527): B1; M1(N(13.9, 10.39); M1B1M1A0 (Max 5/8)	 B2 M1√ M1B1 A1 M1 A1 **8**	Must check numerically B1 for checking one distribution Use normal. May be implied Standardise M0 if variance has divisors cc Solving similar No cc, lose last A1 (n = 22) Wrong cc, lose A1A1

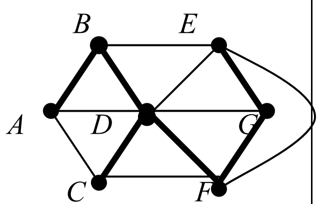
7 (i)	Requires population of (2nd mark – 1st mark) to be normally distributed H_o: $\mu_d = 0$, H_1:$\mu_d > 0$ $T_2 - T_1$: - 1 - 1 2 0 - 2 2 3 2 $\overline{d} = 0.625$, $s^2 = 3.411$ ($3^{23}/_{56}$ or $^{191}/_{56}$) Use 2.998 EITHER: $t = 0.625/\sqrt{(3.411/8)}$ = 0.957 OR: CV(CR), $\overline{d} \geq 2.998\sqrt{3.411/8}$ = 1.958 EITHER 0.957<2.998 OR 0.625 < 1.958 Do not reject H_o, there is insufficient evidence of improvement	B1 M1 B1B1 B1 M1 A1 M1 A1 M1 **8**	 M0 if clearly z With comparison and conclusion
(ii)	Use $E(X_2 - X_1 + k) = 0.625 + k$ Requires $(0.625+k) / \sqrt{(3.411/8)} \geq 2.998$ Giving $k \geq 1.33$ Increase each mark by 2	M1 A1√ A1 **3**	 Allow 1.33
8 (i)	Mean= (20+16+9)/75 = 0.6 $3p = 0.6$, $p = 0.2$ AG	M1 A1 A1 **3**	
(ii)	H_0: B(3,p) fits the data (H_1: B(3,p) does not fit the data) Expected values 38.4 28.8 7.2 0.6 Combine last two cells $\chi^2 = 5.6^2/38.4 + 8.8^2/28.8 + 3.2^2/7.8$ = 4.818 4.818 > 3.841 Reject H_0 and conclude that there is insufficient evidence that B(3,p) fits the data.	B1 M1 A1 A1 B1 M1 A1√ A1 B1√ M1 **10**	Or: X~B(3,p) or B(3,0.2) Not 'Data fits model' Use B(3,0.2)×75 At least 2 correct All correct With one correct At least 2 correct Ft E values Accept 4.82 cao ft 4.818 SR1 If cells not combined: B1M1A1A1B0M1A1A0B1(5.991)M1 SR2:E-values rounded :B1M1A1A1 B1M1A1A0(4.865)B1M1
(iii)	2.74 < 3.841, accept H_0 conclude that B(6, p) fits the data	B1 **1**	Accept with no reason if evidence of method in (ii)

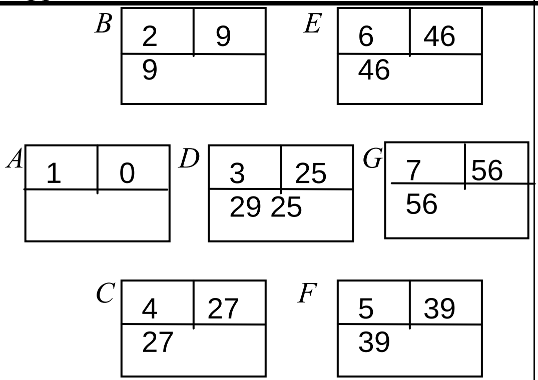
4736 Decision Mathematics 1

1	**(i)**	*A* *B* *C* *D* 614 416 1 198 (*A*=198) 198 891 2 693 (*A*=693) 693 396 3 297	M1 A1 M1 A1	*A*, *B* and *C* correct for first pass *D* = 198 on first pass sca at second <u>and</u> third passes Second and third passes correct	**[4]**
	(ii)	0	B1	0	**[1]**
	(iii)	To make the algorithm terminate	B1	So that it does not get stuck in a loop	**[1]**
				Total =	**6**

