OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Mark Schemes for the Units

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3890-2/7890-2/MS/R/08

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MARK SCHEMES FOR THE UNITS

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4721 Core Mathematics 1

1	**(i)**	$n = -2$	**B1**	
			1	
	(ii)	$n = 3$	**B1**	
			1	
	(iii)		**M1**	$\sqrt{4^3}$ or $64^{\frac{1}{2}}$ or $\left(4^{\frac{1}{2}}\right)^3$ or $\left(4^3\right)^{\frac{1}{2}}$ or $4\times\sqrt{4}$ with brackets correct if used
		$n = \frac{3}{2}$	**A1**	
			2	
2	**(i)**		**M1**	$y = (x \pm 2)^2$
		$y = (x-2)^2$	**A1**	
			2	
	(ii)	$y = -(x^3 - 4)$	**B1**	oe
			1	
3	**(i)**	$\sqrt{2\times100} = 10\sqrt{2}$	**B1**	
			1	
	(ii)	$\frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$	**B1**	
			1	
	(iii)		**M1**	Attempt to express $5\sqrt{8}$ in terms of $\sqrt{2}$
		$10\sqrt{2} - 3\sqrt{2} = 7\sqrt{2}$	**A1**	
			2	
4		$y = x^{\frac{1}{2}}$		
		$2y^2 - 7y + 3 = 0$	**M1***	Use a substitution to obtain a quadratic or factorise into 2 brackets each containing $x^{\frac{1}{2}}$
		$(2y-1)(y-3) = 0$	**M1dep**	Correct method to solve a quadratic
		$y = \frac{1}{2}, y = 3$	**A1**	
			M1	Attempt to square to obtain x
		$x = \frac{1}{4}, x = 9$	**A1**	
			SR	**If first M1 not gained and 3 and ½ given as final answers, award B1**
			5	

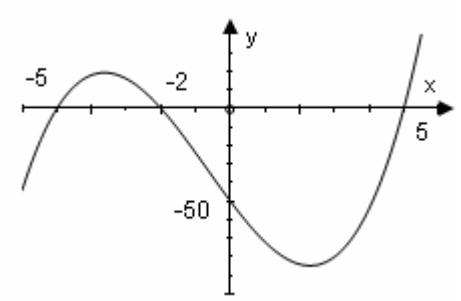
5

	M1	Attempt to differentiate
	A1	$kx^{-\frac{1}{2}}$
$\frac{dy}{dx} = 4x^{-\frac{1}{2}} + 1$	**A1**	
$= 4\left(\frac{1}{\sqrt{9}}\right) + 1$	**M1**	Correct substitution of $x = 9$ into their
$\frac{dy}{dx} = \frac{7}{3}$	**A1**	$\frac{7}{3}$ only
	5	

6 (i)

$(x-5)(x+2)(x+5)$	**B1**	$x^2 - 3x - 10$ or $x^2 + 7x + 10$ or $x^2 - 25$ seen
$= (x^2 - 3x - 10)(x+5)$	**M1**	Attempt to multiply a quadratic by a linear factor
$= x^3 + 2x^2 - 25x - 50$	**A1**	
	3	

(ii)



B1	+ve cubic with 3 roots (not 3 line segments)
B1√	(0, -50) labelled or indicated on y-axis
B1	(–5, 0), (–2, 0), (5, 0) labelled or indicated on x-axis and no other x- intercepts
3	

7 (i)

$8 < 3x - 2 < 11$	**M1**	2 equations or inequalities both dealing with all 3 terms resulting in $a < kx < b$
$10 < 3x < 13$	**A1**	10 and 13 seen
$\frac{10}{3} < x < \frac{13}{3}$	**A1**	
	3	

(ii)

$x(x+2) \geq 0$	**M1**	Correct method to solve a quadratic
	A1	0, –2
	M1	Correct method to solve inequality
$x \geq 0, x \leq -2$	**A1**	
	4	

8 **(i)**	$\frac{dy}{dx} = 3x^2 - 2kx + 1$	**B1**	One term correct
		B1	Fully correct
		2	
(ii)	$3x^2 - 2kx + 1 = 0$ when $x = 1$	**M1**	their $\frac{dy}{dx} = 0$ soi
	$3 - 2k + 1 = 0$	**M1**	$x = 1$ substituted into their $\frac{dy}{dx} = 0$
	$k = 2$	**A1√**	
		3	
(iii)	$\frac{d^2y}{dx^2} = 6x - 4$	**M1**	Substitutes $x = 1$ into their $\frac{d^2y}{dx^2}$ and looks at sign
	When $x = 1$, $\frac{d^2y}{dx^2} > 0$ ∴ min pt	**A1**	States minimum **CWO**
		2	
(iv)	$3x^2 - 4x + 1 = 0$	**M1**	their $\frac{dy}{dx} = 0$
	$(3x - 1)(x - 1) = 0$	**M1**	correct method to solve 3-term quadratic
	$x = \frac{1}{3}, x = 1$		
	$x = \frac{1}{3}$	**A1**	**WWW** at any stage
		3	

9	**(i)**		
	$(x-2)^2+(y-1)^2=100$	**B1**	$(x-2)^2$ and $(y-1)^2$ seen
		B1	$(x\pm 2)^2+(y\pm 1)^2=100$
	$x^2+y^2-4x-2y-95=0$	**B1**	correct form
		3	
	(ii) $(5-2)^2+(k-1)^2=100$	**M1**	$x = 5$ substituted into their equation
	$(k-1)^2=91$ or $k^2-2k-90=0$	**A1**	correct, simplified quadratic in k (or y) obtained
	$k=1+\sqrt{91}$	**A1**	cao
		3	
	(iii) distance from (–3, 9) to (2, 1)		
	$=\sqrt{(2--3)^2+(1-9)^2}$	**M1**	Uses $(x_2-x_1)^2+(y_2-y_1)^2$
	$=\sqrt{25+64}$	**A1**	
	$=\sqrt{89}$		
	$\sqrt{89}<10$ so point is inside	**B1**	compares their distance with 10 and makes consistent conclusion
		3	
	(iv) gradient of radius $=\frac{9-1}{8-2}$	**M1**	uses $\frac{y_2-y_1}{x_2-x_1}$
	$=\frac{4}{3}$	**A1**	oe
	gradient of tangent $=-\frac{3}{4}$	**B1√**	oe
	$y-9=-\frac{3}{4}(x-8)$	**M1**	correct equation of straight line through (8, 9), any non-zero gradient
	$y-9=-\frac{3}{4}x+6$		
	$y=-\frac{3}{4}x+15$	**A1**	oe 3 term equation
		5	

10 (i) $2(x^2-3x)+11$

$=2\left[\left(x-\frac{3}{2}\right)^2-\frac{9}{4}\right]+11$

$=2\left(x-\frac{3}{2}\right)^2+\frac{13}{2}$

B1 $p=2$

B1 $q=-\frac{3}{2}$

M1 $r=11-2q^2$ or $\frac{11}{2}-q^2$

A1 $r=\frac{13}{2}$

4

(ii) $\left(\frac{3}{2},\frac{13}{2}\right)$

B1√

B1√

2

(iii) $36-4\times2\times11$

$=-52$

M1 uses b^2-4ac

A1

2

(iv) 0 real roots

B1 cao

1

(v) $2x^2-6x+11=14-7x$

$2x^2+x-3=0$

$(2x+3)(x-1)=0$

$x=-\frac{3}{2}, x=1$

$y=\frac{49}{2}, y=7$

M1* substitute for *x*/*y* or attempt to get an equation in 1 variable only

A1 obtain correct 3 term quadratic

M1dep correct method to solve 3 term quadratic

A1

A1

SR If A0 A0, one correct pair of values, spotted or from correct factorisation **www B1**

5

4722 Core Mathematics 2

1	$(2 - 3x)^6 = 2^6 + 6.2^5.(-3x) + 15.2^4.(-3x)^2$	**M1**	Attempt (at least) first two terms - product of binomial coefficient and powers of 2 and (-)3x
	$= 64 - 576x + 2160x^2$	**A1**	Obtain $64 - 576x$
		M1	Attempt third term - binomial coefficient and powers of 2 and (-)3x
		A1	Obtain $2160x^2$
	OR		
		M1	Attempt expansion involving all 6 brackets
		A1	Obtain 64
		A1	Obtain $-576x$
		A1	Obtain $2160x^2$
	SR if the expansion is attempted in descending order, and the required terms are never seen, then **B1 B1 B1** for $4860x^4$, $-2916x^5$, $729x^6$		
		4	
2	**(i)** $u_2 = {}^2/_3$ $u_3 = {}^{-1}/_2$ $u_4 = 3$	**B1** **B1√** **B1√**	Obtain correct u_2 Obtain correct u_3 from their u_2 Obtain correct u_4 from their u_3
		3	
	(ii) sequence is periodic / cyclic / repeating	**B1**	Any equivalent comment
		1	
3	**(i)** $\frac{1}{2} \times 8^2 \times \theta = 48$ Hence $\theta = 1.5$ radians	**M1** **A1**	State or imply $(\frac{1}{2})\ 8^2\theta = 48$ Obtain $\theta = 1.5$ (or 0.477π), or equiv
		2	
	(ii) area $= 48 - \frac{1}{2} \times 8^2 \times \sin 1.5$ $= 48 - 31.9$ $= 16.1$	**M1*** **M1d*** **A1**	Attempt area of Δ using $(\frac{1}{2})\ 8^2 \sin\theta$ Attempt 48 – area of Δ Obtain 16.1 cm^2
		3	
4	**(i)** f(3) $= 27a - 36 - 21a + 12 = 0$ $6a = 24$ $a = 4$	**M1*** **M1d*** **A1**	**A**ttempt f(3) Equate attempt at f(3) to 0 and attempt to solve Obtain $a = 4$
	OR		
		M1*	Attempt complete division / matching coeffs
		M1d*	Equate remainder to 0
		A1	Obtain $a = 4$
		3	
	(ii) f(-2) $= -32 - 16 + 56 + 12$ $= 20$	**M1** **A1√**	Attempt f(-2) Obtain 20 (or $6a - 4$, following their a)
		2	

5 (i) $\int x\mathrm{d}y = \int ((y-3)^2 - 2)\mathrm{d}y$ **B1** Show $x = y^2 - 6y + 7$ convincingly

$= \int (y^2 - 6y + 7)\mathrm{d}y$ **A.G.** **B1** State or imply that required area = $\int x\mathrm{d}y$

$3 + \sqrt{(2+2)} = 5$, $3 + \sqrt{(14+2)} = 7$ **B1** Use $x = 2, 14$ to show new limits of $y = 5, 7$

3

(ii) $\left[\frac{1}{3}y^3 - 3y^2 + 7y\right]_5^7$ **M1** Integration attempt, with at least one term correct

$= (^{343}/_3 - 147 + 49) - (\,^{125}/_3 - 75 + 35)$ **A1** All three terms correct

$= 16^1/_3 - 1^2/_3$ **M1** Attempt F(7) – F(5)

$= 14\,^2/_3$ **A1** Obtain $14\,^2/_3$, or exact equiv

4

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6 (i) $ABC = 360 - (150 + 110) = 100^o$ **A.G.** **B1** Show convincingly that angle ABC is 100^o

1

(ii) $CA^2 = 15^2 + 27^2 - 2\times 15\times 27\times \cos 100^0$ **M1** Attempt use of correct cosine rule

$= 1094.655\ldots$

$CA = 33.1$ **A1** Obtain 33.1 km

2

(iii) $\frac{\sin C}{15} = \frac{\sin 100}{33.1}$ or $\frac{\sin A}{27} = \frac{\sin 100}{33.1}$ **M1** Attempt use of sine rule to find angle C or A (or equiv using cosine rule)

A1√ Correct unsimplified eqn, following their CA

$C = 26.5^o$ $A = 53.5^o$ **A1** Obtain $C = 26.5^o$ or $A = 53.5^o$ (allow 53.4^o)

Hence bearing is 263^o **A1√** Obtain 263 or 264 (or 290^o – their angle C / 210 + their angle A)

4

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7 (a) $\int (x^5 - x^4 + 5x^3)\mathrm{d}x$ **M1** Expand brackets and attempt integration, or other valid integration attempt

$= \frac{1}{6}x^6 - \frac{1}{5}x^5 + \frac{5}{4}x^4$ $(+c)$ **A1** Obtain at least one correct term

A1 Obtain a fully correct expression

B1 For $+c$, and no $\int$ or $\mathrm{d}x$ (can be given in **(b)(i)** if not given here)

4

(b) (i) $-6x^{-3}$ $(+c)$ **M1** Obtain integral of the form kx^{-3}

A1 Obtain $-6x^{-3}$ $(+c)$

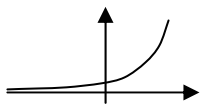
2

(ii) $\left[-6x^{-3}\right]_2^\infty$ **B1*** State or imply that $F(\infty) = 0$ (for kx^n, n -1)

$= ¾$ **B1d*** Obtain ¾ (or equiv)

2

8 (i)



M1 Attempt sketch of exponential graph (1st quad) - if seen in 2nd quad must be approx correct
A1 Correct graph in both quadrants
B1 State or imply (0, 2) only
3

(ii) $8^x = 2 \times 3^x$

$\log_2 8^x = \log_2 (2 \times 3^x)$ **M1** Form equation in x and take logs (to any consistent base, or no base) – could use $\log_8$

$x\log_2 8 = \log_2 2 + x\log_2 3$ **M1** Use $\log a^b = b \log a$
M1 Use $\log ab = \log a + \log b$,or equiv with $\log {}^a/_b$

$3x = 1 + x\log_2 3$ **M1** Use $\log_2 8 = 3$

$x(3 - \log_2 3) = 1$, hence $x = \dfrac{1}{3 - \log_2 3}$ **A.G.** **A1** Show given answer correctly

OR $8^x = 2 \times 3^x$

$2^{3x} = 2 \times 3^x$ **M1** Use $8^x = 2^{3x}$

$2^{(3x-1)} = 3^x$ **M1** Attempt to rearrange equation to $2^k = 3^x$

$\log_2 2^{(3x-1)} = \log_2 3^x$ **M1** Take logs (to any base)

$(3x - 1)\log_2 2 = x \log_2 3$ **M1** Use $\log a^b = b \log a$

$x(3 - \log_2 3) = 1$, hence $x = \dfrac{1}{3 - \log_2 3}$ **A.G.** **A1** Show given answer correctly

5

9 (a) (i) $2\sin x.\dfrac{\sin x}{\cos x} - 5 = \cos x$ **M1** Use $\tan x \equiv \dfrac{\sin x}{\cos x}$

$2\sin^2 x - 5\cos x = \cos^2 x$

$2 - 2\cos^2 x - 5\cos x = \cos^2 x$ **M1** Use $\sin^2 x \equiv 1 - \cos^2 x$

$3\cos^2 x + 5\cos x - 2 = 0$ **A1** Show given equation convincingly
3

(ii) $(3\cos x - 1)(\cos x + 2) = 0$ **M1** Attempt to solve quadratic in $\cos x$

$\cos x = {}^1/_3$ **M1** Attempt to find x from root(s) of quadratic

$x = 1.23$ rad **A1** Obtain 1.23 rad or 70.5°

$x = 5.05$ rad **A1√** Obtain 5.05 rad or 289° (or 2π / 360° - their solution)
SR: B1 B1 for answer(s) only
4

(b) $0.5 \times 0.25 \times \{\cos 0 + 2(\cos 0.25 + \cos 0.5 + \cos 0.75) + \cos 1\}$ **M1** Attempt y-coords for at least 4 of the correct 5 x-coords

M1 Use correct trapezium rule, any h, for their y values to find area between $x = 0$ and $x = 1$

M1 Correct h (soi) for their y values

≈ 0.837 **A1** Obtain 0.837
4

10 (i)	$u_{15} = 2 + 14 \times 0.5$ $= 9$ km	**M1** **A1** **2**	Attempt use of $a + (n-1)d$ Obtain 9 km
(ii)	$u_{20} = 2 \times 1.1^{19} = 12.2$ $u_{19} = 2 \times 1.1^{18} = 11.1$ ***OR***	**B1** **M1** **A1**	State, or imply, $r = 1.1$ Attempt u_{20} , using ar^{n-1} Obtain $u_{20} = 12.2$, and obtain $u_{19} = 11.1$
		B1 **M1** **A1** **3**	State, or imply, $r = 1.1$ Attempt to solve $ar^{n-1} = 12$ Obtain $n = 20$ (allow $n \geq 20$)
(iii)	$\frac{2(1.1^n - 1)}{(1.1-1)} > 200$ $1.1^n > 11$ $n > \frac{\log 11}{\log 1.1}$ $n > 25.2$ ie Day 26	**B1** **M1** **A1** **A1** **4**	State or imply $S_N = \frac{2(1.1^n - 1)}{(1.1-1)}$ Link (any sign) their attempt at S_N (of a GP) to 200 and attempt to solve Obtain 26, or 25.2 or better Conclude $n = 26$ only, or equiv eg Day 26
(iv)	swum = 2 x 30 = 60 km run = ½ x 30 x (4 + 29 x 0.5) $= 277.5$ km cycle = $\frac{2(1.1^{30} - 1)}{(1.1-1)}$ $= 329.0$ km total = 666 km	**B1** **M1** **M1** **A1** **4**	Obtain 60 km, or 2 x 30km Attempt sum of AP, $d = 0.5$, $a = 2$, $n = 30$ Attempt sum of GP, $r = 1.1$, $a = 2$, $n = 30$ Obtain 666 or 667 km

4723 Core Mathematics 3

1 Either: Obtain $x = 0$ **B1** ignoring errors in working
Form linear equation with signs of $4x$ and $3x$ different **M1** ignoring other sign errors
State $4x - 5 = -3x + 5$ **A1** or equiv without brackets
Obtain $\frac{10}{7}$ and no other non-zero value(s) **A1** or exact equiv
4

Or: Obtain $16x^2 - 40x + 25 = 9x^2 - 30x + 25$ **B1** or equiv
Attempt solution of quadratic equation **M1** at least as far as factorisation or use of formula
Obtain $\frac{10}{7}$ and no other non-zero value(s) **A1** or exact equiv
Obtain 0 **B1** ignoring errors in working
4

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2 (i) Show graph indicating attempt at reflection in $y = x$ **M1** with correct curvature and crossing negative y-axis and positive x-axis
Show correct graph with x-coord 2 and y-coord -3 indicated **A1**
2

(ii) Show graph indicating attempt at reflection in x-axis **M1** with correct curvature and crossing each negative axis
Show correct graph with x-coord -3 indicated **A1**
… and y-coord -4 indicated **A1**
[SC: Incorrect curve earning M0 but both correct intercepts indicated B1]
3

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3 Attempt use of product rule **M1** … + … form
Obtain $2x \ln x + x^2 . \frac{1}{x}$ **A1** or equiv
Substitute e to obtain 3e for gradient **A1** or exact (unsimplified) equiv
Attempt eqn of straight line with numerical gradient **M1** allowing approx values
Obtain $y - e^2 = 3e(x - e)$ **A1√** or equiv; following their gradient provided obtained by diffn attempt; allow approx values
Obtain $y = 3ex - 2e^2$ **A1** in terms of e now and in requested form
6

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4 (i) Differentiate to obtain form $kx(2x^2 + 9)^n$ **M1** any constant k; any $n < \frac{5}{2}$
Obtain correct $10x(2x^2 + 9)^{\frac{3}{2}}$ **A1** or (unsimplified) equiv
Equate to 100 and confirm $x = 10(2x^2 + 9)^{-\frac{3}{2}}$ **A1** **AG**; necessary detail required
3

