GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Mark Schemes for the Units

June 2009

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MARK SCHEME FOR THE UNITS

4751 (C1) Introduction to Advanced Mathematics

Section A

1	(0, 14) and (14/4, 0) o.e. isw	4	M2 for evidence of correct use of gradient with (2, 6) eg sketch with 'stepping' or $y - 6 = -4(x - 2)$ seen or $y = -4x + 14$ o.e. or M1 for $y = -4x + c$ [accept any letter or number] and M1 for $6 = -4 \times 2 + c$; A1 for (0, 14) [$c = 14$ is not sufficient for A1] and A1 for (14/4, 0) o.e.; allow when $x = 0$, $y = 14$ etc isw	4
2	$[a =]\frac{2(s-ut)}{t^2}$ o.e. as final answer [condone $[a =]\frac{(s-ut)}{0.5t^2}$]	3	M1 for each of 3 complete correct steps, ft from previous error if equivalent difficulty [eg dividing by t does not count as step – needs to be by t^2] $[a =]\frac{(s-ut)}{\frac{1}{2}t^2}$ gets M2 only (similarly other triple-deckers)	3
3	10 www	3	M1 for f(3) = 1 soi and A1 for $31 - 3k = 1$ or $27 - 3k = -3$ o.e. [a correct 3-term or 2-term equation] long division used: M1 for reaching $(9 - k)x + 4$ in working and A1 for $4 + 3(9 - k) = 1$ o.e. equating coeffts method: M2 for $(x - 3)(x^2 + 3x - 1)$ [+ 1] o.e. (from inspection or division)	3
4	$x < 0$ or $x > 6$ (both required)	2	B1 each; if B0 then M1 for 0 and 6 identified;	2
5	(i) 10 www	2	M1 for $\frac{5\times4\times3}{3\times2(\times1)}$ or $\frac{5\times4}{2(\times1)}$ or for 1 5 10 10 5 1 seen	
	(ii) 80 www or ft 8 × their (i)	2	B2 for $80x^3$; M1 for 2^3 or $(2x)^3$ seen	4

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6	any general attempt at n being odd <u>and</u> n being even	M1	M0 for just trying numbers, even if some odd, some even	
	n odd implies n^3 odd and odd − odd = even	A1	or $n(n^2 - 1)$ used with n odd implies $n^2 - 1$ even and odd × even = even etc [allow even × odd = even]	
	n even implies n^3 even and even − even = even	A1	or A2 for $n(n - 1)(n + 1)$ = product of 3 consecutive integers; at least one even so product even; $\text{odd}^3 - \text{odd} = \text{odd}$ etc is not sufft for A1	
			SC1 for complete general method for only one of odd or even eg $n = 2m$ leading to $2(4m^3 - m)$	3
7	(i) 1	2	B1 for 5^0 or for 25 × 1/25 o.e.	
	(ii) 1000	1		3
8	(i) 2/3 www	2	M1 for 4/6 or for $\sqrt{48} = 2\sqrt{12}$ or $4\sqrt{3}$ or $\sqrt{27} = 3\sqrt{3}$ or $\sqrt{108} = 3\sqrt{12}$ or for $\sqrt{\frac{4}{9}}$	
	(ii) $43 - 30\sqrt{2}$ www as final answer	3	M2 for 3 terms correct of $25 - 15\sqrt{2} - 15\sqrt{2} + 18$ soi, M1 for 2 terms correct	5
9	(i) $(x + 3)^2 - 4$	3	B1 for $a = 3$, B2 for $b = -4$ or M1 for $5 - 3^2$ soi	
	(ii) ft their $(-a, b)$; if error in (i), accept $(-3, -4)$ if evidence of being independently obtained	2	B1 each coord.; allow $x = -3$, $y = -4$; or M1 for $\begin{bmatrix}-3\\-4\end{bmatrix}$ o.e. oe for sketch with −3 and −4 marked on axes but no coords given	5
10	$(x^2 - 9)(x^2 + 4)$	M2	or correct use of quad formula or comp sq reaching 9 and −4; allow M1 for attempt with correct eqn at factorising with factors giving two terms correct, or sign error, or attempt at formula or comp sq [no more than two errors in formula/substn]; for this first M2 or M1 allow use of y etc or of x instead of x^2	
	$x^2 = 9$ [or −4] or ft for integers /fractions if first M1 earned	M1	must have x^2; or M1 for $(x + 3)(x - 3)$; this M1 may be implied by $x = \pm 3$	
	$x = \pm 3$ cao	A1	A0 if extra roots if M0 then allow SC1 for use of factor theorem to obtain both 3 and −3 as roots or $(x + 3)$ and $(x - 3)$ found as factors and SC2 for $x^2 + 4$ found as other factor using factor theorem [ie max SC3]	4

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Section B

11	**i**	$y = 3x$	2	M1 for grad AB = $\frac{1-3}{6}$ or − 1/3 o.e.	2	
	ii	eqn AB is $y = -1/3\,x + 3$ o.e. or ft	M1	need not be simplified; no ft from midpt used in (i); may be seen in (i) but do not give mark unless used in (ii)		
			M1	eliminating x or y , ft their eqns		
		$3x = -1/3x + 3$ or ft $x = 9/10$ or 0.9 o.e. cao	A1	if find y first, cao for y then ft for x		
		$y = 27/10$ oe ft their 3 × their x	A1	ft dep on both Ms earned	4	
	iii	$\left(\frac{9}{10}\right)^2\left(1+3^2\right)$ o.e and completion to given answer	2	or square root of this; M1 for $\left(\frac{9}{10}\right)^2+\left(\frac{27}{10}\right)^2$ or 0.81 + 7.29 soi or ft their coords (inc midpt) <u>or</u> M1 for distance = 3 cos θ and tan θ = 3 and M1 for showing $\sin\theta=\frac{3}{\sqrt{10}}$ and completion	2	
	iv	$2\sqrt{10}$	2	M1 for $6^2 + 2^2$ or 40 or square roots of these	2	
	v	9 www or ft their $a\sqrt{10}$	2	M1 for ½ × 3 × 6 or $\frac{1}{2}\times\text{their}2\sqrt{10}\times\frac{9}{10}\sqrt{10}$	2	12

12	**iA**	expansion of one pair of brackets	M1	eg $[(x + 1)](x^2 - 6x + 8)$; need not be simplified		
		correct 6 term expansion	M1	eg $x^3 - 6x^2 + 8x + x^2 - 6x + 8$; or M2 for correct 8 term expansion: $x^3 - 4x^2 + x^2 - 2x^2 + 8x - 4x - 2x + 8$, M1 if one error allow equivalent marks working backwards to factorisation, by long division or factor theorem etc or M1 for all three roots checked by factor theorem and M1 for comparing coeffts of x^3	2	
	iB	cubic the correct way up x-axis: −1, 2, 4 shown y-axis 8 shown	G1 G1 G1	with two tps and extending beyond the axes at 'ends' ignore a second graph which is a translation of the correct graph	3	
	iC	$[y=](x - 2)(x - 5)(x - 7)$ isw or $(x - 3)^3 - 5(x - 3)^2 + 2(x - 3) + 8$ isw or $x^3 - 14x^2 + 59x - 70$	2	M1 if one slip or for $[y =]$ f$(x - 3)$ or for roots identified at 2, 5, 7 or for translation 3 to the left allow M1 for complete attempt: $(x + 4)(x + 1)(x - 1)$ isw or $(x + 3)^3 - 5(x + 3)^2 + 2(x + 3) + 8$ isw		
		(0, −70) or $y = -70$	1	allow 1 for (0, −4) or $y = -4$ after f$(x + 3)$ used	3	
	ii	$27 - 45 + 6 + 8 = -4$ or $27 - 45 + 6 + 12 = 0$	B1	or correct long division of $x^3 - 5x^2 + 2x + 12$ by $(x - 3)$ with no remainder or of $x^3 - 5x^2 + 2x + 8$ with rem −4		
		long division of f(x) or their f(x) + 4 by $(x - 3)$ attempted as far as $x^3 - 3x^2$ in working	M1	or inspection with two terms correct eg $(x - 3)(x^2$ ……….. $- 4)$		
		$x^2 - 2x - 4$ obtained	A1			
		$[x =]\dfrac{2 \pm \sqrt{(-2)^2 - 4\times(-4)}}{2}$ or $(x - 1)^2 = 5$	M1	dep on previous M1 earned; for attempt at formula or comp square on their other 'factor'		
		$\dfrac{2 \pm \sqrt{20}}{2}$ o.e. isw or $1 \pm \sqrt{5}$	A1		5	13

13	**i**	(5, 2) $\sqrt{20}$ or $2\sqrt{5}$	1 1	0 for $\pm\sqrt{20}$ etc	2
	ii	no, since $\sqrt{20}$ < 5 or showing roots of $y^2 - 4y + 9 = 0$ o.e. are not real	2	or ft from their centre and radius M1 for attempt (no and mentioning $\sqrt{20}$ or 5) or sketch or solving by formula or comp sq $(-5)^2 + (y - 2)^2 = 20$ [condone one error] or SC1 for fully comparing distance from x axis with radius and saying yes	2
	iii	$y = 2x - 8$ or simplified alternative	2	M1 for $y - 2 = 2(x - 5)$ or ft from **(i)** or M1 for $y = 2x + c$ and subst their **(i)** or M1 for ans $y = 2x + k$, $k \neq 0$ or -8	2
	iv	$(x - 5)^2 + (2x)^2 = 20$ o.e.	M1	subst $2x + 2$ for y [oe for x]	
		$5x^2 - 10x + 5[= 0]$ or better equiv.	M1	expanding brackets and rearranging to 0; condone one error; dep on first M1	
		obtaining $x = 1$ (with no other roots) or showing roots equal	M1		
		one intersection [so tangent]	A1	o.e.; must be explicit; or showing line joining (1,4) to centre is perp to $y = 2x + 2$	
		(1, 4) cao	A1	allow $y = 4$	
		<u>alt method</u>			
		$y - 2 = -½ (x - 5)$ o.e.	M1	line through centre perp to $y = 2x + 2$	
		$2x + 2 - 2 = -½ (x - 5)$ o.e.	M1	dep; subst to find intn with $y = 2x + 2$	
		$x = 1$	A1		
		$y = 4$ cao	A1		
		showing (1, 4) is on circle	B1	by subst in circle eqn or finding dist from centre = $\sqrt{20}$ [a similar method earns first M1 for eqn of diameter, 2nd M1 for intn of diameter and circle A1 each for x and y coords and last B1 for showing (1, 4) on line – award onlyA1 if (1, 4) and (9, 0) found without (1, 4) being identified as the soln]	
		<u>alt method</u>			
		perp dist between $y = 2x - 8$ and $y = 2x + 2 = 10 \cos \theta$ where $\tan \theta = 2$	M1		
		showing this is $\sqrt{20}$ so tgt	M1		
		$x = 5 - \sqrt{20} \sin \theta$	M1	or other valid method for obtaining x	
		$x = 1$	A1		
		(1, 4) cao	A1	allow $y = 4$	5

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4752 (C2) Concepts for Advanced Mathematics

Section A

1	using Pythagoras to show that hyp. of right angled isos. triangle with sides a and a is $\sqrt{2}a$ completion using definition of cosine	M1 A1	www a any letter or a number NB answer given	2
2	$2x^6 + 5x$ value at 2 − value at 1 131	M2 M1 A1	M1 if one error ft attempt at integration only	4
3	(i) 193 (ii) divergent + difference between terms increasing o.e.	2 1	M1 for 8 + 15 + …+ 63	3
4	(i) 2.4 (ii) 138	2 2	M1 for 43.2 ÷ 18 M1 for their (i) × $\frac{180}{\pi}$ or $\theta = \frac{43.2\times360}{36\pi}$ o.e. or for other rot versions of 137.50…	4
5	(i)sketch of cos$x$; one cycle, sketch of cos2x; two cycles, both axes scaled correctly (ii) (1-way) stretch parallel to y-axis sf 3	1 1 D1 1 D1		5
6	$y' = 3x^2 - 12x - 15$ use of $y' = 0$, s.o.i. ft $x = 5, -1$ c.a.o. $x < -1$ or $x > 5$ ft	M1 M1 A1 A1 A1	for two terms correct	5
7	use of $\cos^2\theta = 1 - \sin^2\theta$ at least one correct interim step in obtaining $4\sin^2\theta - \sin\theta = 0$. $\theta = 0$ and 180, 14.(47…) 165 − 166	M1 M1 B1 B1 B1	NB answer given r.o.t to nearest degree or better -1 for extras in range	5
8	attempt to integrate $3\sqrt{x} - 5$ $[y=]\ 2x^{\frac{3}{2}} - 5x + c$ subst of (4, 6) in their integrated eqn $c = 10$ or $[y=]\ 2x^{\frac{3}{2}} - 5x + 10$	M1 A2 M1 A1	A1 for two terms correct	5

9	(i) 7	1		
	(ii) 5.5 o.e.	2	M1 for at least one of $5\log_{10}a$ or $\frac{1}{2}\log_{10}a$ or $\log_{10}a^{5.5}$ o.e.	3

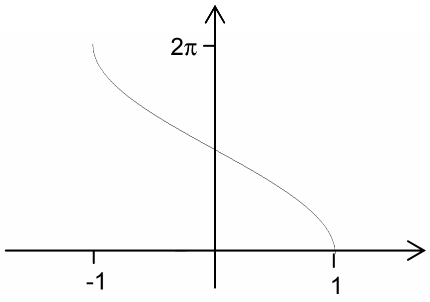
Section B

10	**i**	0.6(0...), 0.8(45...), [1], 1.1(76...) 1.3(0...), 1.6(0...)	T1	Correct to 2 d.p. Allow 0.6, 1.3 and 1.6	
		points plotted correctly ft	P1	tol. 1 mm	
		ruled line of best fit	L1		3
	ii	b = their intercept	M1		
		a = their gradient	M1		
		$-11 \le b \le -8$ *and* $21 \le a \le 23.5$	A1		3
	iii	34 to 35 m	1		1
	iv	29 = ‘22’log$t$ – ‘9’	M1		
		$t = 10^{\text{‘1.727...’}}$	M1		
		55 [years] approx	A1	accept 53 to 59	3
	v	For small t the model predicts a negative height (or $h = 0$ at approx 2.75)	1		
		Hence model is unsuitable	D1		2
11	**iA**	10 + 20 + 30 + 40 + 50 + 60	B1	or $\frac{6}{2}(2\times10+5\times10)$ or $\frac{6}{2}(10+60)$	1
	iB	correct use of AP formula with $a = 10$ and $d = 10$	M1		
		$n(5 + 5n)$ or $5n(n + 1)$ or $5(n^2 + n)$ or $(5n^2 + 5n)$	A1		
		$10n^2 + 10n - 20700 = 0$	M1	Or better	
		45 c.a.o.	A1		4
	iiA	4	1		1
	iiB	£2555	2	M1 for $5(1 + 2 + \ldots 2^8)$ or $5(2^9 - 1)$ o.e.	2
	iiC	correct use of GP formula with $a = 5$, $r = 2$	M1		
		$5(2^n - 1)$ o.e. = 2621435	D M1	‘S’ need not be simplified	
		$2^n = 524288$ www	M1		
		19 c.a.o.	A1		4
12	**i**	6.1	2	M1 for $\frac{(3.1^2-7)-(3^2-7)}{3.1-3}$ o.e.	2

	ii	$\frac{\left((3+h)^2-7\right)-\left(3^2-7\right)}{h}$ numerator = $6h + h^2$ $6 + h$	M1 M1 A1	s.o.i.	3
	iii	as h tends to 0, grad. tends to 6 o.e. f.t.from "6"+h	M1 A1		2
	iv	$y - 2 =$ '6' $(x - 3)$ o.e. $y = 6x - 16$	M1 A1	6 may be obtained from $\frac{dy}{dx}$	2
	v	At P, $x = 16/6$ o.e. or ft At Q, $x = \sqrt{7}$ 0.021 c.a.o.	M1 M1 A1		3

4753 (C3) Methods for Advanced Mathematics

Section A

1 $\int_0^{\frac{\pi}{6}} \sin 3x \, dx = \left[-\frac{1}{3}\cos 3x\right]_0^{\frac{\pi}{6}}$ $= -\frac{1}{3}\cos\frac{\pi}{2} + \cos 0$ $= \frac{1}{3}$	B1 M1 A1cao [3]	$\left[-\frac{1}{3}\cos 3x\right]$ or $\left[-\frac{1}{3}\cos u\right]$ substituting correct limits in ±k cos … 0.33 or better.
2(i) $100 = Ae^0 = A \Rightarrow A = 100$ $50 = 100\,e^{-1500k}$ $\Rightarrow$ $e^{-1500k} = 0.5$ $\Rightarrow$ $-1500k = \ln 0.5$ $\Rightarrow$ $k = -\ln 0.5 \div 1500 = 4.62 \times 10^{-4}$	M1A1 M1 M1 A1 [5]	 $50 = A\,e^{-1500k}$ ft their 'A' if used taking lns correctly 0.00046 or better
(ii) $1 = 100e^{-kt}$ $\Rightarrow$ $-kt = \ln 0.01$ $\Rightarrow$ $t = -\ln 0.01 \div k$ = 9966 years	M1 M1 A1 [3]	ft their A and k taking lns correctly art 9970
3 	M1 B1 A1 [3]	Can use degrees or radians reasonable shape (condone extra range) passes through (-1, 2π), (0, π) and (1, 0) good sketches – look for curve reasonably vertical at (–1, 2π) and (1, 0), negative gradient at (0, π). Domain and range must be clearly marked and correct.
4 $g(x) = 2\lvert x-1\rvert$ $\Rightarrow$ $b = 2\lvert 0-1\rvert = 2$ or (0, 2) $2\lvert x-1\rvert = 0$ $\Rightarrow$ $x = 1$, so $a = 1$ or (1, 0)	B1 M1 A1 [3]	Allow unsupported answers. www $\lvert x\rvert = 1$ is A0 www

5(i) $e^{2y} = 1 + \sin x$ ⇒ $2e^{2y}\frac{dy}{dx} = \cos x$ ⇒ $\frac{dy}{dx} = \frac{\cos x}{2e^{2y}}$	M1 B1 A1 [3]	Their $2e^{2y} \times \frac{dy}{dx}$ $2e^{2y}$ o.e. cao
(ii) $2y = \ln(1 + \sin x)$ ⇒ $y = ½ \ln(1 + \sin x)$ ⇒ $\frac{dy}{dx} = \frac{1}{2}\frac{\cos x}{1+\sin x}$ $= \frac{\cos x}{2e^{2y}}$ as before	B1 M1 B1 E1 [4]	 chain rule (can be within 'correct' quotient rule with $dv/dx = 0$) $1/u$ or $1/(1 + \sin x)$ soi www
6 $f\,f(x) = \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}$ $= \frac{x+1+x-1}{x+1-x+1}$ $= \frac{2x}{2} = x$ * $f^{-1}(x) = f(x)$ Symmetrical about $y = x$.	M1 M1 E1 B1 B1 [5]	correct expression without subsidiary denominators e.g. $= \frac{x+1+x-1}{x-1} \times \frac{x-1}{x+1-x+1}$ stated, or shown by inverting
7(i) $(A)\ (x - y)(x^2 + xy + y^2)$ $= x^3 + x^2y + xy^2 - yx^2 - xy^2 - y^3$ $= x^3 - y^3$ * $(B)\ (x + ½\, y)^2 + ¾\, y^2$ $= x^2 + xy + ¼\, y^2 + ¾\, y^2$ $= x^2 + xy + y^2$	M1 E1 M1 E1 [4]	expanding - allow tabulation www $(x + ½\, y)^2 = x^2 + ½\, xy + ½\, xy + ¼\, y^2$ o.e. cao www
(ii) $x^3 - y^3 = (x - y)[(x + ½\, y)^2 + ¾\, y^2]$ $(x + ½\, y)^2 + ¾\, y^2 > 0$ [as squares ≥ 0] ⇒ if $x - y > 0$ then $x^3 - y^3 > 0$ ⇒ if $x > y$ then $x^3 > y^3$ *	M1 M1 E1 [3]	substituting results of (i)

8(i) A: $1 + \ln x = 0$ $\Rightarrow$ $\ln x = -1$ so A is $(e^{-1}, 0)$ $\Rightarrow$ $x = e^{-1}$ B: $x = 0, y = e^{0-1} = e^{-1}$ so B is $(0, e^{-1})$ C: $f(1) = e^{1-1} = e^0 = 1$ $g(1) = 1 + \ln 1 = 1$	M1 A1 B1 E1 E1 [5]	SC1 if obtained using symmetry condone use of symmetry Penalise $A = e^{-1}$, $B = e^{-1}$, or co-ords wrong way round, but condone labelling errors.
(ii) *Either* by invertion: e.g. $y = e^{x-1}$ $x \leftrightarrow y$ $x = e^{y-1}$ $\Rightarrow$ $\ln x = y - 1$ $\Rightarrow$ $1 + \ln x = y$	M1 E1	taking lns or exps
or by composing e.g. $fg(x) = f(1 + \ln x)$ $= e^{1 + \ln x - 1}$ $= e^{\ln x} = x$	M1 E1 [2]	$e^{1 + \ln x - 1}$ or $1 + \ln(e^{x-1})$
(iii) $\int_0^1 e^{x-1} dx = \left[e^{x-1}\right]_0^1$ $= e^0 - e^{-1}$ $= 1 - e^{-1}$	M1 M1 A1cao [3]	$\left[e^{x-1}\right]$ o.e. or $u = x - 1 \Rightarrow \left[e^u\right]$ substituting correct limits for x or u o.e. not e^0, must be exact.
(iv) $\int \ln x \, dx = \int \ln x \frac{d}{dx}(x) \, dx$ $= x \ln x - \int x . \frac{1}{x} \, dx$ $= x \ln x - x + c$ $\Rightarrow \int_{e^{-1}}^1 g(x) \, dx = \int_{e^{-1}}^1 (1 + \ln x) \, dx$ $= [x + x \ln x - x]_{e^{-1}}^1$ $= [x \ln x)]_{e^{-1}}^1$ $= 1 \ln 1 - e^{-1} \ln(e^{-1})$ $= e^{-1}$ *	M1 A1 A1cao B1ft DM1 E1 [6]	parts: $u = \ln x$, $du/dx = 1/x$, $v = x$, $dv/dx = 1$ condone no 'c' ft their '$x \ln x - x$' (provided 'algebraic') substituting limits dep B1 www
(v) Area = $\int_0^1 f(x) \, dx - \int_{e^{-1}}^1 g(x) \, dx$ $\int_0^1 f(x) \, dx - \int_{e^{-1}}^1 g(x) \, dx$ $= (1 - e^{-1}) - e^{-1}$ $= 1 - \frac{2}{e}$	M1 A1cao	Must have correct limits 0.264 or better.

or Area OCB = area under curve – triangle $= 1 - e^{-1} - \frac{1}{2} \times 1 \times 1$ $= \frac{1}{2} - e^{-1}$	M1	OCA or OCB = $\frac{1}{2} - e^{-1}$
or Area OAC = triangle – area under curve $= \frac{1}{2} \times 1 \times 1 - e^{-1}$ $= \frac{1}{2} - e^{-1}$		
Total area = $2(\frac{1}{2} - e^{-1}) = 1 - \frac{2}{e}$	A1cao [2]	0.264 or better

Section B

9(i) $a = \frac{1}{3}$	B1 [1]	or 0.33 or better
(ii) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(3x-1)2x - x^2.3}{(3x-1)^2}$	M1 A1	quotient rule
$= \frac{6x^2 - 2x - 3x^2}{(3x-1)^2}$		
$= \frac{3x^2 - 2x}{(3x-1)^2}$		
$= \frac{x(3x-2)}{(3x-1)^2}$ *	E1 [3]	www – must show both steps; penalise missing brackets.
(iii) $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x(3x-2) = 0$	M1	if denom = 0 also then M0
⇒ $x = 0$ or $x = \frac{2}{3}$, so at P, $x = \frac{2}{3}$	A1	o.e e.g. $0.\dot{6}$, but must be exact
when $x = \frac{2}{3}$, $y = \frac{(2/3)^2}{3\times(2/3)-1} = \frac{4}{9}$	M1 A1cao	o.e e.g. $0.\dot{4}$, but must be exact
when $x = 0.6$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -0.1875$	B1	-3/16, or -0.19 or better
when $x = 0.8$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0.1633$	B1	8/49 or 0.16 or better
Gradient increasing ⇒ minimum	E1 [7]	o.e. e.g. 'from negative to positive'. Allow ft on their gradients, provided –ve and +ve respectively. Accept table with indications of signs of gradient.
(iv) $\int \frac{x^2}{3x-1}\mathrm{d}x \quad u = 3x-1 \Rightarrow \mathrm{d}u = 3\mathrm{d}x$	B1	$\frac{\frac{(u+1)^2}{9}}{u}$ o.e.
$= \int \frac{\frac{(u+1)^2}{9}}{u}\frac{1}{3}\mathrm{d}u$	M1	$\times\frac{1}{3}$ (d*u*)
$= \frac{1}{27}\int \frac{(u+1)^2}{u}\mathrm{d}u = \frac{1}{27}\int \frac{u^2+2u+1}{u}\mathrm{d}u$	M1	expanding
$= \frac{1}{27}\int (u + 2 + \frac{1}{u})\,\mathrm{d}u$ *	E1	condone missing d*u*'s
Area = $\int_{\frac{2}{3}}^{1} \frac{x^2}{3x-1}\mathrm{d}x$		
When $x = \frac{2}{3}$, $u = 1$, when $x = 1$, $u = 2$		
$= \frac{1}{27}\int_1^2 (u + 2 + \frac{1}{u})\mathrm{d}u$		
$= \frac{1}{27}\left[\frac{1}{2}u^2 + 2u + \ln u\right]_1^2$	B1	$\left[\frac{1}{2}u^2 + 2u + \ln u\right]$
$= \frac{1}{27}\left[(2 + 4 + \ln 2) - (\frac{1}{2} + 2 + \ln 1)\right]$	M1	substituting correct limits, dep integration

$= \frac{1}{27}(3\tfrac{1}{2} + \ln 2) \quad \left[= \frac{7 + 2\ln 2}{54}\right]$	A1cao [7]	o.e., but must evaluate ln 1 = 0 and collect terms.

