OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4733:** Probability and Statistics 2

Mark Scheme for January 2011

Oxford Cambridge and RSA Examinations

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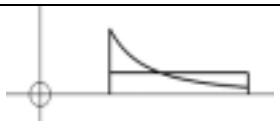
1		$\hat{\mu} = \bar{x} = \frac{468}{9} = 52$ $\frac{24820}{9} - 52^2$ [= 53.78] $\hat{\sigma}^2 = \frac{9}{8} \times 53.78$ = **60.5**	B1 M1 M1 A1 **4**	52 stated Correct method for biased estimator Multiply by 9/8 [if single formula, allow M0 M1 if wrong but divisor 8 seen anywhere] Answer 60.5 or exact equivalent
2		$\frac{53.28 - \mu}{5/\sqrt{n}} = 1.96$ $\frac{\mu - 51.65}{5/\sqrt{n}} = 1.3$ $\sqrt{n} = 10$, n = **100** μ = **52.3**	M1dep A1 B1 depM1 A1 B1 **6**	Standardise with $\sqrt{n}$ once & equate to z, allow sign, square/√ errors twice, signs correct, zs may be wrong Both correct z values seen Solve to get $\sqrt{n}$ or μ, needs first M1 n = 100, not from wrong signs a.r.t. 52.3, right arithmetic needed but $\sqrt{n}$ can be omitted
3		B(200, 0.0228) Po(4.56) $e^{-4.56}(1 + 4.56 + \frac{4.56^2}{2})$ = **0.167** n large *or* $n > 50$; p small *or* $np < 5$	M1 A1 M1 A1 A1 B1 **6**	B(200, 0.0228) stated or implied Po(4.56) stated or implied, allow 4.6 here Correct formula for P(≤ 2) ± 1 term, any λ (tables: M0) Correct formula, 4.56 needed Answer, a.r.t. 0.167 [0.16694] Both, can be merely asserted. If numbers, must be these SR interpolation: clear method M1, answer A2 MR: typically B(200, 0.228) ≈ N(45.6, 3.52): M1A1; standardise correctly, M1; state np, $nq > 5$, B1
4	(i)	*Either* $z = \frac{213.4 - 230}{45/\sqrt{50}}$ = –2.608 –2.608 < –2.576 *or* 0.0047 < 0.005	M1 A1 B1	Standardise z with √50, ignore sign or √ or squaring errors z-value, a.r.t. –2.61, or p in range [0.0044, 0.005) Correctly compare (–)2.576, signs consistent, *or* p explicitly with 0.005
	Or	CV is $230 - 2.576 \times \frac{45}{\sqrt{50}} = 213.6$ 213.4 < 213.6	M1 B1 A1	$230 - z\sigma/\sqrt{50}$, allow √ or squaring errors, allow ± but not just +; z = 2.576 Explicitly compare 213.4 with 213.6
		Reject H_0. Significant evidence that population mean is not 230	M1 A1 FT **5**	"Reject", FT, needs correct method and form of comparison; interpreted, acknowledge uncertainty
	(ii)	Yes, population distribution is not known to be normal	B2 **2**	*Not*, "yes, sample size is large" but ignore "*can* use it as …" SR: Both right and wrong answers: B1 α "Yes as it must be assumed normal": B1
5		H_0: $\lambda = 12$; H_1: $\lambda > 12$ *Either* : P(≥ 19) = 1 – P(≤ 18) = 1 – 0.9626 = 0.0374 < 0.1	B2 M1 A1 B1	Both correct: B2. Allow μ. One error, B1, but *not* x, r etc. Po(12) stated or implied, e.g. 0.9787 0.0374, *or* 0.9626 if compared with 0.9 Explicitly compare P(≥ 19) with 0.1, *or* P(≤ 18) with 0.9
		Or: CR is ≥ 18, p = 0.063 19 ≥ 18	A1 B1	≥ 18 and 0.063 stated Explicit comparison of CV (right-hand CR) with 19
		Reject H_0. Significant evidence of increase in mean number of applicants	M1 A1 FT **7**	"Reject" FT, needs correct method and comparison, e.g. *not* from ≤ 19 or = 19, withhold if inconsistent Interpreted in context, acknowledge uncertainty

