OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE
MATHEMATICS
Mechanics 3

4730

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Thursday 11 June 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

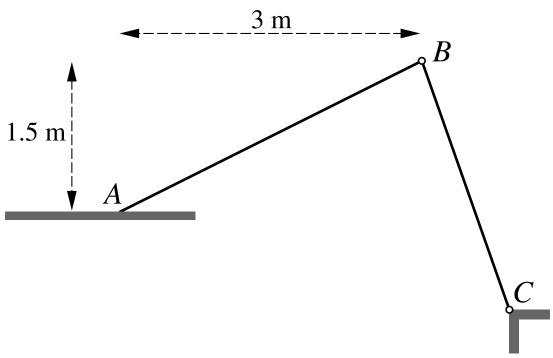
- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 A smooth sphere of mass 0.3 kg bounces on a fixed horizontal surface. Immediately before the sphere bounces the components of its velocity horizontally and vertically downwards are $4\,\mathrm{m\,s^{-1}}$ and $6\,\mathrm{m\,s^{-1}}$ respectively. The speed of the sphere immediately after it bounces is $5\,\mathrm{m\,s^{-1}}$.

(i) Show that the vertical component of the velocity of the sphere immediately after impact is $3\,\mathrm{m\,s^{-1}}$, and hence find the coefficient of restitution between the surface and the sphere. **[3]**

(ii) State the direction of the impulse on the sphere and find its magnitude. **[3]**

2

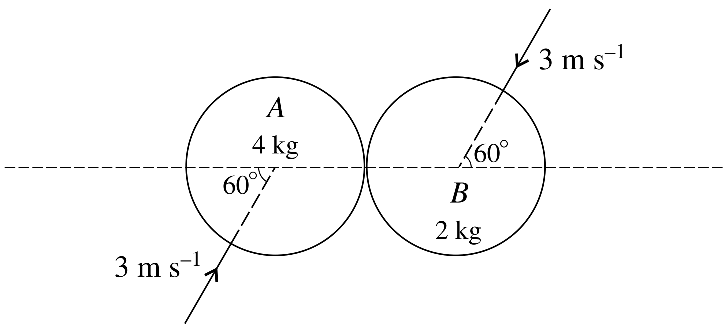


Two uniform rods, AB and BC, are freely jointed to each other at B, and C is freely jointed to a fixed point. The rods are in equilibrium in a vertical plane with A resting on a rough horizontal surface. This surface is 1.5 m below the level of B and the horizontal distance between A and B is 3 m (see diagram). The weight of AB is 80 N and the frictional force acting on AB at A is 14 N.

(i) Write down the horizontal component of the force acting on AB at B and show that the vertical component of this force is 33 N upwards. **[4]**

(ii) Given that the force acting on BC at C has magnitude 50 N, find the weight of BC. **[4]**

3



Two uniform smooth spheres A and B, of equal radius, have masses 4 kg and 2 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision both spheres have speed $3\,\mathrm{m\,s^{-1}}$. The spheres are moving in opposite directions, each at 60° to the line of centres (see diagram). After the collision A moves in a direction perpendicular to the line of centres.

(i) Show that the speed of B is unchanged as a result of the collision, and find the angle that the new direction of motion of B makes with the line of centres. **[8]**

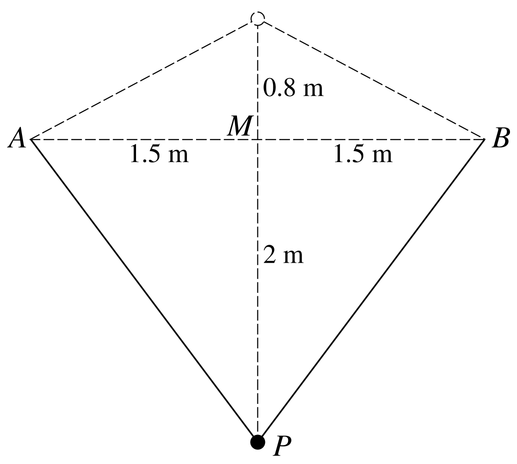
(ii) Find the coefficient of restitution between the spheres. **[2]**