4754 (C4) Applications of Advanced Mathematics

Section A

1 $4\cos\theta - \sin\theta = R\cos(\theta+\alpha)$		
$= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$	M1	correct pairs
$\Rightarrow R\cos\alpha = 4,\ \ R\sin\alpha = 1$	B1	$R = \sqrt{17} = 4.123$
$\Rightarrow R^2 = 1^2 + 4^2 = 17, R = \sqrt{17} = 4.123$	M1	$\tan\theta = ¼$ o.e.
$\tan\theta = ¼$	A1	$\theta = 0.245$
$\Rightarrow \theta = 0.245$		
$\sqrt{17}\cos(\theta + 0.245) = 3$		
$\Rightarrow \cos(\theta + 0.245) = \frac{3}{\sqrt{17}}$	M1	$\theta + 0.245 = \arccos 3/\sqrt{17}$
$\Rightarrow \theta + 0.245 = 0.756, 5.527$		ft their R, α for method (penalise extra solutions in the range (-1))
$\Rightarrow \theta = 0.511, 5.282$	A1A1	
	[7]	
2 $\frac{x}{(x+1)(2x+1)} = \frac{A}{x+1} - \frac{B}{(2x+1)}$	M1	correct partial fractions
$\Rightarrow x = A(2x+1) + B(x+1)$	M1	substituting, equating coeffts or cover-up
$x = -1 \Rightarrow -1 = -A \quad \Rightarrow A = 1$	A1	$A = 1$
$x = -½ \Rightarrow -½ = ½B \quad \Rightarrow B = -1$	A1	$B = -1$
$\Rightarrow \frac{x}{(x+1)(2x+1)} = \frac{1}{x+1} - \frac{1}{(2x+1)}$	B1	$\ln(x+1)$ ft their A
$\Rightarrow \int \frac{x}{(x+1)(2x+1)}\,dx = \int \frac{1}{x+1} - \frac{1}{(2x+1)}\,dx$	B1	$-½\ln(2x+1)$ ft their B
$= \ln(x+1) - ½\ln(2x+1) + c$	A1	cao – must have c
	[7]	
3 $\frac{dy}{dx} = 3x^2y$		
$\Rightarrow \int \frac{dy}{y} = \int 3x^2\,dx$	M1	separating variables
$\Rightarrow \ln y = x^3 + c$	A1	
when $x = 1, y = 1, \Rightarrow \ln 1 = 1 + c \Rightarrow c = -1$	B1	condone absence of c $c = -1$ o.e.
$\Rightarrow \ln y = x^3 - 1$		
$\Rightarrow y = e^{x^3-1}$	A1	o.e.
	[4]	
4 When $x = 0, y = 4$	B1	
$\Rightarrow V = \pi\int_0^4 x^2\,dy$	M1	must have integral, π, x^2 and dy s.o.i.
$= \pi\int_0^4 (4-y)\,dy$	M1	must have π ,their $(4-y)$, their numerical y limits
$= \pi\left[4y - \frac{1}{2}y^2\right]_0^4$	B1	$\left[4y - \frac{1}{2}y^2\right]$
$= \pi(16-8) = 8\pi$	A1	
	[5]	

5 $\frac{dy}{dt} = -a(1+t^2)^{-2}.2t$	M1	
$\frac{dx}{dt} = 3at^2$	A1	$(1+t^2)^{-2} \times kt$ for method
$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2at}{3at^2(1+t^2)^2}$	B1	
$= \frac{-2}{3t(1+t^2)^2}$ *	M1	ft
At (a, ½ a) , $t = 1$	E1	
$\Rightarrow$ gradient = $\frac{-2}{3\times 2^2} = -\frac{1}{6}$	M1 A1 [7]	 finding t
6 $\text{cosec}^2\,\theta = 1 + \cot^2\theta$	E1	clear use of $1 + \cot^2\theta = \text{cosec}^2\,\theta$
$\Rightarrow 1 + \cot^2\theta - \cot\theta = 3$ *		
$\Rightarrow \cot^2\theta - \cot\theta - 2 = 0$	M1	factorising or formula
$\Rightarrow (\cot\theta - 2)(\cot\theta + 1) = 0$	A1	roots 2, -1
$\Rightarrow \cot\theta = 2, \tan\theta = ½, \theta = 26.57°$	M1	cot = 1/tan used
$\cot\theta = -1, \tan\theta = -1, \theta = 135°$	A1	$\theta = 26.57°$
	A1	$\theta = 135°$
	[6]	(penalise extra solutions in the range (-1))

Section B

7(i) $\overrightarrow{\mathrm{AB}} = \begin{pmatrix}-1\\-2\\0\end{pmatrix}$	B1	
$\mathbf{r} = \begin{pmatrix}0\\0\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\0\end{pmatrix}$	B1 [2]	or equivalent alternative
(ii) $\mathbf{n} = \begin{pmatrix}1\\0\\1\end{pmatrix}$	B1	
$\cos\theta = \dfrac{\begin{pmatrix}1\\0\\1\end{pmatrix}.\begin{pmatrix}1\\2\\0\end{pmatrix}}{\sqrt{2}\sqrt{5}} = \dfrac{1}{\sqrt{10}}$	B1 M1 M1	correct vectors (any multiples) scalar product used finding invcos of scalar product divided by two modulae
⇒ $\theta = 71.57°$	A1 [5]	72° or better
(iii) $\cos\phi = \dfrac{\begin{pmatrix}-1\\0\\-1\end{pmatrix}.\begin{pmatrix}-2\\-2\\-1\end{pmatrix}}{\sqrt{2}\sqrt{9}} = \dfrac{2+1}{3\sqrt{2}} = \dfrac{1}{\sqrt{2}}$	M1 A1	ft their **n** for method ±1/√2 o.e. exact
⇒ $\theta = 45°$ *	E1 [3]	
(iv) sin 71.57° = k sin 45°	M1	ft on their 71.57°
⇒ k = sin 71.57° / sin 45° = 1.34	A1 [2]	o.e.
(v) $\mathbf{r} = \begin{pmatrix}0\\0\\2\end{pmatrix} + \mu\begin{pmatrix}-2\\-2\\-1\end{pmatrix}$		
$x = -2\mu,\ z = 2 - \mu$ $x + z = -1$	M1	s.o.i.
⇒ $-2\theta + 2 - \theta = -1$	M1	subst in $x + z = -1$
⇒ $3\theta = 3, \theta = 1$	A1	
⇒ point of intersection is (-2, -2, 1)	A1	
distance travelled through glass = distance between (0, 0, 2) and (-2, -2, 1) $= \sqrt{(2^2 + 2^2 + 1^2)}$ = 3 cm	B1 [5]	www dep on $\mu = 1$

8(i) (*A*) $360° \div 24 = 15°$ $CB/OB = \sin 15°$ ⇒ $CB = 1 \sin 15°$ ⇒ $AB = 2CB = 2 \sin 15°$ *	 M1 E1 [2]	AB = 2AC or 2CB ∠AOC = 15° o.e.
(*B*) $\cos 30° = 1 - 2\sin^2 15°$ $\cos 30° = \sqrt{\frac{3}{2}}$ ⇒ $\sqrt{\frac{3}{2}} = 1 - 2\sin^2 15°$ ⇒ $2\sin^2 15° = 1 - \sqrt{\frac{3}{2}} = (2 - \sqrt{3})/2$ ⇒ $\sin^2 15° = \frac{2-\sqrt{3}}{4}$ ⇒ $\sin 15° = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{1}{2}\sqrt{2-\sqrt{3}}$ *	B1 B1 M1 E1 [4]	 simplifying
(*C*) Perimeter $= 12 \times AB = 24 \times \frac{1}{2}\sqrt{2-\sqrt{3}}$ $= 12\sqrt{2-\sqrt{3}}$ circumference of circle > perimeter of polygon ⇒ $2\pi > 12\sqrt{2-\sqrt{3}}$ ⇒ $\pi > 6\sqrt{2-\sqrt{3}}$	M1 E1 [2]	
(ii) (*A*) $\tan 15° = FE \div OF$ ⇒ $FE = \tan 15°$ ⇒ $DE = 2FE = 2\tan 15°$	M1 E1 [2]	
(*B*) $\tan 30 = \frac{2\tan 15}{1-\tan^2 15} = \frac{2t}{1-t^2}$ $\tan 30 = \frac{1}{\sqrt{3}}$ ⇒ $\frac{2t}{1-t^2} = \frac{1}{\sqrt{3}} \Rightarrow 2\sqrt{3}t = 1-t^2$ ⇒ $t^2 + 2\surd 3\, t - 1 = 0$ *	B1 M1 E1 [3]	
(*C*) $t = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2} = 2 - \sqrt{3}$ circumference < perimeter ⇒ $2\pi < 24(2-\sqrt{3})$ ⇒ $\pi < 12(2-\sqrt{3})$ *	M1 A1 M1 E1 [4]	 using positive root from exact working

(iii) $6\sqrt{2-\sqrt{3}} < \pi < 12(2-\sqrt{3})$ $\Rightarrow \quad 3.106 < \pi < 3.215$	B1 B1 [2]	3.106, 3.215

Comprehension

1. $\frac{1}{4}\times[3+1+(-1)+(-2)]=0.25$ * **M1, E1**

2. (i) b is the benefit of shooting some soldiers from the other side while none of yours are shot**.** w is the benefit of having some of your own soldiers shot while not shooting any from the other side.

   Since it is more beneficial to shoot some of the soldiers on the other side than it is to have your own soldiers shot, $b > w$. **E1**

   (ii) c is the benefit from mutual co-operation (i.e. no shooting).
   d is the benefit from mutual defection (soldiers on both sides are shot).
   With mutual co-operation people don't get shot, while they do with mutual defection. So $c > d$. **E1**

3. $\frac{1\times 2+(-2)\times(n-2)}{n}=-1.999$ **or equivalent** (allow n, n+2) **M1, A1**

   n = 6000 so you have played 6000 rounds. **A1**

4. No. The inequality on line 132, $b + w < 2c$, would not be satisfied since
   $6 + (-3) > 2 \times 1$. **M1**
   $b + w < 2c$ and subst **A1**
   No, $3 > 2$ o.e.

5. (i)

Round	**You**	**Opponent**	**Your score**	**Opponent's score**
1	C	D	-2	3
2	D	C	3	-2
3	C	D	-2	3
4	D	C	3	-2
5	C	D	-2	3
6	D	C	3	-2
7	C	D	-2	3
8	D	C	3	-2
…	…	…	…	…

   M1 Cs and Ds in correct places**, A1** C=-2**, A1** D=3

   (ii) $\frac{1}{2}\times[3+(-2)]=0.5$ **DM1 A1ft** their 3, -2

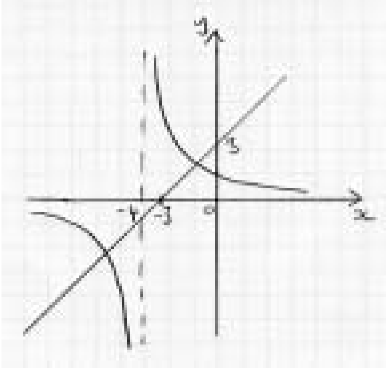
6. (i) All scores are increased by two points per round **B1**

   (ii) The same player wins. No difference/change. The rank order of the players remains the same.**B1**

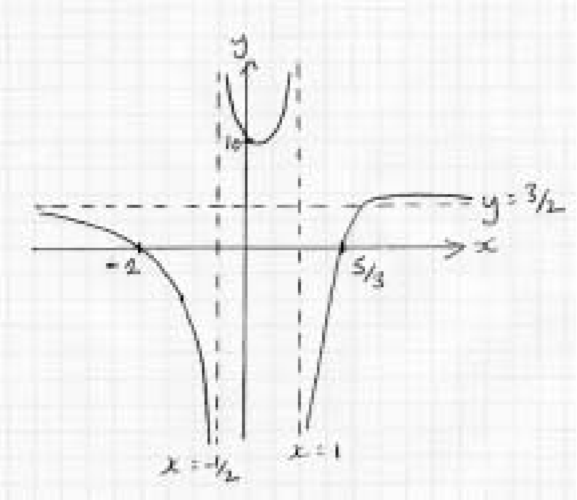
7. (i) They would agree to co-operate by spending less on advertising or by sharing equally. **B1**

   (ii) Increased market share (or more money or more customers). **DB1**

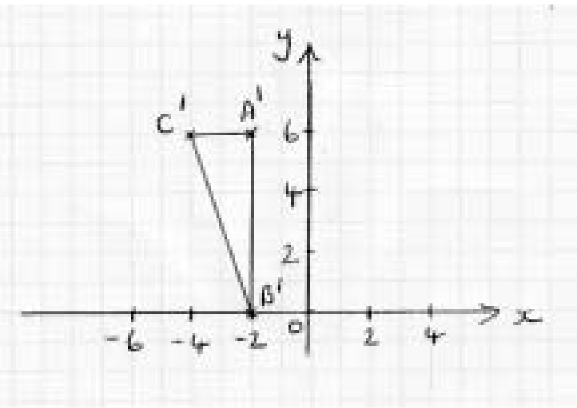
4755 (FP1) Further Concepts for Advanced Mathematics

Section A			
1(i)	$\mathbf{M}^{-1}=\frac{1}{11}\begin{pmatrix}2 & 1\\ -3 & 4\end{pmatrix}$	M1 A1 **[2]**	Dividing by determinant
(ii)	$\frac{1}{11}\begin{pmatrix}2 & 1\\ -3 & 4\end{pmatrix}\begin{pmatrix}49\\ 100\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}=\frac{1}{11}\begin{pmatrix}198\\ 253\end{pmatrix}$ $\Rightarrow x=18, y=23$	M1 A1(ft) A1(ft) **[3]**	Pre-multiplying by their inverse
2	$z^3+z^2-7z-15=(z-3)(z^2+4z+5)$ $z^2+4z+5=0\Rightarrow z=\frac{-4\pm\sqrt{16-20}}{2}$ $\Rightarrow z=-2+\mathrm{j}$ and $z=-2-\mathrm{j}$	B1 M1 A1 M1 A1 **[5]**	Show z = 3 is a root; may be implied Attempt to find quadratic factor Correct quadratic factor Use of quadratic formula or other valid method Both solutions
3(i)		B1 B1 **[2]**	Asymptote at x = -4 Both branches correct
(ii)	$\frac{2}{x+4}=x+3\Rightarrow x^2+7x+10=0$ $\Rightarrow x=-2$ or $x=-5$ $x\geq -2$ or $-4>x\geq -5$	M1 A1 A1 A2 **[5]**	Attempt to find where graphs cross or valid attempt at solution using inequalities Correct intersections (both), or -2 and -5 identified as critical values $x\geq -2$ $-4>x\geq -5$ s.c. A1 for $-4\geq x\geq -5$ or $-4>x>-5$
4	$2w-6w+3w=-\frac{1}{2}$ $\Rightarrow w=\frac{1}{2}$ $\Rightarrow$ roots are 1, -3, $\frac{3}{2}$ $-\frac{q}{2}=\alpha\beta\gamma=-\frac{9}{2}\ \Rightarrow\ q=9$ $\frac{p}{2}=\alpha\beta+\alpha\gamma+\beta\gamma=-6\ \Rightarrow\ p=-12$	M1 A1 A1 M1 A2(ft) **[6]**	Use of sum of roots – can be implied Correct roots seen Attempt to use relationships between roots s.c. M1 for other valid method One mark each for p = -12 and q = 9

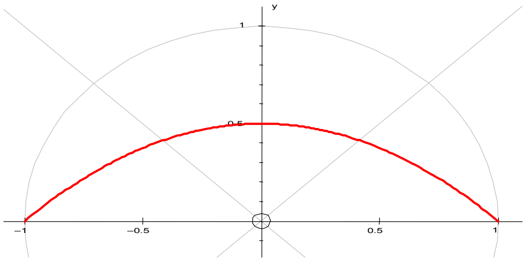
5(i)	$\frac{1}{5r-2}-\frac{1}{5r+3}\equiv\frac{5r+3-5r+2}{(5r+3)(5r-2)}$	M1	Attempt to form common denominator
	$\equiv\frac{5}{(5r+3)(5r-2)}$	A1 **[2]**	Correct cancelling
(ii)	$\sum_{r=1}^{30}\frac{1}{(5r-2)(5r+3)}=\frac{1}{5}\sum_{r=1}^{30}\left[\frac{1}{(5r-2)}-\frac{1}{(5r+3)}\right]$		
	$=\frac{1}{5}\left[\left(\frac{1}{3}-\frac{1}{8}\right)+\left(\frac{1}{8}-\frac{1}{13}\right)+\left(\frac{1}{13}-\frac{1}{18}\right)+\ldots+\left(\frac{1}{5n-7}-\frac{1}{5n-2}\right)+\left(\frac{1}{5n-2}-\frac{1}{5n+3}\right)\right]$	B1 B1 M1	First two terms in full Last term in full Attempt to cancel terms
	$=\frac{1}{5}\left[\frac{1}{3}-\frac{1}{5n+3}\right]=\frac{n}{3(5n+3)}$	A1 **[4]**	
6	When $n = 1$, $\frac{1}{2}n(7n-1)=3$, so true for $n = 1$	B1	
	Assume true for $n = k$ $3+10+17+.....+(7k-4)=\frac{1}{2}k(7k-1)$	E1	Assume true for $n = k$
	$\Rightarrow 3+10+17+.....+(7(k+1)-4)$ $=\frac{1}{2}k(7k-1)+(7(k+1)-4)$ $=\frac{1}{2}[k(7k-1)+(14(k+1)-8)]$	M1	Add $(k+1)$th term to both sides
	$=\frac{1}{2}\left[7k^2+13k+6\right]$ $=\frac{1}{2}(k+1)(7k+6)$	M1	Valid attempt to factorise
	$=\frac{1}{2}(k+1)(7(k+1)-1)$ But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is true for $k + 1$.	A1	c.a.o. with correct simplification
	Since it is true for $n = 1$, it is true for $n = 1, 2, 3$ and so true for all positive integers.	E1 E1 **[7]**	Dependent on previous E1 and immediately previous A1 Dependent on B1 and both previous E marks
			Section A Total: 36

Section B			
7(i)	$(0\ ,10),(-2\ ,0),\left(\frac{5}{3}\ ,\ 0\right)$	**B1** **B1** **B1** **[3]**	
(ii)	$x=\frac{-1}{2}\ ,\ x=1\ ,\ y=\frac{3}{2}$	**B1** **B1** **B1** **[3]**	
(iii)	Large positive x, $y\rightarrow\frac{3^{+}}{2}$ (e.g. consider $x=100$) Large negative x, $y\rightarrow\frac{3^{-}}{2}$ (e.g. consider $x=-100$)	**M1** **B1** **B1** **[3]**	Clear evidence of method required for full marks
(iv)	Curve 3 branches of correct shape Asymptotes correct and labelled Intercepts correct and labelled 	**B1** **B1** **B1** **[3]**	

8 (i)	$\lvert z-(4+2\mathrm{j})\rvert = 2$	**B1** **B1** **B1** **[3]**	Radius = 2 $z-(4+2\mathrm{j})$ or $z-4-2\mathrm{j}$ All correct
(ii)	$\arg\left(z-(4+2\mathrm{j})\right)=0$	**B1** **B1** **B1** **[3]**	Equation involving the argument of a complex variable Argument = 0 All correct
(iii)	$a = 4-2\cos\frac{\pi}{4} = 4-\sqrt{2}$ $b = 2+2\sin\frac{\pi}{4} = 2+\sqrt{2}$ $\mathrm{P} = 4-\sqrt{2}+\left(2+\sqrt{2}\right)\mathrm{j}$	**M1** **A2** **[3]**	Valid attempt to use trigonometry involving $\frac{\pi}{4}$, or coordinate geometry 1 mark for each of *a* and *b* s.c. A1 only for $a = 2.59$, $b = 3.41$
(iv)	$\frac{3}{4}\pi > \arg\left(z-(4+2\mathrm{j})\right) > 0$ and $\lvert z-(4+2\mathrm{j})\rvert < 2$	**B1** **B1** **B1** **[3]**	$\arg\left(z-(4+2\mathrm{j})\right) > 0$ $\arg\left(z-(4+2\mathrm{j})\right) < \frac{3}{4}\pi$ $\lvert z-(4+2\mathrm{j})\rvert < 2$ Deduct one mark if only error is use of inclusive inequalities

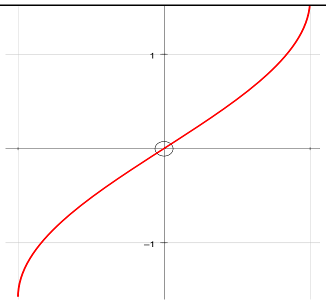
9(i)	Matrix multiplication is associative	**B1** **[1]**	
	$\mathbf{MN}=\begin{pmatrix}3&0\\0&2\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}$	**M1**	Attempt to find **MN** or **QM**
	$\Rightarrow \mathbf{MN}=\begin{pmatrix}0&3\\2&0\end{pmatrix}$	**A1**	or $\mathbf{QM}=\begin{pmatrix}0&-2\\3&0\end{pmatrix}$
	$\mathbf{QMN}=\begin{pmatrix}-2&0\\0&3\end{pmatrix}$	**A1(ft)** **[3]**	
(ii)	M is a stretch, factor 3 in the *x* direction, factor 2 in the *y* direction.	**B1** **B1**	Stretch factor 3 in the *x* direction Stretch factor 2 in the *y* direction
	N is a reflection in the line $y = x$.	**B1**	
	Q is an anticlockwise rotation through 90° about the origin.	**B1** **[4]**	
(iii)	$\begin{pmatrix}-2&0\\0&3\end{pmatrix}\begin{pmatrix}1&1&2\\2&0&2\end{pmatrix}=\begin{pmatrix}-2&-2&-4\\6&0&6\end{pmatrix}$	**M1** **A1(ft)**	Applying their **QMN** to points. Minus 1 each error to a minimum of 0.
		**B2** **[4]**	Correct, labelled image points, minus 1 each error to a minimum of 0. Give B4 for correct diagram with no workings.
			Section B Total: 36
			Total: 72

4756 (FP2) Further Methods for Advanced Mathematics

1(a)(i)	$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}...$		
	$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5}...$	B1	Series for ln(1 – x) as far as x^5 s.o.i.
	$\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$	M1	Seeing series subtracted
	$= 2x + \frac{2x^3}{3} + \frac{2x^5}{5}...$	A1	
	Valid for $-1 < x < 1$	B1 **4**	Inequalities must be strict
(ii)	$\frac{1+x}{1-x} = 3$ $\Rightarrow \quad 1 + x = 3(1-x)$		
	$\Rightarrow \quad 1 + x = 3 - 3x$ $\Rightarrow \quad 4x = 2$	M1	Correct method of solution
	$\Rightarrow \quad x = \frac{1}{2}$	A1	B2 for $x = \frac{1}{2}$ stated
	$\ln 3 \approx 2 \times \frac{2}{3} \times \left(\frac{1}{2}\right)^3 + \frac{2}{5} \times \left(\frac{1}{2}\right)^5$ $= 1 + \frac{1}{12} + \frac{1}{80}$	M1	Substituting their x into their series in (a)(i), even if outside range of validity. Series must have at least two terms *SR*: if >3 correct terms seen in (i), allow a better answer to 3 d.p.
	= 1.096 (3 d.p.)	A1 **4**	Must be 3 decimal places
(b)(i)		G1 G1 G1 **3**	$r(0) = a$, $r(\pi/2) = a/2$ indicated Symmetry in $\theta = \pi/2$ Correct basic shape: flat at $\theta = \pi/2$, not vertical or horizontal at ends, no dimple Ignore beyond $0 \le \theta \le \pi$
(ii)	$r + y = r + r\sin\theta$	M1	Using $y = r\sin\theta$
	$= r(1+\sin\theta) = \frac{a}{1+\sin\theta} \times (1+\sin\theta)$ $= a$	A1 (AG)	
	$\Rightarrow \quad r = a - y$ $\Rightarrow \quad x^2 + y^2 = (a-y)^2$	M1 A1	Using $r^2 = x^2 + y^2$ in $r + y = a$ Unsimplified
	$\Rightarrow \quad x^2 + y^2 = a^2 - 2ay + y^2$ $\Rightarrow \quad 2ay = a^2 - x^2$	A1	A correct final answer, not spoiled
	$\Rightarrow \quad y = \frac{a^2 - x^2}{2a}$	**5**	**16**

2 (i)	$\mathbf{M}-\lambda\mathbf{I}=\begin{pmatrix}3-\lambda&1&-2\\0&-1-\lambda&0\\2&0&1-\lambda\end{pmatrix}$ det($\mathbf{M}-\lambda\mathbf{I}$) $= (3-\lambda)[(-1-\lambda)(1-\lambda)] + 2[2(-1-\lambda)]$ $= (3-\lambda)(\lambda^2-1) + 4(-1-\lambda)$ $\Rightarrow \quad \lambda^3-3\lambda^2+3\lambda+7=0$ det $\mathbf{M}$ = -7	M1 A1 B1 **3**	Attempt at det($\mathbf{M}-\lambda\mathbf{I}$) with all elements present. Allow sign errors Unsimplified. Allow signs reversed. Condone omission of = 0
(ii)	$f(\lambda)=\lambda^3-3\lambda^2+3\lambda+7$ $f(-1) = -1-3-3+7=0 \Rightarrow -1$ eigenvalue $f(\lambda)=(\lambda+1)(\lambda^2-4\lambda+7)$ $\lambda^2-4\lambda+7=(\lambda-2)^2+3\geq 3$ so no real roots $(\mathbf{M}-\lambda\mathbf{I})\mathbf{s}=\mathbf{0}$, $\lambda = -1$ $\Rightarrow \quad \begin{pmatrix}4&1&-2\\0&0&0\\2&0&2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$ $\Rightarrow \quad 4x+y-2z=0$ $2x+2z=0$ $\Rightarrow \quad x=-z$ $y=2z-4x=2z+4z=6z$ $\Rightarrow \quad \mathbf{s}=\begin{pmatrix}-1\\6\\1\end{pmatrix}$ $\begin{pmatrix}3&1&-2\\0&-1&0\\2&0&1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-0.1\\0.6\\0.1\end{pmatrix}$ $\Rightarrow \quad x=0.1, y=-0.6, z=-0.1$	B1 M1 A1 M1 M1 A1 M1 A2 **9**	Showing -1 satisfies a correct characteristic equation Obtaining quadratic factor www $(\mathbf{M}-\lambda\mathbf{I})\mathbf{s}=(\lambda)\mathbf{s}$ M0 below Obtaining equations relating x, y and z Obtaining equations relating two variables to a third. Dep. on first M1 Or any non-zero multiple Solution by any method, e.g. use of multiple of **s**, but M0 if **s** itself quoted without further work Give A1 if any two correct
(iii)	C-H: a matrix satisfies its own characteristic equation $\Rightarrow \quad \mathbf{M}^3-3\mathbf{M}^2+3\mathbf{M}+7\mathbf{I}=\mathbf{0}$ $\Rightarrow \quad \mathbf{M}^3=3\mathbf{M}^2-3\mathbf{M}-7\mathbf{I}$ $\Rightarrow \quad \mathbf{M}^2=3\mathbf{M}-3\mathbf{I}-7\mathbf{M}^{-1}$ $\Rightarrow \quad \mathbf{M}^{-1}=-\tfrac{1}{7}\mathbf{M}^2+\tfrac{3}{7}\mathbf{M}-\tfrac{3}{7}\mathbf{I}$	B1 B1 (AG) M1 A1 **4**	Idea of λ ↔ **M** Must be derived www. Condone omitted **I** Multiplying by $\mathbf{M}^{-1}$ o.e.
(iv)	$\mathbf{M}^2=\begin{pmatrix}3&1&-2\\0&-1&0\\2&0&1\end{pmatrix}\begin{pmatrix}3&1&-2\\0&-1&0\\2&0&1\end{pmatrix}=\begin{pmatrix}5&2&-8\\0&1&0\\8&2&-3\end{pmatrix}$ $-\frac{1}{7}\begin{pmatrix}5&2&-8\\0&1&0\\8&2&-3\end{pmatrix}+\frac{3}{7}\begin{pmatrix}3&1&-2\\0&-1&0\\2&0&1\end{pmatrix}-\frac{3}{7}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$ $=\begin{pmatrix}\frac{1}{7}&\frac{1}{7}&\frac{2}{7}\\0&-1&0\\-\frac{2}{7}&-\frac{2}{7}&\frac{3}{7}\end{pmatrix}$ or $\frac{1}{7}\begin{pmatrix}1&1&2\\0&-7&0\\-2&-2&3\end{pmatrix}$	M1 M1 A1	Correct attempt to find $\mathbf{M}^2$ Using their (iii) SC1 for answer without working

	OR Matrix of cofactors: $\begin{pmatrix} -1 & 0 & 2 \\ -1 & 7 & 2 \\ -2 & 0 & -3 \end{pmatrix}$	M1	Finding at least four cofactors
	Adjugate matrix $\begin{pmatrix} -1 & -1 & -2 \\ 0 & 7 & 0 \\ 2 & 2 & -3 \end{pmatrix}$: det $\mathbf{M} = -7$	M1	Transposing and dividing by determinant. Dep. on M1 above
		3	**19**

3(a)(i)		G1 **1**	Correct basic shape (positive gradient, through (0, 0))
	$y = \arcsin x \Rightarrow \sin y = x$	M1	$\sin y =$ and attempt to diff. both sides
	$\Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \cos y$	A1	Or $\cos y \frac{\mathrm{d}y}{\mathrm{d}x} = 1$
	$\Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$	A1	www. SC1 if quoted without working
	Positive square root because gradient positive	B1 **4**	Dep. on graph of an increasing function
(ii)	$\int_0^1 \frac{1}{\sqrt{2-x^2}}\mathrm{d}x = \left[\arcsin\frac{x}{\sqrt{2}}\right]_0^1$	M1	arcsin function alone, or any sine substitution
		A1	$\frac{x}{\sqrt{2}}$, or $\int 1\,\mathrm{d}\theta$ www without limits
	$= \frac{\pi}{4}$	A1 **3**	Evaluated in terms of π
(b)	$C + jS = e^{j\theta} + \frac{1}{3}e^{3j\theta} + \frac{1}{9}e^{5j\theta} + \dots$	M1	Forming $C + jS$ as a series of powers
	This is a geometric series	M1	Identifying geometric series and attempting sum to infinity or to n terms
	with first term $a = e^{j\theta}$, common ratio $r = \frac{1}{3}e^{2j\theta}$	A1	Correct a and r
	Sum to infinity $= \frac{a}{1-r} = \frac{e^{j\theta}}{1-\frac{1}{3}e^{2j\theta}} \left(= \frac{3e^{j\theta}}{3-e^{2j\theta}}\right)$	A1	Sum to infinity
	$= \frac{3e^{j\theta}}{3-e^{2j\theta}} \times \frac{3-e^{-2j\theta}}{3-e^{-2j\theta}}$	M1*	Multiplying numerator and denominator by $1-\frac{1}{3}e^{-2j\theta}$ o.e. Or writing in terms of trig functions and realising the denominator
	$= \frac{9e^{j\theta} - 3e^{-j\theta}}{9 - 3e^{-2j\theta} - 3e^{2j\theta} + 1}$	M1	Multiplying out numerator and denominator. Dep. on M1*
	$= \frac{9(\cos\theta + j\sin\theta) - 3(\cos\theta - j\sin\theta)}{10 - 3(\cos 2\theta - j\sin 2\theta) - 3(\cos 2\theta + j\sin 2\theta)}$	M1	Valid attempt to express in terms of trig functions. If trig functions used from start, M1 for using the compound angle formulae and