(ii) Attempt relevant calculations with 0.3 and 0.4 **M1**
Obtain at least one correct value **A1**

x	f(x)	x – f(x)	f′(x)
0.3	0.3595	–0.0595	83.4
0.4	0.3515	0.0485	113.8

Obtain two correct values and conclude appropriately **A1** noting sign change or showing 0.3 < f(0.3) and 0.4 > f(0.4) or showing gradients either side of 100
3

	(iii)	Obtain correct first iterate	**B1**	
		Carry out correct process	**M1**	finding at least 3 iterates in all
		Obtain 0.3553	**A1**	answer required to exactly 4 dp
			3	
		[0.3 → 0.35953 → 0.35497 → 0.35534 → 0.35531; 0.35 → 0.35575 → 0.35528 → 0.35532 (→ 0.35531); 0.4 → 0.35146 → 0.35563 → 0.35529 → 0.35532]		
5	**(a)**	Obtain expression of form $\frac{a\tan\alpha}{b+c\tan^2\alpha}$	**M1**	any non-zero constants a, b, c
		State correct $\frac{2\tan\alpha}{1-\tan^2\alpha}$	**A1**	or equiv
		Attempt to produce polynomial equation in $\tan\alpha$	**M1**	using sound process
		Obtain at least one correct value of $\tan\alpha$	**A1**	$\tan\alpha = \pm\sqrt{\frac{4}{5}}$
		Obtain 41.8	**A1**	allow 42 or greater accuracy; allow 0.73
		Obtain 138.2 and no other values between 0 and 180	**A1**	allow 138 or greater accuracy
		[SC: Answers only 41.8 or … B1; 138.2 or … and no others B1]	**6**	
	(b)(i)	State $\frac{7}{6}$	**B1**	
			1	
	(ii)	Attempt use of identity linking $\cot^2\beta$ and $\text{cosec}^2\beta$	**M1**	or equiv retaining exactness; condone sign errors
		Obtain $\frac{13}{36}$	**A1**	or exact equiv
			2	
6		Integrate k_1e^{nx} to obtain k_2e^{nx}	**M1**	any constants involving π or not; any n
		Obtain correct indefinite integral of their k_1e^{nx}	**A1**	
		Substitute limits to obtain $\frac{1}{6}\pi(e^3-1)$ or $\frac{1}{6}(e^3-1)$	**A1**	or exact equiv perhaps involving e^0
		Integrate $k(2x-1)^n$ to obtain $k'(2x-1)^{n+1}$	**M1**	any constants involving π or not; any n
		Obtain correct indefinite integral of their $k(2x-1)^n$	**A1**	
		Substitute limits to obtain $\frac{1}{18}\pi$ or $\frac{1}{18}$	**A1**	or exact equiv
		Apply formula $\int\pi y^2\text{d}x$ at least once	**B1**	for $y=e^{3x}$ and/or $y=(2x-1)^4$
		Subtract, correct way round, attempts at volumes y^2	**M1**	allow with π missing but must involve
		Obtain $\frac{1}{6}\pi e^3-\frac{2}{9}\pi$	**A1**	or similarly simplified exact equiv
			9	
7	**(i)**	State $A=42$	**B1**	
		State $k=\frac{1}{9}$	**B1**	or 0.11 or greater accuracy
		Attempt correct process for finding m	**M1**	involving logarithms or equiv
		Obtain $\frac{1}{9}\ln 2$ or 0.077	**A1**	or 0.08 or greater accuracy
			4	
	(ii)	Attempt solution for t using either formula	**M1**	using correct process (log'ms or T&I or …)
		Obtain 11.3	**A1**	or greater accuracy; allow 11.3 ± 0.1
			2	
	(iii)	Differentiate to obtain form Be^{mt}	**M1**	where B is different from A
		Obtain $3.235e^{0.077t}$	**A1√**	or equiv; following their A and m
		Obtain 47.9	**A1**	allow 48 or greater accuracy
			3	

8	**(i)**	Show at least correct $\cos\theta\cos 60 + \sin\theta\sin 60$ or $\cos\theta\cos 60 - \sin\theta\sin 60$	**B1**	
		Attempt expansion of both with exact numerical values attempted	**M1**	and with $\cos 60 \neq \sin 60$
		Obtain $\frac{1}{2}\sqrt{3}\sin\theta + \frac{5}{2}\cos\theta$	**A1**	or exact equiv
			3	
	(ii)	Attempt correct process for finding R	**M1**	whether exact or approx
		Attempt recognisable process for finding α	**M1**	allowing sin / cos muddles
		Obtain $\sqrt{7}\sin(\theta + 70.9)$	**A1**	allow 2.65 for R; allow 70.9 ± 0.1 for α
			3	
	(iii)	Attempt correct process to find any value of θ + their α	**M1**	
		Obtain any correct value for $\theta + 70.9$	**A1**	–158, –22, 202, 338, …
		Attempt correct process to find θ + their α in 3rd quadrant	**M1**	or several values including this
		Obtain 131	**A1**	or greater accuracy and no other
		[SC for solutions with no working shown: Correct answer only B4; 131 with other answers B2]		
			4	
9	**(i)**	Attempt use of quotient rule	***M1**	or equiv; allow u / v muddles
		Obtain $\dfrac{75 - 15x^2}{(x^2 + 5)^2}$	**A1**	or (unsimplified) equiv; this **M1A1** available at any stage of question
		Equate attempt at first derivative to zero and rearrange to solvable form	**M1**	dep ***M**
		Obtain $x = \sqrt{5}$ or 2.24	**A1**	or greater accuracy
		Recognise range as values less than y-coord of st pt	**M1**	allowing < here
		Obtain $0 \leq y \leq \frac{3}{2}\sqrt{5}$	**A1**	any notation; with ≤ now; any exact equiv
			6	
	(ii)	State $\sqrt{5}$	**B1**√	following their x-coord of st pt; condone answer $x \geq \sqrt{5}$ but not inequality with k
			1	
	(iii)	Equate attempt at first derivative to –1 and attempt simplification	***M1**	and dependent on first **M** in part **(i)**
		Obtain $x^4 - 5x^2 + 100 = 0$	**A1**	or equiv involving 3 non-zero terms
		Attempt evaluation of discriminant or equiv	**M1**	dep ***M**
		Obtain –375 or equiv and conclude appropriately	**A1**	
			4	

4724 Core Mathematics 4

1 (a) $2x^2 - 7x - 4 = (2x+1)(x-4)$ or

$3x^2 + x - 2 = (3x-2)(x+1)$ **B1**

$\frac{2x+1}{3x-2}$ as final answer; this answer only **B1** Do not ISW

2

(b) For correct leading term x in quotient **B1** Identity method

For evidence of correct division process **M1** M1: $x^3 + 2x^2 - 6x - 5 = Q(x^2 + 4x + 1) + R$

Quotient = $x - 2$ **A1** M1: $Q = ax + b$ or $x + b, R = cx + d$ & ≥ 2 ops

[N.B. If $Q = x + b$, this $\Rightarrow$ 1 of the 2 ops]

Remainder = $x - 3$ **A1** A2: $a = 1, b = -2, c = 1, d = -3$ SR: B1 for two

4

---

2 Parts with correct split of $u = \ln x, \frac{dv}{dx} = x^4$ ***M1** obtaining result $f(x) + / - \int g(x)\,dx$

$\frac{x^5}{5}\ln x - \int \frac{x^5}{5}.\frac{1}{x}(dx)$ **A1**

$\frac{x^5}{5}\ln x - \frac{x^5}{25}$ **A1**

Correct method with the limits dep***M1** Decimals acceptable here

$\frac{4e^5}{25} + \frac{1}{25}$ ISW (Not '+c') **A1** Accept equiv fracts; like terms amalgamated

5

---

3 (i) $\frac{d}{dx}(x^2 y) = x^2\frac{dy}{dx} + 2xy$ or $\frac{d}{dx}(xy^2) = 2xy\frac{dy}{dx} + y^2$ ***B1**

Attempt to solve their differentiated equation for $\frac{dy}{dx}$ dep***M1**

$\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$ only **A1** WWW **AG** Must have intermediate line &…

…could imply "=0" on 1st line

3

(ii)(a) Attempt to solve **only** $y^2 - 2xy = 0$ & derive $y = 2x$ **B1** **AG** Any effort at solving $x^2 - 2xy = 0 \rightarrow$ B0

Clear indication why $y = 0$ is not acceptable **B1** Substituting $y = 2x \rightarrow$ B0, B0

2

(b) Attempt to solve $y = 2x$ simult with $x^2 y - xy^2 = 2$ **M1**

Produce $-2x^3 = 2$ or $y^3 = -8$ **A1** AEF

$(-1, -2)$ or $x = -1, y = -2$ **only** **A1**

3

4	**(i)**	For (either point) + t(difference between vectors) $\mathbf{r} = (3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) + t(-2\mathbf{i} + \mathbf{j} + \mathbf{k}$ or $2\mathbf{i} - \mathbf{j} - \mathbf{k})$	**M1** **A1** **2**	't' can be 's', 'λ' etc. '**r**' must be '**r**' but need not be bold Check other formats, e.g. $ta + (1-t)b$
	(ii)	State/imply that their **r** and their $-2\mathbf{i} + \mathbf{j} + \mathbf{k}$ are perpendicular Consider scalar product = 0	***M1** dep***M1**	N.B.This *M1 is dep on M1 being earned in (i)
		Obtain $t = -\frac{1}{6}$ or $\frac{1}{6}$ or $-\frac{5}{6}$ or $\frac{5}{6}$	**A1**	
		Subst their t into their equation of AB	**M1**	
		Obtain $\frac{1}{6}(16\mathbf{i} + 13\mathbf{j} + 19\mathbf{k})$ AEF	**A1** **5**	Accept decimals if clear
5	**(i)**	$(1-x)^{\frac{1}{2}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2$ ignoring x^3 etc	**B2**	SR Allow B1 for $1 - \frac{1}{2}x + kx^2$, $k \neq -\frac{1}{8}$ or 0
		$(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2$ ignoring x^3 etc	**B2**	SR Allow B1 for $1 - \frac{1}{2}x + kx^2$, $k \neq \frac{3}{8}$ or 0
		Product = $1 - x + \frac{1}{2}x^2$ ignoring x^3 etc	**B1** **5**	**AG**; with (at least) 1 intermediate step (cf x^2)
	(ii)	$\sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$ seen	**B1**	
		$\frac{37}{49}$ or $1 - \frac{2}{7} + \frac{1}{2}\left(\frac{2}{7}\right)^2$ seen	**B1**	
		$\frac{\sqrt{5}}{3} \approx \frac{37}{49} \Rightarrow \sqrt{5} \approx \frac{111}{49}$	**B1** **3**	**AG**
6	**(i)**	Produce at least 2 of the 3 relevant equations in t and s Solve for t and s $(t, s) = (4, -3)$ AEF Subst $(4, -3)$ into suitable equation(s) & show consistency	**M1** **M1** ***A1** dep***A1** **4**	$1 + 2t = 12 + s$, $3t = -4s$, $-5 + 4t = 5 - 2s$ Either into "3rd" eqn or into all 3 coordinates. N.B. Intersection coords not asked for
	(ii)	Method for finding magnitude of any vector Method for finding scalar product of any 2 vectors Using $\cos\theta = \frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ AEF for the correct 2 vectors 137 (136.8359) or 43.2(43.164…)	***M1** ***M1** dep***M1** **A1** **4**	Expect $\sqrt{29}$ and $\sqrt{21}$ Expect -18 Should be $-\frac{18}{\sqrt{29}\sqrt{21}}$ 2.39 (2.388236..) or 0.753(0.75335…) rads

7	(i)	Correct (calc) method for dealing with $\frac{1}{\sin x}$ or $(\sin x)^{-1}$	**M1**	
		Obtain $-\frac{\cos x}{\sin^2 x}$ or $-(\sin x)^{-2}\cos x$	**A1**	
		Show manipulation to –cosec x cot x (or vice-versa)	**A1**	WWW **AG** with ≥ 1 line intermed working
			3	
	(ii)	Separate variables, $\int(-)\frac{1}{\sin x \tan x}\,dx = \int \cot t\,dt$	**M1**	or $\int \frac{1}{\sin x \tan x}\,dx = \int(-)\cot t\,dt$
		Style: For the M1 to be awarded, dx and dt must appear on correct sides or there must be $\int$ sign on both sides		
		$\int -\text{cosec } x \cot x\,dx = \text{cosec } x$ (+c)	**A1**	or $\int \text{cosec } x \cot x\,dx = -\text{cosec } x$
		$\int \cot t\,dt = \ln \sin t$ or $\ln\lvert\sin t\rvert$ (+c)	**B1**	or $\int -\cot t\,dt = -\ln \sin t$ or $-\ln\lvert\sin t\rvert$
		Subst $(t,x) = \left(\frac{1}{2}\pi, \frac{1}{6}\pi\right)$ into their equation containing 'c'	**M1**	and attempt to find 'c'
		cosec $x = \ln \sin t + 2$ or $\ln\lvert\sin t\rvert + 2$	**A1**	WWW ISW; cosec $\frac{\pi}{6}$ to be changed to 2
			5	
8	(i)	$A(t+1) + B = 2t$	**M1**	Beware: correct values for A and/or B can be ...
		$A = 2$	**A1**	… obtained from a wrong identity
		B = –2	**A1**	Alt method: subst suitable values into given… …expressions
			3	
	(ii)	Attempt to connect dx and dt	**M1**	But not just dx = dt. As **AG**, look carefully.
		dx = t dt s.o.i. AEF	**A1**	
		$x + \sqrt{2x-1} \rightarrow \frac{t^2+1}{2} + t = \frac{(t+1)^2}{2}$ s.o.i.	**B1**	Any wrong working invalidates
		$\int \frac{2t}{(t+1)^2}\,dt$	**A1**	**AG** WWW The 'dt' must be present
			4	
	(iii)	$\int \frac{1}{t+1}\,dt = \ln(t+1)$	**B1**	Or parts $u = 2t, dv = (t+1)^{-2}$ or subst $u = t + 1$
		$\int \frac{1}{(t+1)^2}\,dt = -\frac{1}{t+1}$	**B1**	
		Attempt to change limits (expect 1 & 3) and use f(t)	**M1**	or re-substitute and use 1 and 5 on g(x)
		$\ln 4 - \frac{1}{2}$	**A1**	AEF (like terms amalgamated); if A0 A0 in (i), then final A0
			4	

9	**(i)**	$A: \theta = \frac{1}{2}\pi$ (accept 90°)	**B1**	
		$B: \theta = 2\pi$ (accept 360°)	**B2**	SR If B0 awarded for point *B*, allow B1 SR for any angle s.t. $\sin\theta = 0$
			3	
	(ii)	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	**M1**	or $\frac{dy}{d\theta}.\frac{d\theta}{dx}$ Must be used, not just quoted
		$\frac{dx}{d\theta} = 2 + 2\cos 2\theta$	**B1**	
		$2 + 2\cos 2\theta = 4\cos^2\theta$ with $\geq$ 1 line intermed work	***B1**	
		$\frac{dy}{dx} = \frac{4\cos\theta}{2 + 2\cos 2\theta}$ s.o.i.	**A1**	This & previous line are interchangeable
		$= \sec\theta$	dep***A1**	WWW **AG**
			5	
	(iii)	Equating $\sec\theta$ to 2 and producing at least one value of θ	**M1**	degrees or radians
		$(x =) -\frac{2}{3}\pi - \frac{\sqrt{3}}{2}$	**A1**	'Exact' form required
		$(y =) -2\sqrt{3}$	**A1**	'Exact' form required
			3	

4725 Further Pure Mathematics 1

1 (i) $\begin{pmatrix} 1 & 1 \\ 5 & -1 \end{pmatrix}$

B1 Two elements correct

B1 All four elements correct

2

(ii) *EITHER*

$\frac{1}{3}\begin{pmatrix} 2 & -1 \\ -5 & 4 \end{pmatrix}$

B1 Both diagonals correct

B1 Divide by determinant

2

OR

B1 Solve sim. eqns. 1st column correct

B1 2nd column correct

---

2 (i) 5

0.927 or 53.1^{o}

B1 Correct modulus

B1 Correct argument, any equivalent form

2

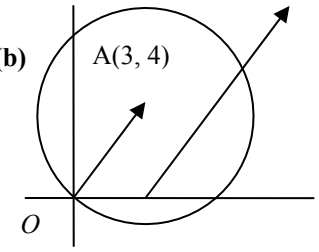
(ii)(a)

B1 Circle centre A (3, 4)

B1 Through O, allow if centre is (4, 3)

2

(b)



B1 Half line with +ve slope

B1 Starting at (3, 0)

B1 Parallel to OA, (implied by correct arg shown)

3

---

3 (i) $\frac{r}{(r+1)!}$

M1 Common denominator of $(r + 1)!$ or $r!(r + 1)!$

A1 Obtain given answer correctly

2

(ii) $1-\frac{1}{(n+1)!}$

M1 Express terms as differences using (i)

A1 At least 1st two and last term correct

M1 Show pairs cancelling

A1 Correct answer a.e.f.