4 A motor-cycle, whose mass including the rider is 120 kg, is decelerating on a horizontal straight road. The motor-cycle passes a point A with speed $40\,\text{m}\,\text{s}^{-1}$ and when it has travelled a distance of x m beyond A its speed is $v\,\text{m}\,\text{s}^{-1}$. The engine develops a constant power of 8 kW and resistances are modelled by a force of $0.25v^2$ N opposing the motion.

(i) Show that $\dfrac{480v^2}{v^3 - 32\,000}\dfrac{\mathrm{d}v}{\mathrm{d}x} = -1$. **[5]**

(ii) Find the speed of the motor-cycle when it has travelled 500 m beyond A. **[6]**

5



Each of two identical strings has natural length 1.5 m and modulus of elasticity 18 N. One end of one of the strings is attached to A and one end of the other string is attached to B, where A and B are fixed points which are 3 m apart and at the same horizontal level. M is the mid-point of AB. A particle P of mass m kg is attached to the other end of each of the strings. P is held at rest at the point 0.8 m vertically above M, and then released. The lowest point reached by P in the subsequent motion is 2 m below M (see diagram).

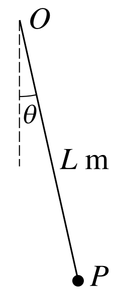
(i) Find the maximum tension in each of the strings during P's motion. **[3]**

(ii) By considering energy,

(a) show that the value of m is 0.42, correct to 2 significant figures, **[5]**

(b) find the speed of P at M. **[3]**

6



A particle P of mass m kg is attached to one end of a light inextensible string of length L m. The other end of the string is attached to a fixed point O. The particle is held at rest with the string taut and then released. P starts to move and in the subsequent motion the angular displacement of OP, at time t s, is θ radians from the downward vertical (see diagram). The initial value of θ is 0.05.

(i) Show that $\dfrac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\dfrac{g}{L}\sin\theta$. **[2]**

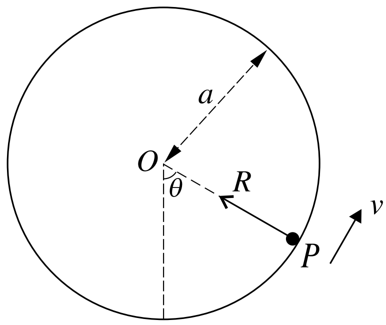
(ii) Hence show that the motion of P is approximately simple harmonic. **[2]**

(iii) Given that the period of the approximate simple harmonic motion is $\frac{4}{7}\pi$ s, find the value of L. **[2]**

(iv) Find the value of θ when $t = 0.7$ s, and the value of t when θ next takes this value. **[4]**

(v) Find the speed of P when $t = 0.7$ s. **[3]**

7



A hollow cylinder has internal radius a. The cylinder is fixed with its axis horizontal. A particle P of mass m is at rest in contact with the smooth inner surface of the cylinder. P is given a horizontal velocity u, in a vertical plane perpendicular to the axis of the cylinder, and begins to move in a vertical circle. While P remains in contact with the surface, OP makes an angle θ with the downward vertical, where O is the centre of the circle. The speed of P is v and the magnitude of the force exerted on P by the surface is R (see diagram).

(i) Find v^2 in terms of u, a, g and θ and show that $R = \dfrac{mu^2}{a} + mg(3\cos\theta - 2)$. **[7]**

(ii) Given that P just reaches the highest point of the circle, find u^2 in terms of a and g, and show that in this case the least value of v^2 is ag. **[4]**

(iii) Given instead that P oscillates between $\theta = \pm\frac{1}{6}\pi$ radians, find u^2 in terms of a and g. **[2]**

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