6	(i)	If one customer arrives, it does not change the probability that another one does so; customers probably arrive in groups of at least 2	B1 B1 **2**	Answer that shows correct understanding of "independent", in context; *not* just equivalent to "singly" Plausible reason, in context, nothing wrong, nothing that suggests "constant average rate"
	(ii)	**0.1730**	M1 A1 **2**	Correct use of tables or formula, e.g. .3007, or .4405 from Po(5) if Po(7) stated; answer 0.173, 0.1730 or better
	(iii)	Po(35) N(35, 35) $1-\Phi\left(\frac{40.5-35}{\sqrt{35}}\right) = 1 - \Phi(0.9297)$ = **0.1763**	B1 M1 A1 M1 A1 A1 **6**	Po(5×7) stated or implied Normal, μ = their λ Both parameters correct, allow 35^2, $\sqrt{35}$ Standardise 40 with λ, $\sqrt{\lambda}$, allow $\sqrt{}$, cc errors Both $\sqrt{\lambda}$ and cc correct Answer, a.r.t. 0.176 [penalise 0.1765]
7	(i)		B1 B1 B1 **3**	Horizontal line above axis Concave decreasing curve above axis Both correct including approx relationship, not extending beyond [1, 3], verticals and scale not needed
	(ii)	$\int_1^3 \frac{a}{x^2}\mathrm{d}x = 1$, $\left[\frac{-a}{x}\right]_1^3 = 1$; $a = \frac{3}{2}$	M1 B1 A1 **3**	Attempt $\int f_X(x)\mathrm{d}x$, limits 1, 3 at some stage, and equate to 1 Correct indefinite integral Correctly obtain 3/2 or 1.5 or exact equivalent
	(iii)	$\int_1^3 \frac{a}{x}\mathrm{d}x = [a\ln x]_1^3$ $= \frac{3}{2}\ln 3$	M1 B1 FT A1 FT **3**	Attempt $\int x f_X(x)\mathrm{d}x$, limits 1, 3 at some stage Correct indefinite integral, FT on a Answer, any exact equivalent or a.r.t 1.65, FT on a, or $a\ln 3$
	(iv)	T is equally likely to take any value between 1 and 3	B1 **1**	Must be "values taken by T" (or "of T") or clear equivalent Any hint that they think T is an *event* gets B0. α "Same chance of occurring anywhere between 1 and 3": 0 β "For values of T between 1 and 3, T is equally likely": 0 γ "Each value of T is equally likely to occur": 1
8	(i)	B(40, 0.225) ≈ N(9, 6.975) $\frac{5.5-9}{\sqrt{6.975}} = -1.325$ **0.9074** $np = 9 > 5$ *or* n large; and $nq = 31 > 5$ *or* p close to 0.5	M1 M1 A1 M1 A1 A1 B2 **8**	B(40, 0.225) stated or implied Normal, mean 9 Variance 6.975 or SD 2.641 or 6.975 Standardise with np and $\sqrt{npq}$, allow npq, no or wrong cc CC and $\sqrt{npq}$ correct, allow from N(3600, 0.225) Answer, in range [0.907, 0.908] Full conditions B2; partial, B1 (assertions OK). Allow npq, allow from e.g. n = 3600
	(ii)	Number list sequentially and select using random numbers If # > 3600, ignore (etc)	B1 B1 B1 **3**	Number list, don't need "sequentially" Mention random numbers (*not* "select numbers randomly") Deal with issue of # > 3600, *or* "ignore repeats" α "Randomly pick numbers from 0 to 3599": (B1) B0 B1
9	(i)	B(14, 0.7) CR is **≥ 13** with probability 0.0475	M1 A1 A1 **3**	B(14, 0.7) stated or implied, e.g. N(9.8, 2.94), can be recovered CV 13, or > 12 or {13, 14}, allow = but no other inequalities Exactly correct CR, and supporting prob .0475 or .9525 seen
	(ii)	H_0: $p = 0.7$, H_1: $p > 0.7$ 12 < 13 Do not reject H_0. Insufficient evidence that proportion who show improvement is greater than 0.7	B2 B1 M1 A1 FT **5**	Both, B2. Allow π. One error, B1, but r, x etc: B0 Compare CV *from correct tail and inequality* with 12, *or* P(≥ 12) = 0.1608 and > 0.05 *or* P(< 12) = 0.8392 and < 0.95 Correct method & conclusion, requires like-with-like; CV method needs ≥ 13 or < 12; p method needs ≥ 12 or < 12 Withhold if inconsistent Contextualised, acknowledge uncertainty [SR: Normal or Po: (i) M1, (ii) B2 maximum] [0.9932 or 0.0068 probably B2 maximum]
	(iii)	B(14, 0.8) P(≤ 12) from B(14, 0.8) **0.8021**	M1 M1 A1 **3**	B(14, 0.8) stated or implied, allow from B(14, 0.75) Attempt prob of acceptance region, e.g. 0.8990, √ on (i) Answer 0.802 or a.r.t. 0.8021

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