4

4

B1 Establish result is true, for $n = 1$ (or 2 or 3)

M1 Attempt to multiply **A** and $\mathbf{A}^{n}$, or vice versa

M1 Correct process for matrix multiplication

A1 Obtain 3^{n+1}, 0 and 1

A1 Obtain $\frac{1}{2}(3^{n+1} - 1)$

A1 Statement of Induction conclusion, only if 5 marks earned, but may be in body of working

6

5			**M1**	Express as difference of two series
			M1	Use standard results
		$\frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1)$	**A1**	Correct unsimplified answer
			M1	Attempt to factorise
			A1	At least factor of $n(n+1)$
		$\frac{1}{12}n(n+1)(3n+2)(n-1)$	**A1**	Obtain correct answer
			6	
6	**(i)**	$3 - i$	**B1**	Conjugate stated
			1	
	(ii)	*EITHER*	**M1**	Use sum of roots
			A1	Obtain correct answer
			M1	Use sum of pairs of roots
			A1	Obtain correct answer
			M1	Use product of roots
		$a = -8,\ b = 22,\ c = -20$	**A1**	Obtain correct answers
			6	
		OR	**M1**	Attempt to find a quadratic factor
			A1	Obtain correct factor
			M1	Expand linear and quadratic factors
		$a = -8, b = 22, c = -20$	**A1A1A1**	Obtain correct answers
		OR		
			M1	Substitute 1 imaginary & the real root into eqn
			M1	Equate real and imaginary parts
			M1	Attempt to solve 3 eqns.
		$a = -8, b = 22, c = -20$	**A1A1A1**	Obtain correct answers
7	**(i)**		**B1**	Enlargement (centre O) scale factor 6
			1	
	(ii)		**B1**	Reflection
			B1	Mirror line is $y = x$
			2	
	(iii)		**B1**	Stretch in y direction
			B1	Scale factor 6, must be a stretch
			2	
	(iv)		**B1**	Rotation
			B1	36.9^o clockwise or equivalent
			2	

8	$\alpha+\beta=-k$	**B1**	State or use correct value
	$\alpha\beta=2k$	**B1**	State or use correct value
		M1	Attempt to express sum of new roots in terms of $\alpha+\beta$, $\alpha\beta$
	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$	**A1**	Obtain correct expression
	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{1}{2}(k-4)$	**A1**	Obtain correct answer a.e.f.
	$\alpha'\beta'=1$	**B1**	Correct product of new roots seen
	$x^2-\frac{1}{2}(k-4)x+1=0$	**B1ft**	Obtain correct answer, must be an eqn.
		7	
			Alternative for last 5 marks
		M1	Obtain expression for $u=\frac{\alpha}{\beta}$ in terms of k and α or k and β
		A1	Obtain a correct expression
		A1	rearrange to get α in terms of u
		M1	Substitute into given equation
		A1	Obtain correct answer
9	**(i)**	**M1**	Attempt to equate real and imaginary parts of $(x + iy)^2$ and $5 + 12i$
	$x^2 - y^2 = 5$ and $xy = 6$	**A1**	Obtain both results
		M1	Eliminate to obtain a quadratic in x^2 or y^2
	$\pm(3 + 2i)$	**M1**	Solve a 3 term quadratic & obtain x or y
		A1	Obtain correct answers as complex nos.
		5	
	(ii) $5 - 12i$	**B1B1**	Correct real and imaginary parts
		2	
	(iii)	**M1**	Attempt to solve a quadratic equation
	$x^2 = 5 \pm 12i$	**A1**	Obtain correct answers
	$x = \pm(3 \pm 2i)$	**A1A1**	Each pair of correct answers a.e.f.
		4	

10 (i)

M1 Find value of det **AB**
A1 Correct value 2 seen
2

(ii)

M1 Show correct process for adjoint entries
A1 Obtain at least 4 correct entries in adjoint
B1 Divide by their determinant

$$(\mathbf{AB})^{-1} = \frac{1}{2}\begin{pmatrix} 0 & 3 & -1 \\ 0 & -1 & 1 \\ 2 & 6-3a & a-6 \end{pmatrix}$$

A1 Obtain completely correct answer

4

(iii) *EITHER*

M1 State or imply $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$
A1 Obtain $\mathbf{B}^{-1} = (\mathbf{AB})^{-1} \times \mathbf{A}$
M1 Correct multiplication process seen
A1 Obtain three correct elements

$$\mathbf{B}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ -6 & 2 & -2 \end{pmatrix}$$

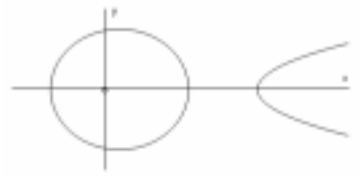
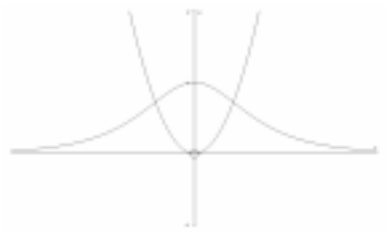
A1 All elements correct

5

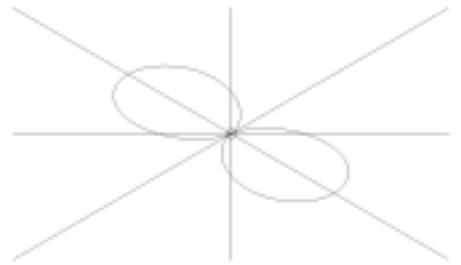
OR

M1 Attempt to find elements of **B**
A1 All correct
M1 Correct process for $\mathbf{B}^{-1}$
A1 3 elements correct
A1 All elements correct

4726 Further Pure Mathematics 2

1	Write as $\frac{A}{x-2a}+\frac{Bx+C}{x^2+a^2}$	**M1**	Accept C=0
	Get $2ax = A(x^2+a^2) + (Bx+C)(x-2a)$	**A1√**	Follow-on for C=0
	Choose values of x and/or equate coeff.	**M1**	Must lead to at least one of their A,B,C
	Get $A = {}^4/_5$, $B = -{}^4/_5$, $C = {}^2/_5 a$	**A1**	For two correct from correct working only
		A1	For third correct
		5	
2		**B1**	Get (4,0), (3,0), (-2,0) only
		B1	Get $(0,\sqrt{5})$ as "maximum"
		B1	Meets x-axis at 90^0 at all crossing points
		B1	Use $-2 \le x \le 3$ and $x \ge 4$ only
		B1	Symmetry in Ox
		5	
3	Quote/derive $dx = \frac{2}{1+t^2}\,dt$	**B1**	
	Replace all x and dx from their expressions	**M1**	Not dx=dt; ignore limits
	Tidy to $2/(3t^2+1)$	**A1**	Not $a/(3t^2+1)$
	Get $k \tan^{-1}(At)$	**M1**	Allow A=1 if from $p/(t^2+1)$ only
	Get $k = \frac{2}{3}\sqrt{3}$, $A = \sqrt{3}$	**A1√**	Allow $k=a/\sqrt{3}$ from line 3; AEEF
	Use limits correctly to ${}^2/_9\sqrt{3}\pi$	**A1**	AEEF
		6	
4 (i)		**B1**	Correct $y = x^2$
		B1	Correct shape/asymptote
		B1	Crossing (0,1)
		3	
(ii)	Define sech $x = 2/(e^x + e^{-x})$	**B1**	AEEF
	Equate their expression to x^2 and attempt to simplify	**M1**	
	Clearly get A.G.	**A1**	
		3	
(iii)	Cobweb	**B1**	
	Values > and then < root	**B1**	Only from cobweb
		2	

5	**(i)**	Factorise to $\tan^{n-2}x(1+\tan^2 x)$	**B1**	Or use $\tan^n x = \tan^{n-2}x.\tan^2 x$
		Clearly use $1+\tan^2 = \sec^2$	**M1**	Allow wrong sign
		Integrate to $\tan^{n-1}x/(n-1)$	**A1**	Quote or via substitution
		Use limits and tidy to A.G.	**A1**	Must be clearly derived
			4	
	(ii)	Get $3(I_4 + I_2) = 1$, $I_2 + I_0 = 1$	**B1**	Write down one correct from reduction formula
		Attempt to evaluate I_0 (or I_2)	**M1**	$I_2 = a\tan x + b$, $a,b \neq 0$
		Get ¼π (or 1 - ¼π)	**A1**	
		Replace to ¼π - ⅔	**A1**	
			4	
6	**(i)**	Attempt to use N-R of correct form with clear f′(x) used	**M1**	
		Get 2.633929, 2.645672	**A1**	For one correct to minimum of 6 d.p.
			A1√	For other correct from their x_2 in correct NR
			3	
	(ii)	√7	**B1**	Allow ±
			1	
	(iii)	Get $e_1 = 0.14575$, $e_2 = 0.01182$	**B1**√	From their values
		Get $e_3 = 0.00008$	**B1**√	
		Verify both ≈ 0.00008	**B1**	From $0.000077..$ or $0.01182^3/0.14575^2$
			3	
7	**(i)**	Attempt quotient/product on bracket	**M1**	
		Get $-3/(2+x)^2$	**A1**	May be implied
		Use Formulae Booklet or derive from $\tanh y = (1-x)/(2+x)$	**M1**	Attempt $\tanh^{-1}$ part in terms of x
		Get $\dfrac{-3}{(2+x)^2}.\dfrac{1}{1-((1-x)/(2+x))^2}$	**A1**√	From their results above
		Clearly tidy to A.G.	**A1**	
		Get $f''(x) = 2/(1+2x)^2$	**B1**	cao
			6	
			SC	Use reasonable ln definition **M1**
				Get $y=\frac{1}{2}\ln((1-k)/(1+k))$ for $k=(1-x)/(1+2x)$ **A1**
				Tidy to $y=\frac{1}{2}\ln(3/(1+2x))$ **A1**
				Attempt chain rule **M1**
				Clearly tidy to A.G. **A1**
				Get $f''(x)$ **B1**
	(ii)	Attempt f(0), f′(0) and f″(0)	**M1**	From their differentiation
		Get $\tanh^{-1}$ ½, -1 and 2	**A1**√	
		Replace $\tanh^{-1}$ ½ = ½ ln3 (=ln√3)	**B1**	Only
		Get $\ln\sqrt{3} - x + x^2$	**A1**	
			4	
			SC	Use standard expansion from ½ln3-½ln(1+2x)

8	**(i)**	Attempt to solve $r = 0$	**M1**	
		Get $\alpha = \frac{1}{4}\pi$	**A1**	From correct method; ignore others; allow θ
			2	
	(ii)	**(a)** Get $1 - \sin((2k+1)\pi - 2\theta)$	**M1**	Attempt $f(\frac{1}{2}(2k+1)\pi - \theta)$, leading to 2θ here
		Expand as $\sin(A+B)$	**M1**	Or discuss periodicity for general k
		Use k as integer so $\sin(2k+1)\pi = 0$,		
		And $\cos(2k+1)\pi = -1$	**A1**	Needs a clear explanation
			3	
		(b) Quote $\frac{1}{4}(2k+1)\pi$	**B1**	For general answer or 2 correct (ignore other answers given)
		Select or give $k = 0,1,2,3$	**B1**	For all 4 correct in $0 \le \theta < 2\pi$
			2	
		(iii) roughly 	**B1**	Correct shape; 2 branches only, as shown
			B1	Clear symmetry in correct rays
			B1	Get max. $r = 2$
			B1	At $\theta = \frac{3}{4}\pi$ and $\frac{7}{4}\pi$; both required (allow correct answers not in $0 \le \theta < 2\pi$ here)
			4	
9	**(i)**	Attempt to use parts	**M1**	Two terms, one yet to be integrated
		Divide out $x/(1+x)$	**M1**	Or use substitution
		Correct answer		
		$x\ln(1+x) - x + \ln(1+x)$	**A1**	
		Limits to correct A.G.	**A1**	
			4	
			SC	Quote $\int \ln x \, dx$ **M1**
				Clear use of limits to A.G. **A1**
			SC	Attempt to diff'ate by product rule **M1**
				Clear use of limits to A.G. **A1**
	(ii)	**(a)** Use sum of areas of rect.<		
		Area under curve (between		
		limits 0 and 70)	**B1**	
		Areas = 1x heights = $1(\ln 2 + \ln 3 + \ldots \ln 70)$	**B1**	Areas to be specified
			2	
		(b) Explain use of 69	**B1**	Allow diagram or use of left shift of 1 unit
		Explain first rectangle	**B1**	
		Areas as above > area under curve	**B1**	
			3	
	(c)	Show/quote $\ln 2 + \ln 3 + \ldots \ln 70 = \ln 70!$	**B1**	
		Use $N = 69, 70$ in (i)	**M1**	No other numbers; may be implied by 228.39.. or 232.65.. seen; allow 228.4, 232.6 or 232.7
		Get 228.3, 232.7	**A1**	
			3	

4727 Further Pure Mathematics 3

1 (a)(i)	e, r^3, r^6, r^9	M1		For stating e, r^m (any m .. **2**), and 2 other different elements in terms of e and r
		A1	**2**	For all elements correct
(ii)	r generates G	B1	**1**	For this or any statement equivalent to: all elements of G are included in a group with e and r *OR* order of r > order of all possible proper subgroups
(b)	m, n, p, mn, np, pm	B1		For any 3 orders correct
		B1	**2**	For all 6 correct and no extras (Ignore 1 and mnp)
			5	

2	METHOD 1			
	$[1, 3, 2] \times [1, 2, -1]$	M1		For attempt to find normal vector, e.g. by finding vector product of correct vectors, or Cartesian equation
	$\mathbf{n} = k[-7, 3, -1]$ *OR* $7x - 3y + z = c\ (=17)$	A1		For correct vector *OR* LHS of equation
		M1√		For using correct vectors for line and plane f.t. from normal
	$\theta = \sin^{-1} \dfrac{\lvert[1, 4, -1] \,.\, [-7, 3, -1]\rvert}{\sqrt{1^2 + 4^2 + 1^2}\sqrt{7^2 + 3^2 + 1^2}}$	M1*		For using scalar product of line and plane vectors
		M1		For calculating both moduli in denominator
	$\theta = \sin^{-1} \dfrac{6}{\sqrt{18}\sqrt{59}} = 10.6°$	A1√ (*dep)		For scalar product. f.t. from their numerator
	$(10.609...°, 0.18517...)$	A1	**7**	For correct angle
	METHOD 2			
	$[1, 3, 2] \times [1, 2, -1]$	M1		For attempt to find normal vector, e.g. by finding vector product of correct vectors, or Cartesian equation
	$\mathbf{n} = k[-7, 3, -1]$ *OR* $7x - 3y + z = c$	A1		For correct vector *OR* LHS of equation
	$7x - 3y + z = 17$	M1√		For attempting to find RHS of equation f.t. from **n** or LHS of equation
	$d = \dfrac{\lvert 21 - 12 + 2 - 17\rvert}{\sqrt{7^2 + 3^2 + 1^2}} = \dfrac{6}{\sqrt{59}}$	M1		For using distance formula from a point on the line, e.g. (3, 4, 2), to the plane
		A1√		For correct distance. f.t. from equation
	$\theta = \sin^{-1} \dfrac{\frac{6}{\sqrt{59}}}{\sqrt{1^2 + 4^2 + 1^2}} = 10.6°$	M1		For using trigonometry
		A1		For correct angle
	$(10.609...°, 0.18517...)$			
			7	

3 (i)	$\dfrac{dz}{dx} = 1 + \dfrac{dy}{dx}$	M1		For differentiating substitution (seen or implied)
	$\dfrac{dz}{dx} - 1 = \dfrac{z+3}{z-1} \Rightarrow \dfrac{dz}{dx} = \dfrac{2z+2}{z-1} = \dfrac{2(z+1)}{z-1}$	A1		For correct equation in z **AEF**
		A1	**3**	For correct simplification to **AG**
(ii)	$\int \dfrac{z-1}{z+1} dz = 2\int dx$	B1		For $\int \dfrac{z-1}{z+1}(dz)$ and $\int (1)\,(dx)$ seen or implied
	$\Rightarrow \int 1 - \dfrac{2}{z+1} dz$ *OR* $\int 1 - \dfrac{2}{u} du = 2x\ (+c)$	M1		For rearrangement of LHS into integrable form *OR* substitution e.g. $u = z+1$ or $u = z-1$
	$\Rightarrow$ $z - 2\ln(z+1)$ *OR* $z + 1 - 2\ln(z+1) = 2x\ (+c)$	A1		For correct integration of LHS as f(z)
	$\Rightarrow -2\ln(x + y + 1) = x - y + c$	A1	**4**	For correct general solution **AEF**

7

4	**(i)**	$\cos^5\theta = \left(\dfrac{e^{i\theta}+e^{-i\theta}}{2}\right)^5$	B1	For $\cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}$ seen or implied z may be used for $e^{i\theta}$ throughout
		$\cos^5\theta = \frac{1}{32}\left(e^{i\theta}+e^{-i\theta}\right)^5$	M1	For expanding $\left(e^{i\theta}+e^{-i\theta}\right)^5$. At least 3 terms and 2 binomial coefficients required *OR* reasonable attempt at expansion in stages
		$\cos^5\theta = \frac{1}{32}\left(e^{5i\theta}+e^{-5i\theta}+5\left(e^{3i\theta}+e^{-3i\theta}\right)+10\left(e^{i\theta}+e^{-i\theta}\right)\right)$	A1	For correct binomial expansion
		$\cos^5\theta = \frac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$	M1 A1 **5**	For grouping terms and using multiple angles For answer obtained correctly **AG**
	(ii)	$\cos\theta = 16\cos^5\theta$	B1	For stating correct equation of degree 5 *OR* $1 = 16\cos^4\theta$ **AEF**
		$\Rightarrow \cos\theta = 0, \quad \cos\theta = \pm\frac{1}{2}$	M1	For obtaining at least one of the values of $\cos\theta$ from $\cos\theta = k\cos^5\theta$ *OR* from $1 = k\cos^4\theta$
		$\Rightarrow \theta = \frac{1}{2}\pi,\ \frac{1}{3}\pi,\ \frac{2}{3}\pi$	A1 A1 **4**	A1 for any two correct values of θ A1 for the 3rd value and no more in $0 \le \theta \le \pi$ Ignore values outside $0 \le \theta \le \pi$
			9	

5 (i) METHOD 1

Lines meet where

$(x =)\ k+2\lambda = k+\mu$
$(y =)\ -1-5\lambda = -4-4\mu$
$(z =)\ 1-3\lambda = -2\mu$

M1 For using parametric form to find where lines meet
A1 For at least 2 correct equations

M1 For attempting to solve any 2 equations

$\Rightarrow \lambda = -1,\ \mu = -2$ A1 For correct values of λ and μ

B1 For attempting a check in 3rd equation *OR* verifying point of intersection is on both lines

$\Rightarrow (k-2, 4, 4)$ A1 **6** For correct point of intersection (allow vector)

SR For finding λ *OR* μ and point of intersection, but no check, award up to M1 A1 M1 A0 B0 A1

METHOD 2

$d = \dfrac{|[0,3,1].[2,-5,-3]\times[1,-4,-2]|}{|\mathbf{b}\times\mathbf{c}|}$ For using $\mathbf{a}.\mathbf{b}\times\mathbf{c}$ with appropriate vectors (division by $|\mathbf{b}\times\mathbf{c}|$ is not essential)

$d = c[0,3,1].[-2,1,-3] = 0$ B1 and showing $d = 0$ correctly

$\Rightarrow$ lines intersect

Lines meet where

$(x =)\ (k+)\ 2\lambda = (k+)\ \mu$
$(y =)\ -1-5\lambda = -4-4\mu$
$(z =)\ 1-3\lambda = -2\mu$

M1 For using parametric form to find where lines meet
A1 For at least 2 correct equations

M1 For attempting to solve any 2 equations

$\Rightarrow \lambda = -1,\ \mu = -2$ A1 For correct value of λ *OR* μ

$\Rightarrow (k-2, 4, 4)$ A1 For correct point of intersection (allow vector)

METHOD 3

e.g. $x-k = \dfrac{2(y+1)}{-5} = \dfrac{y+4}{-4}$ M1 For solving one pair of simultaneous equations

$\Rightarrow y = 4$ A1 For correct value of x, y or z

$\dfrac{z-1}{-3} = \dfrac{y+1}{-5}$ M1 For solving for the third variable

$x = k-2$ *OR* $z = 4$ A1 For correct values of 2 of x, y and z

$x-k = \dfrac{z}{-2}$ checks with $x = k-2,\ z = 4$ B1 For attempting a check in 3rd equation

$\Rightarrow (k-2, 4, 4)$ A1 For correct point of intersection (allow vector)

(ii) METHOD 1

$\mathbf{n} = [2,-5,-3]\times[1,-4,-2]$ M1 For finding vector product of 2 directions

$\mathbf{n} = c[-2,1,-3]$ A1 For correct normal

SR Following Method 2 for **(i),** award M1 A1√ for **n**, f.t. from their **n**

$(1,-1,1)$ *OR* $(1,-4,0)$ *OR* $(-1,4,4)$ M1 For substituting a point in LHS

$\Rightarrow 2x-y+3z = 6$ A1 **4** For correct equation of plane **AEF cartesian**

METHOD 2

$\mathbf{r} = [1,-1,1]+\lambda[2,-5,-3]+\mu[1,-4,-2]$ M1 For using vector equation of plane (*OR* $[1,-4,0]$ for **a)**

$x = 1+2\lambda+\mu$
$y = -1-5\lambda-4\mu$
$z = 1-3\lambda-2\mu$

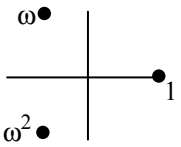
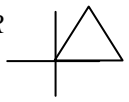
A1 For writing 3 linear equations