2	**(i)**	eg 	M1 A1	Graph need not be simple or planar A graph with five vertices and at least three correct vertex orders A graph with five vertices of orders 1, 2, 2, 3, 4	**[2]**
	(ii)	Semi-Eulerian It has <u>exactly</u> two odd nodes	M1 A1	Unless their graph was not connected, in which case the answer is 'neither' (Unless their graph was not connected, in which case follow this through)	**[2]**
	(iii)	A tree with five vertices would only have four arcs, but this graph has six <u>Or</u> A tree must have at least two vertices of order 1	B2	Give B1 for an incomplete reason, eg 'too many arcs' or 'it has a cycle'	**[2]**
				Total =	**6**

ANSWERED ON INSERT

3	**(i)**	*AB* = 9 *DF* = 14 *BD* = 16 *CD* = 18 *FG* = 20 ~~*CF* = 22~~ *EG* = 23 ~~*EF* = 26~~ ~~*AC* = 27~~ ~~*DE* = 28~~ ~~*AD* = 29~~ ~~*DG* = 31~~ ~~*BE* = 37~~  Total weight = 100	M1 A1 M1 A1 B1	Not selecting *CF* (working seen on list) Selecting correct arcs (working seen on list) A spanning tree drawn Correct (minimum) spanning tree drawn 100 cao	**[5]**

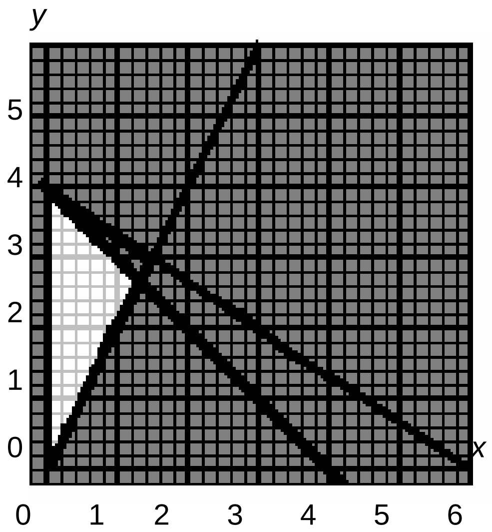
	(ii)	Delete *EG* from spanning tree 100 – 23 = 77 Two shortest arcs from *E* are *EG* and *EF* 77 + 23 + 26 = 126 Lower bound = 126	 B1 M1 A1	Follow through from part (i) if possible Weight of MST on reduced network Adding two shortest arcs to MST 126 cao	**[3]**
	(iii)	*A – B – D – F – G – E* – stall Misses out vertex *C*	M1 A1	*A – B – D – F – G – E* Cannot continue because *B*, *D* and *F* have already been visited	**[2]**
	(iv)	*B – A – C – D – F – G – E – B* Upper bound = 148	M1 A1 B1	Tour starts *B – A – C – D – F –* Correct tour, starting and ending at *B* 148 cao	**[3]**
	(v)	 Weight = 56 Route = *A -B - D - G*	 M1 A1 B1 B1 B1 B1	(Accept correct working starting from *G*, if seen) At least three sets of temporary labels correct, with no extras Temporary labels all correct, with no extras Permanent labels correct Order of labelling (correct or follow through their permanent labels) 56 cao *A - B - D - G* cao	**[4]** **[2]**
	(vi)	*A*, *B*, *C* and *G* are odd *AB* = 9 *AC* = 27 *AG* = 56 *CG* = 42 *BG* = 47 *BC* = 34 51 74 90 Repeat *AB* and *CG* (*C - F - G*) = 51 Weight = 300 + 51 = 351	B1 M1 A1 B1	Identifying or using *A*, *B*, *C*, *G* (seen) At least one correct pairing seen or total seen (not just six weights) All three totals correct, or explanation of how it is known that other pairings are too long 351 cao	**[4]**
				Total =	**23**