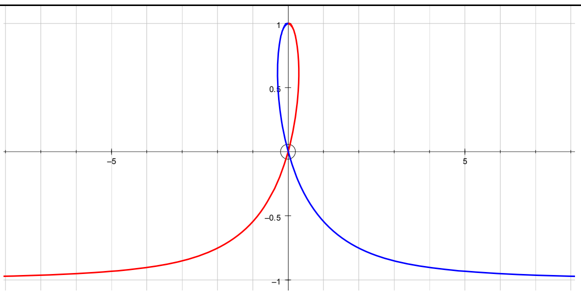
			Pythagoras Dep. on M1*
	$= \frac{6\cos\theta + 12j\sin\theta}{10 - 6\cos 2\theta}$	A1	
	$\Rightarrow \quad C = \frac{6\cos\theta}{10 - 6\cos 2\theta}$	M1	Equating real and imaginary parts. Dep. on M1*
	$= \frac{3\cos\theta}{5 - 3\cos 2\theta}$	A1 (AG)	
	$S = \frac{6\sin\theta}{5 - 3\cos 2\theta}$	A1	o.e.
		11	**19**

4 (i)	$\cosh u = \frac{e^u + e^{-u}}{2}$		
	$\Rightarrow \quad 2\cosh^2 u = \frac{e^{2u} + 2 + e^{-2u}}{2}$	B1	$\left(e^u + e^{-u}\right)^2 = e^{2u} + 2 + e^{-2u}$
	$\Rightarrow \quad 2\cosh^2 u - 1 = \frac{e^{2u} + e^{-2u}}{2}$	B1	$\cosh 2u = \frac{e^{2u} + e^{-2u}}{2}$
	$= \cosh 2u$	B1 (AG) **3**	Completion www
(ii)	$x = \operatorname{arcsinh} y$		
	$\Rightarrow \quad \sinh x = y$		
	$\Rightarrow \quad y = \frac{e^x - e^{-x}}{2}$	M1	Expressing y in exponential form ($\frac{1}{2}$, – must be correct)
	$\Rightarrow \quad e^{2x} - 2ye^x - 1 = 0$		
	$\Rightarrow \quad (e^x - y)^2 - y^2 - 1 = 0$		
	$\Rightarrow \quad (e^x - y)^2 = y^2 + 1$		
	$\Rightarrow \quad e^x - y = \pm\sqrt{y^2 + 1}$		
	$\Rightarrow \quad e^x = y \pm \sqrt{y^2 + 1}$	M1	Reaching e^x by quadratic formula or completing the square. Condone no ±
	Take + because $e^x > 0$	B1	Or argument of ln must be positive
	$\Rightarrow \quad x = \ln(y + \sqrt{y^2 + 1})$	A1 (AG) **4**	Completion www but independent of B1
(iii)	$x = 2\sinh u \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u} = 2\cosh u$	M1	$\frac{\mathrm{d}x}{\mathrm{d}u}$ and substituting for all elements
	$\int \sqrt{x^2 + 4}\,\mathrm{d}x = \int \sqrt{4\sinh^2 u + 4} \times 2\cosh u \,\mathrm{d}u$	A1	Substituting for all elements correctly
	$= \int 4\cosh^2 u \,\mathrm{d}u$		
	$= \int 2\cosh 2u + 2 \,\mathrm{d}u$	M1	Simplifying to an integrable form
	$= \sinh 2u + 2u + c$	A1	Any form, e.g. $\frac{1}{2}e^{2u} - \frac{1}{2}e^{-2u} + 2u$ Condone omission of + c throughout
	$= 2\sinh u \cosh u + 2u + c$		
	$= x\sqrt{1 + \frac{x^2}{4}} + 2 \operatorname{arcsinh} \frac{x}{2} + c$	M1	Using double ‘angle’ formula and attempt to express $\cosh u$ in terms of x
	$= \frac{1}{2}x\sqrt{4 + x^2} + 2 \operatorname{arcsinh} \frac{x}{2} + c$	A1 (AG)	Completion www

		6	

(iv)	$t^2 + 2t + 5 = (t + 1)^2 + 4$ $\int_{-1}^{1}\sqrt{t^2+2t+5}\ \mathrm{d}t = \int_{-1}^{1}\sqrt{(t+1)^2+4}\ \mathrm{d}t$ $= \int_{0}^{2}\sqrt{x^2+4}\ \mathrm{d}x$ $= \left[\tfrac{1}{2}x\sqrt{4+x^2} + 2\,\text{arcsin h}\,\frac{x}{2}\right]_0^2$	B1 M1 A1	Completing the square Simplifying to an integrable form, by substituting $x = t + 1$ s.o.i. or complete alternative method Correct limits consistent with their method seen anywhere
(v)	$= \sqrt{8} + 2 \text{ arcsinh } 1$ $= 2\sqrt{2} + 2\ln(1 + \sqrt{2})$ $= 2(\ln(1 + \sqrt{2}) + \sqrt{2})$	M1 A1 (AG) **5**	Using (iii) or otherwise reaching the result of integration, and using limits Completion www. Condone $\sqrt{8}$ etc. **18**

5 (i)	If $a = 1$, angle OCP = 45° so P is (1 – cos 45°, sin 45°) $\Rightarrow$ P $\left(1 - \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ OR Circle $(x - 1)^2 + y^2 = 1$, line $y = -x + 1$ $(x - 1)^2 + (-x + 1)^2 = 1$ $\Rightarrow$ $x = 1 \pm \frac{1}{\sqrt{2}}$ and hence P Q $\left(1 + \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$	M1 A1 (AG) M1 A1 B1 **3**	Completion www Complete algebraic method to find x
(ii)	cos OCP = $\frac{a}{\sqrt{a^2+1}}$ sin OCP = $\frac{1}{\sqrt{a^2+1}}$ P is ($a - a$ cos OCP, a sin OCP) $\Rightarrow$ P $\left(a - \frac{a^2}{\sqrt{a^2+1}}, \frac{a}{\sqrt{a^2+1}}\right)$ OR Circle $(x - a)^2 + y^2 = a^2$, line $y = -\frac{1}{a}x + 1$ $(x - a)^2 + \left(-\frac{1}{a}x + 1\right)^2 = a^2$ $\Rightarrow$ $x = \dfrac{2a + \frac{2}{a} \pm \sqrt{\left(2a + \frac{2}{a}\right)^2 - 4\left(1 + \frac{1}{a^2}\right)}}{2\left(1 + \frac{1}{a^2}\right)}$	M1 A1 A1 (AG) M1 A1	Attempt to find cos OCP and sin OCP in terms of a Both correct Completion www Complete algebraic method to find x Unsimplified

	$\Rightarrow \quad x = a \pm \frac{a^2}{\sqrt{a^2+1}}$ and hence P Q$\left(a+\frac{a^2}{\sqrt{a^2+1}}, -\frac{a}{\sqrt{a^2+1}}\right)$	A1 B1 **4**	
(iii)	 As $a \to \infty$, P → (0, 1) As $a \to -\infty$, y-coordinate of P → −1 $\frac{a}{\sqrt{a^2+1}} \to \frac{a}{-a} = -1$ as $a \to -\infty$	G1 G1 G1 G1ft B1 B1 M1 A1 **8**	Locus of P (1st & 3rd quadrants) through (0, 0) Locus of P terminates at (0, 1) Locus of P: fully correct shape Locus of Q (2nd & 4th quadrants: dotted) reflection of locus of P in y-axis Stated separately Stated Attempt to consider y as $a \to -\infty$ Completion www
(iv)	POQ = 90° Angle in semicircle Loci cross at 90°	B1 B1 B1 **3**	o.e. **18**

4757 Further Pure 3

1 (i)	Putting $x = 0$, $-3y + 10z = 6$, $-4y - 2z = 8$ $y = -2$, $z = 0$ Direction is given by $\begin{pmatrix}8\\-3\\10\end{pmatrix}\times\begin{pmatrix}3\\-4\\-2\end{pmatrix}$ $=\begin{pmatrix}46\\46\\-23\end{pmatrix}$ Equation of L is $\mathbf{r}=\begin{pmatrix}0\\-2\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\2\\-1\end{pmatrix}$	M1 A1 M1 A1 A1 ft **5**	Finding coords of a point on the line or (2, 0, -1), (1, -1, -$\frac{1}{2}$) etc or finding a second point *Dependent on M1M1* Accept any form Condone omission of '**r** ='
(ii)	$\overrightarrow{AB}\times\mathbf{d}=\begin{pmatrix}7\\-14\\4\end{pmatrix}\times\begin{pmatrix}2\\2\\-1\end{pmatrix}=\begin{pmatrix}6\\15\\42\end{pmatrix}$ $\left[=3\begin{pmatrix}2\\5\\14\end{pmatrix}\right]$ Distance is $\left[\begin{pmatrix}-1\\12\\5\end{pmatrix}-\begin{pmatrix}0\\-2\\0\end{pmatrix}\right]\cdot\hat{\mathbf{n}}=\frac{\begin{pmatrix}-1\\14\\5\end{pmatrix}\cdot\begin{pmatrix}2\\5\\14\end{pmatrix}}{\sqrt{2^2+5^2+14^2}}$ $=\frac{138}{15}=\frac{46}{5}=9.2$	M1 A2 ft M1 A1 ft A1 **6**	Evaluating $\overrightarrow{AB}\times\mathbf{d}$ Give A1 ft if just one error Appropriate scalar product Fully correct expression
(iii)	$\left\|\overrightarrow{AB}\times\mathbf{d}\right\|=\left\|\begin{pmatrix}6\\15\\42\end{pmatrix}\right\|=\sqrt{6^2+15^2+42^2}$ Distance is $\frac{\left\|\overrightarrow{AB}\times\mathbf{d}\right\|}{\left\|\mathbf{d}\right\|}=\frac{\sqrt{6^2+15^2+42^2}}{\sqrt{2^2+2^2+1^2}}$ $=\frac{45}{3}=15$	M1 M1 M1 A1ft A1 **5**	For $\left\|\overrightarrow{AB}\times\mathbf{d}\right\|$ Evaluating magnitude *In this part, M marks are dependent on previous M marks*

(iv)	At D, $\begin{pmatrix}-1\\12\\5\end{pmatrix}+\lambda\begin{pmatrix}k+1\\-12\\-3\end{pmatrix}=\begin{pmatrix}6\\-2\\9\end{pmatrix}+\mu\begin{pmatrix}2\\2\\-1\end{pmatrix}$	M1	*Condone use of same parameter on both sides*
	$12-12\lambda=-2+2\mu$ $5-3\lambda=9-\mu$	A1 ft M1	Two equations for λ and μ Obtaining λ and μ (numerically)
	$\lambda=\frac{1}{3},\quad \mu=5$	M1	Give M1 for λ and μ in terms of k Equation for k
	$-1+\frac{1}{3}(k+1)=6+10$	M1	
	$k=50$	A1	
	D is $(6+2\mu, -2+2\mu, 9-\mu)$ i.e. (16, 8, 4)	M1 A1 **8**	Obtaining coordinates of D

Alternative solutions for Q1

1 (i)	e.g. $23x-23y=46$ $x=t,\ y=t-2$ $3t-4(t-2)-2z=8$ $x=t,\ y=t-2,\ z=-\frac{1}{2}t$	M1A1 M1 A1 ft A1 **5**	Eliminating one of x, y, z
(ii)	$\overrightarrow{PQ}=\begin{pmatrix}-1+7\mu\\12-14\mu\\5+4\mu\end{pmatrix}-\begin{pmatrix}2\lambda\\-2+2\lambda\\-\lambda\end{pmatrix}$	M1 A1 ft	
	$\overrightarrow{PQ}\cdot\mathbf{d}=\overrightarrow{PQ}\cdot\overrightarrow{AB}=0$		
	$2(-1+7\mu-2\lambda)+2(14-14\mu-2\lambda)-(5+4\mu+\lambda)=0$	A1 ft	Two equations for λ and μ
	$7(-1+7\mu-2\lambda)-14(14-14\mu-2\lambda)+4(5+4\mu+\lambda$ $=0$	M1	Expression for shortest distance
	$\lambda=\frac{27}{25},\ \mu=\frac{47}{75}$	A1 ft	
	$\left\|\overrightarrow{PQ}\right\|=\sqrt{\left(\frac{92}{75}\right)^2+\left(\frac{230}{75}\right)^2+\left(\frac{644}{75}\right)^2}=9.2$	A1 **6**	
(iii)	$\overrightarrow{AX}\cdot\mathbf{d}=\begin{pmatrix}6+2\lambda+1\\-2+2\lambda-12\\9-\lambda-5\end{pmatrix}\cdot\begin{pmatrix}2\\2\\-1\end{pmatrix}=0$	M1	
	$2(7+2\lambda)+2(2\lambda-14)-(4-\lambda)=0$ $\lambda=2$	A1ft	
	$\overrightarrow{AX}=\begin{pmatrix}11\\-10\\2\end{pmatrix}$	M1	
	$AX=\sqrt{11^2+10^2+2^2}$ $=15$	M1 A1 **5**	

(iv)	$\begin{pmatrix}7\\-14\\4\end{pmatrix}\cdot\left[\begin{pmatrix}k+1\\-12\\-3\end{pmatrix}\times\begin{pmatrix}2\\2\\-1\end{pmatrix}\right]=0$	M1	Appropriate scalar triple product equated to zero
	$\begin{pmatrix}7\\-14\\4\end{pmatrix}\cdot\begin{pmatrix}18\\k-5\\2k+26\end{pmatrix}=0$		
	$126-14k+70+8k+104=0$	M1	Equation for k
	$k=50$	A1	
	At D, $\begin{pmatrix}-1\\12\\5\end{pmatrix}+\lambda\begin{pmatrix}51\\-12\\-3\end{pmatrix}=\begin{pmatrix}6\\-2\\9\end{pmatrix}+\mu\begin{pmatrix}2\\2\\-1\end{pmatrix}$	M1	*Condone use of same parameter on both sides*
	$-1+51\lambda=6+2\mu$		
	$12-12\lambda=-2+2\mu$	A1 ft	Two equations for λ and μ
	$5-3\lambda=9-\mu$	M1	Obtaining λ and μ
	$\lambda=\frac{1}{3},\quad \mu=5$		
	D is $(6+2\mu,\ -2+2\mu,\ 9-\mu)$	M1	Obtaining coordinates of D
	i.e. (16, 8, 4)	A1 **8**	

2(i)	$\frac{\partial z}{\partial x}=3(x+y)^3+9x(x+y)^2-6x^2+24$	M1 A2	Partial differentiation Give A1 if just one minor error
	$\frac{\partial z}{\partial y}=9x(x+y)^2$	A1 **4**	
(ii)	At stationary points, $\frac{\partial z}{\partial x}=0$ and $\frac{\partial z}{\partial y}=0$	M1	
	$9x(x+y)^2=0 \Rightarrow x=0$ or $y=-x$		
	If $x=0$ then $3y^3+24=0$	M1	
	$y=-2$; one stationary point is $(0,\ -2,\ 0)$	A1A1	
	If $y=-x$ then $-6x^2+24=0$	M1	
	$x=\pm2$; stationary points are $(2,\ -2,\ 32)$	A1	
	and $(-2,\ 2,\ -32)$	A1 **7**	If A0A0, give A1 for $x=\pm2$
(iii)	At P $(1,\ -2,\ 19)$, $\frac{\partial z}{\partial x}=24$, $\frac{\partial z}{\partial y}=9$	B1	
	Normal line is $\mathbf{r}=\begin{pmatrix}1\\-2\\19\end{pmatrix}+\lambda\begin{pmatrix}24\\9\\-1\end{pmatrix}$	M1 A1 ft **3**	For normal vector (allow sign error) Condone omission of '**r** ='
(iv)	$\delta z\approx\frac{\partial z}{\partial x}\delta x+\frac{\partial z}{\partial y}\delta y$	M1	
	$=24\,\delta x+9\,\delta y$	A1 ft	
	$3h\approx24k+9h$	M1	

	$k \approx -\frac{1}{4}h$	A1 **4**	
	OR Tangent plane is $24x+9y-z=-13$	M2	
	$24(1+k)+9(-2+h)-(19+3h) \approx -13$	A1 ft	
	$k \approx -\frac{1}{4}h$	A1	
(v)	$\frac{\partial z}{\partial x}=27$ and $\frac{\partial z}{\partial y}=0$	M1	(Allow M1 for $\frac{\partial z}{\partial x}=-27$)
	$9x(x+y)^2=0 \Rightarrow x=0$ or $y=-x$		
	If $x=0$ then $3y^3+24=27$	M1	
	$y=1,\ z=0$; point is $(0,\ 1,\ 0)$	A1	
	$d=0$	A1	
	If $y=-x$ then $-6x^2+24=27$	M1	
	$x^2=-\frac{1}{2}$; there are no other points	A1 **6**	

3(i)	$\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2=[a(1+\cos\theta)]^2+(a\sin\theta)^2$	M1 A1	Forming $\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2$
	$=a^2(2+2\cos\theta)$		
	$=4a^2\cos^2\frac{1}{2}\theta$	M1	Using half-angle formula
	$s=\int 2a\cos\frac{1}{2}\theta\,\mathrm{d}\theta$	M1	Integrating to obtain $k\sin\frac{1}{2}\theta$
	$=4a\sin\frac{1}{2}\theta+C$	A1	Correctly obtained ($+C$ not needed)
	$s=0$ when $\theta=0 \Rightarrow C=0$	A1(AG) **6**	*Dependent on all previous marks*
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{a\sin\theta}{a(1+\cos\theta)}$	M1	
	$=\frac{2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}{2a\cos^2\frac{1}{2}\theta}=\tan\frac{1}{2}\theta$	M1 A1	Using half-angle formulae
	$\psi=\frac{1}{2}\theta$, and so $s=4a\sin\psi$	A1 **4**	
(iii)	$\rho=\frac{\mathrm{d}s}{\mathrm{d}\psi}=4a\cos\psi$	M1 A1 ft	Differentiating intrinsic equation
	$=4a\cos\frac{1}{2}\theta$	A1(AG) **3**	
	OR $\rho=\frac{\left(4a^2\cos^2\frac{1}{2}\theta\right)^{3/2}}{a(1+\cos\theta)(a\cos\theta)-(-a\sin\theta)(a\sin\theta)}$	M1 A1 ft	Correct expression for ρ or κ
	$=\frac{8a^3\cos^3\frac{1}{2}\theta}{a^2(1+\cos\theta)}=\frac{8a^3\cos^3\frac{1}{2}\theta}{2a^2\cos^2\frac{1}{2}\theta}=4a\cos\frac{1}{2}\theta$	A1(AG)	

(iv)	When $\theta=\frac{2}{3}\pi,\ \psi=\frac{1}{3}\pi,\ x=a(\frac{2}{3}\pi+\frac{1}{2}\sqrt{3}),\ y=\frac{3}{2}a$ $\rho=2a$ $\hat{\mathbf{n}}=\begin{pmatrix}-\sin\psi\\ \cos\psi\end{pmatrix}=\begin{pmatrix}-\frac{1}{2}\sqrt{3}\\ \frac{1}{2}\end{pmatrix}$ $\mathbf{c}=\begin{pmatrix}a(\frac{2}{3}\pi+\frac{1}{2}\sqrt{3})\\ \frac{3}{2}a\end{pmatrix}+2a\begin{pmatrix}-\frac{1}{2}\sqrt{3}\\ \frac{1}{2}\end{pmatrix}$ Centre of curvature is $(\ a(\frac{2}{3}\pi-\frac{1}{2}\sqrt{3})\ ,\ \frac{5}{2}a\ )$	B1 M1 A1 M1 A1A1 **6**	Obtaining a normal vector Correct unit normal *(possibly in terms of θ)* Accept $(1.23a,\ 2.5a)$
(v)	Curved surface area is $\int 2\pi y\,\mathrm{d}s$ $=\int_0^{\pi} 2\pi a(1-\cos\theta)\,2a\cos\frac{1}{2}\theta\,\mathrm{d}\theta$ $=\int_0^{\pi} 8\pi a^2\sin^2\frac{1}{2}\theta\cos\frac{1}{2}\theta\,\mathrm{d}\theta$ $=\Big[\ \frac{16}{3}\pi a^2\sin^3\frac{1}{2}\theta\ \Big]_0^{\pi}$ $=\frac{16}{3}\pi a^2$	M1 A1 ft M1 M1 A1 **5**	Correct integral expression in any form *(including limits; may be implied by later working)* Obtaining an integrable form Obtaining $k\sin^3\frac{1}{2}\theta$ *or equivalent*

4 (i)	In $G,\ 3^2=2,\ 3^3=6,\ 3^4=4,\ 3^5=5,\ 3^6=1$ [or $5^2=4,\ 5^3=6,\ 5^4=2,\ 5^5=3,\ 5^6=1$] In $H,\ 5^2=7,\ 5^3=17,\ 5^4=13,\ 5^5=11,\ 5^6=1$ [or $11^2=13,\ 11^3=17,\ 11^4=7,\ 11^5=5,\ 11^6=1$] *G* has an element 3 (or 5) of order 6 *H* has an element 5 (or 11) of order 6	M1 A1 B1 B1 **4**	All powers of an element of order 6 All powers correct in both groups
(ii)	{1, 6} {1, 2, 4}	B1 B2 **3**	*Ignore {1} and G* *Deduct 1 mark (from B1B2) for each proper subgroup in excess of two*
(iii)	*G* *H* *G* *H* 1 ↔ 1 1 ↔ 1 2 ↔ 7 2 ↔ 13 3 ↔ 5 OR 3 ↔ 11 4 ↔ 13 4 ↔ 7 5 ↔ 11 5 ↔ 5 6 ↔ 17 6 ↔ 17	B4 **4**	Give B3 for 4 correct, B2 for 3 correct, B1 for 2 correct
(iv)	ad(1) = a(3) = 1 ad(2) = a(2) = 3 ad(3) = a(1) = 2, so ad = c da(1) = d(2) = 2 da(2) = d(3) = 1	M1 A1 M1	Evaluating e.g. ad(1) *(one case sufficient; intermediate value must be shown)* For ad = c correctly shown Evaluating e.g. da(1) *(one case sufficient; no need for any working)*

<table>
<tr><td></td><td>da(3) = d(1) = 3, so da = f</td><td>A1
4</td><td></td></tr>
<tr><td>(v)</td><td><i>S</i> is not abelian; <i>G</i> is abelian</td><td>B1
1</td><td><i>or S</i> has 3 elements of order 2;
<i>G</i> has 1 element of order 2
<i>or S</i> is not cyclic <i>etc</i></td></tr>
<tr><td>(vi)</td><td><table><tr><td>Element</td><td>a</td><td>b</td><td>c</td><td>d</td><td>e</td><td>f</td></tr><tr><td>Order</td><td>3</td><td>3</td><td>2</td><td>2</td><td>1</td><td>2</td></tr></table></td><td>B4
4</td><td>Give B3 for 5 correct, B2 for 3 correct,
B1 for 1 correct</td></tr>
<tr><td>(vii)</td><td>{e, c}
{e, d}
{e, f}
{e, a, b}</td><td>B1
B1
B1
B1
4</td><td>Ignore { e } and <i>S</i>
If more than 4 proper subgroups are given, deduct 1 mark for each proper subgroup in excess of 4</td></tr>
</table>

Pre-multiplication by transition matrix

5 (i)	$\mathbf{P}=\begin{pmatrix}0 & 0.1 & 0 & 0.3\\0.7 & 0.8 & 0 & 0.6\\0.1 & 0 & 1 & 0.1\\0.2 & 0.1 & 0 & 0\end{pmatrix}$	B2 **2**	Give B1 for two columns correct
(ii)	$\mathbf{P}^{13}\begin{pmatrix}0.6\\0.4\\0\\0\end{pmatrix}=\begin{pmatrix}0.0810\\0.5684\\0.2760\\0.0746\end{pmatrix}$	M1 A2 **3**	Using $\mathbf{P}^{13}$ (or $\mathbf{P}^{14}$) Give A1 for 2 probabilities correct *(Max A1 if not at least 3dp)* *Tolerance* ± 0.0001
(iii)	$0.5684\times 0.8+0.2760$ $=0.731$	M1M1 A1 ft **3**	For 0.5684×0.8 and 0.2760 *Accept 0.73 to 0.7312*
(iv)	$\mathbf{P}^{30}\begin{pmatrix}0.6\\0.4\\0\\0\end{pmatrix}=\begin{pmatrix}.\\.\\0.4996\\.\end{pmatrix}$, $\mathbf{P}^{31}\begin{pmatrix}0.6\\0.4\\0\\0\end{pmatrix}=\begin{pmatrix}.\\.\\0.5103\\.\end{pmatrix}$ Level 32	M1 A1 A1 **3**	Finding P(C) for some powers of **P** For identifying $\mathbf{P}^{31}$
(v)	Expected number of levels including the next change of location is $\dfrac{1}{0.2}=5$ Expected number of further levels in B is 4	M1 A1 A1 **3**	For $1/(1-0.8)$ or $0.8/(1-0.8)$ For 5 or 4 For 4 as final answer

(vi)	$\mathbf{Q}=\begin{pmatrix}0&0.1&0&0.3\\0.7&0.8&0&0.6\\0.1&0&0.9&0.1\\0.2&0.1&0.1&0\end{pmatrix}$ $\mathbf{Q}^n \to \begin{pmatrix}0.0916&0.0916&0.0916&0.0916\\0.6183&0.6183&0.6183&0.6183\\0.1908&0.1908&0.1908&0.1908\\0.0992&0.0992&0.0992&0.0992\end{pmatrix}$ A: 0.0916 B: 0.6183 C: 0.1908 D: 0.0992	B1 M1 M1 A2 **5**	*Can be implied* Evaluating powers of **Q** *or Obtaining (at least) 3 equations from* $\mathbf{Qp}=\mathbf{p}$ Limiting matrix with equal columns *or Solving to obtain one equilib prob* *or M2 for other complete method* Give A1 for two correct *(Max A1 if not at least 3dp)* *Tolerance* ± 0.0001
(vii)	$\begin{pmatrix}0&0.1&a&0.3\\0.7&0.8&b&0.6\\0.1&0&c&0.1\\0.2&0.1&d&0\end{pmatrix}\begin{pmatrix}0.11\\0.75\\0.04\\0.1\end{pmatrix}=\begin{pmatrix}0.11\\0.75\\0.04\\0.1\end{pmatrix}$ $0.075+0.04a+0.03=0.11$ $0.077+0.6+0.04b+0.06=0.75$ $0.011+0.04c+0.01=0.04$ $0.022+0.075+0.04d=0.1$ $a=0.125,\quad b=0.325,\quad c=0.475,\quad d=0.075$	M1 A1 M1 A2 **5**	Transition matrix and $\begin{pmatrix}0.11\\0.75\\0.04\\0.1\end{pmatrix}$ Forming at least one equation *or* $a+b+c+d=1$ Give A1 for two correct

Post-multiplication by transition matrix

5 (i)	$\mathbf{P}=\begin{pmatrix}0&0.7&0.1&0.2\\0.1&0.8&0&0.1\\0&0&1&0\\0.3&0.6&0.1&0\end{pmatrix}$	B2 **2**	Give B1 for two rows correct
(ii)	$(0.6\quad 0.4\quad 0\quad 0)\,\mathbf{P}^{13}$ $=(0.0810\quad 0.5684\quad 0.2760\quad 0.0746)$	M1 A2 **3**	Using $\mathbf{P}^{13}$ (or $\mathbf{P}^{14}$) Give A1 for 2 probabilities correct *(Max A1 if not at least 3dp)* *Tolerance* ± 0.0001
(iii)	$0.5684\times 0.8+0.2760$ $=0.731$	M1M1 A1 ft **3**	For 0.5684×0.8 and 0.2760 *Accept 0.73 to 0.7312*
(iv)	$(0.6\quad 0.4\quad 0\quad 0)\,\mathbf{P}^{30}=(.\quad .\quad 0.4996\quad .)$ $(0.6\quad 0.4\quad 0\quad 0)\,\mathbf{P}^{31}=(.\quad .\quad 0.5103\quad .)$ Level 32	M1 A1 A1 **3**	Finding P(C) for some powers of **P** For identifying $\mathbf{P}^{31}$
(v)	Expected number of levels including the next change of location is $\dfrac{1}{0.2}=5$ Expected number of further levels in B is 4	M1 A1 A1 **3**	For $1/(1-0.8)$ or $0.8/(1-0.8)$ For 5 or 4 For 4 as final answer