M1 For eliminating λ and μ

$\Rightarrow 2x-y+3z = 6$ A1 For correct equation of plane **AEF cartesian**

10

6	**(i)**	When a, b have opposite signs,	M1	For considering sign of $a\|b\|$ *OR* $b\|a\|$ in general or in a specific case
		$a\|b\| = \pm ab$, $b\|a\| = \mp ba \Rightarrow a\|b\| \neq b\|a\|$	A1 **2**	For showing that $a\|b\| \neq b\|a\|$ Note that $\|x\| = \sqrt{x^2}$ may be used
	(ii)	$(a \circ b) \circ c = (a\|b\|) \circ c = a\|b\|\|c\|$ *OR* $a\|bc\|$	M1	For using 3 distinct elements and simplifying $(a \circ b) \circ c$ *OR* $a \circ (b \circ c)$
			A1	For obtaining correct answer
		$a \circ (b \circ c) = a \circ (b\|c\|) = a\|b\|c\|\| = a\|b\|\|c\|$ *OR* $a\|bc\|$	M1	For simplifying the other bracketed expression
			A1 **4**	For obtaining the same answer
	(iii)		B1*	For stating $e = \pm 1$ *OR* no identity
		EITHER $a \circ e = a \Rightarrow \|e\| = a \Rightarrow e = \pm 1$	M1	For attempting algebraic justification of +1 and –1 for e
		OR $e \circ a = e\|a\| = a$ $\Rightarrow e = 1$ for $a > 0$, $e = -1$ for $a < 0$	A1	For deducing no (unique) identity
		Not a group	B1 (*dep) **4**	For stating not a group
			10	

7 (i)		B1	**1**	Polar or cartesian values of ω and ω^2 may be used anywhere in this question For showing 3 points in approximately correct positions Allow ω and ω^2 interchanged, or unlabelled
(ii)	*EITHER* $1+\omega+\omega^2$ = sum of roots of cubic = 0	M1 A1	**2**	For result shown by any correct method **AG**
	OR $\omega^3=1\Rightarrow(\omega-1)(\omega^2+\omega+1)=0$ $\Rightarrow 1+\omega+\omega^2=0$ (for $\omega\neq 1$)			
	OR sum of G.P. $1+\omega+\omega^2=\frac{1-\omega^3}{1-\omega}\left(=\frac{0}{1-\omega}\right)=0$			
	OR  shown on Argand diagram or explained in terms of vectors			Reference to vectors in part **(i)** diagram may be made
	OR $1+\text{cis}\,\frac{2}{3}\pi+\text{cis}\,\frac{4}{3}\pi=1+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\text{i}\right)+\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\text{i}\right)=0$			
(iii) (a)	$(2+\omega)(2+\omega^2)=4+2(\omega+\omega^2)+\omega^3$ $=4-2+1=3$	M1 A1	**2**	For using $1+\omega+\omega^2=0$ *OR* values of ω, ω^2 For correct answer
(b)	$\frac{1}{2+\omega}+\frac{1}{2+\omega^2}=\frac{2+(\omega+\omega^2)+2}{3}=1$	M1 A1√	**2**	For combining fractions *OR* multiplying top and bottom of 2 fractions by complex conjugates For correct answer f.t. from **(a)**
(iv)	For the cubic $x^3+px^2+qx+r=0$ METHOD 1			
	$\sum\alpha=2+1=3$ $(\Rightarrow p=-3)$	M1		For calculating two of $\sum\alpha$, $\sum\alpha\beta$, $\alpha\beta\gamma$
	$\sum\alpha\beta=\frac{2}{2+\omega}+\frac{2}{2+\omega^2}+\frac{1}{3}=\frac{7}{3}$ $(=q)$	M1		For calculating all of $\sum\alpha$, $\sum\alpha\beta$, $\alpha\beta\gamma$ *OR* all of *p*, *q*, *r*
	$\alpha\beta\gamma=\frac{2}{3}$ $\left(\Rightarrow\ r=-\frac{2}{3}\right)$	A1		For at least two of $\sum\alpha$, $\sum\alpha\beta$, $\alpha\beta\gamma$ correct (or values of *p*, *q*, *r*)
	$\Rightarrow\ 3x^3-9x^2+7x-2=0$	A1	**4**	For correct equation **CAO**
	METHOD 2			
	$\left(x-2\right)\left(x-\frac{1}{2+\omega}\right)\left(x-\frac{1}{2+\omega^2}\right)=0$			
	$x^3+\left(-2-\frac{1}{2+\omega}-\frac{1}{2+\omega^2}\right)x^2$	M1		For multiplying out LHS in terms of ω or $\text{cis}\,\frac{1}{3}k\pi$
	$+\left(\frac{1}{(2+\omega)(2+\omega^2)}+\frac{2}{2+\omega}+\frac{2}{2+\omega^2}\right)x$			
	$-\frac{2}{(2+\omega)(2+\omega^2)}=0$	M1		For simplifying, using parts **(ii)**, **(iii)** or values of ω
	$\Rightarrow\ x^3-3x^2+\frac{7}{3}x-\frac{2}{3}=0$	A1		For at least two of *p*, *q*, *r* correct
	$\Rightarrow\ 3x^3-9x^2+7x-2=0$	A1		For correct equation **CAO**
		11		

8 (i)	$m^2+1=0 \Rightarrow m=\pm \mathrm{i}$	M1		For stating and attempting to solve correct auxiliary equation
	$\Rightarrow$ C.F. $(y=)\ Ce^{\mathrm{i}x}+De^{-\mathrm{i}x}=A\cos x+B\sin x$	A1	**2**	For correct C.F. (must be in trig form) **SR** If some or all of the working is omitted, award full credit for correct answer
(ii)(a)	$y=p(\ln\sin x)\sin x+qx\cos x$	M1		For attempting to differentiate P.I. (product rule needed at least once)
	$\frac{\mathrm{d}y}{\mathrm{d}x}=p\frac{\cos x}{\sin x}\sin x+p(\ln\sin x)\cos x+q\cos x-qx\sin x$	A1		For correct (unsimplified) result **AEF**
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-p\sin x-p(\ln\sin x)\sin x+\frac{p\cos^2 x}{\sin x}$ $-2q\sin x-qx\cos x$	A1		For correct (unsimplified) result **AEF**
	$-p\sin x+\frac{p\cos^2 x}{\sin x}-2q\sin x\equiv\frac{1}{\sin x}$	M1		For substituting their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and y into D.E.
		M1		For using $\sin^2 x+\cos^2 x=1$
	$\Rightarrow p-2(p+q)\sin^2 x\equiv 1$	A1	**6**	For simplifying to **AG** ($\equiv$ may be $=$)
(b)		M1		For attempting to find p and q by equating coefficients of constant and $\sin^2 x$ *AND/OR* giving value(s) to x (allow any value for x, including 0)
	$p=1,\ q=-1$	A1	**2**	For both values correct
(iii)	G.S. $y=A\cos x+B\sin x+(\ln\sin x)\sin x-x\cos x$	B1√		For correct G.S. f.t. from their C.F. and P.I. with 2 arbitrary constants in C.F. (allow given form of P.I. if p and q have not been found)
	$\mathrm{cosec}\,x$ undefined at $x=0,\pi,2\pi$ *OR* $\sin x>0$ in $\ln\sin x$	M1		For considering domain of $\mathrm{cosec}\,x$ *OR* $\sin x\neq 0$ *OR* $\ln\sin x$ term
	$\Rightarrow 0<x<\pi$	A1	**3**	For stating correct range **CAO** **SR** Award B1 for correct answer with justification omitted or incorrect
			13	

4728 Mechanics 1

1(i)	900a = 600 -240	M1	N2L with difference of 2 forces, accept 360
	$a = 0.4$ ms^{-2} AG	A1	
		[2]	
(ii)	$9 = 5 + 0.4t$	M1	$v = u + 0.4t$ or $v = u +$(cv 0.4)t
	$t = 10$ s	A1	
	$9^2 = 5^2 + 2x0.4s$	M1	or $s=(u+v)t/2$ or $s=ut+0.5$xcv$(0.4)t^2$
	$s = 70$ m	A1	
		[4]	

2(i)	Resolves a force in 2 perp. directions	M1*	*Uses vector addition or subtraction*
	Uses Pythagoras $R^2 =$	D*M1	*Uses cosine rule* $R^2 =$
	$(14\sin30)^2 +$	A1	$14^2 + 12^2 -$
	$(12+14\cos30)^2$	A1	$2x14x12\cos150$
	{or $R^2 = (12\sin30)^2 + (14+12\cos30)^2$ }		
	$R = 25.1$ AG	A1	cso (Treat $R^2 = 14^2+12^2 + 2x14x12\cos30$
(ii)		[5]	as correct)
	Trig to find angle in a valid triangle	M1	Angle should be relevant
	tanB=7/24.1,sinB=7/25.1,cosB=24.1/25.	A1	*sinB/14 = sin150/25.1.* Others possible.
	B = 016, (0)16.1° or (0)16.2°	A1	Cosine rule may give (0)16.4, award A1
		[3]	

3(i)	$a = 6/5$	M1	Acceleration is gradient idea, for portion of graph
	$a = 1.2$ ms^{-2}	A1	Accept 6/5
(ii)		[2]	
	$s = (6x10/2)$ {or (6x5/2)	M1	Area under graph idea or a formula used correctly
	x2 x4}	M1	Double {Quadruple} journey
(iii)	$s = 60$ m	A1	
		[3]	
		M1	v=u+at idea, t not equal to 17 (except v=1.2t-24)
	$v = -6 + 1.2(17-15)$	A1	$0 = v +$ cv$(1.2)(20-17)$, $v^2 -2.4v -21.6 = 0$, etc
	$v = -3.6$ ms^{-1}	A1	***SR*** v=3.6 neither A1, but give both A1 if final answer
		[3]	given is -3.6

4(i)		M1	Difference of 2 horizontal components, both < 15
	$F = 15\sin50 - 15\sin30 = 3.99$ N	A1	Not 4 or 4.0
	Left	B1	Accept reference to 30 degree string
		[3]	May be given in ii if not attempted in i
(ii)		M1	Equating 4 vertical forces/components
	$R = f(30, 15\cos50, 15\cos30)$	A1	30g is acceptable
	$R = 30-15\cos50-15\cos30$	A1	=7.36(78..), treat 30g as a misread
	$\mu = 3.99/7.36(78\ )$	M1	Using $F = \mu R$, with cv(3.99) and cv(7.36(78..))
	$\mu = 0.541$ or 0.542 or 0.543	A1	Accept 0.54 from correct work, e.g. 4/7.4
		[5]	

5(i)	2400x5 - 3600x3	B1	Award if g included
	2400v + 3600v	B1	Award if g included
	2400x5 - 3600x3 = 2400v + 3600v	M1	Equating momentums (award if g included)
	$v = 0.2$ ms^{-1}	A1	Not given if g included or if negative.
	B	B1	
		[5]	
(ii)(a)	+/-(-2400v + 3600v)	B1	*No marks in(ii) if g included*
	2400x5 - 3600x3 = -2400v + 3600v	M1	Equating momentums if "after" signs differ
	$v = 1$ ms^{-1}	A1	Do not accept if - sign "lost"
(b)	I = 2400 x (5+/-1) or 3600 x (3+/-1)	M1	Product of either mass and velocity change
	I = 14400 kgms^{-1}	A1	Accept -14400
		[5]	

6(i)	$x = 0.01t^4 - 0.16t^3 + 0.72t^2$.		
	v = dx/dt	M1	Uses differentiation, ignore +c
	$v = 0.04t^3 - 0.48t^2 + 1.44t$.	A1	or $v = 4(0.01t^3) - 3(0.16t^2) + 2(0.72t)$
	$v(2)=1.28$ ms^{-1} AG	A1	Evidence of evaluation needed
		[3]	
(ii)	a = dv/dt	M1	Uses differentiation
	$a = 0.12t^2 - 0.96t + 1.44$	A1	or $a = 3(0.04t^2) - 2(0.48t) + 1.44$
	$t^2 - 8t + 12 = 0$ AG	A1	Simplifies $0.12t^2 - 0.96t + 1.44 = 0$, (or verifies the roots of QE make acceleration zero)
		[3]	
(iii)	(t - 2)(t - 6) = 0	M1	Solves quadratic (may be done in ii <u>if used to find v(6)</u>)
	t = 2	A1	Or *Factorises v into 3 linear factors* M1
	t = 6	A1	$v = 0.04t(t-6)^2$ A1 *Identifies t=6* A1
	v(6) = 0 ms^{-1}	B1	Evidence of evaluation needed
		[4]	
(iv)		B1	Starts at origin
		B1	Rises to single max, continues through single min
		B1	Minimum on t axis, non-linear graph
	Away from A	B1	
		[4]	
(v)	$AB = 0.01x6^4 - 0.16x6^3 + 0.72x6^2$	M1	Or integration of v(t), with limits 0, 6 or substitution,
	AB = 4.32 m	A1	using cv(6) from iii
		[2]	

7(i)	(R=)0.2x9.8cos45	M1	Not F = 0.2x9.8cos45 or 0.2x9.8sin 45 unless followed
	F=1xR=1x.2x9.8cos45=1.386 N AG	A1	by (eg) Fr =1x F = 1.386 when M1A1
		[2]	
(ii)	Any 1 application of N2L // to plane with correct mass and number of forces	M1	Must use component of weight
	0.4a=0.2gsin45+0.2gsin45-1.38(592..)	A1	
	a = 3.465 ms^{-2} AG	A1	
	0.2a = 0.2gsin45 - T or		Accept with 3.465 (or close) instead of a
	0.2a = T +[0.2gsin45 - 1.38(592..)]	M1	Accept omission of [term] for M1
	T = 0.693 N	A1	Accept 0.69
		[5]	
	OR		
	Any 1 application of N2L // to plane with correct mass and number of forces		Must use component of weight
	0.2a = 0.2gsin45 - T or	M1	Either correct
	0.2a = T +[0.2gsin45 - 1.38(592..)]	A1	Both correct. Accept omission of [term] for A1 only
	Eliminates a or T	M1	
	a = 3.465 ms^{-2} AG	A1	
	T = 0.693 N	A1	
(iii)	$v^2 = 2 \times 3.465 \times 0.5$	M1	Using $v^2 = 0^2 + 2xcv(3.465)s$
	v = 1.86 ms^{-1}	A1	
		[2]	
(iv)	For Q		
	(0.2)a = (0.2)gsin45 - (1)(0.2)gcos45.	M1	Attempting equation to find a for Q
	a=0 [AG]	A1	Accept from 0.2gsin45 - 1.386
	T = (3/1.86) =1.6(12)	B1	Accept 2 sf
	For P		
	a = 9.8sin45	B1	a= 6.93
	$2.5 = 1.86(14..)t + 0.5 \times (9.8\sin45)t^2$	M1	Using $2.5 = cv(1.86)t + 0.5cv(6.93)t^2$ [not 9.8 or 3.465]
	t = 0.6(223)	A1	Accept 1sf
	time difference 1.612 - 0.622 = 0.99(0) s	A1	Accept art 0.99 from correct work
		[7]	

4729 Mechanics 2

1	200cos35° 200cos35° x d = 5000 d = 30.5 m	B1 M1 A1 **3**	**3**

2	$0.03R = ½x0.009(250^2 – 150^2)$ 0.03R	M1 B1	$150^2 = 250^2 + 2a$ x 0.03 $a = ±2x10^6/3$ or ±666,667 (A1)
	either K.E. R = 6000 N	B1 A1 ✓ **4**	F = 0.009a (M1) ✓ unit errors **4**

3 (i)	D = 12000/20 12000/20=k x 20 + 600 x 9.8 x 0.1 k = 0.6	B1 M1 A1 **3**	**AG**
(ii)	16000/v = 0.6v + 600 x 9.8 x 0.1 $0.6 v^2 + 588v – 16000 = 0$ $v = 26.5$ m s^{-1}	M1 M1 A1 **3**	attempt to solve quad. (3 terms)
(iii)	16000/32 – 0.6 x 32 = 600a $a = 0.801$ m s^{-2}	M1 A1 A1 **3**	0.80 or 0.8 **9**

4 (i)	$0 = 35\sin\theta$ x t – $4.9t^2$ t = 35sinθ/4.9 50sinθ/7 R = 35cosθ x t aef $R = 35^2\sin\theta.\cos\theta/4.9$ R = 125sin2θ	M1 A1 B1 M1 A1 **5**	$R=u^2\sin2\theta/g$ only ok if proved or 70sinθ/g aef their t eliminate t **AG**
(ii)	110 = 125sin2θ θ = 30.8° or 59.2° t = 3.66 s or 6.13 s	M1 A1+1 A1+1 **5**	**10**

5 (i)	3/8 x 3 (1.125) 0.53d = 5x0.02 + (10 + 3/8x3) x 0.5 d = 10.7	B1 M1 A1 A1 **4**	c.o.m. hemisphere 0.53e=3x5/8x0.5+8x0.02+13x .01 0.53f=3x3/8x0.5-5x0.02-10x0.01 **AG (**e = 2.316 f = 0.684**)**
(ii)	Attempt to calc a pair relevant to P,G OP=0.9 (pair) , p= 73.3° q=16.7° r=76.9° (77.2°) , s=13.1°(12.8°) AC=0.86 , BC=0.67 , AD=10.4 BD=10.2 r > p , s < q , p + s < 90 , 0.67 < 0.86 , 10.2 < 10.4 it is in equilibrium	M1 A1 M1 A1 **4**	distance / angle not a complimentary pair make relevant comparison 0.7 < 0.9 (OG < OP) 10.7 < 10.9 **8**

6 (i)	$T\cos60° = S\cos60° + 4.9$	M1		Resolving vertically nb for M1:
		A1		(must be components – all 4 cases)
	$T\sin60° + S\sin60° = 0.5 \times 3^2/0.4$	M1		Res. Horiz. $mr\omega^2$ ok if $\omega \neq 3$
		A1		If equal tensions 2T=45/4 M1 only
	$(S + 9.8)\sin60° + S\sin60° = 45/4$	M1		
	$S = 1.60$ N	A1		
	$T = 11.4$ N	A1	**7**	
(ii)	$T\cos60° = 4.9$	M1		Resolving vertically (component)
	$T = 9.8$	A1		
	$T\sin60° = 0.5 \times 0.4\omega^2$	M1		Resolving horiz. (component)
		A1		
	$\omega = 6.51$ rad s^{-1}	A1	**5**	or 6.5 **12**

7 (i)	$u = 3$ m s^{-1}	B1		
	$6 = 2x + 3y$	M1		
		A1		
	$e = (y - x)/3$	M1		
		A1		($e = ⅔$) (equs must be consistent)
	$y = 2$	A1	**6**	**AG**
(ii)	$v_h = 2$	B1		or (B1) $½mx2^2$
	$v_v^2 = 2 \times 9.8 \times 4$	M1		(B1) $½mxv^2$
	$v_v = 8.85$ $(14\sqrt{10}/5)$	A1		
				(B1) $mx9.8x4$
	speed $= \sqrt{(8.85^2 + 2^2)}$	M1		$v = \sqrt{(2^2 + 2x9.8x4)}$
	9.08 m s^{-1}	A1		
	$\tan^{-1}(8.85/2)$	M1		or $\cos^{-1}(2/9.08)$
	77.3° to horizontal	A1	**7**	12.7° to vertical **13**

8 (i)	com of Δ 3 cm right of C	B1		
	$(48+27)\bar{x} = 48x4 + 27x11$	M1		
		A1		
	$\bar{x} = 6.52$	A1		
	com of Δ 2 cm above AD	B1		
	$(48+27)\bar{y} = 48x3 + 27x2$	M1		
		A1		
	$\bar{y} = 2.64$	A1	**8**	
(ii)	14F	B1		can be implied e.g. 7/sin30°. F
	$3g\cos30° \times 6.52$	B1		7.034 (AG) or (6.52-2.64tan30°)
	$3g\sin30° \times 2.64$	B1		52.0° (GAH) or (above)xcos30° (5.00)xcos30° (4.33)
	$14F = 3g\cos30° x6.52 - 3g\sin30° x2.64$	M1		$14F = 3x9.8x7.034x\cos52.0°$
	$F = 9.09$ N	A1	**5**	**13**

4730 Mechanics 3

1	(i) $T = (1.35mg)(3 - 1.8) \div 1.8$	B1		
	[$0.9mg = ma$]	M1		For using $T = ma$
	Acceleration is $8.82ms^{-2}$	A1	3	
	(ii) Initial EE = $(1.35mg)(3 - 1.8)^2 \div (2x1.8)$	B1		
	[$\frac{1}{2} mv^2 = 0.54mg$]	M1		For using $\frac{1}{2} mv^2$ = Initial EE
	Speed is $3.25ms^{-1}$	A1	3	

2	**(i)**	M1		For using NEL vertically
	Component is $8esin27^o$	A1		
	Component is $2.18ms^{-1}$	A1	3	
	(ii) Change in velocity vertically = $8sin27^o(1 + e)$	B1ft		ft $8sin27^o$ + candidate's ans. in (i)
	$\|I\| = 0.2 \times 5.81$	M1		For using $\|I\|$ = m x change in velocity
	Magnitude of Impulse is $1.16\ kgms^{-1}$	A1ft	3	ft incorrect ans. in (i) providing both M marks are scored.