ANSWERED ON INSERT

4	**(i)**	8	B1	cao	**[1]**
	(ii)	1 comparison and 1 swap	B1	1 and 1	**[1]**
	(iii)	76 65 21 13 88 62 67 28 34 2 comparisons and 1 swap	B1 B1	Correct list (complete) 2 and 1	**[2]**
	(iv)	C S 76 65 21 13 88 62 67 28 34 1 0 88 76 65 21 13 62 67 28 34 4 4 88 76 65 62 21 13 67 28 34 3 2 88 76 67 65 62 21 13 28 34 5 4 88 76 67 65 62 28 21 13 34 3 2 88 76 67 65 62 34 28 21 13 4 3	 M1 M1 A1 M1 A1 A1	Underlined values correct in 3rd and 4th passes, values not underlined may be left blank Similarly for 5th and 6th passes, follow through slips in previous passes Similarly for 7th and 8th passes, but cao (Dependent on both M marks) Reasonable attempt at Comp and Swap 1 4 3 5 3 4 cao in figures 0 4 2 4 2 3 cao in figures	**[3]** **[3]**

	(v)	Shuttle sort uses 23 comparisons and 17 swaps Shuttle sort is more efficient because although it uses the same number of swaps as bubble sort it uses fewer comparisons	 M1 A1	Follow through their totals if possible Choosing shuttle sort with a reason or with totals seen (here) Correct reason stated (comparisons and swaps both compared, in words)	**[2]**
				Total =	**12**

5	**(i)**	Katie must spend at least 8 minutes preparing the first batch of cookies so she has at most 52 minutes of baking time. $52 \div 12 = 4.3$, hence at most 4 batches	M1 A1	Identifying why there is less than 60 minutes of baking time (or seeing 52) Explaining why 4 is the greatest possible number of batches	**[2]**
	(ii)	The last batch takes 12 minutes to bake, so Katie has (at most) 48 minutes of preparation time $8x + 12y + 10z \leq 48 \Rightarrow 4x + 6y + 5z \leq 24$ as given	B1 B1	Explaining why total time for preparation cannot exceed 48 minutes $8x + 12y + 10z \leq 48$ seen or explicitly referred to	**[2]**
	(iii)	Must be integer valued	B1	Integers	**[1]**
	(iv)	$P = 5x + 4y + 3z$ Assumes that she sells all the cookies (batches) that she makes	B1 B1	$5x + 4y + 3z$ or any positive multiple of this Assumes she sells them all	**[2]**
	(v)	P x y z s t *1 -5 -4 -3 0 0 0* *0 1 1 1 1 0 4* *0 4 6 5 0 1 24*	M1 A1 A1	Correct use of slack variable columns Objective row correct (cao) Constraint rows correct (cao)	**[3]**
		$4 \div 1 = 4$, $24 \div 4 = 6$, $4 < 6$ Pivot on the 1 in the x column	B1	Working need not be seen Correct pivot choice (row 2) (cao)	
		P x y z s t *1 0 1 2 5 0 20* *0 1 1 1 1 0 4* *0 0 2 1 -4 1 8* Row 1 = R1 + 5×R2 Row 2 = R2 ÷ 1 Row 3 = R3 - 4×R2	 M1 A1 B1	Follow through their tableau and pivot choice, if possible sca pivoting (x, t cols, P not decreased) Correct tableau (final column contains no negative values) Showing valid method, may imply row 2	**[4]**
		$x = 4$, $y = 0$, $z = 0$, $P = 20$ Katie should make 4 batches of plain cookies, and no chocolate chip or fruit cookies, to give a profit of £20.	 M1 A1 A1	Follow through their tableau, if reasonable (non-negative variables) Reading off values from tableau (may be implied from answer) Interpretation: 4 batches of plain cookies (may imply none of others) Interpretation: £20	**[3]**

	(vi)		M1 A1 A1	At least two of the lines $y = 2x$, $x+y = 4$ and $4x + 6y = 24$ drawn correctly All three lines drawn correctly and graph has both scales and labels Feasible region identified and correct Follow through their feasible region if possible	[3]
		Vertices of feasible region are (0, 0), (0, 4) and $(1\frac{1}{3}, 2\frac{2}{3})$	M1 A1	At least two correct All (three) correct (1 dp or better)	[2]
		$x = 0, y = 4 \Rightarrow P = 16$ $x = 1, y = 3 \Rightarrow P = 17$ $(x = 1\frac{1}{3}, y = 2\frac{2}{3} \Rightarrow P = 17\frac{1}{3})$	M1 A1	Or a line of constant profit drawn (or gradient discussed) and used correctly on integer-valued coordinates For (1, 3) or 17 chosen (cao)	
		Make 1 batch of plain cookies and 3 batches of chocolate chip cookies	B1	Interpretation: 1 batch of plain, 3 batches of chocolate chip (cao)	[3]
				Total =	25