(vi)	$\mathbf{Q} = \begin{pmatrix} 0 & 0.7 & 0.1 & 0.2 \\ 0.1 & 0.8 & 0 & 0.1 \\ 0 & 0 & 0.9 & 0.1 \\ 0.3 & 0.6 & 0.1 & 0 \end{pmatrix}$	B1	*Can be implied*
	$\mathbf{Q}^n \to \begin{pmatrix} 0.0916 & 0.6183 & 0.1908 & 0.0992 \\ 0.0916 & 0.6183 & 0.1908 & 0.0992 \\ 0.0916 & 0.6183 & 0.1908 & 0.0992 \\ 0.0916 & 0.6183 & 0.1908 & 0.0992 \end{pmatrix}$	M1 M1	Evaluating powers of **Q** *or Obtaining (at least) 3 equations from* $\mathbf{pQ} = \mathbf{p}$ Limiting matrix with equal rows *or Solving to obtain one equilib prob* *or M2 for other complete method*
	A: 0.0916 B: 0.6183 C: 0.1908 D: 0.0992	A2 **5**	Give A1 for two correct *(Max A1 if not at least 3dp)* *Tolerance* ±0.0001
(vii)	$\begin{pmatrix} 0.11 & 0.75 & 0.04 & 0.1 \end{pmatrix} \begin{pmatrix} 0 & 0.7 & 0.1 & 0.2 \\ 0.1 & 0.8 & 0 & 0.1 \\ a & b & c & d \\ 0.3 & 0.6 & 0.1 & 0 \end{pmatrix}$	M1	Transition matrix and $\begin{pmatrix} 0.11 & 0.75 & 0.04 & 0.1 \end{pmatrix}$
	$= \begin{pmatrix} 0.11 & 0.75 & 0.04 & 0.1 \end{pmatrix}$	A1	
	$0.075 + 0.04a + 0.03 = 0.11$ $0.077 + 0.6 + 0.04b + 0.06 = 0.75$ $0.011 + 0.04c + 0.01 = 0.04$ $0.022 + 0.075 + 0.04d = 0.1$	M1	Forming at least one equation *or* $a + b + c + d = 1$
	$a = 0.125$, $b = 0.325$, $c = 0.475$, $d = 0.075$	A2 **5**	Give A1 for two correct

4758 Differential Equations

1(i)	$\alpha^2+25=0$	M1	Auxiliary equation
	$\alpha=\pm 5$	A1	
	CF $y=A\cos 5t+B\sin 5t$	F1	CF for their roots
	PI $y=at\cos 5t+bt\sin 5t$	B1	
	$\dot{y}=a\cos 5t-5at\sin 5t+b\sin 5t+5bt\cos 5t$		
	$\ddot{y}=-10a\sin 5t-25at\cos 5t+10b\cos 5t-25bt\sin 5t$	M1	Differentiate twice
	In DE $\Rightarrow 10b\cos 5t-10a\sin 5t=20\cos 5t$	M1	Substitute and compare coefficients
	$\Rightarrow b=2,\ a=0$	A1	
	PI $y=2t\sin 5t$		
	GS $y=2t\sin 5t+A\cos 5t+B\sin 5t$	F1	
		8	
(ii)	$t=0,\ y=1\Rightarrow A=1$	B1	From correct GS
	$\dot{y}=2\sin 5t+10t\cos 5t-5A\sin 5t+5B\cos 5t$	M1	Differentiate
	$t=0,\ \dot{y}=0\Rightarrow B=0$	M1	Use condition on $\dot{y}$
	$y=2t\sin 5t+\cos 5t$	A1	
		4	
(iii)	Curve through (0, 1)	B1	
	Curve with zero gradient at (0, 1)	B1	
	Oscillations	B1	
	Oscillations with increasing amplitude	B1	
		4	
(iv)	$y=2\sin 5t,\quad \dot{y}=10\cos 5t,\quad \ddot{y}=-50\sin 5t$		
	$\ddot{y}+2\dot{y}+25y=-50\sin 5t+20\cos 5t+50\sin 5t$	M1	Substitute into DE
	$=20\cos 5t$	E1	
	$\alpha^2+2\alpha+25=0$	M1	Auxiliary equation
	$\alpha=-1\pm \mathrm{i}\sqrt{24}$	A1	
	CF $\mathrm{e}^{-t}\left(C\cos\sqrt{24}t+D\sin\sqrt{24}t\right)$	F1	CF for their complex roots
	GS $y=2\sin 5t+\mathrm{e}^{-t}\left(C\cos\sqrt{24}t+D\sin\sqrt{24}t\right)$	F1	Their PI + their CF with two arbitrary constants
		6	
(v)	Oscillations of amplitude 2	B1	or bounded oscillations; or both oscillate
	Compared to unbounded oscillations in first model	B1	o.e. or one bounded, one unbounded
		2	
2(i)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}+\dfrac{3}{x}y=\dfrac{\sin x}{x^2}$	M1	Rearrange
	$I=\mathrm{e}^{\int\frac{3}{x}\,\mathrm{d}x}$	M1	Attempting integrating factor
	$=\mathrm{e}^{3\ln x}$	A1	

	$= x^3$	A1	Correct and simplified
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3 y\right) = x\sin x$	M1	Multiply and recognise derivative
	$x^3 y = \int x\sin x\ \mathrm{d}x = -x\cos x + \int \cos x\ \mathrm{d}x$	M1	Integrate
		A1	
	$= -\cos x + \sin x + A$	A1	All correct
	$y = \frac{-x\cos x + \sin x + A}{x^3}$	F1	Must include constant
		9	
(ii)	$y \approx \frac{-x\left(1 - \frac{1}{2}x^2\right) + x - \frac{1}{6}x^3 + A}{x^3}$	M1	Substitute given approximations
		F1	
	$= \frac{1}{3} + \frac{A}{x^3}$	M1	Use finite limit to deduce *A*
	$A = 0$	A1	
	$y = \frac{\sin x - x\cos x}{x^3}$	B1	Correct particular solution
	$\lim_{x \to 0} y = \frac{1}{3}$	B1	Correct limit
		6	
(iii)	$y = 0 \quad \Rightarrow \quad \sin x - x\cos x = 0$	M1	Equate to zero and attempt to get tan*x*
	$\Rightarrow \quad \tan x = x$	E1	Convincingly shown
		2	
(iv)	$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{3}{x}y = \frac{1}{x} - \frac{1}{6}x$, multiply by $I = x^3$	M1	Rearrange and multiply by IF
		B1	Same IF as in (i) or correct IF
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3 y\right) = x^2 - \frac{1}{6}x^4$	A1	Recognise derivative and RHS correct
	$x^3 y = \frac{1}{3}x^3 - \frac{1}{30}x^5 + B$	M1	Integrate
	$y = \frac{1}{3} - \frac{1}{30}x^2 + \frac{B}{x^3}$	A1	c.a.o
	Finite limit $\Rightarrow B = 0$	M1	Use condition to find constant
	$\lim_{x \to 0} y = \frac{1}{3}$	E1	Show correct limit (or same limit as (ii))
		7	
3(a)(i)	$2\alpha + 4 = 0 \quad \Rightarrow \quad \alpha = -2$	M1	Find root of auxiliary equation
	CF $A\mathrm{e}^{-2t}$	A1	
	PI $I = a\cos 2t + b\sin 2t$	B1	
	$\dot{I} = -2a\sin 2t + 2b\cos 2t$	M1	Differentiate
	$-4a\sin 2t + 4b\cos 2t + 4a\cos 2t + 4b\sin 2t = 3\cos 2t$	M1	Substitute
	$-4a + 4b = 0,\ 4b + 4a = 3 \quad \Rightarrow \quad a = b = \frac{3}{8}$	M1	Compare coefficients and solve

	PI $I = \frac{3}{8}(\cos 2t + \sin 2t)$	A1	
	GS $I = Ae^{-2t} + \frac{3}{8}(\cos 2t + \sin 2t)$	F1	Their PI + their CF with *one* arbitrary constant
		8	
(ii)	$t = 0, l = 0 \quad \Rightarrow \quad 0 = A + \frac{3}{8} \quad \Rightarrow \quad A = -\frac{3}{8}$	M1	Use condition
	$I = \frac{3}{8}(\cos 2t + \sin 2t - e^{-2t})$	A1	c.a.o
		2	
(iii)	For large t, $I \approx \frac{3}{8}(\cos 2t + \sin 2t)$	M1	Consider behaviour for large t (may be implied)
	Amplitude = $\frac{3}{8}\sqrt{1^2 + 1^2} = \frac{3}{8}\sqrt{2}$	A1	
	Curve with oscillations with constant amplitude	B1	
	Their amplitude clearly indicated	B1	
		4	
(b)(i)	(A) $t = 0, y = 0 \quad \Rightarrow \quad \frac{dy}{dt} = 2 - 2(0) + e^0$	M1	Substitute into DE
	Gradient =3	A1	
	(B) At stationary point, $\frac{dy}{dt} = 0, y = \frac{9}{8}$	M1	Substitute into DE
	$\Rightarrow \quad 0 = 2 - 2\left(\frac{9}{8}\right) + e^{-t} \quad \Rightarrow \quad e^{-t} = \frac{1}{4}$	M1	Solve for t
	$\Rightarrow \quad t = \ln 4$	A1	
	(C) $\frac{dy}{dt} \to 0, e^{-t} \to 0$	M1	Substitute into DE
	Giving $0 = 2 - 2y + 0$, so $y \to 1$	A1	
		7	
(ii)	Curve through origin with positive gradient	B1	
	With maximum at (ln4, $\frac{9}{8}$)	B1	Follow their ln4
	With $y \to 1$ as $x \to \infty$	B1	Follow their (C)
		3	
4(i)	$\ddot{x} = 7\dot{x} + 6\dot{y} - 6e^{-3t}$	M1	Differentiate
	$= 7\dot{x} + 6(-12x - 10y + 5\sin t) - 6e^{-3t}$	M1	Substitute for $\dot{y}$
	$y = \frac{1}{6}\left(\dot{x} - 7x - 2e^{-3t}\right)$	M1	y in terms of $x, \dot{x}, t$
	$\ddot{x} = 7\dot{x} - 72x - 10(\dot{x} - 7x - 2e^{-3t}) + 30\sin t - 6e^{-3t}$	M1	Substitute for y
	$\ddot{x} + 3\dot{x} + 2x = 14e^{-3t} + 30\sin t$	E1	Complete argument
		5	
(ii)	$x = ae^{-3t} - 9\cos t + 3\sin t$		
	$\dot{x} = -3ae^{-3t} + 9\sin t + 3\cos t$		
	$\ddot{x} = 9ae^{-3t} + 9\cos t - 3\sin t$	M1	Differentiate twice
	In DE gives	M1	Substitute
	$9ae^{-3t} + 9\cos t - 3\sin t$ $+ 3(-3ae^{-3t} + 9\sin t + 3\cos t)$		

	$+2(ae^{-3t}-9\cos t+3\sin t)$		
	$=2ae^{-3t}+30\sin t$		
	So PI with $2a = 14$	E1	Correct form shown
	$\Rightarrow \quad a=7$	A1	
	AE $\alpha^2+3\alpha+2=0$	M1	Auxiliary equation
	$\alpha=-1,-2$	A1	
	CF $Ae^{-t}+Be^{-2t}$	F1	CF for their roots
	GS $x=Ae^{-t}+Be^{-2t}+7e^{-3t}-9\cos t+3\sin t$	F1	Their PI + their CF with two arbitrary constants
		8	
(iii)	$x=\frac{1}{6}(\dot{x}-7x-2e^{-3t})$	M1	y in terms of $x, \dot{x}, t$
	$\dot{x}=-Ae^{-t}-2Be^{-2t}-21e^{-3t}+9\sin t+3\cos t$	M1	Differentiate GS for x
		F1	Follow their GS
	$y=-\frac{4}{3}Ae^{-t}-\frac{3}{2}Be^{-2t}-12e^{-3t}+11\cos t-2\sin t$	A1	c.a.o
		4	
(iv)	$x\approx 3\sin t-9\cos t$	B1	Follow their x
	$y\approx 11\cos t-2\sin t$	B1	Follow their y
	$x=y \quad \Rightarrow \quad 11\cos t-2\sin t\approx 3\sin t-9\cos t$	M1	Equate
	$\Rightarrow \quad 20\cos t\approx 5\sin t \quad \Rightarrow \quad \tan t\approx 4$	A1	Complete argument
		4	
(v)	Amplitude of $x\approx\sqrt{3^2+9^2}=3\sqrt{10}$	M1	Attempt both amplitudes
	Amplitude of $y\approx\sqrt{11^2+2^2}=5\sqrt{5}$	A1	One correct
	Ratio is $\frac{5}{6}\sqrt{2}$	A1	c.a.o (accept reciprocal)
		3	

4761 Mechanics 1

Q 1		**Mark**	**Comment**	**Sub**
(i)	0.5 × 8 × 10 = 40 m	M1	Attempt to find whole area or … If *suvat* used in 2 parts, accept any *t* value $0 \le t \le 8$ for max.	
		A1	c.a.o.	
				2
(ii)	$0.5 \times (T-8) = 10$	M1	$0.5 \times 5 \times k = 10$ seen. Accept ±5 and ±10 only. If *suvat* used need whole area; if in 2 parts, accept any *t* value $8 \le t \le T$ for min.	
		B1	Attempt to use $k = T - 8$.	
	$T = 12$	A1	c.a.o. [Award 3 if $T = 12$ seen]	
				3
(iii)	40 – 10 = 30 m	B1	ft **their** 40.	
				1
		6		

Q 2		**Mark**	**Comment**	**Sub**
(i)	$\sqrt{10^2 + 24^2} = 26$ so 26 N	B1		
	$\arctan \frac{10}{24}$	M1	Using arctan or equiv. Accept $\arctan \frac{24}{10}$ or equiv.	
	= 22.619… so 22.6° (3 s.f.)	A1	Accept 157.4°.	
				3
(ii)	$\mathbf{W} = -w\mathbf{j}$	B1	Accept $\begin{pmatrix} 0 \\ -w \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -w\mathrm{j} \end{pmatrix}$	
				1
(iii)	$\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{W} = \mathbf{0}$	M1	Accept in any form and recovery from $\mathbf{W} = w\mathbf{j}$. Award if not explicit and part (ii) and **both** *k* and *w* correct.	
	$k = -10$	B1	Accept from wrong working.	
	$w = 34$	B1	Accept from wrong working but not −34. [Accept $-10\mathbf{i}$ *or* $34\mathbf{j}$ but not both]	
				3
		7		

Q 3		Mark	Comment	Sub
(i)	The line is not straight	B1	Any valid comment	1
(ii)	$a = 3 - \frac{6t}{8}$	M1	Attempt to differentiate. Accept 1 term correct but not $3 - \frac{3t}{8}$.	
	$a(4) = 0$	F1		
	The sprinter has reached a steady speed	E1	Accept 'stopped accelerating' but not just $a = 0$. Do not ft $a(4) \neq 0$.	3
(iii)	We require $\int_1^4 \left(3t - \frac{3t^2}{8}\right) \mathrm{d}t$	M1	Integrating. Neglect limits.	
	$= \left[\frac{3t^2}{2} - \frac{t^3}{8}\right]_1^4$	A1	One term correct. Neglect limits.	
	$= (24 - 8) - \left(\frac{3}{2} - \frac{1}{8}\right)$	M1	Correct limits subst in integral. Subtraction seen. If arb constant used, evaluated to give $s = 0$ when $t = 1$ and then sub $t = 4$.	
	$= 14\frac{5}{8}$ m (14.625 m)	A1	c.a.o. Any form. [If trapezium rule used M1 use of rule (must be clear method and at least two regions) A1 correctly applied M1 At least 6 regions used A1 Answer correct to at least 2 s.f.)]	4
		8		

Q 4		Mark	Comment	Sub
(i)	$32\cos\alpha t$	B1		1
(ii)	$32\cos\alpha\times5=44.8$	M1	ft **their** x.	
	so $160\cos\alpha=44.8$ and $\cos\alpha=0.28$	E1	Shown. Must see some working e.g cosθ = 44.8 ÷ 160 or 160 cosθ = 44.8. If 32 × 0.28 × 5 = 44.8 seen then this needs a statement that 'hence cosθ = 0.28'.	2
(iii)	$\sin\alpha=0.96$	B1	Need not be explicit e.g. accept sin(73.73…) seen.	
	either			
	$0=(32\times0.96)^2-2\times9.8\times s$	M1	Allow use of 'u' = 32, $g = \pm$ (10, 9.8, 9.81).	
		A1	Correct substitution.	
	s = 48.1488… so 48.1 m (3 s. f.)	A1	c.a.o.	
	or			
	Time to max height is given by 32 × 0.96 − 9.8 T = 0 so T = 3.1349….	B1	Could use ½ total time of flight to the horizontal.	
	$y = 32 \times 0.96\, t - 4.9\, t^2$	M1	Allow use of 'u' = 32, $g = \pm$ (10, 9.8, 9.81) May use $s=\dfrac{(u+v)}{2}t$.	
	putting $t = T$, y = 48.1488 so 48.1 m (3 s. f.)	A1	c.a.o.	4
		7		

Q 5		Mark	Comment	Sub
(i)	$\mathbf{v}=\mathbf{i}+(3-2t)\mathbf{j}$	M1	Differentiating **r**. Allow 1 error. Could use const accn.	
		A1		
	v(4) = **i** – 5**j**	F1	Do not award if $\sqrt{26}$ is given as vel (accept if **v** given and v given as well called speed or magnitude).	3
(ii)	**a** = – 2**j**	B1	Diff **v**. ft **their v**. Award if – 2**j** seen & isw.	
	Using N2L **F** = 1.5 × (–2**j**)	M1	Award for 1.5 × (± **a** or a) seen.	
	so –3**j** N	A1	c.a.o. Do not award if final answer is not correct. [Award M1 A1 for –3**j** WW]	3
(iii)	$x=2+t$ and $y=3t-t^2$	B1	Must have both but may be implied.	
	Substitute $t=x-2$			
	so $y=3(x-2)-(x-2)^2$ $[=(x-2)(5-x)]$	B1	c.a.o. isw. Must see the form y = ….	2
		8		

Q 6		**Mark**	**Comment**	**Sub**
(i)	Up the plane $T - 4g\sin 25 = 0$	M1	Resolving parallel to the plane. If any other direction used, all forces must be present. Accept $s \leftrightarrow c$. Allow use of *m*. No extra forces.	
	T = 16.5666… so 16.6 N (3 s. f.)	A1		
				2
(ii)	Down the plane, (4 + *m*)g sin25 – 50 = 0	M1	No extra forces. Must attempt resolution in at least 1 term. Accept $s \leftrightarrow c$. Accept *M*g sin25. Accept use of mass.	
		A1	Accept *Mg*sin25	
	m = 8.0724… so 8.07 (3 s. f.)	A1		
				3
(iii)	Diagram	B1	Any 3 of weight, friction normal reaction and *P* present in approx correct directions with arrows.	
		B1	All forces present with suitable directions, labels and arrows. Accept *W*, *m*g, 4g and 39.2.	
				2
(iv)	Resolving up the plane	M1	Or resolving parallel to the plane. All forces must be present. Accept $s \leftrightarrow c$. Allow use of *m*. At least one resolution attempted and accept wrong angles. Allow sign errors.	
		B1	*P* cos15 term correct. Allow sign error.	
	$P\cos 15 - 20 - 4g\sin 25 = 0$	B1	Both resolutions correct. Weight used. Allow sign errors. ft use of *P* sin 15.	
		A1	All correct but ft use of *P* sin 15.	
	P = 37.8565…. so 37.9 N (3 s. f.)	A1		
				5
(v)	Resolving perpendicular to the plane	M1	May use other directions. All forces present. No extras. Allow $s \leftrightarrow c$. Weight not mass used. Both resolutions attempted. Allow sign errors.	
	$R + P\sin 15 - 4g\cos 25 = 0$	B1	Both resolutions correct. Allow sign errors. Allow use of *P*cos15 if *P*sin15 used in (iv).	
		F1	All correct. Only ft **their** *P* and their use of *P*cos15.	
	R = 25.729… so 25.7 N	A1	c.a.o.	
				4
		16		

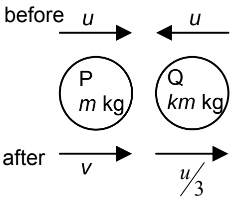
If there is a consistent $s \leftrightarrow c$ error in the weight term throughout the question, penalise only two marks for this error. In the absence of other errors this gives
(i) 35.52… (ii) 1.6294… (iv) 57.486… (v) 1.688…

For use of mass instead of weight lose maximum of 2.

Q 7		Mark	Comment	Sub
	With the 11.2 N resistance acting to the left			
(i)	N2L $F - 11.2 = 8 \times 2$	M1	Use of N2L (allow $F = mga$). Allow 11.2 omitted; no extra forces.	
		A1	All correct	
	$F = 27.2$ so 27.2 N	A1	c.a.o.	
				3
(ii)	The string is inextensible	E1	Allow 'light inextensible' but not other irrelevant reasons given as well (e.g. smooth pulley).	
				1
(iii)		B1	One diagram with all forces present; no extras; correct arrows and labels accept use of words.	
		B1	Both diagrams correct with a common label.	
				2
(iv)	**Method (1)**	M1	For either box or sphere, $F = ma$. Allow omitted force and sign errors but not extra forces. Need correct mass. Allow use of mass not weight.	
	Box N2L $\rightarrow 105 - T - 11.2 = 8a$	A1	Correct and in any form.	
	Sphere N2L $\uparrow$ $T - 58.8 = 6a$	A1	Correct and in any form. [box and sphere equns with consistent signs]	
	Adding $35 = 14a$	M1	Eliminating 1 variable from 2 equns in 2 variables.	
	$a = 2.5$ so 2.5 ms^{-2}	E1		
	Substitute $a = 2.5$ giving $T = 58.8 + 15$	M1	Attempt to substitute in either box or sphere equn.	
	$T = 73.8$ so 73.8 N	A1		
	Method (2)			
	$105 - 11.2 - 58.8 = 14a$	M1	For box and sphere, $F = ma$. Must be correct mass. Allow use of mass not weight.	
	$a = 2.5$	A1		
		E1	Method made clear.	
		M1	For **either** box **or** sphere, $F = ma$. Allow omitted force and sign errors but not extra forces. Need correct mass. Allow use of mass not weight.	
	either: box N2L $\rightarrow 105 - T - 11.2 = 8a$			
	or: sphere N2L $\uparrow$ $T - 58.8 = 6a$	A1	Correct and in any form.	
	Substitute $a = 2.5$ in either equn	M1	Attempt to substitute in either box or sphere equn.	
	$T = 73.8$ so 73.8 N	A1		
			[If AG used in either equn award M1 A1 for that equn as above and M1 A1 for finding T. For full marks, both values must be shown to satisfy the second equation.]	
				7

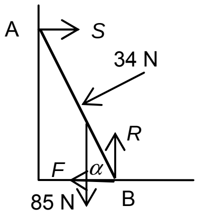
(v)(A)	g downwards	B1	Accept ±g, ±9.8, ±10, ±9.81	1
(B)	Taking ↑ + ve, $s = -1.8$, $u = 3$ and $a = -9.8$ so $-1.8 = 3T - 4.9T^2$	M1	Some attempt to use $s = ut + 0.5at^2$ with $a = \pm 9.8$ etc $s = \pm 1.8$ and $u = \pm 3$. Award for $a = g$ even if answer to (A) wrong.	
	and so $4.9T^2 - 3T - 1.8 = 0$	E1	Clearly shown. No need to show +ve required.	2
(C)	Time to reach 3 ms^{-1} is given by $3 = 0 + 2.5t$ so $t = 1.2$	B1		
	remaining time is root of quad	M1	Quadratic solved and + ve root added to time to break.	
	time is 0.98513… s	B1	Allow 0.98. [Award for answer seen WW]	
	Total 2.1851…so 2.19 s (3 s. f.)	A1	c.a.o.	4
	With the 11.2 N resistance acting to the right			
(i)	$F + 11.2 = 8 \times 2$ so $F = 4.8$		The same scheme as above	
(iii)			The 11.2 N force may be in either direction, otherwise the same scheme	
(iv)	The same scheme with + 11.2 N instead of – 11.2 N acting on the box **Method (1)** Box N2L $\rightarrow 105 - T + 11.2 = 8a$ Sphere as before **Method (2)** $105 + 11.2 - 58.8 = 14a$ These give $a = 4.1$ and $T = 83.4$		Allow 2.5 substituted in box equation to give $T = 96.2$ If the sign convention gives as positive the direction of the sphere descending, $a = -4.1$. Allow substituting $a = 2.5$ in the equations to give $T = 43.8$ (sphere) or 136.2 (box).	
(v)			In (C) allow use of $a = 4.1$ to give time to break as 0.73117..s. and total time as 1.716…s	4
		20		

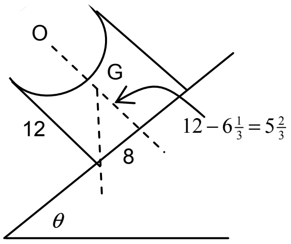
4762 Mechanics 2

Q 1		**Mark**	**Comment**	**Sub**
(a)(i)		B1		1
(ii)	$mu - kmu = mv + km\frac{u}{3}$	M1	PCLM applied	
		A1	Either side correct (or equiv)	
	$v = \left(1-\frac{4k}{3}\right)u$	E1	Must at least show terms grouped	3
(iii)	Need $v < 0$	E1	Accept $\frac{4k}{3} > 1$ without reason	
	so $k > \frac{3}{4}$	B1		
			[SC1: $v = 0$ used and inequality stated without reason]	2
(iv)	$\frac{\frac{1}{3}u - v}{-u-u} = -\frac{1}{2}$	M1	Use of NEL	
		A1		
	so $v = -\frac{2u}{3}$	E1		
	$-\frac{2u}{3} = u\left(1-\frac{4k}{3}\right)$	M1		
	so $k = 1.25$	A1	c.a.o.	5
(b)(i)	$9\begin{pmatrix}1\\-2\end{pmatrix} + 5\begin{pmatrix}3\\2\end{pmatrix} = 8\mathbf{V}$	M1	Use of PCLM	
		B1	Use of mass 8 in coalescence	
		M1	Use of $\mathbf{I} = \mathbf{F}t$	
	$\mathbf{V} = \begin{pmatrix}3\\-1\end{pmatrix}$	E1		4
(ii)	**i** cpt $3 \to -3\times\frac{1}{2}$	M1	Allow wrong sign	
	j cpt unchanged	B1	May be implied	
	New velocity $\begin{pmatrix}-1.5\\-1\end{pmatrix}$ ms^{-1}	A1	c.a.o. [Award 2/3 if barrier taken as $\begin{pmatrix}1\\0\end{pmatrix}$]	3
		18		

Q 2		**Mark**	**Comment**	**Sub**
(a)(i)(A)	Yes. Only WD is against conservative forces.	E1	Accept only WD is against gravity or no work done against friction.	