3		M1		For using the principle of conservation of momentum in the **i** direction
	$0.8x12cos60^o = 0.8a + 2b$	A1		
		M1		For using NEL
	$0.75x12cos60^o = b - a$	A1		
	[$4.8 = 0.8a + 2(a + 4.5)$]	DM1		For eliminating b; depends on at least one previous M mark
	$a = -1.5$	A1		
	Comp. of vel. perp. to l.o.c. after impact is $12sin60^o$	B1		
		M1		For correct method for speed or direction
	The speed of A is $10.5ms^{-1}$	A1ft		ft $v^2 = a^2 + 108$
	Direction of A is at 98.2^o to l.o.c.	A1ft	10	Accept $\theta = 81.8^o$ if θ is clearly and appropriately indicated; ft $\tan^{-1}\theta = (12sin60^o)/\|a\|$)

4	**(i)** $[mg\sin\alpha - 0.2mv = ma]$	M1		For using Newton's second law
	$5\frac{dv}{dt} = 28 - v$	A1		AG
	$[\int \frac{5}{28-v}dv = \int dt]$	M1		For separating variables and integrating
	(C) - 5ln(28 –v) = t	A1		
		M1		For using v = 0 when t = 0
	ln[(28 – v)/28] = -t/5	A1ft		ft for ln[(28 – v)/28] = t/A from C + Aln(28 –v) = t previously
	$[28 - v = 28e^{-t/5}]$	M1		For expressing v in terms of t
	$v = 28(1 - e^{-t/5})$	A1ft	8	ft for $v = 28(1 - e^{t/A})$ from ln[(28 – v)/28] = t/A previously
	(ii)			
	$[a = 28e^{-2}/5]$	M1		For using a = (28 – v(t))/5 or a = $d(28 - 28e^{-t/5})dt$ and substituting t = 10.
	Acceleration is $0.758ms^{-2}$	A1ft	2	ft from incorrect v in the form $a + be^{ct}$ ($b \neq 0$); Accept $5.6/e^2$

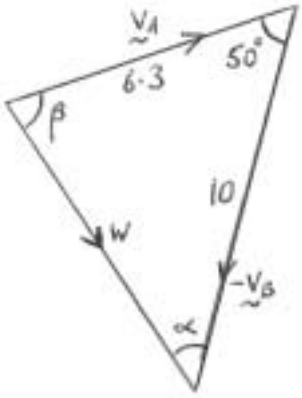
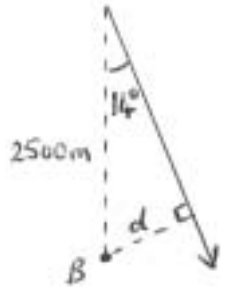
5	**(i)**			
		M1		For taking moments about B or about A for the whole or For taking moments about X for the whole and using $R_A + R_B = 280$ and $F_A = F_B$
	$1.4R_A = 150x0.95 + 130x0.25$ or $1.4R_B = 130x1.15 + 150x0.45$ or $1.2F - 0.9(280 - R_B) + 0.45x150 - 1.2F + 0.5R_B - 0.25x130 = 0$	A1		
	$R_A = 125N$	A1		AG
	$R_B = 155N$	B1	4	
	(ii)			
		M1		For taking moments about X for XA or XB
	$1.2F_A = -150x0.45 + 0.9R_A$ or $1.2F_B = 0.5R_B - 130x0.25$	A1		
	F_A or $F_B = 37.5N$	A1ft		$F_B = (1.25R_B - 81.25)/3$
	F_B or $F_A = 37.5N$	B1ft	4	
	(iii) Horizontal component is 37.5N to the left	B1ft		ft H = F or H = 56.25 – 0.75V or 12H = 325 + 5V
	$[Y + R_A = 150]$	M1		For resolving forces on XA vertically
	Vertical component is 25N upwards	A1ft	3	ft 3V = 225 – 4H or V = 2.4H -65

6	**(i)**			For applying Newton's second law
	[0.36 – 0.144x = 0.1a]	M1		
	$\ddot{x} = 3.6 - 1.44x$	A1		
	$\ddot{y} = -1.44y$ ➔ SHM or $d^2(x-2.5)/dt^2 = -1.44(x-2.5)$ ➔SHM	B1		
		M1		For using $T = 2\pi/n$
	Of period 5.24s	A1	5	AG
	(ii) Amplitude is 0.5m	B1		
		M1		For using $v^2 = n^2(a^2 - y^2)$
	$0.48^2 = 1.2^2(0.5^2 - y^2)$	A1ft		
	Possible values are 2.2 and 2.8	A1	4	
	(iii) [$t_0 = (\sin^{-1}0.6)/1.2$; $t_1 = (\cos^{-1}0.6)/1.2$]	M1		For using $y = 0.5\sin 1.2t$ to find t_0 or $y = 0.5\cos 1.2t$ to find t_1
	$t_0 = 0.53625$ .... or $t_1 = 0.7727$…..	A1		Principal value may be implied
	(a)			For using $\Delta t = 2t_0$ or $\Delta t = T/2 - 2t_1$
	[$2(\sin^{-1}0.6)/1.2$ or $(\pi - 2\cos^{-1}0.6)/1.2$]	M1		
	Time interval is 1.07s	A1ft		ft incorrect t_0 or t_1
	(b)			From $\Delta t = T/2 - 2t_0$ or $\Delta t = 2t_1$; ft 2.62 – ans(a) or incorrect t_0 or t_1
	Time interval is 1.55s	B1ft	5	

7	**(i)**	M1		For using KE gain = PE loss
	½ $mv^2 = mga(1 - \cos\theta)$	A1		
	$aw^2 = 2g(1 - \cos\theta)$	B1	3	AG From v = wr
	(ii)			For using Newton's second law radially (3 terms required) with accel = v^2/r or w^2r
	$mv^2/a = mg\cos\theta - R$ or $maw^2 = mg\cos\theta - R$	M1		
		A1		
	[$2mg(1 - \cos\theta) = mg\cos\theta - R$]	DM1		For eliminating v^2 or w^2; depends on at least one previous M1
	$R = mg(3\cos\theta - 2)$	A1ft	4	ft sign error in N2 equation
	(iii)			
	[$mg\sin\theta$ = m(accel.) or $2a(\dot{\theta})\ddot{\theta} = 2g\sin\theta(\dot{\theta})$]	M1		For using Newton's second law tangentially or differentiating $aw^2 = 2g(1 - \cos\theta)$ w.r.t. t
	Accel. (=$a\ddot{\theta}$) = $g\sin\theta$	A1		
	[$\theta = \cos^{-1}(2/3)$]	M1		For using R = 0
	Acceleration is $7.30 ms^{-2}$	A1ft	4	ft from incorrect R of the form $mg(A\cos\theta + B)$, $A \neq 0$, $B \neq 0$; accept $g\sqrt{5}/3$
	(iv)			
		M1		For using rate of change = $(dR/d\theta)(d\theta/dt)$
	$dR/dt = (-3mg\sin\theta)\sqrt{2g(1-\cos\theta)/a}$	A1ft		ft from incorrect R of the form $mg(A\cos\theta + B)$, $A \neq 0$
		M1		For using $\cos\theta = 2/3$
	Rate of change is $-mg\sqrt{\frac{10g}{3a}}$ Ns^{-1}	A1ft	4	Any correct form of $\dot{R}$ with $\cos\theta = 2/3$ used; ft with θ from incorrect R of the form $mg(A\cos\theta + B)$, $A \neq 0$, $B \neq 0$

4731 Mechanics 4

1	By conservation of angular momentum $1.5\times21+I_G\times36=1.5\times28+I_G\times34$ $I_G=5.25\ \text{kg m}^2$	M1 A1A1 A1 **4**	Give A1 for each side of the equation or $1.5(28-21)=I_G(36-34)$
2 (i)	Using $\omega_1^2=\omega_0^2+2\alpha\theta$, $0^2=8^2+2\alpha(2\pi\times16)$ $\alpha=-\frac{1}{\pi}=-0.318$ Angular deceleration is $0.318\ \text{rad s}^{-2}$	M1 A1 **2**	*Accept* $-\frac{1}{\pi}$
(ii)	Using $\omega_1^2=\omega_0^2+2\alpha\theta$, $\omega^2=8^2+2\alpha(2\pi\times15)$ $\omega=2\ \text{rad s}^{-1}$	M1 A1 ft **2**	or $0^2=\omega^2+2\alpha(2\pi)$ ft is $\sqrt{64-60\pi\lvert\alpha\rvert}$ or $\sqrt{4\pi\lvert\alpha\rvert}$ *Allow A1 for* $\omega=2$ *obtained using* $\theta=16$ *and* $\theta=15$ *(or* $\theta=1$*)*
(iii)	Using $\omega_1=\omega_0+\alpha t$, $0=\omega+\alpha t$ $t=2\pi=6.28\ \text{s}$	M1 A1 ft **2**	or $2\pi=0t-\frac{1}{2}\alpha t^2$ ft is $\frac{\omega}{\lvert\alpha\rvert}$ or $\sqrt{\frac{4\pi}{\lvert\alpha\rvert}}$ *Accept* 2π
3	$A=\int_0^3(2x+x^2)\,\text{d}x$ $=\left[\ x^2+\frac{1}{3}x^3\ \right]_0^3=18$ $A\bar{x}=\int_0^3 x(2x+x^2)\,\text{d}x$ $=\left[\ \frac{2}{3}x^3+\frac{1}{4}x^4\ \right]_0^3=\frac{153}{4}=38.25$ $\bar{x}=\frac{38.25}{18}=\frac{17}{8}=2.125$ $A\bar{y}=\int_0^3\frac{1}{2}(2x+x^2)^2\,\text{d}x$ $=\int_0^3(2x^2+2x^3+\frac{1}{2}x^4)\,\text{d}x$ $=\left[\ \frac{2}{3}x^3+\frac{1}{2}x^4+\frac{1}{10}x^5\ \right]_0^3=82.8$ $\bar{y}=\frac{82.8}{18}=4.6$	M1 A1 M1 M1 A1 M1 M1 M1 A1 **9**	*Definite integrals may be evaluated by calculator (i.e with no working shown)* Integrating and evaluating *(dependent on previous M1)* or $\int_0^{15}\left(3-(\sqrt{y+1}-1)\right)y\,\text{d}y$ Arranging in integrable form Integrating and evaluating *SR If ½ is missing, then M0M1M1A0 can be earned for* $\bar{y}$

4 (i)		B1	Correct velocity triangle
	$w^2 = 6.3^2 + 10^2 - 2\times 6.3\times 10\cos 50°$	M1	
	$w = 7.66 \text{ m s}^{-1}$	A1	
	$\dfrac{\sin\alpha}{6.3} = \dfrac{\sin 50°}{w}$	M1	*This mark cannot be earned from work done in part (ii)*
	$\alpha = 39.04°$ $(\beta = 90.96°)$		
	Bearing is $205 - \alpha = 166°$	A1	
		5	
	OR $\begin{pmatrix} 6.3\sin 75 \\ 6.3\cos 75 \end{pmatrix} - \begin{pmatrix} 10\sin 25 \\ 10\cos 25 \end{pmatrix} = \begin{pmatrix} 1.859 \\ -7.433 \end{pmatrix}$ M1A1		
	M1		Finding magnitude or direction
	$w = \sqrt{1.859^2 + 7.433^2} = 7.66$ A1		
	Bearing is $180 - \tan^{-1}\dfrac{1.859}{7.433} = 166°$ A1		
(ii)	As viewed from B 	B1 ft	Diagram showing path of A as viewed from B *May be implied* Or B1 for a correct (ft) expression for d^2 in terms of t
	$d = 2500\sin 14.04$	M1	or other complete method
	$= 607$ m	A1	Accept 604.8 to 609
		3	*SR* If $\beta = 89°$ is used, give A1 for 684.9 to 689.1

5 (i)	$V = \int_a^{4a} \pi(a\,x)\,\mathrm{d}x$	M1	*(Omission of π is an accuracy error)*
	$= \left[\ \frac{1}{2}\pi a\,x^2\ \right]_a^{4a} = \frac{15}{2}\pi a^3$	M1	
	Hence $m = \frac{15}{2}\pi a^3\rho$	M1	
	$I = \sum \frac{1}{2}(\rho\pi y^2\delta x)y^2 = \int \frac{1}{2}\rho\pi y^4\,\mathrm{d}x$	M1 A1	For $\int y^4\,\mathrm{d}x$
	$= \int_a^{4a} \frac{1}{2}\rho\pi a^2x^2\,\mathrm{d}x$	A1 ft	Substitute for y^4 and correct limits
	$= \left[\ \frac{1}{6}\rho\pi a^2x^3\ \right]_a^{4a} = \frac{21}{2}\rho\pi a^5$	A1	
	$= \frac{7}{5}(\frac{15}{2}\pi a^3\rho)a^2 = \frac{7}{5}ma^2$	A1 (ag) **8**	
(ii)	MI about axis, $I_A = \frac{7}{5}ma^2 + ma^2$	M1	Using parallel axes rule
	$= \frac{12}{5}ma^2$	A1	
	Period is $2\pi\sqrt{\dfrac{I}{mgh}}$	M1	
	$= 2\pi\sqrt{\dfrac{\frac{12}{5}ma^2}{mga}} = 2\pi\sqrt{\dfrac{12a}{5g}}$	A1 ft **4**	ft from any I with $h = a$
6 (i)	$I = \frac{1}{3}m\{a^2 + (\frac{3}{2}a)^2\} + m(\frac{1}{2}a)^2$	M1 M1	MI about perp axis through centre Using parallel axes rule
	$= \frac{13}{12}ma^2 + \frac{1}{4}ma^2 = \frac{4}{3}ma^2$	A1 (ag) **3**	
(ii)	By conservation of energy	M1	Equation involving KE and PE
	$\frac{1}{2}(\frac{4}{3}ma^2)\omega^2 - \frac{1}{2}(\frac{4}{3}ma^2)\dfrac{9g}{10a} = mg(\frac{1}{2}a - \frac{1}{2}a\times\frac{3}{5})$	A1	
	$\frac{2}{3}ma^2\omega^2 - \frac{3}{5}mga = \frac{1}{5}mga$		
	$\omega^2 = \dfrac{6g}{5a}$	A1 (ag) **3**	
(iii)	$mg\cos\theta - R = m(\frac{1}{2}a)\omega^2$	M1	Acceleration $r\omega^2$ and three terms (one term must be R)
	$mg\times\frac{3}{5} - R = \frac{3}{5}mg$	A1	
	$R = 0$	A1 (ag)	*SR* $mg\cos\theta + R = m(\frac{1}{2}a)\omega^2 \Rightarrow R = 0$ earns M1A0A1
	$mg(\frac{1}{2}a\sin\theta) = I\alpha$	M1A1	Applying $L = I\alpha$
	$\alpha = \dfrac{3g}{10a}$	A1	
	$mg\sin\theta - S = m(\frac{1}{2}a)\alpha$	M1A1	Acceleration $r\alpha$ and three terms (one term must be S)
	$S = \frac{4}{5}mg - \frac{3}{20}mg$		
	$= \frac{13}{20}mg$	A1 **9**	or $S(\frac{1}{2}a) = I_G\,\alpha = \frac{13}{12}ma^2\alpha$

7 (i)	$U = 3mgx + 2mg(3a - x)$ $\quad + \frac{mg}{2a}(x-a)^2 + \frac{2mg}{2a}(2a-x)^2$ $= \frac{mg}{2a}(3x^2 - 8ax + 21a^2)$ $\frac{\mathrm{d}U}{\mathrm{d}x} = 3mg - 2mg + \frac{mg}{a}(x-a) - \frac{2mg}{a}(2a-x)$ $= \frac{3mgx}{a} - 4mg$ When $x = \frac{4}{3}a$, $\frac{\mathrm{d}U}{\mathrm{d}x} = 4mg - 4mg = 0$ so this is a position of equilibrium $\frac{\mathrm{d}^2U}{\mathrm{d}x^2} = \frac{3mg}{a}$ > 0, so equilibrium is stable	B1B1 B1B1 M1 A1 A1 (ag) M1 A1 (ag) **9**	*Can be awarded for terms listed separately* Obtaining $\frac{\mathrm{d}U}{\mathrm{d}x}$ *(or any multiple of this)*
(ii)	KE is $\frac{1}{2}(3m)v^2 + \frac{1}{2}(2m)v^2$ Energy equation is $U + \frac{5}{2}mv^2 = \text{constant}$ Differentiating with respect to t $\left(\frac{3mgx}{a} - 4mg\right)\frac{\mathrm{d}x}{\mathrm{d}t} + 5mv\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ $\frac{3gx}{a} - 4g + 5\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 0$ Putting $x = \frac{4}{3}a + y$, $\frac{3gy}{a} + 5\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = 0$ $\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -\frac{3g}{5a}y$ Hence motion is SHM with period $2\pi\sqrt{\frac{5a}{3g}}$	M1A1 M1 A1 ft A1 ft M1A1 ft A1 (ag) A1 **9**	Differentiating the energy equation (with respect to t or x) *Condone $\ddot{x}$ instead of $\ddot{y}$* *Award M1 even if KE is missing* *Must have $\ddot{y} = -\omega^2 y$ or other satisfactory explanation*

4732 Probability & Statistics 1

Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to ≥ 3sfs, ISW for later rounding
Penalise over-rounding only once in paper.