4737 Decision Mathematics 2

1 (i)

Stage	State	Action	Working	Maximin
1	0	0	10	**10**
	1	0	11	**11**
	2	0	14	**14**
	3	0	15	**15**
2	0	0	min(12, **10**)=**10**	
		2	min(10, **14**)=**10**	**10**
	1	0	min(13, **10**)=**10**	
		1	min(10, **11**)=**10**	
		2	min(11, **14**)=**11**	**11**
	2	1	min(9, **11**)=**9**	
		2	min(10, **14**)=**10**	**10**
		3	min(7, **15**)=**7**	
	3	1	min(8, **11**)=**8**	
		3	min(12, **15**)=**12**	**12**
3	0	0	min(15, **10**)=**10**	
		1	min(14, **11**)=**11**	
		2	min(16, **10**)=**10**	
		3	min(13, **12**)=**12**	**12**

Mark	Guidance	
	Answered on insert	
M1	Transferring maximin values from stage 1 correctly	
M1	Completing working column for stage 2 (method)	
M1	Calculating maximin values for stage 2 (method)	
A1	Maximin values correct for stage 2 (cao)	
M1	Transferring maximin values from stage 2 correctly	
A1	Working column for stage 3 correct (cao)	**[6]**

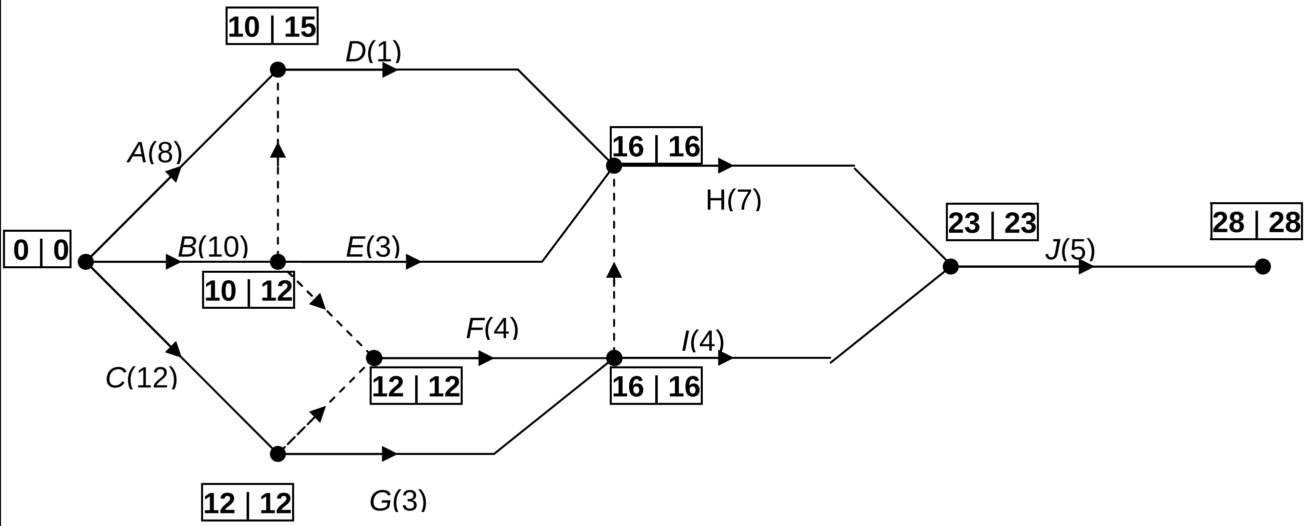
(ii)

Answer	Mark	Guidance	
Maximin value = **12**	B1	12 (cao)	
Maximin route = **(0; 0) – (1; 3) – (2; 3) – (3; 0)**	M1	Route, or in reverse, follow through their table if possible, condone omission of (0; 0)	
	A1	Correct route, including (0; 0) (cao)	**[3]**

