OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE **4763/01**

MATHEMATICS (MEI)

Mechanics 3

FRIDAY 23 MAY 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

[Turn over

1 **(a)** **(i)** Write down the dimensions of velocity, acceleration and force. [3]

A ball of mass m is thrown vertically upwards with initial velocity U. When the velocity of the ball is v, it experiences a force λv^2 due to air resistance where λ is a constant.

(ii) Find the dimensions of λ. [2]

A formula approximating the greatest height H reached by the ball is

$$H \approx \frac{U^2}{2g} - \frac{\lambda U^4}{4mg^2}$$

where g is the acceleration due to gravity.

(iii) Show that this formula is dimensionally consistent. [4]

A better approximation has the form $H \approx \frac{U^2}{2g} - \frac{\lambda U^4}{4mg^2} + \frac{1}{6}\lambda^2 U^{\alpha} m^{\beta} g^{\gamma}$.

(iv) Use dimensional analysis to find α, β and γ. [5]

(b) A girl of mass 50 kg is practising for a bungee jump. She is connected to a fixed point O by a light elastic rope with natural length 24 m and modulus of elasticity 2060 N. At one instant she is 30 m vertically below O and is moving vertically upwards with speed 12 m s^{-1}. She comes to rest instantaneously, with the rope slack, at the point A. Find the distance OA. [4]

2 A particle P of mass 0.3 kg is connected to a fixed point O by a light inextensible string of length 4.2 m.

Firstly, P is moving in a horizontal circle as a conical pendulum, with the string making a constant angle with the vertical. The tension in the string is 3.92 N.

(i) Find the angle which the string makes with the vertical. [2]

(ii) Find the speed of P. [4]

P now moves in part of a vertical circle with centre O and radius 4.2 m. When the string makes an angle θ with the downward vertical, the speed of P is v m s^{-1} (see Fig. 2). You are given that $v = 8.4$ when $\theta = 60°$.

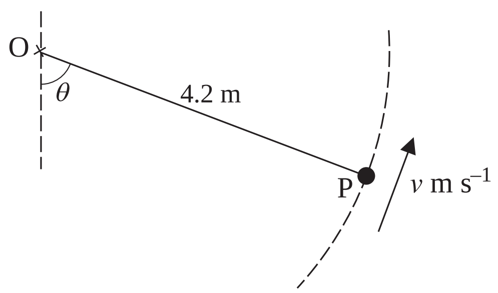

Fig. 2

(iii) Find the tension in the string when $\theta = 60°$. [3]

(iv) Show that $v^2 = 29.4 + 82.32 \cos\theta$. [4]

(v) Find θ at the instant when the string becomes slack. [5]

3 A small block B has mass 2.5 kg. A light elastic string connects B to a fixed point P, and a second light elastic string connects B to a fixed point Q, which is 6.5 m vertically below P.

The string PB has natural length 3.2 m and stiffness $35\,\mathrm{N\,m^{-1}}$; the string BQ has natural length 1.8 m and stiffness $5\,\mathrm{N\,m^{-1}}$.

The block B is released from rest in the position 4.4 m vertically below P. You are given that B performs simple harmonic motion along part of the line PQ, and that both strings remain taut throughout the motion. Air resistance may be neglected. At time t seconds after release, the length of the string PB is x metres (see Fig. 3).

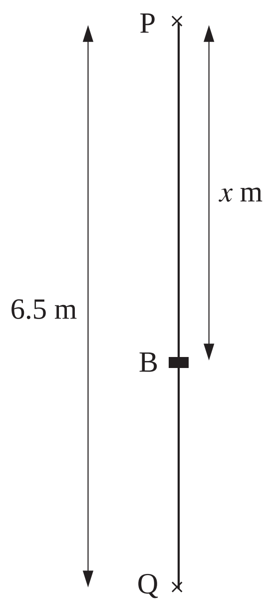

Fig. 3

(i) Find, in terms of x, the tension in the string PB and the tension in the string BQ. [3]

(ii) Show that $\dfrac{d^2x}{dt^2} = 64 - 16x$. [4]

(iii) Find the value of x when B is at the centre of oscillation. [2]

(iv) Find the period of oscillation. [2]

(v) Write down the amplitude of the motion and find the maximum speed of B. [3]

(vi) Find the time after release when B is first moving *downwards* with speed $0.9\,\mathrm{m\,s^{-1}}$. [4]

[Question 4 is printed overleaf.]

4 **(a)** A uniform solid of revolution is obtained by rotating through 2π radians about the y-axis the region bounded by the curve $y = 8 - 2x^2$ for $0 \leqslant x \leqslant 2$, the x-axis and the y-axis.

(i) Find the y-coordinate of the centre of mass of this solid. [7]

The solid is now placed on a rough plane inclined at an angle θ to the horizontal. It rests in equilibrium with its circular face in contact with the plane as shown in Fig. 4.

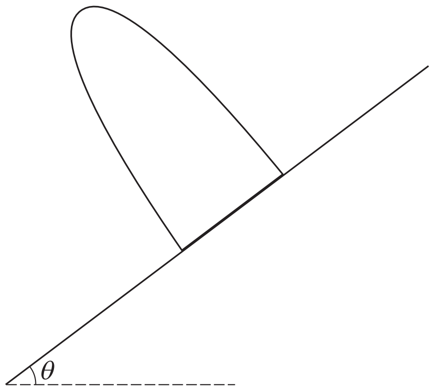

Fig. 4

(ii) Given that the solid is on the point of toppling, find θ. [4]

(b) Find the y-coordinate of the centre of mass of a uniform lamina in the shape of the region bounded by the curve $y = 8 - 2x^2$ for $-2 \leqslant x \leqslant 2$, and the x-axis. [7]