1(i)	(a) -1 (b) 0	B1 B1 2	allow ≈ -1 or close to -1 not "strong corr'n", not -0.99 allow ≈ 0 or close to 0 not "no corr'n"
(ii)	4 3 2 1 or 1 2 3 4 1 3 4 2 4 2 1 3 Σd^2 (= 14) $1 - \frac{6\Sigma d^2}{4(4^2-1)}$ = -0.4 oe	M1 A1 M1 M1 A1 5	Ranks attempted, even if opp Dep M1 or $S_{xy} = 23-{}^{100}/_{4}$ or $S_{xx} = S_{yy} = 30-{}^{100}/_{4}$ Dep 2nd M1 $S_{xy}/\sqrt{(S_{xx}S_{yy})}$
Total		**7**	
2(i)	$\frac{{}^{7}C_2 \times {}^{8}C_3}{{}^{15}C_5}$ = ${}^{56}/_{143}$ or ${}^{1176}/_{3003}$ or 0.392 (3sfs)	M1 M1 A1 3	${}^{7}C_2 \times {}^{8}C_3$ or 1176 : M1 (Any C or P)/${}^{15}C_5$: M1 (dep < 1) or $\frac{7}{15}\times\frac{6}{14}\times\frac{8}{13}\times\frac{7}{12}\times\frac{6}{11}$ or 0.0392: M1 $\times {}^{5}C_2$ or × 10 : M1 (dep ≥4 probs mult) if 2↔3, treat as MR max M1M1
(ii)	3! x 2! or ${}^{3}P_3 \times {}^{2}P_2$ not in denom = 12	M1 A1 2	BABAB seen: M1 120-12: M1A0 NB ${}^{4!}/_{2!}$ = 12: M0A0
Total		**5**	
3(i)(a)	0.9368 or 0.937	B1 1	
(b)	0.7799 – 0.5230 or ${}^{8}C_5 \times 0.45^3 \times 0.55^5$ = 0.2569 or 0.2568 or 0.257	M1 A1 2	Allow 0.9368 – 0.7799
(c)	0.7799 seen – 0.0885 (not 1 – 0.0885) = 0.691 (3 sfs)	M1 M1 A1 3	${}^{8}C_5 \times 0.45^3 \times 0.55^5 + {}^{8}C_4 \times 0.45^4 \times 0.55^4 + {}^{8}C_3 \times 0.45^5 \times 0.55^3$: M2 1 term omitted or wrong or extra: M1
(ii)(a)	${}^{10}C_2 \times ({}^{7}/_{12})^8 \times ({}^{5}/_{12})^2$ seen = 0.105 (3 sfs)	M1 A1 2	or 0.105 seen, but not ISW for A1
(b)	$2{}^{31}/_{72}$ or ${}^{175}/_{72}$ or 2.43 (3 sfs)	B1 1	NB ${}^{12}/_{5}$ = 2.4: B0
Total		**9**	
4(i)	${}^{1}/_{20} \times {}^{1}/_{10}$ or ${}^{1}/_{200}$ or 0.005 x 2 = ${}^{1}/_{100}$ or 0.01	M1 M1dep A1 3	
(ii)	$E(X) = 0+50\times{}^{1}/_{10}+500\times{}^{1}/_{20}$ or $0+0.5\times{}^{1}/_{10}+5\times{}^{1}/_{20}$ = 30p = £0.30 or ${}^{3}/_{10}$ Charge "30p" + 20p or 0.3 + 0.2 = 50p or 0.50 or 0.5	M1 A1 M1 A1 4	or eg 20 goes: 2 × £0.50 + £5.00 = £6.00 ("£6.00" + 20 × £0.20) ÷ 20 condone muddled units eg 0.3 + 20 x = 20, 70, 520 : M1A1 $20\times{}^{17}/_{20} + 70\times{}^{1}/_{10} + 520\times{}^{1}/_{20}$: M1 = 50 A1 x, $(x-50)$, $(x-500)$: M1A1 $x\times{}^{17}/_{20}+(x-50)\times{}^{1}/_{10}+(x-500)\times{}^{1}/_{20} = 20$: M1 x = 50 : A1 Ignore "£" or "p"
Total		**7**	

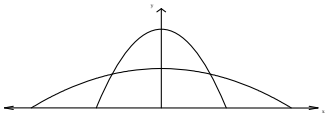
5(i)	$^{12}/_{22}$ x $^{11}/_{21}$ $= {}^{2}/_{7}$ oe or 0.286 (3 sfs)	M1 A1 2	or $^{12}C_2$ / $^{22}C_2$
(ii)	$^{7}/_{15}$ x $^{6}/_{14}$ x $^{8}/_{13}$ or $^{8}/_{65}$ oe × 3 oe $= {}^{24}/_{65}$ or 0.369 (3 sfs)	M1 M1 A1 3	Numerators any order $^{7}C_2$ x $^{8}C_1$:M1 3 x prod any 3 probs (any C or P)/$^{15}C_3$:M1 (dep <1) $1-(^{8}/_{15}x^{7}/_{14}x^{6}/_{13}+3\times{}^{8}/_{15}x^{7}/_{14}x^{7}/_{13}+{}^{7}/_{15}x^{6}/_{14}x^{5}/_{13})$: M2 one prod omitted or wrong: M1
(iii)	$\frac{x}{45}\times\frac{x-1}{44}=\frac{1}{15}$ oe $x^2 - x - 132 = 0$ or $x(x-1) = 132$ $(x-12)(x+11) = 0$ or $x = \frac{1 \pm \sqrt{(1^2 - 4\times(-132))}}{2}$ No. of Ys = 12	M1 A1 M1 A1 4	not $\frac{x}{45}\times\frac{x}{44}=\frac{1}{15}$ or $\frac{x}{45}\times\frac{x}{45}=\frac{1}{15}$ or $\frac{x}{45}\times\frac{x-1}{45}=\frac{1}{15}$ oe ft 3-term QE for M1 condone signs interchanged allow one sign error Not $x = 12$ or -11 ans 12 from less wking, eg $12\times11=132$ or T & I: full mks Some incorrect methods: $\frac{x}{45}\times\frac{x-1}{44}=\frac{1}{15}$ oe M1 $x^2 + x = 132$ A0 $x = 11$ M1A0 $12 \times 11 = 132$ M1A1M1 $x = 12$ and (or "or") 11 A0 NB 12 from eg 12.3 rounded, check method
Total		**9**	

6(i)(a)	256	B1 1	
			(i)(b) & (ii)(abc): ISW ie if correct seen, ignore extras
(b)	Total unknown or totals poss diff or Y13 may be smaller or similar or size of pie chart may differ	B1 1	pie chart shows only proportions oe or no. of students per degree may differ not "no. of F may be less" not "Y13 may be larger"
(ii)(a)	B&W does not show frequencies oe	B1 1	or B&W shows spread or shows mks or M lger range
(b)			1 mk about overall standard, based on median or on F's IQR being "higher" 1 mk about spread (or range or IQR) or about skewness. must be overall, not indiv mks must be comparison, not just figures Examples:
	F generally higher or median higher F higher on average or F better mks F IQR is above M IQR	B1	not F higher mean
	F more compact M wide(r) range or gter IQR or gter variation or gter variance or more spread or less consistent M evenly spread or F skewed	B1 2	not M have hiest and lowest mks condone F +ve skew
(c)	<u>Advantage</u>: B&W shows med or Qs or IQR or range or hiest & lowest or key values	B1	not B&W shows skewness not B&W shows info at a glance not B&W easier to compare data sets not B&W shows mean not B&W shows spread not B&W easier to calculate or easier to read
	<u>Disadvantage</u>: B&W loses info' B&W shows less info' B&W not show freqs B&W not show mode B&W: outlier can give false impression hist shows more info hist shows freqs or fds hist shows modal class (allow mode) hist shows distribution better can calc mean from hist	B1 2	not B&W does not give indiv (or raw) data not B&W does not show mean not hist shows freq for each mark not hist shows all the results not hist shows total allow adv of hist as disadv of B&W
(iii)	102 x 51 + 26 x 59 ÷ 128 = 52.6 (3 sfs)	M1 M1dep A1 3	or 5202 + 1534 or 6736
Total		**10**	

7(i)	Geo stated 0.7^3 x 0.3 $^{1029}/_{10000}$ oe or 0.103 (3 sfs)	M1 M1 A1 3	or implied by 0.7^rx0.3 or 0.3^rx0.7 Allow 0.7^4 x 0.3
(ii)	0.7^6 alone = 0.118 (3 sfs)	M1 A1 2	1-(0.3+0.3×0.7+...+ 0.3×0.7^5) not 1-0.7^6
(iii)	0.7^9 $1 - 0.7^9$ 0.960 (3 sfs)	M1 M1 A1 3	not 0.3 × 0.7^9 allow $1 - 0.7^{10}$ or 0.972 for M1 allow 0.96, if no incorrect wking seen 0.3 + 0.7 x 0.3 + …. + 0.7^8 x 0.3: M2 1 term omitted or wrong or "correct" extra: M1
(iv)	Bin stated $^5C_2$ x 0.7^3 x 0.3^2 or 0.8369 – 0.5282 = 0.3087 or 0.309 (3 sfs)	M1 M1 A1 3	or implied by table or ^nC_r or $0.7^3 \times 0.3^2$ or 0.0309
Total		**11**	
8(i)	$\dfrac{168.6 - \frac{88\times16.4}{8}}{\sqrt{(1136 - \frac{88^2}{8})(34.52 - \frac{16.4^2}{8})}}$ = -0.960 (3 sfs)	M2 A1 3	$(= \dfrac{-11.8}{\sqrt{168\times0.9}})$ M1: correct subst in any correct *S* formula M2: correct substn in any correct *r* formula allow -0.96, if no incorrect wking seen
(ii)	must refer to, or imply, external constraint on *x* e.g *x* is controlled or values of *x* fixed or chosen allow *x* is fixed	B1 1	not *x* is not random not *x* affects *y* not *x* not affected by *y* not *x* goes up same amount each time not charge affects no. of vehicles not *x* not being measured
(iii)	$\dfrac{168.6 - \frac{88\times16.4}{8}}{1136 - \frac{88^2}{8}}$ = -0.0702 (3 sfs) or $-^{59}/_{840}$ or $-^{11.8}/_{168}$ $y - {}^{16.4}/_8 =$ "-0.0702"$(x - {}^{88}/_8)$ y = -0.07x + 2.8 or better	M1 A1 M1 A1 4	ft their S_{xy} and S_{xx} incl $^{168.6}/_{1136}$ if used in (i) or -0.07 if no incorrect wking or $a = {}^{16.4}/_8 -$ ("-0.0702") x $^{88}/_8$ or $^{2371}/_{840}$ oe eg $y = -{}^{59}/_{840}x + {}^{2371}/_{840}$
(iv)(a)	"-0.07" x 20 + "2.8" = 1.4(2) million (2 sfs)	M1 A1 2	 no ft
(b)	*r* close to -1 or corr'n is high just outside given data, so reliable	B1 B1 2	or good corr'n or pts close to line but not if "close to -1, hence unreliable" if *r* low in (i), ft: "*r* low" or "poor corr'n" etc or outside given data so unreliable not "reliable as follows trend" not "reliable as follows average" no ft from (iv)(a)
(v)	*y* on *x* *x* is indep	B1 B1 2	 or *x* controlled or *y* depends on *x* or *y* not indep dep on not "*x* on *y*" *r* close to -1 so makes little difference: B2
Total		**14**	

4733 Probability & Statistics 2

General: Conclusions to hypothesis tests must acknowledge uncertainty. Thus "time is unchanged" is A0. Similarly, "Significant evidence that time is unchanged" is also A0.

1	**(i)**	Biased in favour of those with strong political interest	B2	**2**	"Biased", "unrepresentative", "not indept" or equiv [but *not* "not random"] stated, with sensible reason. [SR: partial answer, B1]
	(ii)	Obtain list of all pupils Allocate numbers sequentially Choose using random numbers	B1 B1 B1	**3**	List, can be implied; number serially or randomly, not just "number pupils" Select consistently with method of numbering, not just "select randomly" [SR: systematic: List B1, every n^{th} B1, random start B1] [SR: names in a hat: B2]
2	**(i)**	$\Phi\left(\frac{24-30}{12}\right) - \Phi\left(\frac{20-30}{12}\right)$ $= \Phi(-0.5) - \Phi(-0.833)$ $= (1 - 0.6915) - (1 - 0.7976) =$ **0.1061**	M1 A1 M1 A1	**4**	Standardise one, allow $\sqrt{12}$, 12^2, $\sqrt{n}$ Both standardisations correct, allow cc here Correct handling of tails [0.3085 – 0.2024] Answer, a.r.t. 0.106, c.a.o.
	(ii)	Not symmetrical (skewed) Therefore inappropriate	M1 A1	**2**	Any comment implying not symmetric Conclude "not good model" [Partial answer: B1]
3		$H_0 : \mu = 28$ $H_1 : \mu \neq 28$ $\sigma^2 = 37.05 \times 40/39$ [= 38]	B2 M1		Both hypotheses correctly stated; one error, allow wrong or no letter, but not x or t or $\bar{x}$, B1 Multiply 37.05 or $\sqrt{37.05}$ by $n/(n-1)$ or $\sqrt{[n/(n-1)]}$
	α	$z = \frac{26.44 - 28}{\sqrt{38/40}} = -1.601$ Compare –1.645, or 0.0547 with 0.05	M1 A1 B1		Standardise with $\sqrt{n}$, allow $\sqrt{}$ errors, cc, + Correct z, a.r.t –1.60, or $p \in$ [0.0547, 0.0548] Explicit comparison of z with –1.645 or p with 0.05
	β	Critical value $28 - z\sigma/\sqrt{n}$ [= 26.397] $z = 1.645$ Compare 26.44 with 26.40	M1 B1 A1√		Allow "±", √ errors, cc, ignore other tail $z = 1.645$ in CV expression, and compare 26.44 CV, √ on their z, rounding to 3 SF correct
		Do not reject H_0 [can be implied] Insufficient evidence that time taken has changed.	M1 A1√	**8**	Needs $\sqrt{n}$, correct method & comparison, *not* $\mu = 26.44$ Conclusion interpreted in context, √ on z,
4	**(i)**	$\frac{53-50}{\sigma/\sqrt{10}} < 2.326$ $\sigma >$ **4.08** **AG** [Allow ≥]	M1 A1 B1 A1	**4**	Standardise with 10 or $\sqrt{10}$ and Φ^{-1} Both sides same sign, $\sqrt{10}$, don't worry about < 2.326 or 2.33 seen Convincingly obtain $\sigma > 4.08$ to 3 SF, one other step [SR: Substitution: standardise & substitute 4.08 M1; 0.0101 A1; 4.07 or 4.075 tried, M1; full justification A1]
	(ii)	P(Type I) = 0.01 used, e.g. Geo(0.01) $0.99^4 \times 0.01$ = **0.0096**	M1 M1 A1	**3**	Not enough merely to state $p = 0.01$ $p^4 \times q$ Answer, a.r.t. 0.0096
5	**(i)**	$\int_{-1}^{1} \frac{3}{4}(x^2 - x^4)dx = \frac{3}{4}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_{-1}^{1}$ [= 1/5] $1/5 - 0^2$ = **1/5**	M1 A1 B1 A1	**4**	Attempt $\int_{-1}^{1} x^2 f(x)dx$ Correct indefinite integral Mean 0 clearly indicated Answer 1/5 or a.r.t. 0.200, don't need $\mu = 0$
	(ii)	(a)  (b) Areas equal, more spread out, so g_{max} lower (c) W greater as more spread out	B1 M1 A1 B1dep depB1	**5**	Correct graph, don't need f(x) as well. Don't allow if graph goes further below axis than "pips". Don't worry too much about exact shape Mention areas or total probability Convincing argument, not just "flatter" W greater… …with convincing reason

6	(a)	Po(2.375) $e^{-2.375}\left(\frac{2.375^3}{3!}+\frac{2.375^4}{4!}\right)$ [= 0.2079 + 0.1233] = **0.3310**	M1 M1 A1 A1	**4**	Po(19/8) stated or implied One correct Poisson formula, *not* tables Complete correct expression, including addition Answer, a.r.t. 0.331 [SR: Po(2) or Po(2.4) and tables, M1]
	(b)	(i) *n* large OR $n > 50$ *p* small OR $np < 5$	B1 B1	**2**	Or equivalent [Allow ≤ and ≥ throughout] Or equivalent, e.g. $np \approx npq$, or $p < 0.1$ [Treat "$np < 5$, $npq < 5$" as single wrong statement]
		(ii) B(108, $\frac{1}{36}$) ≈ Po(3) 1 – P(≤ 3) = 1 – 0.6472 = **0.3528**	M1 M1 A1 M1 A1	**5**	Correct binomial distribution stated or implied Po(*np*), √ on their *n*, *p* Po(3) Use Po tables, "1 –", *or* correct formula, ± 1 term, e.g. 0.1847; a.r.t. 0.353, allow from exact Binomial
7	(i)	Dropped catches must occur independently of one another and at constant average rate	B1 B1	**2**	"independently", in context, allow "random" "Constant average rate", in context ["Singly" doesn't gain B1]
Marks can be awarded in either part	(ii)	Use: "Reject H_0 when correct" Po(10) P(≥ 16) = 1 – P(≤ 15) = 1 – 0.9513 Probability 0.0487	M1 M1 M1 A1		Find P(≥ *r*) where $r > \lambda$, e.g. P(≥ 6) from Po(2) Po(10) stated or implied [can be recovered in (iii)] Seek biggest prob < 0.05, e.g. 0.0835 or 0.0166, allow 0.0293 but no other LH tail Answer in range [0.0487, 0.0488], cwd, cwo
	(iii)	$H_0 : \lambda = 10$ or 2 [or μ] $H_1 : \lambda > 10$ or 2 [or μ] α: P(≥ 14) = 1 – 0.8645 = 0.1355 > 0.05	B2 A1 B1		Hypotheses fully correct, allow λ or μ [SR: one error, B1, but *r* or *R* or *x* or $\bar{x}$: B0] $p \in$ [0.135, 0.136] from Po(10) Compare explicitly with 0.05 or 0.0487
		β: Critical region $r \geq 16$, $p = 0.0487$ Compare $r = 14$	A1√ B1√		√ on answer from (ii)
		Do not reject H_0 [can be implied] Insufficient evidence of an increase in the number of dropped catches	M1 A1√	**10**	Method correct, √ on *p*, must be upper tail and "≥" Conclusion interpreted in context [SR: P(≤ 14) = 0.9165 < 0.95: (B2 M1) A0 B1 M0A0; same for P(> 14) or P(= 14)] [SR: N(10,10): (ii) 0.05 M0. (iii) (B2) M1 A0 B1 M0A0]
8	(i)	$H_0 : p = 0.4$ or $\mu = 4.8$ $H_1 : p > 0.4$ or $\mu > 4.8$ B(12, 0.4) P(≥ 9) = 1 – 0.9847 = 0.0153 < 0.05 Reject H_0 [can be implied] Significant evidence of increase in proportion of audience members who know sponsor's name	B2 M1 A1 B1√ M1 A1√	**7**	Both fully correct, B2. [SR: one error, B1, but *x* or *R* or *r* or $\bar{x}$: B0] B(12, 0.4) stated or implied, e.g. 0.9972 or 0.9847 Or: CR is ≥ 9 and $p \in$ [0.015, 0.0153] Explicitly compare with 0.05, or 9 with ≥ 9, √ on < Reject H_0, √ on probability, must be "≥" Conclusion interpreted in context [SR: P(≤ 9) or P(= 9) or P(> 9): (B2 M1) A0 B1 M0A0] [SR: N(4.8, 2.88): (B2) M1 A0 B0 M0A0]
	(ii)	N(160, 96) $\frac{(x-0.5)-160}{\sqrt{96}} = 1.645$ Solve to find *x* [= 176.6] Minimum value is **177**	B1 B1 M1 A1 B1 M1 A1	**7**	Normal, mean 160 Variance (or SD) 96 [96/400: B2M0] Standardise unknown with *np* and √*npq* or *npq,* & equate to Φ^{-1}; √96 and signs correct, ignore cc RHS = 1.645 Solve [implied by 177 or 176.6 or 176.1] 177 only, from 176.6, CWO [cc error: 6 ex 7]

4734 Probability & Statistics 3

1 (i)	$\frac{1}{99}(6115.04-\frac{761.2^2}{100})$	M1		AEF
	=3.240	A1	**2**	
(ii)	$761.2/100 \pm z\sqrt{(3.24/100)}$	M1		z= 1.282, 1.645, or 1.96
	z = 1.96	B1		
	(7.26,7.96)	A1	**3**	Allow from σ^2=3.21; allow 7.97 but not from wrong σ. Allow 4 or 5 SF but no more**.**
(iii)	None necessary, since sample size large enough for sample mean to have a normal distribution	B1	**1**	OR:None necessary, n large enough for Central Limit theorem to apply
			[6]	
2	$(\bar{x}-12.6)/\sqrt{0.1195/10}$	M1		Any variable, correct mean, /10, ignore z
		A1		All correct
	1.383 seen	B1		
	Solve for variable	M1		Allow any symbol (<,>,=)
	$\bar{x} \geq 12.75$	A1	**5**	Allow > ; 12.7 or 12.8 No z seen
			[5]	
3(i)	Choice of newspaper is independent of level of income	B1	**1**	Or equivalent
(ii)	Use df=4	B1		May be implied by 13.28 seen or 0.0152
	EITHER: CV 13.28, from df=4 or sig. level	M1		From tables
	Largest significance level is 1%	**B1**		Accept 0.01
	OR: Use$P(\chi^2 > 12.32)$			Use of calculator
	Largest significance level is 1.52%	**B2**	**3**	Accept 0.0152
			[4]	
				SR: from df=6: CV 12.59 used ; SL=5% : B0M1B1
4(i)	$\int_0^1 \frac{4}{3}x^3 dx + \int_1^2 \frac{4}{3x^3} dx$ Limits seen anywhere	M1		For both integrals OR 1 - $\int_2^\infty \frac{4}{3x^3} dx$
	$\left[\frac{x^4}{3}\right]_0^1 + \left[-\frac{2}{3x^2}\right]_1^2$	A1		For both OR $1-\left[-\frac{2}{3x^2}\right]_2^\infty$
	$^5/_6$	A1	**3**	
(ii)	EITHER: $\int_0^1 \frac{4}{3}x^3 dx = \frac{1}{3}$	M1		
	< ½	A1		
	Median must exceed 1	A1		
	OR:	M1		Attempt to find median
	m=√(4/3)	A1		M0 for $1.5^{1/4}$
	> 1 AG	A1	**3**	Accept 1.15..