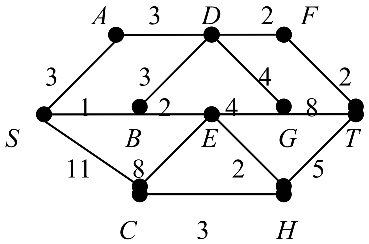
Total = 9

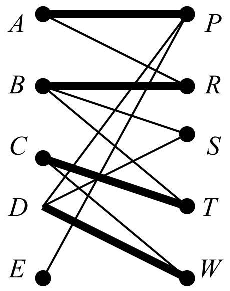
Q	Part	Activity	Duration (days)	Immediate predecessors	Mark	Guidance	Total
2	(i)					**Answered on insert**	
		A	8	-			
		B	10	-			
		C	12	-			
		D	1	**A B**	B1	Precedences correct for *D* and *E*	
		E	3	**B**			
		F	4	**B C**	B1	Precedences correct for *F* and *G*	
		G	3	**C**			
		H	7	**D E F G**	B1	Precedences correct for *H*, *I* and *J*	
		I	4	**F G**			**[3]**
		J	5	**H I**			

(ii)



Q	Part	Answer	Mark	Guidance	Total
	(ii)		M1	Substantially correct attempt at forward pass (at most one independent error)	
			M1	Substantially correct attempt at backward pass (at most one independent error) No follow through, 28 given in question	
			A1	Both passes wholly correct	
		Critical activities ***C F H J***	B1	*C F H J* and no others (no follow through)	**[4]**
	(iii)	A B C D E F G H I J ***1 1 3 2 1 1 2 2 3 4***	B1	*J* correct	
			B1	*H* and *I* correct	
			B1	*F* and *G* correct	
			B1	*D* and *E* correct	
			B1	*B* and *C* correct	
			B1	*A* correct	**[6]**
	(iv)	Minimum delay **1** day	B1	1	
		Maximum delay **3** days	B1	3	**[2]**
				Total =	15

3	**(i)**	 4+3-2+8-2+7 = **18** litres per second	 M1 A1	**Answered on insert** Imply method mark from 18, 20 or 22 cao	 **[2]**
	(ii)	3 litres per second flow out of *B* (arc *BD*) so only 2 litres per second can enter *B* from *E* and only 1 litre per second can enter *B* from *S*. At least 4 litres per second flow out of *E* to *G*, 2 litres per second from *E* to *B* and 2 litres per second from *E* to *H*, so 8 litres per second must flow into *E* from *C*. 8 litres per second flows from *C* to *E* and at most 11 litres per second enters *C* from *S*, so at most 3 litres per second flows from *C* to *H*. Also, 2 litres per second flow from *E* to *H* so the most that can enter *H* is 5 litres per second. But at least 5 litres per second leave *H* along *HT*, hence the flow in *HT* is 5 litres per second.	B1 B1 M1 A1	At B: 3 out and 1 + 2 in At *E*: (at least) 4 + 2 + 2 out Considering *C* to show flow in *CH* is <u>at most</u> 3 Must explicitly refer to ≤ 3, or 2 ≤ flow ≤ 3, not just stating 3 At *H*: 2 + 3 in	**[4]**
	(iii)	 Flow augmenting route: ***S A D F T*** or ***S A D G T*** Cut: $X = \{S, B\}$, $Y = \{A, C, D, E, F, G, H, T\}$ Or $X = \{S, A, B\}$, $Y = \{C, D, E, F, G, H, T\}$	M1 A1 B1 B1	Substantially correct attempt (at least 12 correct) (Not shown as excess capacities and potential backflows) All correct (cao) Either of these (correct) flow augmenting routes Either of these (correct) cuts described in any way, or marked clearly on diagram	**[4]**
	(iv)	*B* would have at most 3 litres per second entering it and at least 5 litres per second leaving.	M1 A1	Identifying that problem is at *B* A correct explanation	**[2]**
				Total =	**12**