				1
(B)	Block has no displacement in that direction	E1		
				2
(ii)	$0.5\times 50\times 1.5^2 = 20gx - 5gx$	M1 B1 M1 A1	Use of WE with KE. Allow $m = 25$. Use of 50 At least 1 GPE term GPE terms correct signs	
	$x = 0.38265\ldots$ so 0.383 m (3 s.f.)	A1	c.a.o.	
				5
(iii)	$0.5\times 50\times V^2 - 0.5\times 50\times 1.5^2$	M1	WE equation with WD term. Allow GPE terms missing	
		B1	Both KE terms. Accept use of 25.	
	$= 2 \times 20g - 2 \times 5g - 180$	B1	Either GPE term	
		B1	180 with correct sign	
	$V = 2.6095\ldots$ so 2.61 ms^{-1}	A1	c.a.o.	
				5
(b)	Force down the slope is $2000 + 450g \sin 20$	M1	Both terms. Allow mass not weight	
		B1	Weight term correct	
	Using $P = Fv$	M1		
	$P = (2000 + 450g \sin 20) \times 2.5$	F1	ft **their** weight term	
	$P = 8770.77\ldots$ so 8770 W (3 s.f.)	A1	c.a.o.	
				5
		17		

Q 3		Mark	Comment	Sub
(i)	c.w. moments about A	M1	Moments equation.	
	$5R_{\text{B}} - 3 \times 85$ so $R_{\text{B}} = 51$ giving 51 N ↑	A1	Accept no direction given	
	Either a.c. moments about B or resolve ↑	M1		
	$R_{\text{A}} = 34$ so 34 N ↑	F1	Accept no direction given	
				4
(ii)	c.w. moments about A	M1	Moments with attempt to resolve at least one force. Allow $s \leftrightarrow c$.	
	$85\times3\cos\alpha-27.2\times5\sin\alpha=0$	B1	Weight term	
		B1	Horiz force term	
	so $\tan\alpha=\dfrac{3\times85}{27.2\times5}=\dfrac{15}{8}$	E1	Must see some arrangement of terms or equiv	
				4
(iii)		B1	All forces present and labelled	
	a.c. moments about B	M1	Moments with attempt to resolve forces and all relevant forces present	
	$85\times2\times\cos\alpha+34\times2.5-5S\times\sin\alpha=0$	B1	34×2.5	
		A1	All other terms correct. Allow sign errors.	
	$S=37.4$	A1	All correct	
	Resolving horizontally and vertically	M1	Either attempted	
	→ $S-F-34\sin\alpha=0$ so $F = 7.4$	E1		
	↑ $R-85-34\cos\alpha=0$	A1	$R = 101$ need not be evaluated here [Allow A1 for the two expressions if correct other than $s \leftrightarrow c$]	
	Using $F=\mu R$	M1		
	$\mu=\dfrac{7.4}{101}$ = 0.07326… so 0.0733(3 s.f.)	A1	c.a.o.	
				10
		18		

Q 4		**Mark**	**Comment**	**Sub**
(i)	Taking y-axis vert downwards from O		Allow areas used as masses	
	$2\pi\sigma\times 8^2\times 4+2\pi\sigma\times 8\times k\times\frac{k}{2}$	M1	Method for c.m.	
		B1	'4' used	
		B1	$16\pi k$	
		B1	$\frac{k}{2}$ used	
	$=\left(2\pi\sigma\times 8^2+2\pi\sigma\times 8k\right)\bar{y}$	B1	Masses correct	
	so $\bar{y}=\frac{64+k^2}{16+2k}$	E1	Must see some evidence of simplification Need no reference to axis of symmetry	
				6
(ii)	$k = 12$ gives OG as 5.2 and mass as $320\pi\sigma$	B1	Allow for either. Allow $\sigma=1$	
	$320\pi\sigma\times 5.2+\pi\sigma\times 8^2\times 12$	M1	Method for c.m. combining with (i) or starting again	
		B1	One term correct	
		B1	Second term correct	
	$=(320\pi\sigma+64\pi\sigma)\bar{y}$			
	$\bar{y}=6\frac{1}{3}$	E1	Some simplification shown	
				5
(iii)		B1 B1 B1	G above edge of base $12-6\frac{1}{3}=5\frac{2}{3}$ seen here or below 8 seen here or below	
	$\tan\theta=\frac{8}{5\frac{2}{3}}$	M1	Accept $\frac{5\frac{2}{3}}{8}$ or attempts based on $6\frac{1}{3}$ and 8.	
	$\theta = 54.6887\ldots$ so 54.7° (3 s. f.)	A1	c.a.o.	
				5
(iv)	Slips when $\mu=\tan\theta$	M1	Or ….	
	$\frac{8}{5\frac{2}{3}}=1.4117...$	B1		
	< 1.5 so does not slip	A1	There must be a reason	
				3
		19		

4763 Mechanics 3

1 (i)	$\frac{1}{2}m(v^2-1.4^2)=m\times 9.8(2.6-2.6\cos\theta)$ $v^2-1.96=50.96-50.96\cos\theta$ $v^2=52.92-50.96\cos\theta$	M1 A1 E1 **3**	Equation involving KE and PE
(ii)	$0.65\times 9.8\cos\theta-R=0.65\times\frac{v^2}{2.6}$ $6.37\cos\theta-R=0.25(52.92-50.96\cos\theta)$ $6.37\cos\theta-R=13.23-12.74\cos\theta$ $R=19.11\cos\theta-13.23$	M1 A1 M1 A1 **4**	Radial equation involving $\frac{v^2}{r}$ Substituting for v^2 *Dependent on previous M1* *Special case:* $R=13.23-19.11\cos\theta$ *earns* M1A0M1SC1
(iii)	Leaves surface when $R = 0$ $\cos\theta=\frac{13.23}{19.11}\ (=\frac{9}{13})\quad(\theta=46.19°)$ $v^2=52.92-50.96\times\frac{9}{13}$ Speed is 4.2 m s^{-1}	M1 A1 M1 A1 **4**	(ft if $R = a + b\cos\theta$ and $0<-\frac{a}{b}<1$) *Dependent on previous M1*
(iv)	$T\sin\alpha+R\cos\alpha=0.65\times 9.8$ $T\cos\alpha-R\sin\alpha=0.65\times\frac{1.2^2}{2.4}$	M1 A1 M1 A1	Resolving vertically (3 terms) Horiz eqn involving $\frac{v^2}{r}$ or $r\omega^2$
	OR $T-mg\sin\alpha=m\left(\frac{1.2^2}{2.4}\right)\cos\alpha$ $mg\cos\alpha-R=m\left(\frac{1.2^2}{2.4}\right)\sin\alpha$	M1A1 M1A1	
	$\sin\alpha=\frac{2.4}{2.6}=\frac{12}{13},\quad\cos\alpha=\frac{5}{13}\quad(\alpha=67.38°)$ Tension is 6.03 N Normal reaction is 2.09 N	M1 M1 A1 A1 **8**	Solving to obtain a value of *T* or *R* *Dependent on necessary M1s* *(Accept 6 , 2.1)* *Treat* $\omega=1.2$ *as a misread, leading to* $T=6.744,\ R=0.3764$ *for 7 / 8*

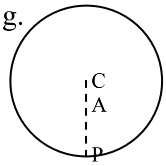
2 (i)	$\frac{1}{2}\times 5000x^2 = \frac{1}{2}\times 400\times 3^2$ Compression is 0.849 m	M1 A1 A1 **3**	Equation involving EE and KE Accept $\frac{3\sqrt{2}}{5}$
(ii)	Change in PE is $400\times 9.8\times(7.35+1.4)\sin\theta$ $= 400\times 9.8\times 8.75\times\frac{1}{7}$ $= 4900$ J Change in EE is $\frac{1}{2}\times 5000\times 1.4^2$ $= 4900$ J Since Loss of PE = Gain of EE, car will be at rest	M1 A1 M1 E1 **4**	Or $400\times 9.8\times 1.4\sin\theta$ *and* $\frac{1}{2}\times 400\times 4.54^2$ Or $784+4116$ M1M1A1 can also be given for a correct equation in x (compression): $2500x^2-560x-4116=0$ Conclusion required, or solving equation to obtain $x=1.4$
(iii)	WD against resistance is $7560(24+x)$ Change in EE is $\frac{1}{2}\times 5000x^2$ Change in KE is $\frac{1}{2}\times 400\times 30^2$ Change in PE is $400\times 9.8\times(24+x)\times\frac{1}{7}$	B1 B1 B1 B1	($=181440+7560x$) ($=2500x^2$) ($=180000$) ($=13440+560x$)
	OR Speed 7.75 m s^{-1} when it hits buffer, then WD against resistance is $7560x$ Change in EE is $\frac{1}{2}\times 5000x^2$ Change in KE is $\frac{1}{2}\times 400\times 7.75^2$ Change in PE is $400\times 9.8\times x\times\frac{1}{7}$	B1 B1 B1 B1	($=2500x^2$) ($=12000$) ($=560x$)
	$-7560(24+x)=\frac{1}{2}\times 5000x^2-\frac{1}{2}\times 400\times 30^2$ $-400\times 9.8\times(24+x)\times\frac{1}{7}$ $-7560(24+x)=2500x^2-180\,000-560(24+x)$ $-3.024(24+x)=x^2-72-0.224(24+x)$ $x^2+2.8x-4.8=0$ $x=\frac{-2.8+\sqrt{2.8^2+19.2}}{2}$ $=1.2$	M1 F1 M1 A1 M1 A1 **10**	Equation involving WD, EE, KE, PE Simplification to three term quadratic

3(a)(i)	[Velocity] $= \mathrm{L\,T}^{-1}$	B1	*Deduct 1 mark for ms^{-1} etc*
	[Force] $= \mathrm{M\,L\,T}^{-2}$	B1	
	[Density] $= \mathrm{M\,L}^{-3}$	B1	
		3	
(ii)	$\mathrm{M\,L\,T}^{-2} = (\mathrm{M\,L}^{-3})^{\alpha}\,(\mathrm{L\,T}^{-1})^{\beta}\,(\mathrm{L}^{2})^{\gamma}$		
	$\alpha = 1$	B1	
	$\beta = 2$	B1	
	$-3\alpha + \beta + 2\gamma = 1$	M1A1	(ft if equation involves α, β and γ)
	$\gamma = 1$	A1	
		5	
(b)(i)	$\frac{2\pi}{\omega} = 4.3$	M1	
	$\omega = \frac{2\pi}{4.3}$ $(= 1.4612)$	A1	
	$\dot{\theta}^2 = 1.4612^2(0.08^2 - 0.05^2)$	M1	Using $\omega^2(A^2 - \theta^2)$
		F1	For RHS
	Angular speed is $0.0913\ \mathrm{rad\,s}^{-1}$	A1	(b.o.d. for $v = 0.0913\ \mathrm{m\,s}^{-1}$)
		5	
	OR $\dot{\theta} = 0.08\omega\cos\omega t$	M1	Or $\dot{\theta} = (-)\ 0.08\omega\sin\omega t$
	$= 0.08 \times 1.4612\cos 0.6751$	F1	$= (-)\ 0.08 \times 1.4612\sin 0.8957$
	$= 0.0913$	A1	
(ii)	$\theta = 0.08\sin\omega t$	B1	or $\theta = 0.08\cos\omega t$
	When $\theta = 0.05$, $0.08\sin\omega t = 0.05$	M1	Using $\theta = (\pm)0.05$ to obtain an equation for t
	$\omega t = 0.6751$	A1 cao	*B1M1 above can be earned in (i)*
	$t = 0.462$		or $t = 0.613$ from $\theta = 0.08\cos\omega t$ or $t = 1.537$ from $\theta = 0.08\cos\omega t$
	Time taken is 2×0.462	M1	Strategy for finding the required time (2×0.462 or $\frac{1}{2} \times 4.3 - 2 \times 0.613$
	$= 0.924$ s	A1 cao	or $1.537 - 0.613$) *Dep on first M1* *For $\theta = 0.05\sin\omega t$, max B0M1A0M0* *(for $0.05 = 0.05\sin\omega t$)*
		5	

4(a)	Area is $\int_0^{\ln 3} \mathrm{e}^x \,\mathrm{d}x = \Big[\mathrm{e}^x \Big]_0^{\ln 3}$	M1	
	$= 2$	A1	
	$\int x\,y\,\mathrm{d}x = \int_0^{\ln 3} x\,\mathrm{e}^x\,\mathrm{d}x$	M1	
	$= \Big[x\,\mathrm{e}^x - \mathrm{e}^x \Big]_0^{\ln 3}$	M1 A1	Integration by parts For $x\,\mathrm{e}^x - \mathrm{e}^x$
	$= 3\ln 3 - 2$		
	$\bar{x} = \dfrac{3\ln 3 - 2}{2} = \tfrac{3}{2}\ln 3 - 1$	A1	ww full marks (B4) Give B3 for 0.65
	$\int \tfrac{1}{2} y^2 \,\mathrm{d}x = \int_0^{\ln 3} \tfrac{1}{2}(\mathrm{e}^x)^2\,\mathrm{d}x$	M1	For integral of $(\mathrm{e}^x)^2$
	$= \Big[\tfrac{1}{4}\mathrm{e}^{2x} \Big]_0^{\ln 3}$		
	$= 2$	A1	For $\tfrac{1}{4}\mathrm{e}^{2x}$
	$\bar{y} = \dfrac{2}{2} = 1$	A1	If area wrong, SC1 for $\bar{x} = \dfrac{3\ln 3 - 2}{area}$ and $\bar{y} = \dfrac{2}{area}$
		9	
(b)(i)	Volume is $\int \pi y^2 \,\mathrm{d}x = \int_2^a \pi \dfrac{36}{x^4}\,\mathrm{d}x$	M1	*π may be omitted throughout*
	$= \pi\Big[-\dfrac{12}{x^3} \Big]_2^a = \pi\left(\dfrac{3}{2} - \dfrac{12}{a^3}\right)$	A1	
	$\int \pi x y^2 \,\mathrm{d}x = \int_2^a \pi \dfrac{36}{x^3}\,\mathrm{d}x$	M1	
	$= \pi\Big[-\dfrac{18}{x^2} \Big]_2^a = \pi\left(\dfrac{9}{2} - \dfrac{18}{a^2}\right)$	A1	
	$\bar{x} = \dfrac{\int \pi x y^2 \,\mathrm{d}x}{\int \pi y^2 \,\mathrm{d}x}$	M1	
	$= \dfrac{\pi\left(\dfrac{9}{2} - \dfrac{18}{a^2}\right)}{\pi\left(\dfrac{3}{2} - \dfrac{12}{a^3}\right)} = \dfrac{3(a^3 - 4a)}{a^3 - 8}$	E1	
		6	
(ii)	Since $a > 2$, $4a > 8$	M1	*Condone* $\geq$ *instead of* $>$ *throughout*
	so $a^3 - 4a < a^3 - 8$	A1	
	Hence $\bar{x} = \dfrac{3(a^3 - 4a)}{a^3 - 8} < 3$ i.e. CM is less than 3 units from O	E1	Fully acceptable explanation *Dependent on M1A1*
		3	
	OR As $a \to \infty$, $\bar{x} = \dfrac{3(1 - 4a^{-2})}{1 - 8a^{-3}} \to 3$	M1A1	Accept $\bar{x} \approx \dfrac{3a^3}{a^3} \to 3$, etc
	Since $\bar{x}$ increases as *a* increases, $\bar{x}$ is less than 3	E1	(M1 for $\bar{x} \to 3$ stated, but A1 requires correct justification)

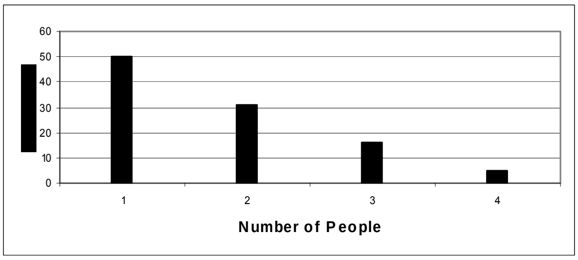
4764 MEI Mechanics 4

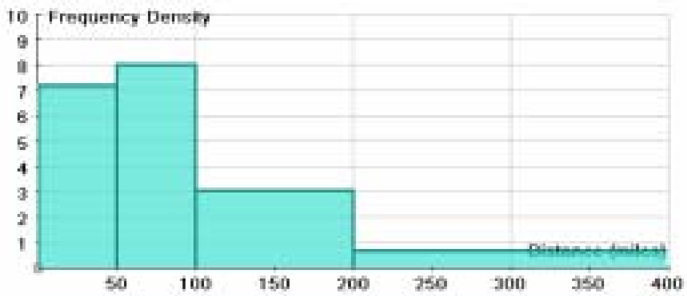
1(i)	$\frac{\mathrm{d}}{\mathrm{d}t}(mv) = mg$	B1	Seen or implied
	$\Rightarrow \quad \frac{\mathrm{d}m}{\mathrm{d}t}v + m\frac{\mathrm{d}v}{\mathrm{d}t} = mg$	M1	Expand
	$\Rightarrow \quad \frac{mg}{2(v+1)}v + m\frac{\mathrm{d}v}{\mathrm{d}t} = mg$	M1	Use $\frac{\mathrm{d}m}{\mathrm{d}t}v = \frac{mg}{2(v+1)}$
	$\Rightarrow \quad \frac{\mathrm{d}v}{\mathrm{d}t} = g\left(1 - \frac{v}{2(v+1)}\right) = g\left(\frac{v+2}{2(v+1)}\right)$		
	$\Rightarrow \quad \left(\frac{v+1}{v+2}\right)\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{2}g$	M1	Separate variables (oe)
	$\Rightarrow \quad \left(1 - \frac{1}{v+2}\right)\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{2}g$	E1	
			5
(ii)	$\int\left(1 - \frac{1}{v+2}\right)\mathrm{d}v = \int \frac{1}{2}g \ \mathrm{d}t$	M1	Integrate
	$v - \ln\lvert v+2\rvert = \frac{1}{2}gt + c$	A1	LHS
	$t = 0,\ v = 0 \quad \Rightarrow \quad -\ln 2 = c$	M1	Use condition
	$v - \ln\lvert v+2\rvert = \frac{1}{2}gt - \ln 2$		
	$t = \frac{2}{g}\left(v - \ln\lvert v+2\rvert + \ln 2\right)$	A1	
	$v = 10 \quad \Rightarrow \quad t \approx 1.68$	B1	
			5
(iii)	As t gets large, v gets large	M1	
	So $\frac{\mathrm{d}v}{\mathrm{d}t} \to \frac{1}{2}g$ (i.e. constant)	A1	Complete argument
			2
2(i)	$V = -mg \cdot 2a\sin\theta + \frac{\frac{1}{2}mg}{2a}(4a\sin\theta - a)^2$	B1	GPE
		M1	Reasonable attempt at EPE
		A1	EPE correct
	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = -2mga\cos\theta + \frac{mg}{2a}(4a\sin\theta - a)\cdot 4a\cos\theta$	M1	Differentiate
	$= -2mga\cos\theta + 2mga\cos\theta(4\sin\theta - 1)$		
	$= 4mga\cos\theta(2\sin\theta - 1)$	E1	Complete argument
			5
(ii)	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0$	M1	Set derivative to zero
	$\Leftrightarrow \quad \cos\theta = 0 \text{ or } \sin\theta = \frac{1}{2}$	M1	Solve
	$\Leftrightarrow \quad \theta = \frac{1}{2}\pi \text{ or } \frac{1}{6}\pi$	A1	Both
	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 4mga\cos\theta(2\cos\theta) - 4mga\sin\theta(2\sin\theta - 1)$	M1	Second derivative (or alternative method)
	$V''\left(\frac{1}{2}\pi\right)(= -4mga) < 0 \quad \Rightarrow$ unstable	M1	Consider sign
		A1	One correct conclusion validly

			shown
	$V''\left(\frac{1}{6}\pi\right)\left(=4mga\cdot\frac{\sqrt{3}}{2}\left(\sqrt{3}\right)\right)>0 \quad\Rightarrow\quad$ stable	A1	Complete argument
			7
3(i)	Mass of 'ring' $\approx 2\pi r\delta r\rho$	B1	May be implied
	$\Rightarrow \quad I_C=\int_0^a r^2\cdot 2\pi\rho r\,\mathrm{d}r$	M1	Set up integral
		A1	All correct
	$=\left[2\pi\rho\cdot\frac{1}{4}r^4\right]_0^a=\frac{1}{2}\pi a^4\rho$	M1	Integrate
	$M=\pi a^2\rho$	M1	Use relationship between ρ and M
	$\Rightarrow \quad I_C=\frac{1}{2}Ma^2$	E1	Complete argument
			6
(ii)	$I_A=I_C+M\left(\frac{1}{10}a\right)^2$	M1	Use parallel axis theorem
	$=\frac{1}{2}Ma^2+\frac{1}{100}Ma^2=0.51Ma^2$	E1	Convincingly shown
			2
(iii)	$I_A\overline{\theta}=-Mg\cdot\frac{1}{10}a\sin\theta$	B1	LHS
		B1	RHS
	$\Rightarrow \quad \overline{\theta}=-\frac{g}{5.1a}\sin\theta$	M1	Expression for $\overline{\theta}$
	θ small $\quad\Rightarrow\quad \sin\theta\approx\theta$	M1	Use small angle approximation
	$\Rightarrow \quad \overline{\theta}=-\frac{g}{5.1a}\theta$, i.e. SHM	E1	Complete argument and conclude SHM
	Period $2\pi\sqrt{\frac{5.1a}{g}}\approx 4.53\sqrt{a}$	F1	Follow their SHM equation
			6
(iv)	e.g. 	B1	Show PAC in straight line (in any direction)
	$mg\cdot\frac{9}{10}a=Mg\cdot\frac{1}{10}a$	M1	Moments or $\left(\sum m\right)\bar{x}=\sum mx$ (oe)
	$\Rightarrow \quad m=\frac{1}{9}M$	A1	Method may be implied
	$I=0.51Ma^2+m\left(\frac{9}{10}a\right)^2$	M1	
	$=0.6Ma^2$	E1	Convincingly shown
			5
(v)	KE $=\frac{1}{2}I\omega^2=\frac{1}{2}(0.6Ma^2)\omega^2$	M1	Attempt to find KE
	$=0.3Ma^2\omega^2$	A1	
	$C\cdot n\cdot 2\pi=0.3Ma^2\omega^2$	M1 A1	Work-energy equation Correct equation
	$\Rightarrow \quad C=\frac{0.3Ma^2\omega^2}{2n\pi}$	A1	
			5

4(i)	At terminal velocity, $\sum F = 0 \quad \Rightarrow \quad k \cdot 60^2 = 90g$	M1	Equilibrium of forces
	$\Rightarrow \quad k = \frac{1}{40}g$	E1	Convincingly shown
			2
(ii)	$90v\dfrac{dv}{dx} = 90g - \frac{1}{40}gv^2$	M1 A1	N2L
	$\displaystyle\int \frac{90v}{90g - \frac{1}{40}gv^2}dv = \int dx$	M1	Separate and integrate
	$-\dfrac{1800}{g}\ln\left\|90g - \frac{1}{40}gv^2\right\| = x + c_1$	A1	LHS
	$90 - \frac{1}{40}gv^2 = Ae^{-\frac{gx}{1800}}$		
	$v^2 = \dfrac{40}{g}\left(90g - Ae^{-\frac{gx}{1800}}\right)$	M1	Rearrange, dealing properly with constant
	$x = 0, v = 0 \quad \Rightarrow \quad A = 90g$	M1	Use condition
	$v^2 = 3600\left(1 - e^{-\frac{9x}{1800}}\right)$	E1	Complete argument
			7
(iii)	WD against $R = \displaystyle\int_0^{1800} kv^2 dx$	B1	
	$= \displaystyle\int_0^{1800} 90g\left(1 - e^{-\frac{gx}{1800}}\right)dx$		
	$= \left[90g\left(x + \dfrac{1800}{g}e^{-\frac{gx}{1800}}\right)\right]_0^{1800}$	M1	Integrate
	$= 162000\left(g + e^{-g} - 1\right)$	A1	
	$x = 1800 \quad \Rightarrow \quad v^2 = 3600\left(1 - e^{-g}\right)$	B1	
	Loss in energy $= 90g \cdot 1800 - \frac{1}{2} \cdot 90 \cdot 3600\left(1 - e^{-g}\right)$	M1 M1	GPE KE
	$= 162000(g + e^{-g} - 1) =$ WD against R	E1	Convincingly shown (including signs)
			7
(iv)	$v = 60\sqrt{1 - e^{-g}} \approx 59.9983$	B1	
			1
(v)	$90\dfrac{dv}{dt} = 90g - 90v$	M1 A1	N2L
	$\displaystyle\int \frac{dv}{g - v} = \int dt \quad \left[\text{or} \int_{59.9983}^{10} \frac{dv}{g - v} = \int_0^t dt\right]$	M1	Separate and integrate
	$-\ln\|g - v\| = t + c_2$	A1	
	$t = 0, v = 59.9983 \quad \Rightarrow \quad c_2 = -3.91598$	M1	Use condition (or limits)
	$v = 10 \quad \Rightarrow \quad t = -\ln 0.2 + 3.91598$	M1	Calculate t
	≈ 5.53 s	A1	
			7

4766 Statistics 1

Q1 (i)	Median = 2 Mode = 1	B1 cao B1 cao	**2**
(ii)		S1 labelled linear scales on both axes H1 heights	**2**
(iii)	Positive	B1	**1**
		TOTAL	**5**
Q2 (i)	$\binom{25}{5}$ different teams = 53130	M1 for $\binom{25}{5}$ A1 cao	**2**
(ii)	$\binom{14}{3}\times\binom{11}{2} = 364\times 55 = 20020$	M1 for either combination M1 for product of both A1 cao	**3**
		TOTAL	**5**
Q3 (i)	Mean $= \frac{126}{12} = 10.5$ $S_{xx} = 1582 - \frac{126^2}{12} = 259$ $s = \sqrt{\frac{259}{11}} = 4.85$	B1 for mean M1 for attempt at S_{xx} A1 cao	**3**
(ii)	New mean = 500 + 100 × 10.5 = 1550 New s = 100 × 4.85 = 485	B1 ANSWER GIVEN M1A1 ft	**3**
(iii)	On average Marlene sells more cars than Dwayne. Marlene has less variation in monthly sales than Dwayne.	E1 E1 ft	**2**
		TOTAL	**8**

Q4 (i)	E(X) = 25 because the distribution is symmetrical. Allow correct calculation of Σrp	E1 ANSWER GIVEN	**1**
(ii)	E(X^2) = $10^2 \times 0.2 + 20^2 \times 0.3 + 30^2 \times 0.3 + 40^2 \times 0.2 = 730$ Var(X) = $730 - 25^2 = 105$	M1 for $\Sigma r^2 p$ (at least 3 terms correct) M1dep for -25^2 A1 cao	**3**
		TOTAL	**4**
Q5 (i)	Distance freq width f dens 0- 360 50 7.200 50- 400 50 8.000 100- 307 100 3.070 200-400 133 200 0.665 	M1 for fds A1 cao Accept any suitable unit for fd such as eg freq per 50 miles. L1 linear scales on both axes and label W1 width of bars H1 height of bars	**5**
(ii)	Median = 600th distance Estimate = 50 + $^{240}/_{400} \times 50 = 50 + 30 = 80$	B1 for 600th M1 for attempt to interpolate A1 cao	**3**
		TOTAL	**8**
Q6 (i)	(A) P(at most one) = $\frac{83}{100} = 0.83$ (B) P(exactly two) = $\frac{10+2+1}{100} = \frac{13}{100} = 0.13$	B1 aef M1 for (10+2+1)/100 A1 aef	**1** **2**
(ii)	P(all at least one) = $\frac{53}{100} \times \frac{52}{99} \times \frac{51}{98} = \frac{140556}{970200} = 0.145$	M1 for $\frac{53}{100} \times$ M1*dep* for product of next 2 correct fractions A1 cao	**3**
		TOTAL	**6**

Q7 (i)	$a = 0.8,\ b = 0.85,\ c = 0.9.$	B1 for any one B1 for the other two	**2**
(ii)	P(Not delayed) = $0.8 \times 0.85 \times 0.9 = 0.612$ P(Delayed) = $1 - 0.8 \times 0.85 \times 0.9 = 1 - 0.612 = 0.388$	M1 for product A1 cao M1 for 1 – P(delayed) A1 ft	**4**
(iii)	P(just one problem) $= 0.2{\times}0.85{\times}0.9 + 0.8{\times}0.15{\times}0.9 + 0.8{\times}0.85{\times}0.1$ $= 0.153 + 0.108 + 0.068 = 0.329$	B1 one product correct M1 three products M1 sum of 3 products A1 cao	**4**
(iv)	P(Just one problem | delay) $= \frac{\text{P(Just one problem)}}{\text{P(Delay)}} = \frac{0.329}{0.388} = 0.848$	M1 for numerator M1 for denominator A1 ft	**3**
(v)	P(Delayed | No technical problems) *Either* $= 0.15 + 0.85 \times 0.1 = 0.235$ *Or* $= 1 - 0.9 \times 0.85 = 1 - 0.765 = 0.235$ *Or* $= 0.15 \times 0.1 + 0.15 \times 0.9 + 0.85 \times 0.1 = 0.235$ *Or (using conditional probability formula)* $\frac{\text{P(Delayed and no technical problems)}}{\text{P(No technical problems)}}$ $= \frac{0.8{\times}0.15{\times}0.1 + 0.8{\times}0.15{\times}0.9 + 0.8{\times}0.85{\times}0.1}{0.8}$ $= \frac{0.188}{0.8} = 0.235$	M1 for 0.15 + M1 for second term A1 cao M1 for product M1 for 1 – product A1 cao M1 for all 3 products M1 for sum of all 3 products A1 cao M1 for numerator M1 for denominator A1 cao	**3**
(vi)	Expected number = $110 \times 0.388 = 42.7$	M1 for product A1 ft	**2**
		TOTAL	**18**