(iii)	$\int_0^1 \frac{4}{3}x^4\,dx + \int_1^\infty \frac{4}{3x^2}\,dx$	M1		Correct form for at least one integral
	$[4x^5/15] + [-4/(3x)]$	B1		Both integrals correct without limits
	1.6	A1	**3**	AEF
(iv)	$E(X^2) = \ldots + \int_1^\infty \frac{4}{3x}\,dx$	M1		For second integral
	Second integral $= \left[\frac{4}{3}\ln x\right]_1^\infty$	A1		
	This is not finite, (so variance not finite)	A1	**3**	AEF
		[12]		

5 (i) Justify a relevant Poisson approximation — M1 — Using $n>50$ or n large; $np<5$ or p small (<0.1) or $np \approx npq$

$E(A)=75\times0.022$ ($=1.65$), $E(B) = 90\times0.025$ ($=2.25$) — B1B1

Sum of two independent Poisson variables X has a Poisson distribution — A1

Mean m $=3.9$ — B1 — **5** — Accept Po(3.9)

(ii) $1 - P(\leq 5)$ — M1 — Or From Po(m) Accept ≤ 4; OR Exact 1 – sum of at least 5 correct terms

0.1994 — A1 — **2** — From calculator or tables, art 0.20

[7]

6 (i) Use $p_s \pm zs$ — M1

$z=2.326$ — B1

$s = \sqrt{(0.12\times0.88/50)}$ — A1 — Or /49

(0.013,0.227) Allow limits if penalised in Q1 — A1 — **4** — Or (0.012,0.228) from 49

(ii) $z(0.12\times0.88/n)^{1/2}$ — M1 — Any z

< 0.05 — A1 — Allow =

Solve to obtain — M1 — Must contain $\sqrt{n}$

$n > 228.5$ — A1 — Accept =

$n \approx 229$ or 230 — A1 — **5** — Must be integer

[9]

7 (i) Each population of test scores should have normal distributions with equal variances — B1 B1 B1 — **2** — OR: Variances equal and normal distns B1; Context B1

(ii) EITHER:Cannot test for normality from data OR: Sample variances are close enough to accept population variances equal — B1 — **1** — Not variances are not equal

(iii)	$H_0: \mu_B = \mu_G$, $H_1: \mu_B > \mu_G$	B1	For both. No other variables. Allow words
	$s^2 = (23\times86.79+17\times93.01)/40$	M1	Finding pooled estimate of variance
	$=89.4335$	A1	May be implied by later value of t
	$t = (1238.4/18 - 1526.8/24)/ [s^2(18^{-1}+24^{-1})]^{1/2}$	M1	With pooled estimate of variance
		A1	All correct
	$= 1.758$	A1	art 1.76, or -
	Use CV of 1.684	B1	Consistent
	$1.758 > 1.684$	M1	Compare correctly with their CV (t value)
	Reject H_0 and accept there is sufficient evidence at the 5% significance level that teenage boys worry more, on average than teenage girls.	A1√ **9**	Not assertive Ft on their 1.758 SR:Using $s^2 = 93.01/18+86.79/24$: B1M0A0M1A0A1(for 1.749) B1M1(from 1.645 or 1.684)A1 Max 6/9
		12]	
8 (i)	$\Sigma xf/80 = 1.9$ AG	B1	With evidence
	$\Sigma x^2 f/80 - 1.9^2$	M1	Or $\times$ 80/79
	1.365 or 1.382	A1 **3**	
(ii)	Poisson distribution requires equal mean and variance	B1	May be indicated
	EITHER: No, mean and variance differ significantly OR: Yes, indicated by sample statistics taking into account sampling error	B1 **2**	
(iii)	$e^{-1.9}1.9^3/3!$	B1	Or from tables
	$\times 80$	B1 **2**	
(iv)	Considering sample as random selection of all similar matches	B1	
	H_0: Poisson suitable model		
	Combine last two cells	B1	
	$0.97^2/11.97+7.73^2/22.73+11.40^2/21.60$	M1	Any two correct
	$+ 2.32^2/13.68+5.02^2/10.02$	A1	All correct
	$=11.63$	A1	art 11.6
	CV 7.815	B1 *dep	OR p=0.00875
	$11.63 > 7.815$	M1dep*	OR $0.00875 < 0.05$
	There is sufficient evidence that a Poisson distribution is not a suitable model confirming (or not) the answer to part (ii)	A1√ **8**	Ft (ii) SR: If last cells not combined:$\chi^2 = 12.3$ M1A1A1 CV=9.448 or p = 0.0152, B1*dep the M1dep*
(v)	E-values or probabilities would change	B1	Or other valid observation
	df would increase by 1	B1 **2**	Or CV would change
		[17]	

4735 Statistics 4

1	**(i)**	Use P(A) + P(B) - P($A \cap B$) ≤ 1, P($A \cap B$)=0	B1	**1**	AEF
	(ii)	Use P($A|B$)=P($A \cap B$) / P(B)	M1		AEF e.g. Inependent if ($A \cap B$)=P(A)P(B)=x^2,
		Use P($A \cap B$) = 0 with argument with $x \neq 0$	A1	**2**	P($A \cap B$)=0, $x \neq 0$, so A and B are not indep.
	(iii)	Use P($A \cup B \cup C$) = P(A)+P(B)+P(C)– P($A \cap B$) – P($B \cap C$) – P($C \cap A$) + P($A \cap B \cap C$)	M1		Or equivalent. Allow one sign error
		Use P($A \cap B$)= 0; P($A \cap B \cap C$)=0	A1		For both
		P($B \cap C$)=$2x^2$; P($C \cap A$)=$2x^2$	A1		For both
		Substitute and obtain required result AG	A1	4 (7)	
2	**(i)**	Wilcoxon test requires a symmetric distribution not supported by the diagram	B1	**1**	Or equivalent
	(ii)	H_0: m = 1.80, H_1: m > 1.80		B1	Needs "population median" if words
		Use sign test	M1		
		Number exceeding 1.8 = 20	A1		
		Use B(30,0.5), P(≥20) Or P(≤10)	M1		
		0.0494	A1		
		Compare with 0.05 correctly	M1		OR: 1.645 if N(15,7.5) , z =1.643, 1.816, 2.008
		Conclude there is significant evidence that the median time exceeds 1.80 sec	A1√	**7 (8)**	used; OR CR ($X \geq 20$) ft p or z
3	**(i)**	Marginal distribution of X			
		x 0 1 2 3			
		p 0.27 0.23 0.32 0.18	B1		
		1×0.23+2×0.32+ 3×0.18	M1		
		=1.41	A1	**3**	
	(ii)	P($Y>X$)=0.08+0.05+0.03+0.08+0.06 +0.07	M1		
		= 0.37	A1	**2**	
	(iii)	Use P($Y > X \cap X > 0$)/ P($X > 0$)	M1		
		P($X > 0$)= 0.73	A1		From marginal distribution
		P($Y>X \cap X > 0$)=0.08+0.06+0.07	A1		
		21/73	A1	**4**	AEF
	(iv)	The director cannot conclude independence from cov. So director's conclusion incorrect.	M1 A1		Idea that independence implies cov = 0 but not the reverse
		OR: Eg P(X=0 ∩Y=0)=0.11, P(X=0)P(Y=0)=0.27× 0.29≠ P(X=0 ∩Y=0)	M1 A1	**2 (11)**	

4 (i)	Variances seem not to be equal	B1	**1**	
(ii)	H_0: $m_M = m_A$, H_1: $m_M \neq m_A$		B1	Both hypotheses, AEF. Not "average"
	$R_m = 40$, $m(m+n+1) - R_m = 72$	M1		Both found
	$W = 40$	A1		A0 if no or wrong 72
	CR: $W \leq 38$	B1		
	40 not in CR, so do not reject H_0	M1		Or equivalent
	Insufficient evidence that median times differ	A1	**6 (7)**	In context. B1 if no M1 but conclusion correct Allow average here
5 (i)	$a+b = ¾$	B1		From M(0)=1
	$M'(0) = 3^3/_8$	M1		
	$½ + 3a + 4b = 3⅜$	A1		AEF
	Solve simultaneously	M1		Elimination or substitution
	$a = ⅛$ AG	A1		
	$b = ⅝$	A1	**6**	
(ii)	$M''(t) = e^{2t} + {}^9/_8 e^{3t} + 10e^{4t}$	B1		
	$M''(0) - (M'(0))^2$	M1		
	${}^{97}/_8 - (3⅜)^2$; ${}^{47}/_{64}$	A1A1	**4**	
(iii)	$x = 2, 3, 4$	B1	**1 (11)**	
6 (i)	$P(Y > y) = 1 - F(y)$	M1		Allow any variables
	$= a^3/y^3$	A1		
	$P(S > s) = P(\text{all 3 values} > s) = (a/s)^9$ AG	A1		
	$\mathbf{f}(s) = \frac{d}{ds}(1 - (a/s)^9)$	M1		
	$= \begin{cases} 9\dfrac{a^9}{s^{10}} & s \geq a, \\ 0 & s < a \end{cases}$	A1	**5**	
(ii)	$\int_a^\infty \frac{a^9}{s^9} ds$	M1		
	$= 9a/8$	A1		
	S not unbiased since this not equal to a	M1		
	$T_1 = 8S/9$	B1√	**4**	Ft E(S)
(iii)	$\text{Var}(T_1) = a^2/63$, $\text{Var}\ T_2 = a^2/9$	M1		Correct method
		A1 for both		
	$\text{Var}(T_1) < \text{Var}(T_2)$, T_1 is more efficient	A1√	**3**	Comparison, completion.. √ one variance correct with same dimensions
(iv)	$t_1 = 4.0$, $t_2 = 5.4$	B1		Both
	From data $a \leq 4.5$ and $t_2 > 4.5$	B1B1	**3 (15)**	AEF

7 (i) $G(1) = 1$ M1
$a = 2$ A1 **2**

(ii) $(1+2t)/(4-t) = c\,(1+2t)(1-\frac{1}{4}t)^{-1}$ M1 $c = \frac{1}{4}$ or 4
$= \frac{1}{4}(1+2t)(1+\frac{1}{4}t+(\frac{1}{4}t)^2+\ldots)$ A1
Coefficient of $t^3 = \frac{1}{4}[(\frac{1}{4})^3 + 2(\frac{1}{4})^2]$ M1√ With 2 terms from previous line
$= {}^{9}/_{256}$ A1 **4**

(iii) $H(t) = \left(\frac{1+2t}{4-t}\right)^3$ B1

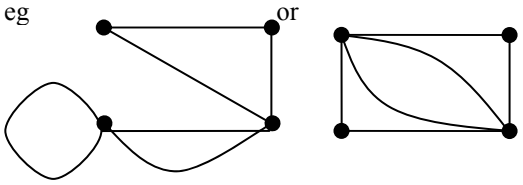
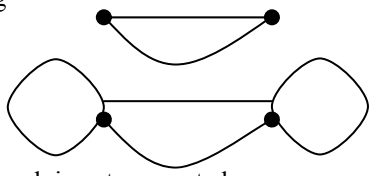
$H'(t) = 3\left(\frac{1+2t}{4-t}\right)^2\left[\frac{2(4-t)+1+2t}{(4-t)^2}\right]$ M1A1

$E(Y) = H'(1)$ M1
$=3$ A1 **5**

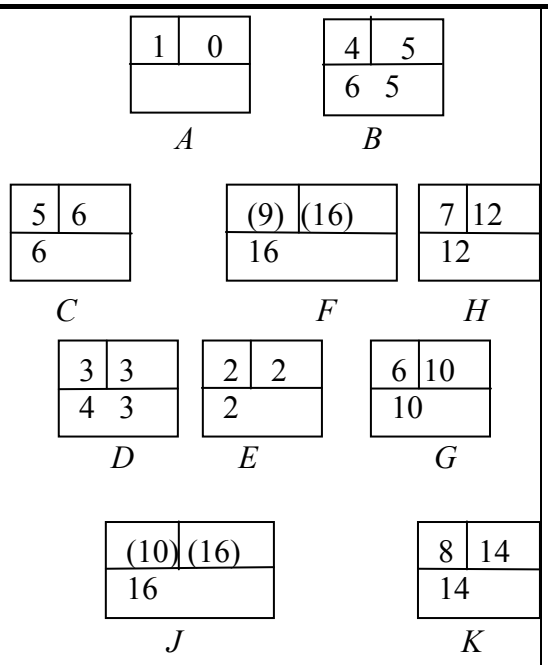
(iv) $H(1) = p_0 + p_1 + p_2 + p_3 + p_4 + \ldots = 1$
$H(-1) = p_0 - p_1 + p_2 - p_3 + p_4 - \ldots = -{}^{1}/_{125}$
Add: $2(p_0+p_2+p_4+\ldots) = 1 - {}^{1}/_{125}$ M1 With sufficient detail
${}^{1}/_{2}(1 - {}^{1}/_{125})$ AG A1 **2 (13)**

4736 Decision Mathematics 1

1	**(i)**	Biggest/largest/last number (only) (Not showing effect on a specific list)	B1	Accept bubbling to left unless inconsistent with part (ii): Smallest/first number	[1]
	(ii)	2 1 3 4 5 horizontally or vertically (may see individual comparisons/swaps) [For reference: original list was 3 2 1 5 4] 4 comparisons and 3 swaps (both correct)	M1 A1	Or bubbling to left: 1 3 2 4 5 Watch out for shuttle sort used If not stated, assume that comparisons come first	[2]
	(iii)	1 2 3 4 5 One (more pass after this)	M1 A1	FT from their first pass with their bubbling if possible Watch out for 'One swap (in 2nd pass)'	[2]
	(iv)	$(3000 \div 500)^2 \times 0.2$ = 7.2 seconds	M1 A1	$6^2 \times 0.2$ or $8\times10^{-7} \times 9\times10^6$ or any equivalent calculation cao UNITS	[2]
				Total =	**7**

2	**(i)**	eg or  - Graph is not simple - Two of the vertices are joined by two arcs (if appropriate) - It has a 'loop' (if appropriate) - For a simple graph each vertex must have order 3 or less	M1 A1 B1	A graph with four vertices of orders 2, 2, 4, 4 (ignore any vertex labels) A connected graph Recognition that their graph is not simple (although it is connected). Need not use the word 'simple'.	[3]
	(ii)	eg  Graph is not connected	M1 A1 B1	Any graph with four vertices of orders 2, 2, 4, 4 (that is topologically different from that in part (i)) A graph that is not connected Recognition in words that their graph is not connected	[3]
				Total =	**6**

3	**(i)**	$y \leq x + 2$ $x + 2y \geq 6$ $\quad (y \geq -\frac{1}{2}x + 3)$ $2x + y \leq 12$ $\quad (y \leq -2x + 12)$	M1 M1 M1 A1	Line $y = x + 2$ in any form Line $x + 2y = 6$ in any form Line $2x + y = 12$ in any form All inequalities correct	**[4]**
	(ii)	$x + 2y = 6$ and $y = x + 2 \Rightarrow (\frac{2}{3}, 2\frac{2}{3})$ $y + 2x = 12$ and $y = x + 2 \Rightarrow (3\frac{1}{3}, 5\frac{1}{3})$ $y + 2x = 12$ and $x + 2y = 6 \Rightarrow (6, 0)$	M1 A1 A1 B1	Follow through if possible Calculating from their lines or implied from either A mark $(\frac{2}{3}, \frac{8}{3})$ (art (0.7, 2.7)) $(\frac{10}{3}, \frac{16}{3})$ (art (3.3, 5.3)) (6, 0) cao	**[4]**
	(iii)	$(\frac{2}{3}, 2\frac{2}{3}) \Rightarrow 11\frac{1}{3}$ $(3\frac{1}{3}, 5\frac{1}{3}) \Rightarrow 32\frac{2}{3}$ $(6, 0) \Rightarrow 30$ At optimum, $x = 3\frac{1}{3}$ and $y = 5\frac{1}{3}$ Maximum value = $32\frac{2}{3}$	M1 A1 A1	Follow through if possible Testing vertices or using a line of constant profit (may be implied) Accept $(3\frac{1}{3}, 5\frac{1}{3})$ identified (ft) $32\frac{2}{3}$ (air 32.6 to 32.7) (ft)	**[3]**
	(iv)	$5\times3\frac{1}{3} + k\times5\frac{1}{3} \geq 5\times6 + k\times0$ $\Rightarrow k \geq 2.5$	M1 M1 A1	$5\times3\frac{1}{3} + k\times5\frac{1}{3}$ (ft) or implied $5\times6 + k\times0$ or 30 or implied Greater than or equal to 2.5 (cao)	**[3]**
				Total =	**14**

4	**(i)**	 Route = $A - E - B - G - H - K$ Length = 14 metres	M1 M1 A1 B1 B1 B1 B1	Both 6 and 5 shown at B All temporary labels correct including F and J No extra temporary labels All permanent labels correct (may omit F and/or J) cao Order of labelling correct (may omit F and/or J, may reverse F and J) cao $A - E - B - G - H - K$ cao 14 cao	**[7]**
	(ii)	Without using CJ: Route = $A - E - B - G - F - J$ Length = 21 metres	 B1 B1	Follow through their **(i)** $A - E - B - G - F - J$ 21	**[2]**
	(iii)	More than 2 metres (Answer of 'more than 7 metres' or '7 metres' ⇒ M1, A0)	M1 A1	2 (cao) More than, or equivalent (Answer of 3 or ≥ 3 ⇒ SC1)	**[2]**
					Total = 11

5 (i)

	E	W
A	x	$3 - x$
B	y	$3 - y$
C	$4 - x - y$	$x + y - 1$

Total cost = £$(250x + 250(3-x)$
$+ 200y + 140(3-y)$
$+ 300(4-x-y) + 280(x+y-1))$
= £$(2090 - 20x + 40y)$ (AG)

B1 $AW = 3 - x$
B1 $BW = 3 - y$
B1 $CE = 4 - x - y$, in any form
M1 An appropriate calculation for their table
A1 Leading to given result **[5]**