Q	Part	Answer	Mark	Guidance	Total
4	(i)		B1 B1	Bipartite graph correct Incomplete matching correct (clearly shown, or shown on a separate bipartite graph)	[2]
	(ii)	***E – P – A – R – B – S*** Anya = restaurant review Ben = sports news Connie = theatre review Derek = weather report Emma = problem page	M1 A1 B1	A valid alternating path from *E* to *S*, written out This path written out (not just shown on diagram) $A = R \quad B = S \quad C = T \quad D = W \quad E = P$ (cao)	[3]
	(iii)	Add a dummy column (see table 1 below) Reduce rows (see table 2 below) Then reduce columns (see table 3 below)	B1 M1 M1 A1	Adding a dummy column of equal 'costs' of at least 60 minutes Substantially correct attempt at reducing rows (at most one error) Substantially correct attempt at reducing columns (at most one error) Correct reduced cost matrix, with rows reduced first (cao)	[4]

Add a dummy column

	P	R	S	T	W	X
J	*56*	*56*	*51*	*57*	*58*	*60*
K	*53*	*52*	*53*	*54*	*54*	*60*
L	*57*	*55*	*52*	*58*	*60*	*60*
M	*59*	*55*	*53*	*59*	*57*	*60*
N	*57*	*57*	*53*	*59*	*60*	*60*
O	*58*	*56*	*51*	*56*	*57*	*60*

Reduce rows

5	*5*	*0*	*6*	*7*	*9*
1	*0*	*1*	*2*	*2*	*8*
5	*3*	*0*	*6*	*8*	*8*
6	*2*	*0*	*6*	*4*	*7*
4	*4*	*0*	*6*	*7*	*7*
7	*5*	*0*	*5*	*6*	*9*

Then reduce columns

4	*5*	*0*	*4*	*5*	*2*
0	*0*	*1*	*0*	*0*	*1*
4	*3*	*0*	*4*	*6*	*1*
5	*2*	*0*	*4*	*2*	*0*
3	*4*	*0*	*4*	*5*	*0*
6	*5*	*0*	*3*	*4*	*2*

Cross out 0's using 3 (minimum number of) lines

4	5	0	4	5	2
0	0	1	0	0	1
4	3	0	4	6	1
5	2	0	4	2	0
3	4	0	4	5	0
6	5	0	3	4	2

Augment by 2

2	3	0	2	3	2
0	0	3	0	0	3
2	1	0	2	4	1
3	0	0	2	0	0
1	2	0	2	3	0
4	3	0	1	2	2

M1 Follow through their reduced cost matrix for crossing through 0's and augmenting (without errors)

A1 Augment by 2 in a single augmentation (cao)

Cross out 0's using 4 (minimum number of) lines

2	3	0	2	3	2
0	0	3	0	0	3
2	1	0	2	4	1
3	0	0	2	0	0
1	2	0	2	3	0
4	3	0	1	2	2

Augment by 1

1	2	0	1	2	2
0	0	4	0	0	4
1	0	0	1	3	1
3	0	1	2	0	1
0	1	0	1	2	0
3	2	0	0	1	2

Alternative

2	3	0	2	3	2
0	0	3	0	0	3
2	1	0	2	4	1
3	0	0	2	0	0
1	2	0	2	3	0
4	3	0	1	2	2

1	2	0	1	2	1
0	0	4	0	0	3
1	0	0	1	3	0
3	0	1	2	0	0
1	2	1	2	3	0
3	2	0	0	1	1

M1 Follow through their matrix for crossing through 0's and augmenting (correct for theirs)

A1 (Either) correct final matrix (cao)

[4]

To get a complete allocation

1	2	0	1	2	2
0	0	4	0	0	4
1	0	0	1	3	1
3	0	1	2	0	1
0	1	0	1	2	0
3	2	0	0	1	2

1	2	0	1	2	1
0	0	4	0	0	3
1	0	0	1	3	0
3	0	1	2	0	0
1	2	1	2	3	0
3	2	0	0	1	1

Jeremy	Kath	Laura	Mohammed	Ollie
Sports	Problems	Restaurant	Weather	Theatre

B1 $J = S \quad K = P \quad L = R \quad M = W \quad O = T$

51 + 53 + 55 + 57 + 56 = 272

M1 Correct method

272 × £0.25 = £68

A1 £68 (cao) with units

[3]