Q8 (i)	X ~ B(15, 0.2) (*A*) $P(V = 3) = \begin{pmatrix}15\\3\end{pmatrix} \times 0.2^3 \times 0.8^{12} = 0.2501$	M1 $0.2^3 \times 0.8^{12}$ M1 $\binom{15}{3} \times p^3 q^{12}$ A1 cao	**3**
	Or from tables $0.6482 - 0.3980 = 0.2502$	Or: M2 for 0.6482 – 0.3980 A1 cao	
	(*B*) $P(X \geq 3) = 1 - 0.3980 = 0.6020$	M1 P(X≤2) M1 1-P(X≤2) A1 cao	**3**
	(*C*) E(X) = $np = 15 \times 0.2 = 3.0$	M1 for product A1 cao	**2**
(ii)	(*A*) Let p = probability of a randomly selected child eating at least 5 a day H_0: $p = 0.2$ H_1: $p > 0.2$ (*B*) H_1 has this form as the proportion who eat at least 5 a day is expected to <u>increase</u>.	B1 for definition of p in context B1 for H_0 B1 for H_1 E1	**4**
(iii)	Let $X \sim B(15, 0.2)$ $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.8358 = 0.1642 > 10\%$ $P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.9389 = 0.0611 < 10\%$ So critical region is {6,7,8,9,10,11,12,13,14,15} 7 lies in the critical region, so we reject null hypothesis and we conclude that there is evidence to suggest that the proportion who eat at least five a day has increased.	B1 for 0.1642 B1 for 0.0611 M1 for at least one comparison with 10% A1 cao for critical region *dep* on M1 and at least one B1 M1 *dep* for comparison A1 *dep* for decision and conclusion **in context**	**6**
		TOTAL	**18**

4767 Statistics 2

Question 1

(i)	**EITHER:** $S_{xy} = \Sigma xy - \frac{1}{n}\Sigma x \Sigma y = 316345 - \frac{1}{50} \times 2331.3 \times 6724.3$ $= 2817.8$ $S_{xx} = \Sigma x^2 - \frac{1}{n}(\Sigma x)^2 = 111984 - \frac{1}{50} \times 2331.3^2 = 3284.8$ $S_{yy} = \Sigma y^2 - \frac{1}{n}(\Sigma y)^2 = 921361 - \frac{1}{50} \times 6724.3^2 = 17036.8$ $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{2817.8}{\sqrt{3284.8 \times 17036.8}} = 0.377$ **OR:** $\text{cov}(x, y) = \frac{\Sigma xy}{n} - \overline{xy} = \frac{316345}{50} - 46.626 \times 134.486$ $= 56.356$ $\text{rmsd}(x) = \sqrt{\frac{S_{xx}}{n}} = \sqrt{\frac{3284.8}{50}} = \sqrt{65.696} = 8.105$ $\text{rmsd}(y) = \sqrt{\frac{S_{yy}}{n}} = \sqrt{\frac{17036.8}{50}} = \sqrt{340.736} = 18.459$ $r = \frac{\text{cov}(x,y)}{\text{rmsd}(x)\text{rmsd}(y)} = \frac{56.356}{8.105 \times 18.459} = 0.377$	M1 for method for S_{xy} M1 for method for at least one of S_{xx} or S_{yy} A1 for at least one of S_{xy}, S_{xx} or S_{yy} correct M1 for structure of r A1 (AWRT 0.38) M1 for method for cov(x,y) M1 for method for at least one msd A1 for at least on of cov(x,y), rmsd(x) or rmsd(y) correct M1 for structure of r A1 (AWRT 0.38)	5
(ii)	H_0: $\rho = 0$ H_1: $\rho \neq 0$ (two-tailed test) where ρ is the population correlation coefficient For $n = 50$, 5% critical value = 0.2787 Since 0.377 > 0.2787 we can reject H_0: There is sufficient evidence at the 5% level to suggest that there is correlation between oil price and share cost	B1 for H_0, H_1 in symbols B1 for defining ρ B1FT for critical value M1 for sensible comparison leading to a conclusion A1 for result B1 FT for conclusion in context	6
(iii)	Population The scatter diagram has a roughly elliptical shape, hence the assumption is justified.	B1 B1 elliptical shape E1 conclusion	3
(iv)	Because the alternative hypothesis should be decided without referring to the sample data and there is no suggestion that the correlation should be positive rather than negative.	E1 E1	2
		TOTAL	**16**

Question 2

(i)	Meteors are seen randomly and independently There is a uniform (mean) rate of occurrence of meteor sightings	B1 B1	**2**
(ii)	(*A*) *Either* $P(X = 1) = 0.6268 - 0.2725 = 0.3543$ *Or* $P(X = 1) = e^{-1.3}\dfrac{1.3^1}{1!} = 0.3543$ (*B*) Using tables: $P(X \geq 4) = 1 - P(X \leq 3)$ $= 1 - 0.9569$ $= 0.0431$	M1 for appropriate use of tables or calculation A1 M1 for appropriate probability calculation A1	**4**
(iii)	$\lambda = 10 \times 1.3 = 13$ $P(X = 10) = e^{-13}\dfrac{13^1}{1!} = 0.0859$	B1 for mean M1 for calculation A1 CAO	**3**
(iv)	Mean no. per hour = $60 \times 1.3 = 78$ Normal approx. to the Poisson, $X \sim N(78, 78)$ $P(X \geq 100) = P\left(Z > \dfrac{99.5 - 78}{\sqrt{78}}\right)$ $= P(Z > 2.434) = 1 - \Phi(2.434)$ $= 1 - 0.9926 = 0.0074$	B1 for Normal approx. B1 for correct parameters (SOI) B1 for continuity corr. M1 for correct Normal probability calculation using correct tail A1 CAO**, (**but FT wrong or omitted CC)	**5**
(v)	*Either* P(At least one) = $1 - e^{-\lambda}\dfrac{\lambda^0}{0!} = 1 - e^{-\lambda} \geq 0.99$ $e^{-\lambda} \leq 0.01$ $-\lambda \leq \ln 0.01$, so $\lambda \geq 4.605$ $1.3\,t \geq 4.605$, so $t \geq 3.54$ Answer $t = 4$ *Or* $t = 1, \lambda = 1.3$, P(At least one) = $1 - e^{-1.3} = 0.7275$ $t = 2, \lambda = 2.6$, P(At least one) = $1 - e^{-2.6} = 0.9257$ $t = 3, \lambda = 3.9$, P(At least one) = $1 - e^{-3.9} = 0.9798$ $t = 4, \lambda = 5.2$, P(At least one) = $1 - e^{-5.2} = 0.9944$ Answer $t = 4$	M1 formation of equation/inequality using $P(X \geq 1) = 1 - P(X = 0)$ with Poisson distribution. A1 for correct equation/inequality M1 for logs A1 for 3.54 A1 for *t* (correctly justified) M1 at least one trial with any value of *t* A1 correct probability. M1 trial with either $t = 3$ or $t = 4$ A1 correct probability of $t = 3$ and $t = 4$ A1 for *t*	**5**
		TOTAL	**19**

Question 3

(i)	$X \sim \text{N}(1720, 90^2)$ $\text{P}(X < 1700) = \text{P}\left(Z < \frac{1700-1720}{90}\right)$ $= \text{P}(Z < -0.2222)$ $= \Phi(-0.2222) = 1 - \Phi(0.2222)$ $= 1 - 0.5879$ $= 0.4121$	M1 for standardising A1 M1 use of tables (correct tail) A1CAO **NB ANSWER GIVEN**	**4**
(ii)	P(2 of 4 below 1700) $= \binom{4}{2} \times 0.4121^2 \times 0.5879^2 = 0.3522$	M1 for coefficient M1 for $0.4121^2 \times 0.5879^2$ A1 FT (min 2sf)	**3**
(iii)	Normal approx with $\mu = np = 40 \times 0.4121 = 16.48$ $\sigma^2 = npq = 40 \times 0.4121 \times 0.5879 = 9.691$ $\text{P}(X \geq 20) = \text{P}\left(Z \geq \frac{19.5-16.48}{\sqrt{9.691}}\right)$ $= \text{P}(Z \geq 0.9701) = 1 - \Phi(0.9701)$ $= 1 - 0.8340 = 0.1660$	B1 B1 B1 for correct continuity corr. M1 for correct Normal probability calculation using correct tail A1 CAO**, (**but FT wrong or omitted CC)	**5**
(iv)	H_0: $\mu = 1720$; H_1 is of this form since the consumer organisation suspects that the mean is below 1720 μ denotes the mean intensity of 25 Watt low energy bulbs made by this manufacturer.	B1 E1 B1 for definition of μ	**3**
(v)	Test statistic = $\frac{1703-1720}{\frac{90}{\sqrt{20}}} = \frac{-17}{20.12}$ $= -0.8447$ Lower 5% level 1 tailed critical value of $z = -1.645$ $-0.8447 > -1.645$ so not significant. There is not sufficient evidence to reject H_0 There is insufficient evidence to conclude that the mean intensity of bulbs made by this manufacturer is less than 1720	M1 must include √20 A1FT B1 for –1.645 No FT from here if wrong. Must be –1.645 unless it is clear that absolute values are being used. M1 for sensible comparison leading to a conclusion. FT only candidate's test statistic A1 for conclusion in words in context	**5**
		TOTAL	**20**

Question 4

<table>
<tr>
<td>(i)</td>
<td>
H_0: no association between type of car and sex;
H_1: some association between type of car and sex;
<table>
<tr><td>EXPECTED</td><td>Male</td><td>Female</td></tr>
<tr><td>Hatchback</td><td>83.16</td><td>48.84</td></tr>
<tr><td>Saloon</td><td>70.56</td><td>41.44</td></tr>
<tr><td>People carrier</td><td>51.66</td><td>30.34</td></tr>
<tr><td>4WD</td><td>17.01</td><td>9.99</td></tr>
<tr><td>Sports car</td><td>29.61</td><td>17.39</td></tr>
</table>
<table>
<tr><td>CONTRIBUTION</td><td>Male</td><td>Female</td></tr>
<tr><td>Hatchback</td><td>1.98</td><td>3.38</td></tr>
<tr><td>Saloon</td><td>0.59</td><td>1.00</td></tr>
<tr><td>People carrier</td><td>3.61</td><td>6.15</td></tr>
<tr><td>4WD</td><td>0.23</td><td>0.40</td></tr>
<tr><td>Sports car</td><td>1.96</td><td>3.33</td></tr>
</table>
$X^2 = 22.62$

Refer to Ξ_4^2
Critical value at 5% level = 9.488

22.62 > 9.488
Result is significant
There is evidence to suggest that there is some association between sex and type of car.

NB if H_0 H_1 reversed, or ‘correlation’ mentioned, do not award first B1or final A1
</td>
<td>
B1
M1 A2 for expected values (to 2 dp)
(allow A1 for at least one row or column correct)

M1 for valid attempt at $(O - E)^2/E$
A1 for all correct
NB These M1A1 marks cannot be implied by a correct final value of X^2

M1 for summation
A1 for X^2 CAO

B1 for 4 deg of f

B1 CAO for cv

M1 sensible comparison leading to a conclusion
A1
</td>
<td>12</td>
</tr>
<tr>
<td>(ii)</td>
<td>
• In hatchbacks, male drivers are more frequent than expected.
• In saloons, male drivers are slightly more frequent than expected.
• In people carriers, female drivers are much more frequent than expected.
• In 4WDs the numbers are roughly as expected
• In sports cars, female drivers are more frequent than expected.
</td>
<td>E1
E1
E1
E1
E1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TOTAL</td>
<td>17</td>
</tr>
</table>

4768 Statistics 3

Q1	W ~ N(14, 0.552) $G \sim \text{N}(144,\ 0.9^2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.	
(i)	$\text{P}(G < 145) = \text{P}\left(Z < \frac{145-144}{0.9} = 1.1111\right)$	M1 A1	For standardising. Award once, here or elsewhere.	
	= 0·8667	A1	c.a.o.	3
(ii)	$W + G \sim \text{N}(14 + 144 = 158,$	B1	Mean.	
	$\sigma^2 = 0.55^2 + 0.9^2 = 1.1125\ \ )$	B1	Variance. Accept sd (= 1.0547…).	
	P(this > 160) = $\text{P}\left(Z > \frac{160-158}{1.0547} = 1\cdot 896\right) = 1 - 0\cdot 9710 = 0.0290$	A1	c.a.o.	3
(iii)	$H = W_1 + ... + W_7 + G_1 + ... + G_6 \sim \text{N}(962,$	B1	Mean.	
	$\sigma^2 = 0.55^2 + ... + 0.55^2 + 0.9^2 + ... + 0.9^2 = 6.9775)$	B1	Variance. Accept sd (= 2.6415).	
	P(960 < this < 965) = $\text{P}\left(\frac{960-962}{2\cdot 6415} = -0.7571 < Z < \frac{965-962}{2\cdot 6415} = 1.1357\right)$	M1	Two-sided requirement.	
	= 0·8720 – (1 – 0.7755) = 0.6475	A1	c.a.o.	
	Now want P(B(4, 0.6475) ≥ 3)	M1	Evidence of attempt to use binomial. ft c's *p* value.	
	$= 4 \times 0.6475^3 \times 0.3525 + 0.6475^4$	M1	Correct terms attempted. ft c's *p* value. Accept 1 – P(… ≤ 2)	
	= 0.38277 + 0.17577 = 0.5585	A1	c.a.o.	7
(iv)	$D = H_1 - H_2 \sim \text{N}(0,$	B1	Mean. (May be implied.)	
	$6.9775 + 6.9775 = 13.955)$	B1	Variance. Accept sd (= 3.7356). Ft 2 × c's 6.9775 from (iii).	
	Want *h* s.t. P(−*h* < *D* < *h*) = 0.95	M1	Formulation of requirement as 2-sided.	
	i.e. P(*D* < *h*) = 0 975 $\therefore h = \sqrt{13.955} \times 1.96 \quad = 7.32$	B1 A1	For 1.96. c.a.o.	5
				18

Q2				
(i)	$H_0: \mu = 1$ $H_1: \mu < 1$	B1	Both hypotheses. Hypotheses in words only must include "population".	
	where μ is the mean weight of the cakes.	B1	For adequate verbal definition. Allow absence of "population" if correct notation μ is used, but do NOT allow " $\overline{X} = \ldots$ " or similar unless $\overline{X}$ is clearly and explicitly stated to be a <u>population</u> mean.	
	$\bar{x} = 0.957375 \quad s_{n-1} = 0.07314(55)$	B1	$s_n = 0.06842$ but do <u>NOT</u> allow this here or in construction of test statistic, but FT from there.	
	Test statistic is $\dfrac{0.957375 \ -1}{\dfrac{0.07314}{\sqrt{8}}}$	M1	Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: $1 + (\text{c's } -1.895) \times \dfrac{0.07314}{\sqrt{8}}$ (= 0.950997) for subsequent comparison with $\bar{x}$. (Or $\bar{x} - (\text{c's } -1.895) \times \dfrac{0.07314}{\sqrt{8}}$ (= 1.006377) for comparison with 1.)	
	= −1.648(24).	A1	c.a.o. but ft from here in any case if wrong. Use of $1 - \bar{x}$ scores M1A0, but ft.	
	Refer to t_7.	M1	No ft from here if wrong. $P(t < -1.648(24)) = 0.0716$.	
	Single-tailed 5% point is −1.895.	A1	Must be minus 1.895 unless absolute values are being compared. No ft from here if wrong.	
	Not significant.	A1	ft only c's test statistic.	
	Insufficient evidence to suggest that the cakes are underweight on average.	A1	ft only c's test statistic.	9
(ii)	CI is given by 0.957375 ± 2·365 $\times \dfrac{0.07314}{\sqrt{8}}$	M1 B1 M1		
	= 0.957375 ± 0.061156= (0.896(2), 1.018(5))	A1	c.a.o. Must be expressed as an interval. ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_7 is OK.	4
(iii)	$\bar{x} \ \pm 1.96 \times \sqrt{\dfrac{0.006}{n}}$	M1 B1 A1	Structure correct, incl. use of Normal. 1.96. All correct.	3

(iv)	$2 \times 1.96 \times \sqrt{\frac{0.006}{n}} < 0.025$ $n > \left(\frac{2 \times 1.96}{0.025}\right)^2 \times 0.006 = 147.517$ So take $n = 148$	M1 M1 A1	Set up appropriate in equation. Condone an equation. Attempt to rearrange and solve. c.a.o. (expressed as an integer). S.C. Allow max M1A1(c.a.o.) when the factor "2" is missing. ($n > 36.879$)	3
				19

Q3				
(i)	For a systematic sample • she needs a list of all staff • with no cycles in the list. All staff equally likely to be chosen if she • chooses a random start between 1 and 10 • then chooses every 10th. Not simple random sampling since not all samples are possible.	 E1 E1 E1 E1 E1		5
(ii)	Nothing is known about the background population ... ... of differences between the scores. H_0: $m = 0$ H_1: $m \neq 0$ where m is the population median difference for the scores.	E1 E1 B1 B1	Any reference to unknown distribution or "non-parametric" situation. Any reference to pairing/differences. Both hypotheses. Hypotheses in words only must include "population". For adequate verbal definition.	4

(iii)

Diff	−0.8	−2.6	8.6	6.2	6.0	−3.6	−2.4	−0.4	−4.0	5.6	6.6	2.2
Rank	2	5	12	10	9	6	4	1	7	8	11	3

		M1	For differences. ZERO in this section if differences not used.	
		M1	For ranks.	
		A1	ft from here if ranks wrong.	
	$W_- = 1 + 2 + 4 + 5 + 6 + 7 = 25$	B1	(or $W_+ = 3 + 8 + 9 + 10 + 11 + 12 = 53$)	
	Refer to tables of Wilcoxon paired (/single sample) statistic for $n = 12$.	M1	No ft from here if wrong.	
	Lower (or upper if 53 used) 2½ % tail is 13 (or 65 if 53 used).	A1	i.e. a 2-tail test. No ft from here if wrong.	
	Result is not significant.	A1	ft only c's test statistic.	
	No evidence to suggest a preference for one of the uniforms.	A1	ft only c's test statistic.	8
				17

Q4	$\mathrm{f}(x)=\frac{2x}{\lambda^2}$ for $0<x<\lambda$, $\lambda>0$			
(i)	$\mathrm{f}(x)>0$ for all x in the domain. $\int_0^{\lambda}\frac{2x}{\lambda^2}\,\mathrm{d}x=\left[\frac{x^2}{\lambda^2}\right]_0^{\lambda}=\frac{\lambda^2}{\lambda^2}=1$	E1 M1 A1	 Correct integral with limits. Shown equal to 1.	3
(ii)	$\mu=\int_0^{\lambda}\frac{2x^2}{\lambda^2}\,\mathrm{d}x=\left[\frac{2x^3/3}{\lambda^2}\right]_0^{\lambda}=\frac{2\lambda}{3}$ $\mathrm{P}(X<\mu)=\int_0^{\mu}\frac{2x}{\lambda^2}\,\mathrm{d}x=\left[\frac{x^2}{\lambda^2}\right]_0^{\mu}$ $=\frac{\mu^2}{\lambda^2}=\frac{4\lambda^2/9}{\lambda^2}=\frac{4}{9}$ which is independent of λ.	M1 A1 M1 A1	Correct integral with limits. c.a.o. Correct integral with limits. Answer plus comment. ft c's μ provided the answer does not involve λ.	4
(iii)	Given $\mathrm{E}(X^2)=\frac{\lambda^2}{2}$ $\sigma^2=\frac{\lambda^2}{2}-\frac{4\lambda^2}{9}=\frac{\lambda^2}{18}$	M1 A1	Use of Var(X) = E(X^2) – E(X)^2. c.a.o.	2
(iv)				

Probability	0.18573	0.25871	0.36983	0.18573
Expected f	9.2865	12.9355	18.4915	9.2865

	  X^2 = 3.0094 + 0.2896 + 0.1231 + 3.5152 = 6.937(3) Refer to χ_3^2. Upper 5% point is 7.815. Not significant. Suggests model fits the data for these jars. But with a 10% significance level (cv = 6.251) a different conclusion would be reached.	M1 A1 M1 A1 M1 A1 A1 A1 E1	Probs × 50 for expected frequencies. All correct. Calculation of X^2. c.a.o. Allow correct df (= cells – 1) from wrongly grouped table and ft. Otherwise, no ft if wrong. P(X^2 > 6.937) = 0.0739. No ft from here if wrong. ft only c's test statistic. ft only c's test statistic. Any valid comment which recognises that the test statistic is close to the critical values.	9
				18

4769 Statistics 4

Q1	Follow-through all intermediate results in this question, unless obvious nonsense.			
(i)	$P(X \geq 2) = 1 - \theta - \theta\,(1 - \theta) = \quad (1 - \theta)^2$ [o.e.]	M1 A1		
	$L = [\theta]^{n_0}\,[\theta(1 - \theta)]^{n_1}\,[(1\text{-}\theta)^2]^{n-n_0-n_1}$	M1 A1	Product form Fully correct	
	$= \theta^{n_0+n_1}\,(1 - \theta)^{2n-2n_0-n_1}$	A1	BEWARE PRINTED ANSWER	5
(ii)	$\ln L = (n_0 + n_1)\ln\theta + (2n - 2n_0 - n_1)\ln(1 - \theta)$	M1 A1		
	$\frac{d\ln L}{d\theta}$	M1		
	$= \frac{n_0 + n_1}{\theta} - \frac{2n - 2n_0 - n_1}{1 - \theta}$	A1		
	$= 0$	M1		
	$\Rightarrow (1 - \hat{\theta})\,(n_0 + n_1) = \hat{\theta}\,(2n - 2n_0 - n_1)$			
	$\Rightarrow \hat{\theta} = \frac{n_0 + n_1}{2n - n_0}$	A1		6
(iii)	$E(X) = \sum_{x=0}^{\infty} x\theta(1 - \theta)^x$	M1		
	$= \theta\,\{0 + (1 - \theta) + 2(1 - \theta)^2 + 3(1 - \theta)^3 + \ldots\}$		Divisible, for algebra; e.g. by "GP of GPs"	
	$= \frac{1 - \theta}{\theta}$	A2	BEWARE PRINTED ANSWER	
	So could sensibly use (method of moments)			
	$\tilde{\theta}$ given by $\frac{1 - \tilde{\theta}}{\tilde{\theta}} = \overline{X}$	M1		
	$\Rightarrow \tilde{\theta} = \frac{1}{1 + \overline{X}}$	A1	BEWARE PRINTED ANSWER	
	To use this, we need to know the exact numbers of faults for components with "two or more".	E1		+ 6
(iv)	$\bar{x} = \frac{14}{100} = 0 \cdot 14$	B1		
	$\tilde{\theta} = \frac{1}{1 + 0.14} = 0 \cdot 8772$	B1		
	Also, from expression given in question,			
	$\text{Var}(\tilde{\theta}) = \frac{0 \cdot 8772^2(1 - 0 \cdot 8772)}{100}$			
	$= 0 \cdot 000945$	B1		
	CI is given by $0 \cdot 8772 \pm 1 \cdot 96 \text{ x } \sqrt{0 \cdot 000945} =$ $(0 \cdot 817, 0 \cdot 937)$	M1 B1 M1 A1	For 0.8772 For 1.96 For $\sqrt{0 \cdot 000945}$	7

Q2				
(i)	Mgf of Z = $\mathrm{E}(\mathrm{e}^{tZ}) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{tz-\frac{z^2}{2}}\, dz$	M1		
	Complete the square	M1		
	$tz - \frac{z^2}{2} = -\frac{1}{2}(z-t)^2 + \frac{1}{2}t^2$	A1		
		A1		
	$= \mathrm{e}^{\frac{t^2}{2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{(z-t)^2}{2}}\, dt \; = \mathrm{e}^{\frac{t^2}{2}}$	M1	For taking out factor $\mathrm{e}^{\frac{t^2}{2}}$	
		M1	For use of pdf of N(t,1)	
	Pdf of N(t,1)	M1	For ∫ pdf = 1	
	$\therefore \int = 1$	A1	For final answer $\mathrm{e}^{\frac{t^2}{2}}$	8
(ii)	Y has mgf $M_Y(t)$			
	Mgf of $aY+b$ is $\mathrm{E}[\mathrm{e}^{t(aY+b)}]$	M1	For factor e^{bt}	
	$= \mathrm{e}^{bt}\mathrm{E}[\mathrm{e}^{(at)Y}] = \mathrm{e}^{bt}M_Y(at)$	1	For factor $\mathrm{E}[\mathrm{e}^{(at)Y}]$	
		1	For final answer	
		1		4
(iii)	$Z = \frac{X-\mu}{\sigma}$, so $X = \sigma Z + \mu$	M1		
		1	For factor $\mathrm{e}^{\mu t}$	
	$\therefore M_X(t) = \mathrm{e}^{\mu t}.\mathrm{e}^{\frac{(\sigma t)^2}{2}} = \mathrm{e}^{\mu t + \frac{\sigma^2 t^2}{2}}$	1	For factor $\mathrm{e}^{\frac{(\sigma t)^2}{2}}$	
		1	For final answer	4
(iv)	$W = \mathrm{e}^X$			
	$\mathrm{E}(W^k) = \mathrm{E}[(\mathrm{e}^X)^k] = \mathrm{E}(\mathrm{e}^{kX}) = M_X(k)$	M1	For $\mathrm{E}[(\mathrm{e}^X)^k]$	
		A1	For $\mathrm{E}(\mathrm{e}^{kX})$	
		A1	For $M_X(k)$	
	$\therefore \mathrm{E}(W) = M_X(1) = \mathrm{e}^{\mu + \frac{\sigma^2}{2}}$	M1 A1		
	$\mathrm{E}(W^2) = M_X(2) = \mathrm{e}^{2\mu + 2\sigma^2}$			
	$\therefore \mathrm{Var}(W) = \mathrm{e}^{2\mu+2\sigma^2} - \mathrm{e}^{2\mu+\sigma^2} [= \mathrm{e}^{2\mu+\sigma^2}(\mathrm{e}^{\sigma^2} - 1)]$	M1 A1		
		A1		8

Q3			
(i)	$\bar{x}=126\cdot 2$ $s=8{\cdot}7002$ $s^2=75\cdot 69\dot{3}$ $\bar{y}=133\cdot 9$ $s=10{\cdot}4760$ $s^2=109\cdot 74\dot{6}$	A1	A1 if all correct. [No mark for use of s_n, which are 8·2537 and 9·6989 respectively.]
	$H_0:\mu_A=\mu_B$ $H_0:\mu_A\neq\mu_B$ Where μ_A,μ_B are the population means.	1 1	Do not accept $\overline{X}=\overline{Y}$ or similar.
	Pooled s^2 $=\dfrac{9\times 75\cdot 69\dot{3}+6\times 109\cdot 47\dot{6}}{15}=\dfrac{681\cdot 24+658\cdot 48}{15}$ $=89\cdot 314\dot{6}$ [√ = 9·4506]	B1	
	Test statistic is $\dfrac{126\cdot 2-133\cdot 9}{\sqrt{89\cdot 314\dot{6}}\sqrt{\dfrac{1}{10}+\dfrac{1}{7}}}=-\dfrac{7\cdot 7}{4\cdot 6573}=-1\cdot 653$	M1 A1	
	Refer to t_{15}	1	No FT if wrong
	Double-tailed 10% point is 1·753	1	No FT if wrong
	Not significant	1	
	No evidence that population mean concentrations differ.	1	10
(ii)	There may be consistent differences between days (days of week, types of rubbish, ambient conditions, …) which should be allowed for.	E1 E1	
	Assumption: Normality of population of <u>differences.</u>	1	
	Differences are 7·4 -1·2 11·1 5·5 6·2 3·7 –0·3 1·8 3·6 [$\bar{d}$ = 4·2, s = 3·862 (s^2 =14·915)] Use of $s_n(=3\cdot 641)$ is <u>not</u> acceptable, even in a denominator of $s_n/\sqrt{n-1}$]	M1	A1 Can be awarded here if NOT awarded in part (i)
	Test statistic is $\dfrac{4\cdot 2-0}{3\cdot 862/\sqrt{9}}$ = 3·26	M1 A1	
	Refer to t_8	1	No FT if wrong
	Double-tailed 5% point is 2·306	1	No FT if wrong
	Significant	1	
	Seems population means differ	1	10