(ii)

$2090 - 20x + 40y \leq 2150$
$\Rightarrow -20x + 40y \leq 60$
$\Rightarrow -x + 2y \leq 3$ (AG)

B1 Showing where the given inequality comes from **[1]**

(iii)

$50(3-x) + 40(3-y) + 60(x+y-1)$
$= 210 + 10x + 20y$
So need to maximise $x + 2y$ (AG)

M1 Follow through their table
A1 Correct expression $210 + 10x + 20y$ **[2]**

(iv)

P	x	y	s	t	-
1	-1	-2	0	0	0
0	-1	**2**	1	0	3
0	1	1	0	1	3

Rows and columns may be in any order
B1 -1 -2 in objective row
B1 Constraint rows correct **[2]**

(v)

Pivot on the 2 in the y column

1	-2	0	1	0	3
0	-0.5	1	0.5	0	1.5
0	1.5	0	-0.5	1	1.5

Pivot on 1.5 in the x column

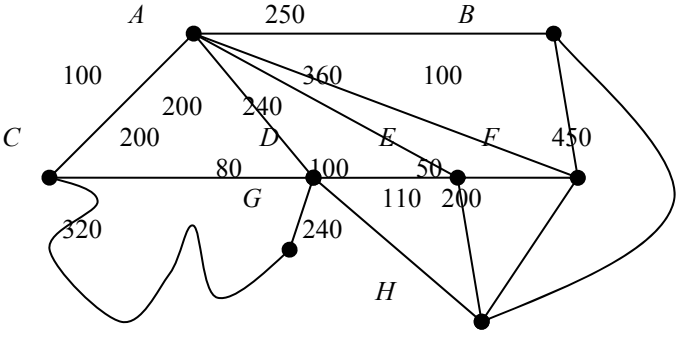
1	0	0	$\frac{1}{3}$	$1\frac{1}{3}$	5
0	0	1	$\frac{1}{3}$	$\frac{1}{3}$	2
0	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	1

$x = 1, y = 2$

B1 Correct choice of pivot from y column
Follow through their tableau and valid pivot if possible
M1 Pivot row correct
A1 Other rows correct
M1 Correct choice of pivot
Follow through their tableau and valid pivot if possible
A1 Correct tableau
B1 Correct answer only **[6]**

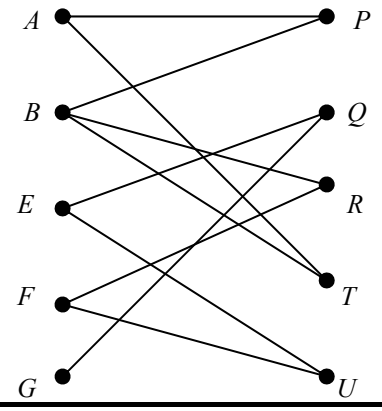
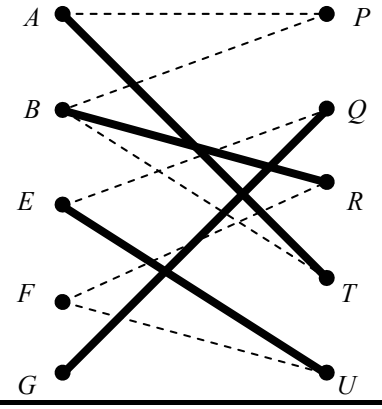
Total = 16

6	(a)(i)	Route Inspection (problem)	B1	Or Chinese postman (problem)	[1]
	(ii)	Odd nodes are A, B, C and D $AB = 250$ $AC = 100$ $AD = 200$ $CD = \underline{200}$ $BD = \underline{250}$ $BC = \underline{350}$ 450 350 550 Repeat AC and $BFED = 350$ Length of shortest route = 3350 metres	B1 M1 A1 B1	Identifying odd nodes (may be implied from working) Pairing odd nodes (all three pairings considered) M mark may not be implied 350 as minimum 3350 m or 3.35 km UNITS	[4]
	(iii)	C is an odd node, so we can end at another odd node. $AB = 250$ $AD = 200$ $BD = 250$ Repeat $AD = 200$ Length of route = 3200 metres Route ends at B	 M1 A1 B1	Working need not be seen May be implied from answer 3200 B	[3]
	(b)(i)	$D – G – C – A – E – F – B – H – D$ 1580 metres $A – C – D$ - G then method stalls	M1 A1 B1	Correct cycle If drawn then arcs must be directed 1580 Identifying the stall	[3]
	(ii)	$BF = 100$ $FE = 50$ $ED = 100$ $DG = 80$ $EH = 110$ $DC = 200$ Order of adding nodes: $B\ F\ E\ D\ G\ H\ C$ Total weight of tree = 640 metres	M1 A1 B1 A1 B1	Use of Prim's algorithm to build tree (e.g. an attempt at list of arcs or order of adding vertices). NOT Kruskal Correct arcs chosen (listed or seen on tree) A correct tree with vertices labelled Order stated or clearly implied 640	[5]
	(iii)	Lower bound = 640 + 100 + 200 = 940 940 metres $\leq$ shortest tour $\leq$ 1580 metres	M1 A1	300 + weight of their tree their 940 $\leq$ length $\leq$ their 1580 (condone use of < here)	[2]
				Total =	**18**



For reference:

4737 Decision Mathematics 2

<table>
<tr><td>1(a)</td><td>(i)</td><td> 
</td><td>B1</td><td>A correct bipartite graph</td><td>[1]</td></tr>
<tr><td></td><td>(ii)</td><td> 
</td><td>B1</td><td>A second bipartite graph showing the incomplete matching correctly</td><td>[1]</td></tr>
<tr><td></td><td>(iii)</td><td>F – R – B – P

A = T B = P E = U F = R G = Q</td><td>B1

B1</td><td>This path in any reasonable form

This complete matching</td><td>[2]</td></tr>
<tr><td></td><td>(iv)</td><td>A = P B = T E = U F = R G = Q</td><td>B1</td><td>This complete matching</td><td>[1]</td></tr>
<tr><td>(b)</td><td>(i)</td><td>Hungarian algorithm finds the minimum cost matching, subtract from 10 to convert a maximising problem into a minimising problem.

Column X is a dummy column (dummy task) to make the table square</td><td>B1

B1</td><td>An appropriate reference to maximising/minimising

‘Dummy’ or ‘square table’ or equivalent</td><td>[2]</td></tr>
<tr><td></td><td>(ii)</td><td>
<table>
<tr><td></td><td>C</td><td>D</td><td>L</td><td>S</td><td>X</td></tr>
<tr><td>H</td><td>1</td><td>2</td><td>4</td><td>4</td><td>10</td></tr>
<tr><td>I</td><td>2</td><td>4</td><td>7</td><td>6</td><td>10</td></tr>
<tr><td>J</td><td>4</td><td>6</td><td>5</td><td>9</td><td>10</td></tr>
<tr><td>K</td><td>3</td><td>8</td><td>7</td><td>7</td><td>10</td></tr>
<tr><td>N</td><td>3</td><td>7</td><td>7</td><td>5</td><td>10</td></tr>
</table>
</td><td></td><td>For reference only</td><td></td></tr>
</table>

Reduce columns

0	**0**	**0**	**0**	**0**
1	2	3	2	0
3	4	1	5	0
2	6	3	3	0
2	5	3	1	0

M1 Either reducing columns or reducing rows of 5×5 matrix

Rows are already reduced

A1 This reduced matrix
Correct answer only

Augment by 1

0	**0**	0	0	1
0	1	2	1	0
2	3	**0**	4	0
1	5	2	2	**0**
1	4	2	**0**	0

M1 A reasonable attempt to augment

A1 This final matrix
Correct answer only

H = *D* Harry is the director
I = *C* Iannos operates the camera
J = *L* Jack is in charge of lighting
N = *S* Nadia is in charge of sound
(and Kerry is not used)

B1 This matching, indicated in any way

Total score =
(10-2) + (10-2) + (10-5) + (10-5) + (10-10)
= 26

M1 A reasonable attempt,
A1 14 or 24 ⇒ M1, A0
26 **[7]**

(iii)

	C	*D*	*L*	*S*
I	2	4	7	6
J	4	6	5	9
K	3	8	7	7
N	3	7	7	5

B1 This 4×4 matrix (need not have row and column labels)

Reduce columns

0	0	2	1
2	2	0	4
1	4	2	2
1	3	2	0

Or reduce rows

0	2	5	4
0	2	1	5
0	5	4	4
0	4	4	2

M1

Then reduce rows

0	**0**	2	1
2	2	**0**	4
0	3	1	1
1	3	2	**0**

Then reduce columns

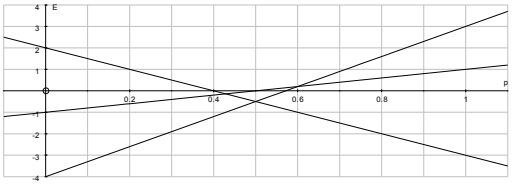
0	**0**	4	2
0	0	**0**	3
0	3	3	2
0	2	3	**0**

A1

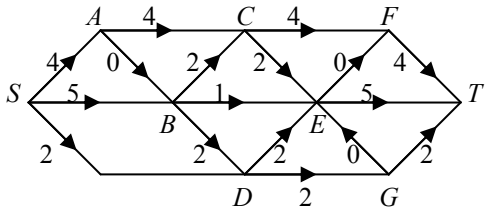
I = *D* *J* = *L* *K* = *C* *N* = *S*

B1 This matching indicated in any way
Correct answer only **[4]**

Total = 18

2	**(i)**	-2	B1	Accept 'loses 2' or equivalent	**[1]**
	(ii)	Column *W* is dominated by column *Y*. If Rowena plays *P*, Collette loses 2 with *W* but 1 with *Y*. If Rowena plays *Q*, Collette loses 1 with *W* but gains 1 with *Y*.	B1 B1	Stating *Y* (but not *W* dominates *Y*) Correct comparisons explained, 2 > 1 and 1 > -1, or equivalent	**[2]**
	(iii)	Collete [*W*] *X* *Y* *Z* Row min Rowena *P* [2] -3 1 3 -3 *Q* [1] 2 -1 -4 -4 Col max [2] 2 1 3 Play-safe for Rowena is *P* Play-safe for Collette is *Y*	M1 A1 A1	Determining row minima and column maxima, or equivalent. Must be correct, including *W* if shown. May not be implied from answers. *P* stated *Y* stated	**[3]**
	(iv)	$-3p + 2(1-p) = 2-5p$ *Y* gives $2p-1$ *Z* gives $7p-4$	B1 B1	$2-5p$ in simplified form Both $2p-1$ and $7p-4$ in any form	**[2]**
	(v)	 $7p - 4 = 2 - 5p \Rightarrow p = 0.5$ $E = -0.5$	 B1 M1 A1 B1	Graph must be on graph paper Their lines drawn correctly on a reasonable scale Solving the correct pair of equations (only) or using graph correctly 0.5, correct answer only -0.5, correct answer only	**[4]**
	(vi)	Add 4 throughout matrix to make all values non-negative On this augmented matrix, if Collette plays *Y* Rowena expects $4p_1 + 3p_2 + 6p_3$, and if Collette plays *Z* Rowena expects $7p_1 + 0p_2 + 2p_3$ We are solving a <u>maximin</u> problem. *m* is less than or equal to each of these values since we need find the maximum value of the worst possible augmented expected pay-off for each value of *p*	B1 B1 B1	'Add 4', or new matrix written out or equivalent Relating to columns *Y* and *Z* respectively. Note: $4p_1 + 3p_2 + 6p_3$ and $7p_1 + 2p_3$ are given in question Or shown on a diagram. For <u>each value of *p*</u> we look at the minimum output.	**[3]**
	(vii)	We use an inequality instead of an equality because this is needed to enable the Simplex algorithm to pivot on a row that will increase the value of *M*	B1	So that we can use the Simplex algorithm.	**[1]**
	(viii)	$p_3 = \frac{3}{7}$ $E = \frac{6}{7}$	B1 B1	$\frac{3}{7}$ $\frac{6}{7}$	**[2]**
					Total = 18

ANSWERED ON INSERT

3	**(i)**	$\{S\ A, B, D, G\}$, $\{C, E, F, T\}$ (given) $AC = 4$, $BC = 2$, $BE = 1$, $DE = 2$, $GE = 5$, $GT = 6$ 4+2+1+2+5+6 = 20 litres per minute	M1 A1	Identifying the correct arcs, on a diagram or list or by using 4, 2, 1, 2, 5, 6 20 from a correct calculation	**[2]**
	(ii)	At most 2 litres per minute can enter G so the arc GE can carry at most 2 litres per minute	B1	Maximum into $G = 2$	**[1]**
	(iii)	At most 8 litres per minute can flow into E Flow shown on diagram on insert Flow in = flow out for each vertex except S, T A feasible flow of 8 litres per minute through E	B1 M1 A1	8 A flow of the rate they have claimed through E (irrespective of whether it is feasible) (directions may not be changed, assume a blank means 0) No pipe capacities exceeded and flow through $E = 8$	**[3]**
	(iv)	Arrows labelled on diagram $SA = 0$ $AC = 0$ $CF = 0$ $FT = 1$ $AS = 4$ $CA = 4$ $FC = 4$ $TF = 4$ $AB = 3$ $BC = 2$ $CE = 3$ $EF = 4$ $BA = 0$ $CB = 0$ $EC = 0$ $FE = 0$ $SB = 4$ $BE = 0$ $ET = 5$ $BS = 1$ $EB = 1$ $TE = 1$ $BD = 3$ $DE = 2$ $EG = 0$ $DB = 0$ $ED = 0$ $GE = 5$ $SD = 0$ $DG = 0$ $GT = 4$ $DS = 2$ $GD = 2$ $TG = 2$	 M1 M1 A1	Assume blanks mean 0 Arrows on arcs on one of the routes $SACFT$, $SBET$, $SDGT$ labelled correctly, or all labels on the route reversed Arrows on all three routes labelled correctly or all reversed All arrows labelled correctly, not reversed	**[3]**
	(v)	Amount that flows along $SBDET$ = 2 litres per min $SB = 4\ 2$ $BD = 3\ 1$ $DE = 2\ 0$ $ET = 5\ 3$ $BS = 1\ 3$ $DB = 0\ 2$ $ED = 0\ 2$ $TE = 1\ 3$	B1 M1 A1	2 For arrows on route $SBDET$: Labels updated consistently These all labelled correctly (and not reversed)	**[3]**
	(vi)	Route used = $SBCET$ $SB = 4\ 2\ 0$ $BC = 2\ 0$ $CE = 3\ 1$ $ET = 5\ 3\ 1$ $BS = 1\ 3\ 5$ $CB = 0\ 2$ $EC = 0\ 2$ $TE = 1\ 3\ 5$	B1 M1 A1	$SBCET$ listed For arrows on route $SBCET$: Labels updated consistently These all labelled correctly (and not reversed)	**[3]**
	(vii)		B1	Follow through their (v) and (vi) if possible Assume blanks mean 0	**[1]**
	(viii)	Eg cut through arcs SA, SB, SD Or arcs AC, BC, BE, DE, DG	M1 A1	A suitable cut chosen, indicated in any way Indicated by listing arcs cut	**[2]**
					Total = 18

PART (a) ANSWERED ON INSERT

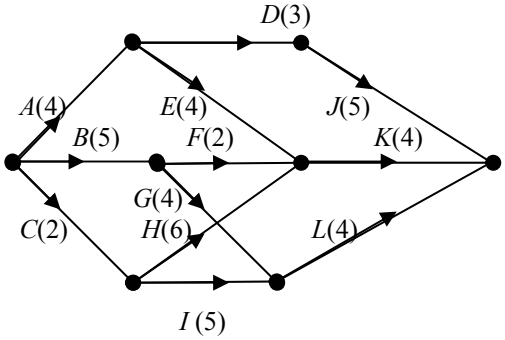
4 (a)

Stage	State	Action	Working	Suboptimal maximum
2	0	0	5	**5**
	1	0	4	**4**
	2	0	4	**4**
1	0	0	3+ **5 = 8**	**8**
		1	4+ **4 = 8**	**8**
	1	1	2+ **4 = 6**	
		2	4+ **4 = 8**	**8**
	2	1	6+ **4 = 10**	**10**
		2	5+ **4 = 9**	
0	0	0	4+ **8 = 12**	
		1	5+ **8 = 13**	**13**
		2	2+ **10 = 12**	

Length of longest path = 13
Route = (0;0) – (1;1) – (2;2) – (3;0)

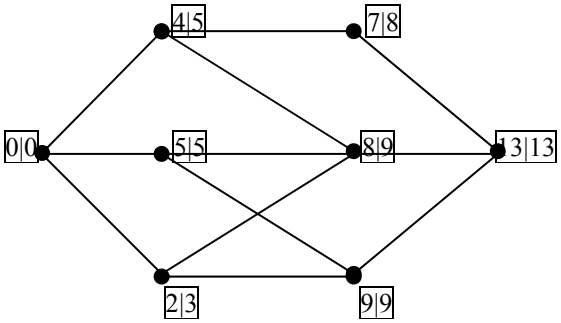
B1 5, 4, 4 identified as suboptimal maxima for stage 2
M1 Transferring suboptimal maxima from stage 2 to stage 1 correctly
A1 Correct additions or totals seen for all rows in stage 1
B1 8, 8, 10 identified as suboptimal maxima for stage 1 (cao)
M1 Transferring suboptimal maxima from stage 1 to stage 0 correctly
A1 Correct additions or totals seen for all rows in stage 0
B1 13
B1 Correct route or in reverse (including (0; 0) and (3; 0)) **[8]**

(b)(i)



M1 A reasonable attempt, arcs should be labelled
A1 Any correct form
Condone extra dummies provided precedences are not violated, accept networks with multiple end vertices
Arc weights may be shown but are not necessary **[2]**

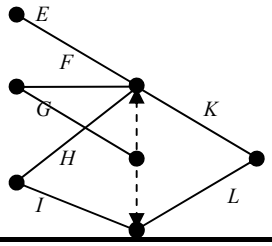
(ii)



Minimum project completion time = 13 days
Critical activities *B*, *G*, *L*

M1 Follow through their network if possible
Values at vertices may be recorded using any consistent notation
A1 Forward pass with no more than one independent error
M1 Forward pass correct
A1 Backward pass with no more than one independent error (follow through their 13)
B1 Backward pass correct
B1 13 stated, cao
B, *G*, *L* correct answer only **[6]**

(iii)



Not follow through
B1 A directed dummy from end of *G* to start of *K*
B1 A directed dummy from end of *G* to start of *L*
Condone extra dummies provided precedences are not violated
Watch out for *K* following *I* **[2]**

Total = 18

Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)
June 2008 Examination Series

Unit Threshold Marks

7892		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	63	55	47	39	32	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	55	47	40	33	26	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	56	49	43	37	31	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	49	43	37	31	25	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	54	47	41	35	29	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	61	53	45	37	29	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	56	47	38	29	20	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	56	47	38	29	21	0
	UMS	100	80	70	60	50	40	0
4731	Raw	72	59	50	42	34	26	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	60	52	45	38	31	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	55	48	41	34	28	0
	UMS	100	80	70	60	50	40	0
4735	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	53	46	39	32	26	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	61	54	47	40	34	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	33.3	50.4	65.4	77.0	86.6	100	14679
3891	100	100	100	100	100	100	1
3892	57.2	76.7	88.2	94.1	97.6	100	1647
7890	45.4	67.3	82.4	92.1	97.8	100	10512
7891	33.3	66.7	100	100	100	100	6
7892	56.5	77.9	90.0	95.4	98.2	100	1660

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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