Total = 16

<table>
<tr><td>5</td><td>(i)</td><td>5

(10 - 4) ÷ 2
= 3</td><td>B1

M1
A1</td><td>5

3 or 7
3</td><td>[3]</td></tr>
<tr><td></td><td>(ii)</td><td><table><tr><th></th><th>D</th><th>E</th><th>F</th><th>row min</th></tr><tr><td>S</td><td>0</td><td>4</td><td>-2</td><td>-2</td></tr><tr><td>T</td><td>-4</td><td>2</td><td>-4</td><td>-4</td></tr><tr><td>U</td><td>2</td><td>-6</td><td>0</td><td>-6</td></tr><tr><td>col max</td><td>2</td><td>4</td><td>0</td><td></td></tr></table>
Play-safe for rugby club (rows) is Sanjeev
Play-safe for cricket club (cols) is Fiona

Not stable because $-2 \neq 0$</td><td>M1

M1

A1
A1

B1</td><td>Calculating row minima

Calculating col maxima (or equivalent)

Sanjeev or S (not just -2 or identifying row)
Fiona or F (not just 0 or identifying column)

Any correct explanation</td><td>[5]</td></tr>
<tr><td></td><td>(iii)</td><td>

Fiona
Ursula</td><td>

B1
B1</td><td>Follow through their play-safe strategies if possible
F
U</td><td>[2]</td></tr>
<tr><td></td><td>(iv)</td><td>Sanjeev's row dominates Tom's row

Doug

Fiona's column dominates Doug's (once Tom's row has been removed)</td><td>B1

M1

A1</td><td>This or any equivalent statement about Tom and Sanjeev (note: Tom is named in the question)

Doug

This or any equivalent statement about Doug and Fiona</td><td>[3]</td></tr>
<tr><td></td><td>(v)</td><td>E: $4p - 6(1-p) = 10p - 6$
F: $-2p$

$10p - 6 = -2p$
⇒ $p = 0.5$</td><td>M1

A1</td><td>Follow through their choice from part (iv)
Both expressions seen in any form
(note: D gives $2(1-p) = 2 - 2p$)

$p = 0.5$ (cao)</td><td>[2]</td></tr>
<tr><td></td><td>(vi)</td><td>Delete T row
$\begin{matrix} 0 & 4 & -2 \\ 2 & -6 & 0 \end{matrix}$

Multiply entries by -1 to show scores for Cricket club
$\begin{matrix} 0 & -4 & 2 \\ -2 & 6 & 0 \end{matrix}$

Add 4 to make entries non-negative
$\begin{matrix} 4 & 0 & 6 \\ 2 & 10 & 4 \end{matrix}$

Choose Doug with probability x, Euan with probability y and Fiona with probability z.

If Sanjeev plays, expected score = $4x + 6z$
If Ursula plays, expected score = $2x + 10y + 4z$</td><td>B1

B1

B1</td><td>Delete T row <u>and</u> multiply entries by -1

Add 4 to make entries non-negative

Identifying meaning of x, y, z or implied by reference to S for $4x + 6z$ and U for $2x + 10y + 4z$</td><td>[3]</td></tr>
<tr><td></td><td>(vii)</td><td>$z = \frac{5}{6}$ □ maximum value for $m = 5$
Hence, maximum value for $M = 1$</td><td>M1
A1</td><td></td><td>[2]</td></tr>
<tr><td colspan="5" align="right">Total =</td><td>20</td></tr>
</table>

Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)
January 2009 Examination Series

Unit Threshold Marks

7892		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	57	50	43	37	31	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	59	51	44	37	30	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	55	48	41	34	28	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	62	54	46	38	31	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	49	44	39	34	30	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	54	47	40	33	27	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	62	54	46	38	30	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	61	51	41	31	21	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	57	48	40	32	24	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	58	50	43	36	29	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	58	49	41	33	25	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	50	43	37	31	25	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	58	51	45	39	33	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	60	53	46	39	33	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	24.1	50.4	72.7	85.8	95.1	100	960
3892	28.1	59.4	78.1	90.6	93.8	100	32
7890	26.8	58.1	84.4	92.2	96.6	100	205
7892	33.3	75.0	91.7	91.7	100	100	12

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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