(iii)	Wilcoxon rank sum test	B1		
	Wilcoxon signed rank test	B1		
	H_0: $\text{median}_A = \text{median}_B$	1	[Or more formal	
	H_1: $\text{median}_A \neq \text{median}_B$	1	statements]	4
Q4				
(i)	Description must be in <u>context</u>. If no context given, mark according to scheme and then give half-marks, rounded down.			
	Clear description of "rows".	E1		
		E1		
	And "columns"	E1		
		E1		
	As extraneous factors to be taken account of in the	E1		
	design, with "treatments" to be compared.	E1		
	Need same numbers of each	E1		
	Clear contrast with situations for completely	E1		
	randomised design and randomised trends.	E1		9
(ii)	$e_{ij} \sim \text{ind N}(0, \sigma^2)$	1	Allow uncorrelated	
		1	For 0	
		1	For σ^2	
	α_i is population mean effect by which ith treatment	1		
	differs from overall mean	1		
				5

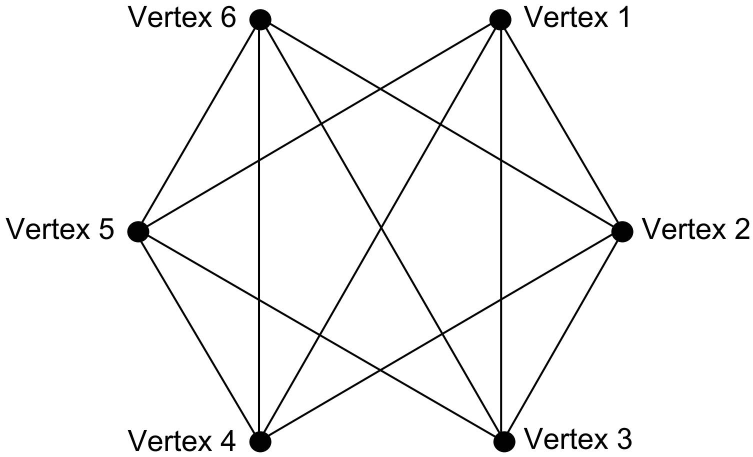
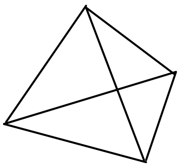
(iii)

Source of Variation	SS	df	MS	MS ratio	
					1 1
Between Treatments	92·30	4	23·075	5·034	1
Residual	68·76	15	4·584		1 1
Total	161·06	19			1 1

Refer to $F_{4,15}$	1	No FT if wrong	
Upper 1% point is 4·89	1	No FT if wrong	
Significant, seems treatments are not all the same	1		10

4771 Decision Mathematics 1

Question 1

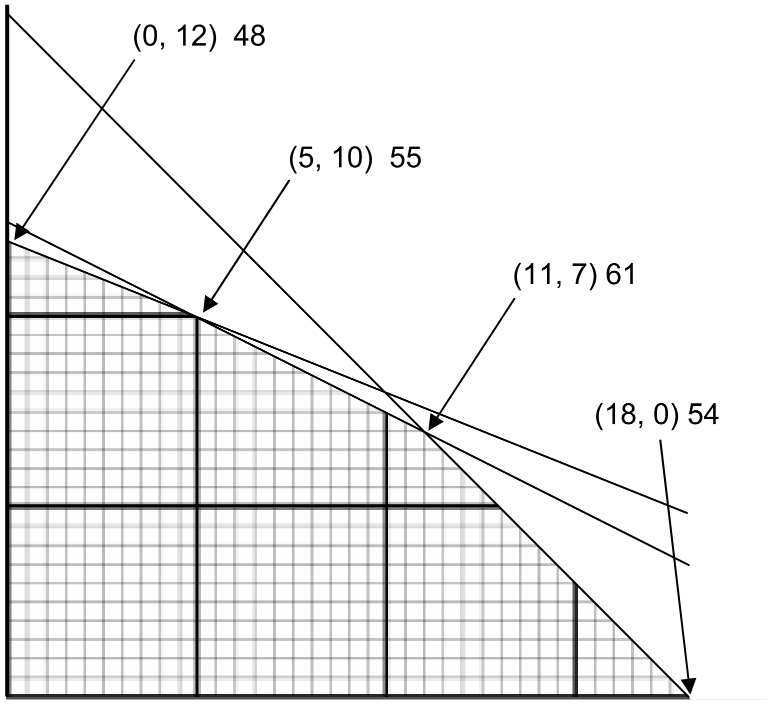
(i) 1 and 6, 2 and 5, 3 and 4	B1
(ii) 	M1 10 to 14 edges A4 (–1 each edge error)
(iii) 	B1 identification B1 sketch

Question 2.

(i) A's c takes 2, leaving 3. You have to take 1. A's c takes one and you lose.	M1 A1 A1
(ii) A's c takes 3 leaving 3. Then as above.	M1 A1
(iii) A's c takes 3 leaving 4. You can then take 1, leading to a win.	M1 A1 A1

Question 3.

<table>
<tr><td>(i)

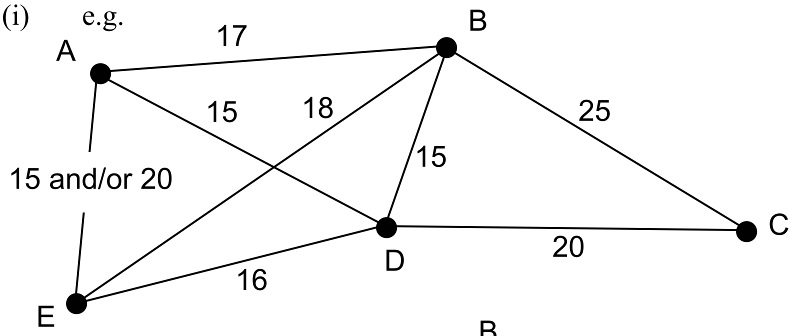
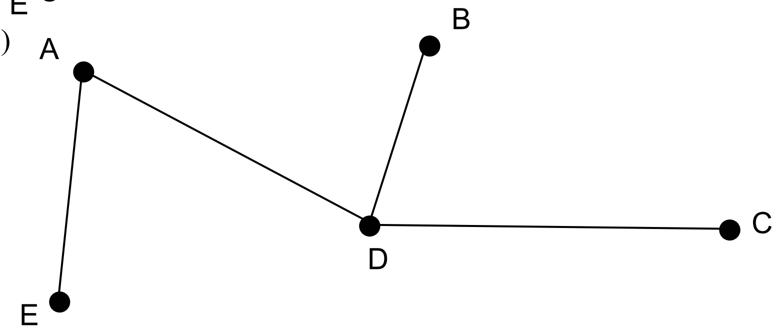
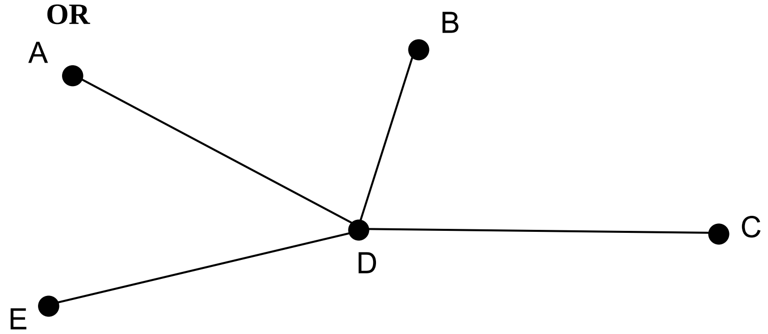
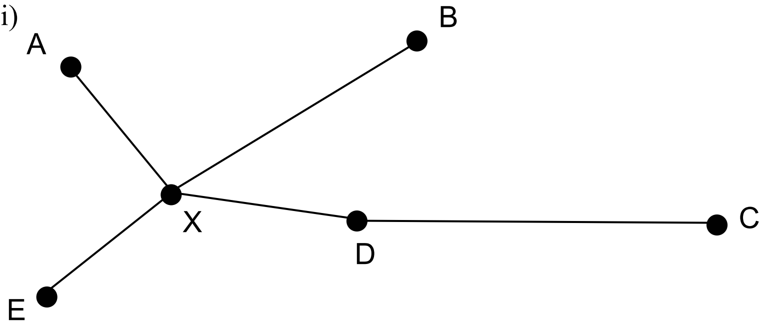


</td><td>B3 lines
B1 shading</td></tr>
<tr><td>61 at (11, 7)</td><td>M1 optimisation
A1</td></tr>
<tr><td>(ii) Intersection of $2x+5y=60$ and $x+y=18$ is at (10,8)</td><td>M1</td></tr>
<tr><td>$10 + 2\times8 = 26$</td><td>A1</td></tr>
</table>

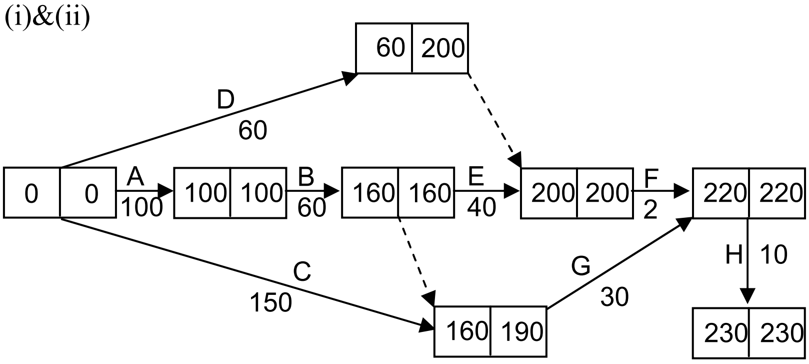
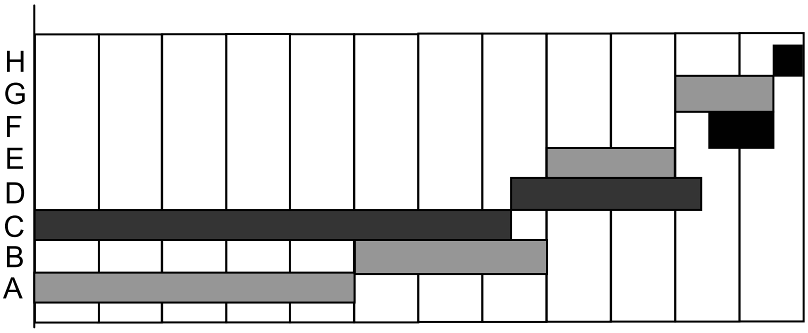
Question 4.

(i)	e.g. 0 – 4 exit 5 – 9 other vertex	B1 B1
(ii)	e.g. 1 A E×A 2 A B A B A B E×B 3 A E×A 4 A B A B A E×A 5 A B E×B 6 A B A B A B E×B 7 A B A B E×B 8 A E×A 9 A B E×B 10 A E×A	M1 process with exits A1
	0.5, 0.5, 1.9 (Theoretical answers: 2/3, 1/3, 2) (Gambler's ruin)	B1 probabilities M1 duration A1
(iii)	e.g. 0–2 exit 3–5 next vertex in cycle 6–8 other vertex 9–ignore and re-draw	M1 ignore DM1 conditionality A1 equal prob A1 efficient
(iv)	e.g. 1 A B A B A E×A 2 A C A E×A 3 A E×A 4 A B C B C E×C 5 A E×A 6 A C A B E□B 7 A E×A 8 A B C E×C 9 A E×A 10 A E×A	M1 A2
	0.7, 0.1, 0.2 (Theoretical probs are 0.5, 0.25, 0.25) (Markov chain)	M1 A1

Question 5.

(i) e.g. 	M1 A1 connectivity A1 lengths
(ii)  Order: AE; AD; DB; DC or AD; AE; DB; DC or AD; DB; AE; DC Length: 65 km **OR**  Order: AD; DB; DE; DC Length: 66 km	M1 connected tree A2 (–1 each error) A1 B1
(iii)  Length: 53 km Advice: Close BC, AE and BD	M1 connected tree A2 (–1 each error) B1 B3
(iv) facility (e.g. anglers) distances (e.g. B to C)	B1

Question 6.

(i)&(ii) 	M1 sca (activity on arc) A1 single start & end A1 dummy A1 rest M1 forward pass A1 M1 backward pass A1
time – 230 minutes	B1
critical – A; B; E; F; H	B1 cao
(iii) e.g. 	M1 cascade A2 B1 Joan/Keith
Least time = 240 mins	B1
Minimum project completion times assumes no resource constraints.	B1

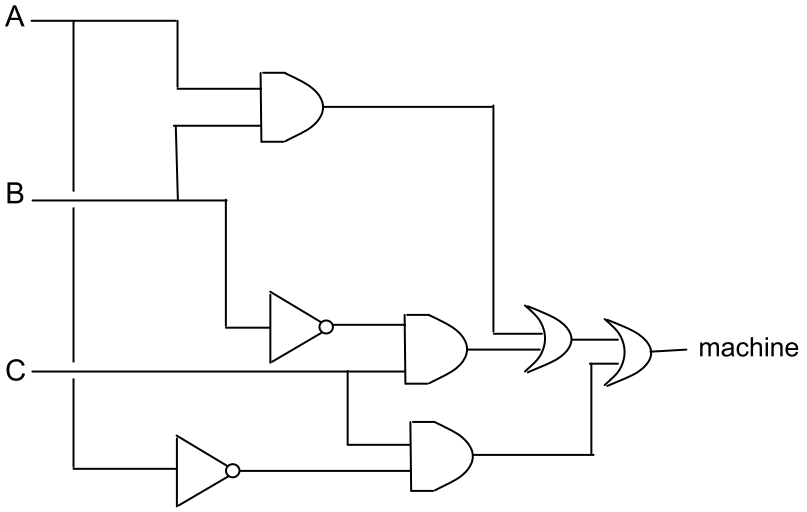
4772 Decision Mathematics 2

Question 1.

(a) e.g.
"It is easy to overestimate the effect that your contribution will make."

M1 remove double negatives
A1 same meaning

(b) e.g.



M1 combinatorial
A1 "ands"
A1 negations
A1 "ors"
A3 one for each alternative

(c) e.g.

((a	∧	b)	∨	(~a	∧	c))	∨	(~b	∧	c)
1	1	1	**1**	0	0	1	𝟙	0	0	1
1	1	1	**1**	0	0	0	𝟙	0	0	0
1	0	0	**0**	0	0	1	𝟙	1	1	1
1	0	0	**0**	0	0	0	𝟘	1	0	0
0	0	1	**1**	1	1	1	𝟙	0	0	1
0	0	1	**0**	1	0	0	𝟘	0	0	0
0	0	0	**1**	1	1	1	𝟙	1	1	1
0	0	0	**0**	1	0	0	𝟘	1	0	0

M1 8 lines
A1 a, b, c
A1 negations
A1 "and"s
A1 "or"s

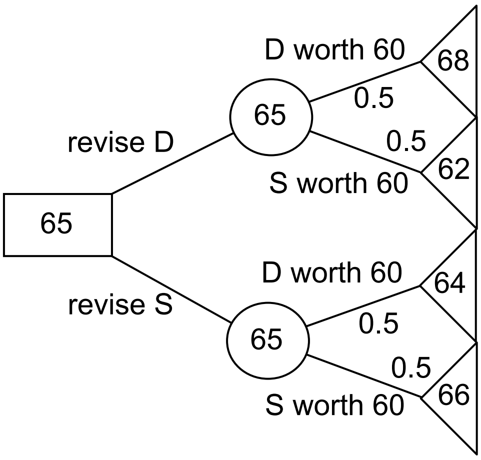
~	((~a	∧	~c)	∨	(~b	∧	~c))
𝟙	0	0	0	**0**	0	0	0
𝟙	0	0	1	**0**	0	0	1
𝟙	0	0	0	**0**	1	0	0
𝟘	0	0	1	**1**	1	1	1
𝟙	1	0	0	**0**	0	0	0
𝟘	1	1	1	**1**	0	0	1
𝟙	1	0	0	**0**	1	0	0
𝟘	1	1	1	**1**	1	1	1

M1
A1

Question 2.

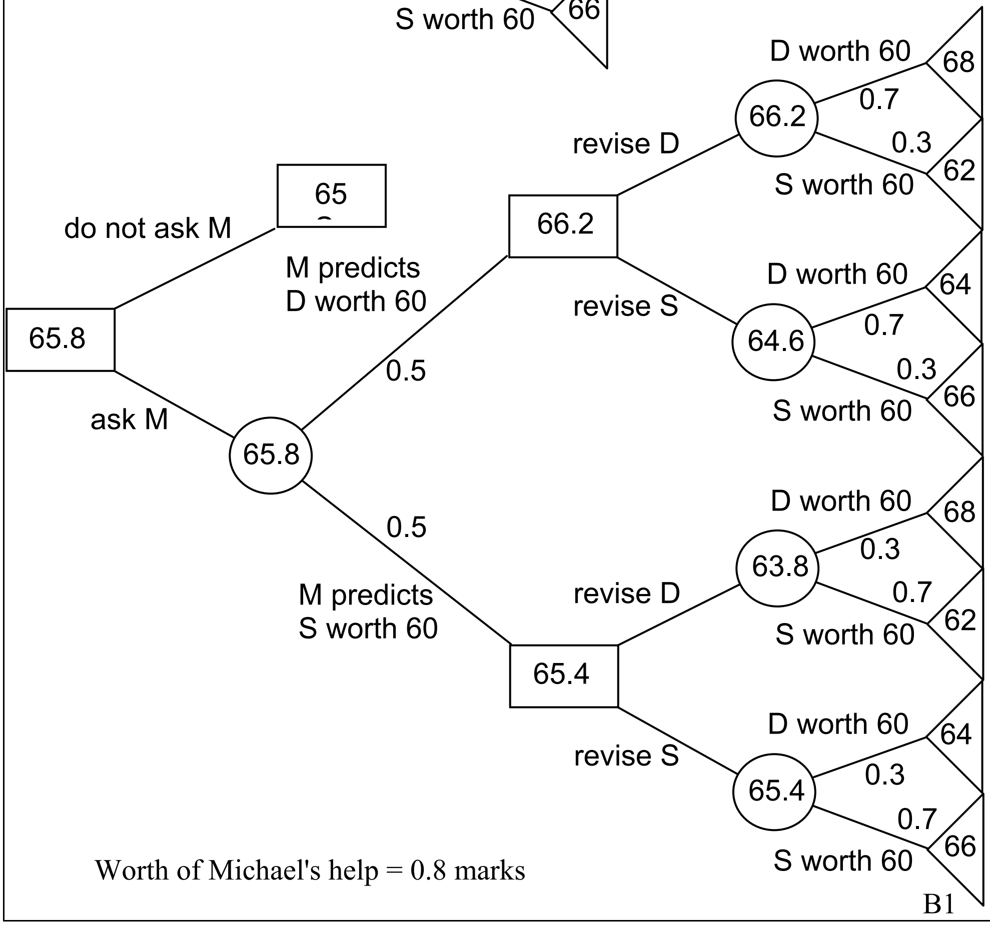
(i)	revised	60marks	score	M1
	D	D	48+20 = 68	A1
	D	S	32+30 = 62	A1
	S	D	36+28 = 64	A1
	S	S	24+42 = 66	A1

(ii)



M1 chance node
A1
A1

M1 decision node
A1



Worth of Michael's help = 0.8 marks

B1

M1 chances
A1

M1decisions
A1 revise
A1 ask/don't

Question 3.

(i) a is the number of acres of land put to crop A, etc — B1
a + b ≤ 20 is equivalent to a + b ≤ c + d — B1
Given that a + b + c + d ≤ 40, the maximisation will ensure that a + b + c + d = 40 (and it's easier to solve using simplex). — B1

(ii)

P	a	b	c	d	s1	s2	RHS
1	–50	–40	–40	–30	0	0	0
0	1	1	0	0	1	0	20
0	1	1	1	1	0	1	40
1	0	10	–40	–30	50	0	1000
0	1	1	0	0	1	0	20
0	0	0	1	1	–1	1	20
1	0	10	0	10	10	40	1800
0	1	1	0	0	1	0	20
0	0	0	1	1	–1	1	20

M1
A1
A1
A1

M1 A1

M1 A1

20 acres to A and 20 acres to C, giving profit of £1800 — B1 B1

(iii) Max 50a + 40b + 40c + 30d
st a + b ≤ 20
a + b + c + d ≤ 40
a + b + c + d ≥ 40 — B1

A	P	a	b	c	d	s1	s2	sur	art	R
1	0	1	1	1	1	0	0	–1	0	40
0	1	–50	–40	–40	–30	0	0	0	0	0
0	0	1	1	0	0	1	0	0	0	20
0	0	1	1	1	1	0	1	0	0	40
0	0	1	1	1	1	0	0	–1	1	40

B1 new obj
B1 surplus
B1 artificial

B1 3 constraints

Minimise A (to zero) then drop A row and art column and continue normally — B1 B1

OR — **OR**

P	a	b	c	d	s1	s2	sur	art	R
1	–50 –M	–40 –M	–40 –M	–30 –M	0	0	M	0	–40M
0	1	1	0	0	1	0	0	0	20
0	1	1	1	1	0	1	0	0	40
0	1	1	1	1	0	0	–1	1	40

M1
A1

B1 surplus
B1 artificial

Proceed as per simplex, regarding M as a large fixed number. — B1 B1

Question 4.

(a) (i),(ii) and (iii)

	1	2	3	4	5			1	2	3	4	5
1	∞	22	∞	15	15		**1**	1	2	3	4	5
2	22	∞	20	5	23		**2**	1	2	3	4	5
3	∞	20	∞	40	∞		**3**	1	2	3	4	5
4	15	5	40	∞	16		**4**	1	2	3	4	5
5	15	23	∞	16	∞		**5**	1	2	3	4	5

M1 distance
A1 1 to 5 etc
A1 rest

B1 route

	1	2	3	4	5			1	2	3	4	5
1	∞	22	∞	15	15		**1**	1	2	3	4	5
2	22	44	20	5	23		**2**	1	1	3	4	5
3	∞	20	∞	40	∞		**3**	1	2	3	4	5
4	15	5	40	30	16		**4**	1	2	3	1	5
5	15	23	∞	16	30		**5**	1	2	3	4	1

Not part of the question

	1	2	3	4	5			1	2	3	4	5
1	44	22	42	15	15		**1**	2	2	2	4	5
2	22	44	20	5	23		**2**	1	1	3	4	5
3	42	20	40	25	43		**3**	2	2	2	2	2
4	15	5	25	10	16		**4**	1	2	2	2	5
5	15	23	43	16	30		**5**	1	2	2	4	1

Not part of the question

	1	2	3	4	5			1	2	3	4	5
1	44	22	42	15	15		**1**	2	2	2	4	5
2	22	44	20	5	23		**2**	1	1	3	4	5
3	42	20	40	25	43		**3**	2	2	2	2	2
4	15	5	25	10	16		**4**	1	2	2	2	5
5	15	23	43	16	30		**5**	1	2	2	4	1

Not part of the question

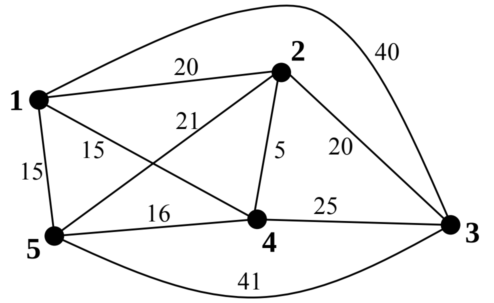
	1	2	3	4	5			1	2	3	4	5
1	30	20	40	15	15		**1**	4	4	4	4	5
2	20	10	20	5	21		**2**	4	4	3	4	4
3	40	20	40	25	41		**3**	2	2	2	2	2
4	15	5	25	10	16		**4**	1	2	2	2	5
5	15	21	41	16	30		**5**	1	4	4	4	1

M1
A1 10 changed dists

M1 2's in r3 of route
A1 rest of route

	1	2	3	4	5			1	2	3	4	5
1	30	20	40	15	15		**1**	4	4	4	4	5
2	20	10	20	5	21		**2**	4	4	3	4	4
3	40	20	40	25	41		**3**	2	2	2	2	2
4	15	5	25	10	16		**4**	1	2	2	2	5
5	15	21	41	16	30		**5**	1	4	4	4	1

Shortest distance from **3** to **1** is 40 — B1
(1st row and 3rd column of distance matrix) — B1

	Shortest route is **3 2 4 1** **3** followed by route matrix (3,1) = **2** followed by route matrix (2,1) = **4** followed by route matrix (4,1) = **1**	B1 M1 A1
(iv)		M1 A1
(v)	**2** (5) **4** (15) **1** (15) **5** (41) **3** (20) **2** Total length = 96	B1 B1
	2 4 1 5 (4 2) 3 2	M1 A1
	Finds a (hopefully short) route visiting every vertex and returning to the start, **or**, upper bound to the TSP	B1

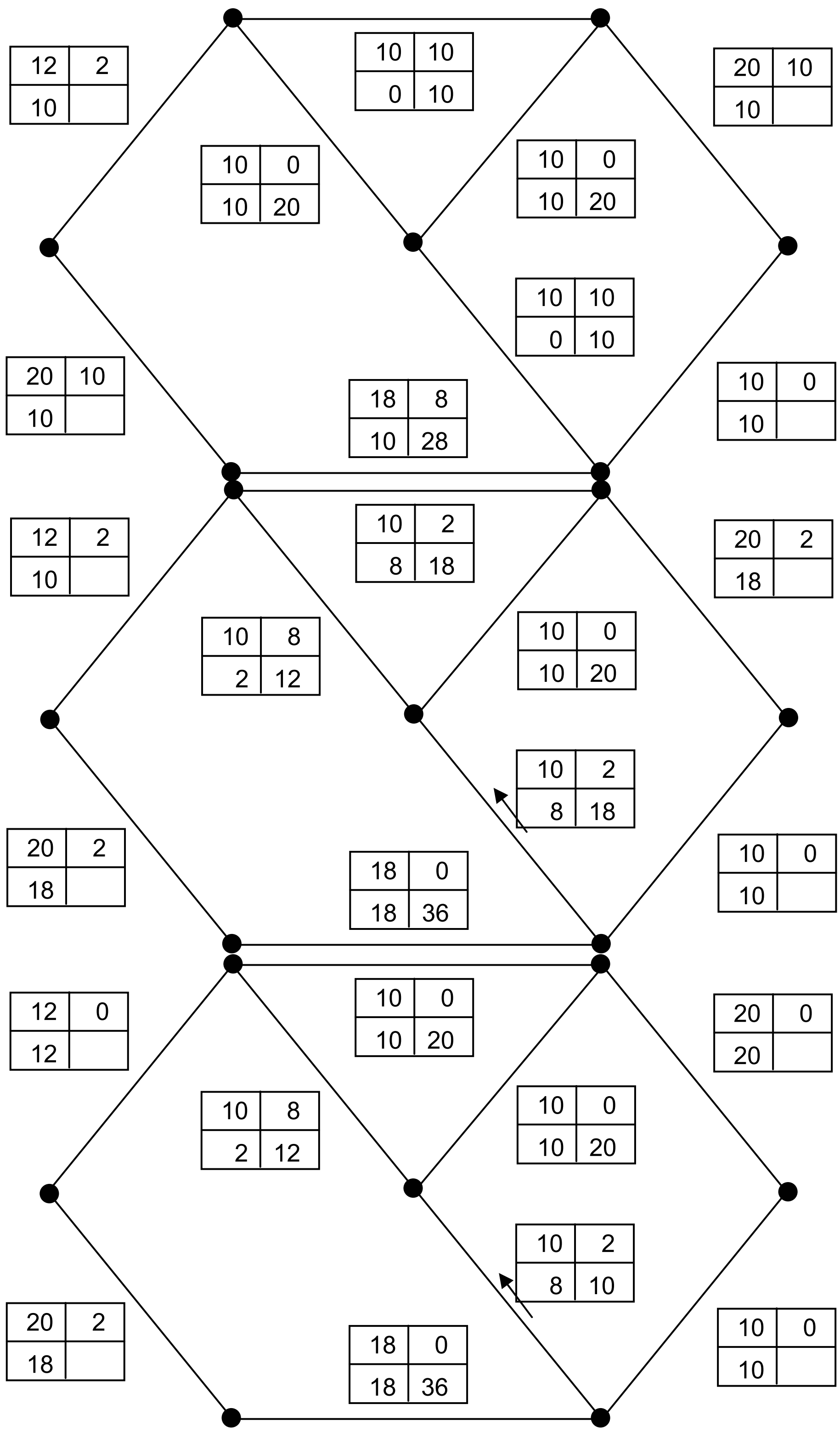
4773 Decision Mathematics Computation

Question 1.

(i) $B_{n+2} = B_{n+1} + (0 - B_n)$	M1 A1
(ii) Oscillation: 2, 4, 2, –2, –4, –2, 2, 4, …	M1 A1 B1
(iii) $B_{n+2} - B_{n+1} + \frac{1}{2} B_n = 0$ 2, 4, 3, 1, –0.5, –1, …, 0.00391, –0.00195 Oscillatory convergence	B1 B1 B1
(iv) 2, 4, 3.5, 2.5, 1.625, 1, …, 0.00022, 0.00012 Faster and uniform convergence	B1 B1 B1
(v) Auxiliary eqn: $x^2 - x + \frac{1}{4} = 0$ $x = \frac{1}{2}$ $B_n = A\left(\frac{1}{2}\right)^n + Bn\left(\frac{1}{2}\right)^n$ $2 = A$ $4 = 1 + \frac{1}{2}B$ giving $B = 6$ $B_n = (2+6n)\left(\frac{1}{2}\right)^n$ or $(1+3n)\left(\frac{1}{2}\right)^{n-1}$ $(2+6n)\left(\frac{1}{2}\right)^n$ or $(1+3n)\left(\frac{1}{2}\right)^n$ "the same"	B1 B1 B1 B1 B1 B1 B1

Question 2.

(i) e.g.



M1
A1

M1 reversal
A1

B1 rest

M1
A1

(ii) {S, A, C, D, E} / {B, T}	M1 A1
(iii) e.g. Max SA + SE	M1 variables A1 objective
st SA + BA + CA – AB – AC = 0 AB + CB – BA – BC – BT = 0 AC + DC + BC – CA – CB – CD = 0 SE + DE – ED = 0 CD + ED – DC – DE – DT = 0 SA < 12 SE < 20 AB < 10	 M1 balancing A1
BA < 10 AC < 10 CA < 10 BC < 10 CB < 10 CD < 10 DC < 10 ED < 18 DE < 18 BT < 20 DT < 10 end	M1 capacities A1 forwards A1 backwards
(iv)	B1 running

```
OBJECTIVE FUNCTION VALUE
1)      30.00000
VARIABLE        VALUE           REDUCED COST
SA              12.000000       0.000000
SE              18.000000       0.000000
BA              0.000000        0.000000
CA              0.000000        0.000000
AB              10.000000       0.000000
AC              2.000000        0.000000
CB              10.000000       0.000000
BC              0.000000        0.000000
BT              20.000000       0.000000
DC              8.000000        0.000000
CD              0.000000        0.000000
DE              0.000000        1.000000
ED              18.000000       0.000000
DT              10.000000       0.000000
```

Solution as per part (i)	B1

Question 3.

<table>
<tr><td>

(i) e.g.

<table>
<tr><td></td><td>5</td></tr>
<tr><td>-1</td><td>4</td></tr>
<tr><td>1</td><td>5</td></tr>
<tr><td>1</td><td>6</td></tr>
<tr><td>1</td><td>7</td></tr>
<tr><td>1</td><td>8</td></tr>
<tr><td>1</td><td>9</td></tr>
<tr><td>-1</td><td>8</td></tr>
</table>

= if(rand()<0.55,1,–1)

= B1 + A2

etc.

</td><td>

M1
A1 "if" or equivalent
A1 accumulation

</td></tr>
<tr><td>

(ii) repeating until a player is ruined
repeating 10 times
estimating the probability (theoretical value is 0.2683)

</td><td>

M1
A1
M1 A1

</td></tr>
<tr><td>

(iii) e.g.

<table>
<tr><td></td><td>5</td></tr>
<tr><td>-1</td><td>4</td></tr>
<tr><td>1</td><td>5</td></tr>
<tr><td>-1</td><td>4</td></tr>
<tr><td>-1</td><td>3</td></tr>
<tr><td>-1</td><td>2</td></tr>
<tr><td>-1</td><td>1</td></tr>
<tr><td>-1</td><td>0</td></tr>
</table>

= if(rand()<0.45,1,–1)

= B1 + A2

etc.

estimating the run length
The theoretical value is 50, so there should be some long runs seen.

</td><td>

B1 change of parameter
M1 count to ruin
A1 repetitions

M1
A1

</td></tr>
<tr><td>

(iv) e.g.

<table>
<tr><td></td><td>0</td></tr>
<tr><td>-1</td><td>-1</td></tr>
<tr><td>1</td><td>0</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>-1</td><td>0</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>1</td><td>2</td></tr>
</table>

= if(rand()<0.55,1,–1)

= B1 + A2

etc

repetitions + probability estimate
(theoretical answer = 0.599)

</td><td>

M1
A1 termination condition

B1 B1

B1

</td></tr>
<tr><td>

(v) As above
How can one tell when a simulation is not emptying the pot?

</td><td>

B1

</td></tr>
</table>

Question 4.

(i) max 4s1m0+7s1m1+8s1m2+9s1m3+11s1m4+3s2m0	M1
+6s2m1+10s2m2+12s2m3+14s2m4+3s3m0	A1
+7s3m1+8s3m2+13s3m3+15s3m4	
st s1m0+s1m1+s1m2+s1m3+s1m4=1	B1
s2m0+s2m1+s2m2+s2m3+s2m4=1	B1
s3m0+s3m1+s3m2+s3m3+s3m4=1	B1
s1m1+2s1m2+3s1m3+4s1m4+s2m1+2s2m2+3s2m3	M1
+4s2m4 +s3m1+2s3m2+3s3m3+4s3m4=4	A1
end	
int 15	B1
(ii) LP OPTIMUM FOUND AT STEP 14	
OBJECTIVE VALUE = 24.0000000	
NEW INTEGER SOLUTION OF 24.0000000 AT BRANCH 0	
PIVOT 14	
RE-INSTALLING BEST SOLUTION...	
OBJECTIVE FUNCTION VALUE	
1) 24.00000	
VARIABLE VALUE REDUCED COST	
S1M0 0.000000 -4.000000	
S1M1 1.000000 -7.000000	M1
S1M2 0.000000 -8.000000	A1
S1M3 0.000000 -9.000000	
S1M4 0.000000 -11.000000	
S2M0 0.000000 -3.000000	
S2M1 0.000000 -6.000000	
S2M2 1.000000 -10.000000	
S2M3 0.000000 -12.000000	
S2M4 0.000000 -14.000000	
S3M0 0.000000 -3.000000	
S3M1 1.000000 -7.000000	
S3M2 0.000000 -8.000000	
S3M3 0.000000 -13.000000	
S3M4 0.000000 -15.000000	
ROW SLACK OR SURPLUS DUAL PRICES	B3
2) 0.000000 0.000000	B1
3) 0.000000 0.000000	
4) 0.000000 0.000000	M1
5) 0.000000 0.000000	A1
Invest £1 million at site 1, £2 million at site 2 and £1 million at site 3.	M1
Revenue = £24 million.	A1
(iii) (a) £2 million at site 2 and £1 million at site 3.	
Revenue = £21 million.	
(b) £1 million at site 1, £3 million at site 2 and £4 million at site 3	
(or £1m, £4m and £3m).	
Revenue = £34 million.	

4776 Numerical Methods

1(i) f(x) = 1.6(x − 0.4)(x − 1)/(-0.4)(-1) + 2.4x(x − 1)/0.4(0.4 − 1) + 1.8x(x − 0.4)/1(1 − 0.4) **[M1A1,1,1]**
= 4(x^2 − 1.4x + 0.4) − 10(x^2 − x) + 3(x^2 − 0.4x) **[A1]**
= -3x^2 + 3.2x + 1.6 **[A1]**

(ii) Newton's formula requires equally spaced data **[E1]**

[TOTAL 7]

2

x	1	2		
$x^2 + 1/x - 3$	-1	1.5	(change of sign so root)	**[M1A1]**

f(x) = x^2 + 1/x − 3 so f '(x) = 2x − 1/x^2 hence NR formula **[M1A1]**

r	0	1	2	3		
x_r	1.5	1.532609	1.532089	1.532089	**1.53209**	**[M1A1A1]**

[TOTAL 7]

3(i)

term	X	X+Y	X − Y	10X + 20Y	
mpe	0.0005	0.001	0.001	0.015	**[B1B1B1B1]**

(ii)

term	X	Y	XY	X/Y	
mpre	0.000184	0.000159	0.000343	0.000343	**[B1B1B1B1]**

[TOTAL 8]

4(i) to 6 dp:

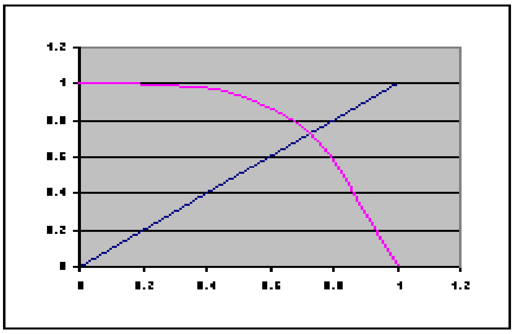
sin A	sin B	LHS	RHS	
0.846832	0.841471	0.5361	0.536088	**[B1B1]**

(ii) It is an approximate equality. LHS involves subtraction of nearly equal numbers.
LHS involves 2 trig functions, RHS just 1. **[E1E1]**

(iii) Subtraction of nearly equal quantities is a bigger problem as the difference decreases.
RHS involves no such problem. **[E1E1]**

[TOTAL 6]

5



r	x_r	
0	0.6	**[G2]**
1	0.8704	
2	0.426048	**[M1A1A1]**
3	0.967052	

cobweb diagram showing spiralling out from root **[M1A1A1]**

[TOTAL 8]

6(i)

x	f(x)				
0	1.732051				
0.8	1.777639	T1 =	1.403876	M	**[M1]**

0.4	1.8	M1 =	1.44	T2 =	1.421938	*T*	**[M1]**
0.2	1.777639					*values*	**[A1,1,1,1]**
0.6	1.8	M2 =	1.431056				

[subtotal 6]

(ii) S1 = 1.427959 (a.g.) **[M1]**

S2 = 1.428016 **[M1A1]**

[subtotal 3]

(iii) S4 = (2 M4 + T4) / 3 = 1.428020 **[M1A1]**

[subtotal 2]

(iv)

M	1.44	1.431056	1.428782		
diffs		-0.00894	-0.00227		
ratio			0.254186	approx 0.25	**[M1A1]**

S	1.427959	1.428016	1.428020		
diffs		5.77E-05	3.99E-06		
ratio			0.069037	(approx 0.0625)	**[M1A1A1]**

Reasoning to: integral is secure as 1.42802(0) **[M1B1]**

[subtotal 7]

[TOTAL 18]

7(i)

x	f(x)	1st diff	2nd diff	
1	0.6			
1.2	-0.1	-0.7		
1.4	0.4	0.5	1.2	**[M1A1]**

$f(x) = 0.6 + (-0.7)(x - 1) / 0.2 + 1.2(x - 1)(x - 1.2) /(2\ (0.2)^2)$ **[M1A1A1A1**

$= 0.6 - 3.5x + 3.5 +15x^2 - 33x + 18$

$= 15x^2 - 36.5x + 22.1$ **[M1A1]**

[subtotal 8]

(ii) $f'(x) = 30x - 36.5$ $f'(1.2) = 36 - 36.5 = -0.5$ **[M1A1]**

Central difference: $(0.4 - 0.6)/(1.4 - 1) = -0.2/0.4 = -0.5$ **[M1A1]**

Suggests central difference is accurate for quadratics. **[E1]**

[subtotal 5]

(iii) $f'(1) = 30 - 36.5 = -6.5$ **[B1]**

Forward difference: $(-0.1 - 0.6)/(1.2 - 1) = -0.7/0.2 = -3.5$ **[M1A1]**

Shows that forward difference is not exact for quadratics. **[E1]**

Quadratic estimate (-6.5) is likely to be more accurate. (Allow comments saying that we cannot be sure.) **[E1]**

[subtotal 5]

[TOTAL 18]

4777 MEI Numerical Computation

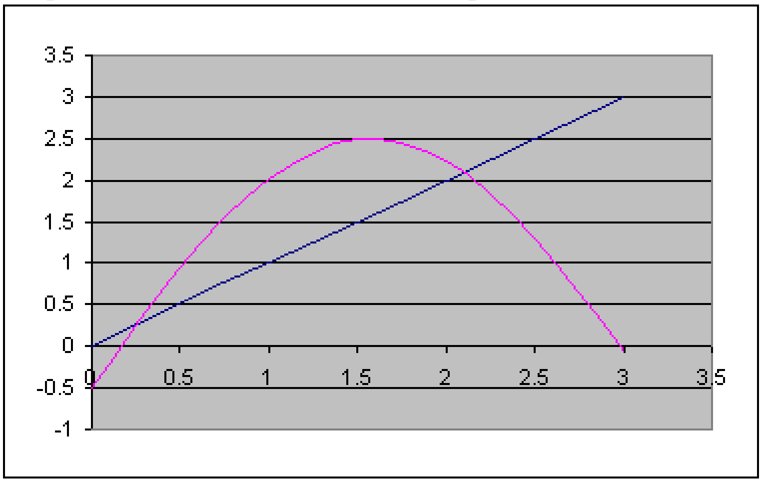
1(i) $-1 < g'(\alpha) < 1$ **[B1]**

E.g. Multiply both sides of $x = g(x)$ by λ and add $(1 - \lambda)x$ to both sides. **[M1A1]**
Derivative of rhs set to zero at root: $\lambda g'(\alpha) + 1 - \lambda = 0$ **[M1A1]**
algebra to obtain given result **[A1]**

In practice use an initial estimate x_0 in place of α **[A1]**

[subtotal 7]

(iii)



**[G3
]**

Roots approximately 0.25, 2.1 **[B1B1]**

Eg:

r	x_r	x_r	x_r	x_r	x_r	x_r
0	0	0.2	0.4	2	2.2	2.4
1	-0.5	0.096008	0.668255	2.227892	1.925489	1.52639
2	-1.93828	-0.21242	1.358852	1.875308	2.31326	2.497043
3	-3.29971	-1.13247	2.432871	2.36198	1.710416	1.302517
4	-0.02763	-3.21639	1.452591	1.609012	2.470807	2.392685
5	-0.58289	-0.2758	2.479066	2.49781	1.364805	1.542517
6	-2.15131	-1.31696	1.345334	1.300676	2.436576	2.498801
7	-3.00855	-3.40387	2.424072	2.391217	1.444139	1.298298
8	-0.89795	0.277847	1.472555	1.545741	2.475969	2.389305
9	-2.84615	0.322857	2.485535	2.499058	1.352649	1.549934
10	-1.37349	0.451832	1.329994	1.297679	2.4289	2.499347

No convergence in each case **[M1A1A1]**

Let $g(x) = 3 \sin x - 0.5$
Then $g'(x) = 3 \cos x$
So $\lambda = 1 / (1 - 3 \cos a)$ **[M1A1]**

Smaller root: $\lambda =$ -0.52446 (approx -0.5) Larger root: $\lambda =$ 0.397687 (approx 0.4) **[M1A1A1]**

r	x_r
0	0.25
1	0.253894
2	0.254078

NB: must
be using
relaxatio

r	x_r
0	2.1
1	2.095851
2	2.095866

n

3	0.254087	3	2.095866	
4	0.254088	4	2.095866	**[M1A1]**
5	0.254088	5	2.095866	**[M1A1]**

[subtotal 17]
[TOTAL 24]

2(i)	f(x) = 1	$2h = 2a + b$	**[M1A1]**
	f(x) = x, x^3 give 0 = 0		**[M1A1]**
	f(x) = x^2	$2h^3/3 = 2a\alpha^2$	**[A1]**
	f(x) = x^4	$2h^5/5 = 2a\alpha^4$	**[A1]**
	Convincing algebra to verify given results		**[A1A1]**
			[subtotal 8]

(ii)

	L	R	m	h	×1	×2		
	0	0.785398	0.392699	0.392699	0.088516	0.696882		
function values			1.189207		1.043431	1.35535		**setup:**
weights			0.349066		0.218166	0.218166		**[M3A3]**
integral			0.415112		0.227641	0.295691	**0.938444**	**[A1]**

	L	R	m	h	×1	×2		
	0	0.392699	0.19635	0.19635	0.044258	0.348441		
function values			1.094949		1.021903	1.167589		
weights			0.174533		0.109083	0.109083		
integral			0.191105		0.111472	0.127364	0.429941	
	0.392699	0.785398	0.589049	0.19635	0.436957	0.74114		
function values			1.29158		1.211226	1.383901		**repeat:**
								[M2
weights			0.174533		0.109083	0.109083		**]**
integral			0.225423		0.132124	0.15096	0.508508	
							0.938449	**[A1]**

Either repeat with h halved to verify that 0.938449 is correct to 6 dp **[M1A1]**
Or observe that the method is converging so rapidly that 0.938449 will be correct to 6dp **or [E1A1]**
[subtotal 12]

(iii) Use routine known to deliver 6dp and vary k:

	L	R	m	h	×1	×2		
						k =	1.46572	
	0	0.392699	0.19635	0.19635	0.044258	0.348441		
function values			1.136464		1.031946	1.237918		
weights			0.174533		0.109083	0.109083		
integral			0.19835		0.112568	0.135036		0.445954
	0.392699	0.785398	0.589049	0.19635	0.436957	0.74114		**modify**
function values			1.406898		1.297918	1.530164		**[M1A1]**
weights			0.174533		0.109083	0.109083		
integral			0.24555		0.141581	0.166915	0.554046	
							1.000000	

k	1.465	1.466	1.467	**find k**
integral	0.999908	1.000036	1.000163	**[M1A1]**

hence k = 1.466

[subtotal 4]
[TOTAL 24]

3(i) Use central difference formulae for 2nd and 1st derivatives to obtain first given result **[M1A1A1]**

Hence obtain $y_1 = h^2 - y_{-1}$ **[M1A1]**

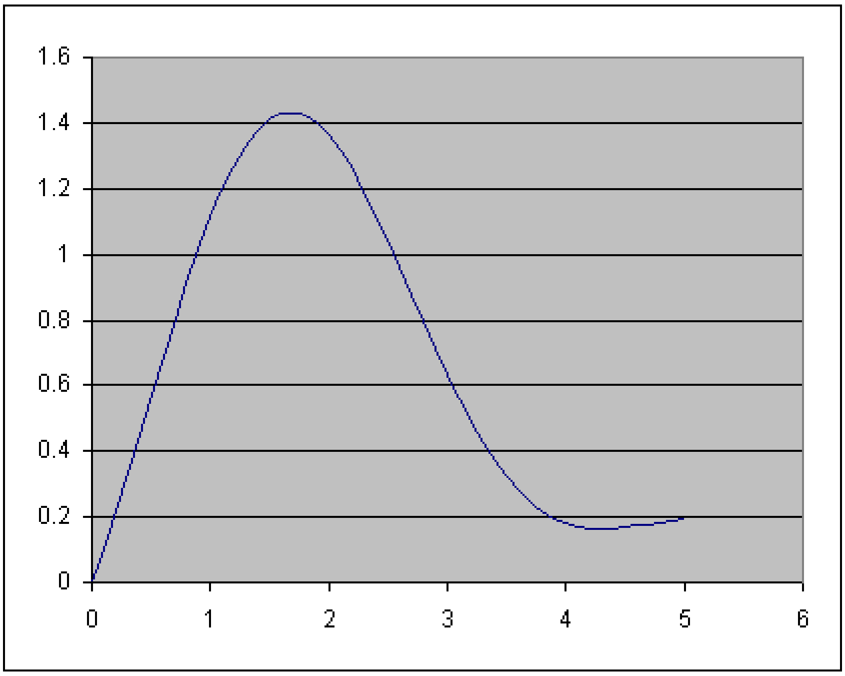
Use central difference to obtain $y_1 - y_{-1} = 2h$ **[M1A1]**

Hence given result for y_1 **[M1]**

[subtotal 8]

(ii)

h	*x*	*y*
0.1	0	0
	0.1	0.105
	0.2	0.216472
	0.3	0.332426
	0.4	0.450961
	0.5	0.570174
	0.6	0.68815
	0.7	0.802981
	0.8	0.912793
	0.9	1.015786
	1	1.11027
	1.1	1.194705
	1.2	1.26774
	1.3	1.328248
	1.4	1.375354
	1.5	1.40846
	1.6	1.42726
	1.7	1.431751
	1.8	1.42223
	1.9	1.399287
	2	1.363785
	2.1	1.316838
	2.2	1.259773
	2.3	1.194096
	2.4	1.121445
	2.5	1.04354
	2.6	0.962141
	2.7	0.878993
	2.8	0.79578
	2.9	0.714082
	3	0.635337
	3.1	0.560807
	3.2	0.491549
	3.3	0.428404
	3.4	0.371982
	3.5	0.322662
	3.6	0.280597
	3.7	0.245729
	3.8	0.217808
	3.9	0.196416
	4	0.180999
	4.1	0.170894
	4.2	0.165365
	4.3	0.163635
	4.4	0.164915
	4.5	0.168435
	4.6	0.173469



4.7	0.179352
4.8	0.185502
4.9	0.191424
5	0.196725

setup	numbers	graph
[M3]	**[A3]**	**[A3]**

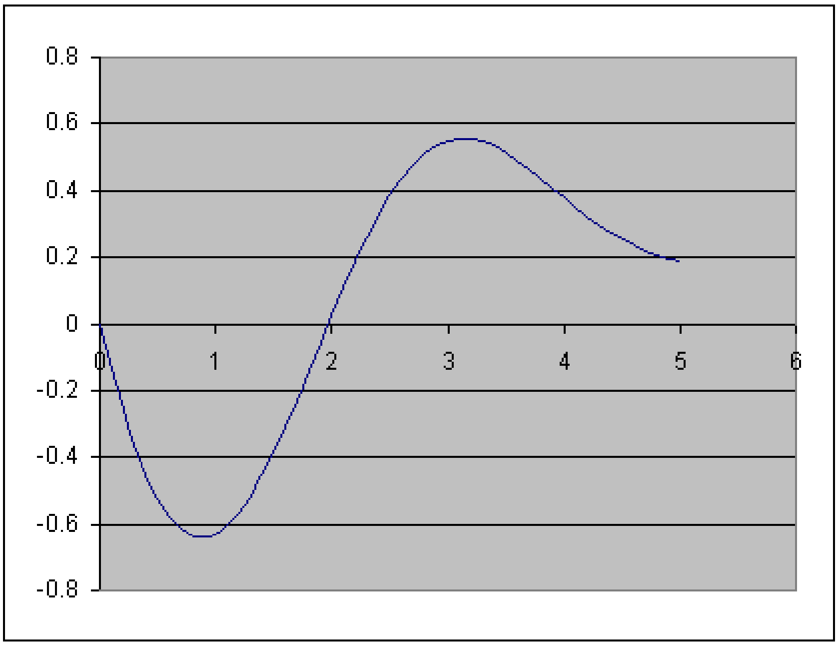
[subtotal 9]

(ii) Obtain formula $y_1 = ah + 0.5h^2$ **[M1A1]**

Modify routine **[M1A1]**

Trial on a to obtain a = -1.4 or -1.5 **[M1A1G1]**

h	x	y
0.1	0	0
a	0.1	-0.135
-1.4	0.2	-0.25582
	0.3	-0.36107
	0.4	-0.44993
	0.5	-0.5219
	0.6	-0.57677
	0.7	-0.6146
	0.8	-0.63565
	0.9	-0.64047
	1	-0.6298
	1.1	-0.60462
	1.2	-0.56614
	1.3	-0.51572
	1.4	-0.45494
	1.5	-0.3855
	1.6	-0.3092
	1.7	-0.22792
	1.8	-0.14356
	1.9	-0.05802
	2	0.026884
	2.1	0.109408
	2.2	0.187962
	2.3	0.26113
	2.4	0.327696
	2.5	0.386672
	2.6	0.437316
	2.7	0.479135
	2.8	0.51189
	2.9	0.535589
	3	0.550471
	3.1	0.556986
	3.2	0.555768
	3.3	0.547604
	3.4	0.533401
	3.5	0.514147
	3.6	0.490876
	3.7	0.464631
	3.8	0.43643
	3.9	0.40724
	4	0.377942
	4.1	0.349319
	4.2	0.322033



4.3	0.296623
4.4	0.27349
4.5	0.252909
4.6	0.235026
4.7	0.219875
4.8	0.207386
4.9	0.197404
5	0.189706

[subtotal 7]
[TOTAL24]

4(i) Diagonal dominance: the magnitude of the diagonal element in any row is greater than or equal to the sum of the magnitudes of the other element. **[E1]**
| *a* | > | *b* | + 2 will ensure convergence. (> required as dominance has to be strict) **[E1E1]**
[subtotal 3]

(ii)

					a	b
4	1	2	1	1	4	2
1	4	1	2	0		
2	1	4	1	0		
1	2	1	4	0		

0	0	0	0	
0.25	-0.0625	-0.10938	-0.00391	**setup**
0.321289	-0.05103	-0.14691	-0.01808	**[M3A3]**
0.340733	-0.03941	-0.15599	-0.02648	
0.344469	-0.03388	-0.15715	-0.02989	
0.344515	-0.0319	-0.15681	-0.03098	
0.344124	-0.03134	-0.15648	-0.03124	
0.343886	-0.03123	-0.15633	-0.03127	
0.343789	-0.03123	-0.15627	-0.03127	
0.343758	-0.03124	-0.15625	-0.03126	
0.34375	-0.03125	-0.15625	-0.03125	
0.343749	-0.03125	-0.15625	-0.03125	
0.34375	-0.03125	-0.15625	-0.03125	**values**
0.34375	-0.03125	-0.15625	-0.03125	**[A3]**

					a	*b*
2	1	4	1	1	2	4
1	2	1	4	0		
4	1	2	1	0		
1	4	1	2	0		

0	0	0	0	
0.5	-0.25	-0.875	0.6875	
2.03125	-1.95313	-3.42969	4.605469	
6.033203	-10.5127	-9.11279	22.56519	
12.69934	-46.9236	-13.2195	94.10735	
3.347054	-183.278	37.89147	345.9377	
-156.613	-632.515	456.5137	1115.079	**values**
-1153.81	-1881.51	2690.835	2994.509	**[A3]**
-5937.67	-4365.6	12560.88	5419.593	

[subtotal 12]

(iii) No convergence when *a* = 2, *b* = 0 **[M1A1]**
Indicates that non-strict diagonal dominance is not sufficient **[E1E1]**

[subtotal 4]

(iv) Use RHSs 1,0,0,0 0,1,0,0 0,0,1,0 0,0,0,1 to obtain inverse as **[M1]**

0.34375	-0.03125	-0.15625	-0.03125	**[A1]**
-0.03125	0.34375	-0.03125	-0.15625	**[A1]**
-0.15625	-0.03125	0.34375	-0.03125	**[A1]**
-0.03125	-0.15625	-0.03125	0.34375	**[A1]**

[subtotal 5]

[TOTAL 24]

Grade Thresholds

Advanced GCE MEI Mathematics 7895-8 3895-8
June 2009 Examination Series

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	59	52	45	39	33	0
4752	Raw	72	51	44	38	32	26	0
4753	Raw	72	57	52	47	42	37	0
4753/02	Raw	18	15	13	11	9	8	0
4754	Raw	90	67	59	51	43	35	0
4755	Raw	72	53	45	37	30	23	0
4756	Raw	72	51	45	39	33	27	0
4757	Raw	72	60	51	42	34	26	0
4758	Raw	72	61	55	49	43	36	0
4758/02	Raw	18	15	13	11	9	8	0
4761	Raw	72	57	48	39	30	21	0
4762	Raw	72	47	40	33	26	20	0
4763	Raw	72	55	46	38	30	22	0
4764	Raw	72	61	52	43	34	26	0
4766/G241	Raw	72	60	53	46	40	34	0
4767	Raw	72	57	50	44	38	32	0
4768	Raw	72	55	48	41	34	28	0
4769	Raw	72	56	49	42	35	28	0
4771	Raw	72	63	56	49	42	36	0
4772	Raw	72	57	51	45	39	33	0
4773	Raw	72	51	44	37	30	24	0
4776	Raw	72	62	53	45	37	28	0
4776/02	Raw	18	14	12	10	8	7	0
4777	Raw	72	55	47	39	32	25	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895-7898	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	44.1	65.4	81.4	92.1	97.9	100	10375
7896	57.2	78.0	88.9	95.4	98.9	100	1807
7897	87.1	93.55	100	100	100	100	31
7898	0	0	100	100	100	100	1
3895	35.3	52.9	67.4	79.1	88.1	100	16238
3896	52.1	70.2	82.4	90.4	95.7	100	2888
3897	80.4	88.2	91.2	96.1	97.1	100	102
3898	6.3	12.5	18.8	25.0	68.8	100	16

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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