GCE

Mathematics

Advanced GCE **4724/01**

Core Mathematics 4

Mark Scheme for June 2010

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1	First 2 terms in expansion = $1-5x$	B1	(simp to this, now or later)
	3rd term shown as $\dfrac{-\frac{5}{3}.-\frac{8}{3}}{2}(3x)^2$	M1	$-\dfrac{8}{3}$ can be $-\dfrac{5}{3}-1$ $(3x)^2$ can be $9x^2$ or $3x^2$
	$= +\ 20x^2$	A1	
	4th term shown as $\dfrac{-\frac{5}{3}.-\frac{8}{3}.-\frac{11}{3}}{2.3}(3x)^3$	M1	$-\dfrac{11}{3}$ can be $-\dfrac{5}{3}-2$ $(3x)^3$ can be $27x^3$ or $3x^3$
	$= -\dfrac{220}{3}x^3$ ISW	A1	Accept $-\dfrac{440}{6}x^3$ ISW

N.B. If 0, SR B2 to be awarded for $1-\frac{5}{3}x+\frac{20}{9}x^2-\frac{220}{81}x^3$. Do not mark $(1+x)^{-5/3}$ as a MR.

$\boxed{5}$

2	Attempt quotient rule [Show fraction with denom $(1-\sin x)^2$ & num $+/-(1-\sin x)+/-\sin x+/-\cos x+/-\cos x$]	M1	
	Numerator = $(1-\sin x).-\sin x-\cos x.-\cos x$ { Product symbols must be clear or implied by further work }	A1	terms in any order
	Reduce correct numerator to $1-\sin x$	B1	or $-\sin x+\sin^2 x+\cos^2 x$
	Simplify to $\dfrac{1}{1-\sin x}$ ISW	A1	Accept $-\dfrac{1}{\sin x-1}$

$\boxed{4}$

3	$\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x-2}$	M1	For correct format
	$A(x-1)(x-2)+B(x-2)+C(x-1)^2 \equiv x^2$	M1	
	$A=-3$	A1	
	$B=-1$	A1	(B1 if cover-up rule used)
	$C=4$	A1	(B1 if cover-up rule used)

[NB1: Partial fractions need not be written out; correct format + correct values sufficient.
NB2: Having obtained B & C by cover-up rule, candidates may substitute into general expression & algebraically manipulate; the M1 & A1 are then available if deserved.]

$\boxed{5}$

These special cases using different formats are the only other ones to be considered		Max
$\dfrac{A}{x-1}+\dfrac{Bx+C}{(x-1)^2}+\dfrac{D}{x-2}$;	M1 M1; A0 for any values of A, B & C, A1 or B1 for $D=4$	3
$\dfrac{Ax+B}{(x-1)^2}+\dfrac{C}{x-2}$;	M0 M1; A1 for $A=-3$ and $B=2$, A1 or B1 for $C=4$	3

4		Att by diff to connect dx & du or find $\frac{dx}{du}$ or $\frac{du}{dx}$ (not dx=du)	M1	no accuracy; <u>not 'by parts'</u>
		$dx = 2u\,du$ or $\frac{du}{dx} = \frac{1}{2}(x+2)^{-\frac{1}{2}}$ AEF	A1	
		Indefinite integral $\rightarrow \int 2(u^2-2)^2 (\frac{u}{u})(du)$	A1	May be implied later
		{If relevant, cancel u/u and} attempt to square out	M1	
		{dep $\int kI(du)$ where $k=2$ or $\frac{1}{2}$ or 1 and I = $(u^2-2)^2$ or $(2-u^2)^2$ or $(u^2+2)^2$ }		
		Att to change limits if working with f(u) after integration	M1	or re-subst into integral attempt and use -1 & 7
		Indef integ = $\frac{2}{5}u^5 +/- \frac{8}{3}u^3 + 8u$ or $\frac{1}{10}u^5 +/- \frac{2}{3}u^3 + 2u$	A1	or $\frac{1}{5}u^5 +/- \frac{4}{3}u^3 + 4u$
		$\frac{652}{15}$ or $43\frac{7}{15}$ ISW but no '+c'	A1	
			$\boxed{7}$	
5		$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$ s.o.i.	B1	Implied by e.g., $4x\frac{dy}{dx} + y$
		$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1	
		Diff eqn(=0 can be implied)(solve for $\frac{dy}{dx}$ and) put $\frac{dy}{dx} = 0$	M1	
		Produce <u>only</u> $2x + 4y = 0$ (though AEF acceptable)	*A1	<u>without any error seen</u>
		Eliminate x or y from curve eqn & eqn(s) just produced	M1	
		Produce either $x^2 = 36$ or $y^2 = 9$	dep* A1	Disregard other solutions
		$\underline{(\pm 6, \mp 3)}$ AEF, as the only answer ISW	dep* A1	Sign aspect must be clear
			$\boxed{7}$	
6	**(i)**	State/imply scalar product of any two vectors = 0	M1	
		Scalar product of correct two vectors = 4 + 2a −6	A1	($4 + 2a - 6 = 0 \rightarrow$ M1A1)
		$\underline{a = 1}$	A1 **3**	
	(ii)	**(a)** Attempt to produce at least two relevant equations	M1	e.g. $2t = 3 + 2s$ …..
		Solve two not containing 'a' for s and t	M1	
		Obtain at least one of $s = -\frac{1}{2}$, $t = 1$	A1	
		Substitute in third equation & produce $\underline{a = -2}$	A1 **4**	
		(b) Method for finding magnitude of <u>any</u> vector	M1	possibly involving 'a'
		Using $\cos\theta = \frac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ for the pair of direction vectors	M1	possibly involving 'a'
		<u>107, 108 (107.548) or 72, 73, 72.4, 72.5 (72.4516)</u> c.a.o.	A1 **3**	<u>1.87, 1.88 (1.87707) or 1.26</u>
			$\boxed{10}$	

7	**(i)**	Differentiate x as a quotient, $\frac{v\,\mathrm{d}u - u\,\mathrm{d}v}{v^2}$ or $\frac{u\,\mathrm{d}v - v\,\mathrm{d}u}{v^2}$	M1	or product clearly defined	
		$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{(t+1)^2}$ or $\frac{-1}{(t+1)^2}$ or $-(t+1)^{-2}$	A1	WWW → 2	
		$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{2}{(t+3)^2}$ or $\frac{-2}{(t+3)^2}$ or $-2(t+3)^{-2}$	B1		
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$	M1	quoted/implied and used	
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2(t+1)^2}{(t+3)^2}$ or $\frac{2(t+3)^{-2}}{(t+1)^{-2}}$ (dep 1st 4 marks)	*A1	ignore ref $t=-1, t=-3$	
		State <u>squares</u> +ve or $(t+1)^2$ & $(t+3)^2$ + ve $\therefore \frac{\mathrm{d}y}{\mathrm{d}x}$ +ve	dep*A1 **6**	or $\left(\frac{t+1}{t+3}\right)^2$ + ve . Ignore ≥ 0	
	(ii)	Attempt to obtain t from either the x or y equation	M1	No accuracy required	
		$t = \frac{2-x}{x-1}$ AEF <u>or</u> $t = \frac{2}{y} - 3$ AEF	A1		
		Substitute in the equation not yet used in this part	M1	or equate the 2 values of t	
		Use correct meth to eliminate ('double-decker') fractions	M1		
		Obtain <u>$2x + y = 2xy + 2$</u> ISW AEF	A1 **5**	but not involving fractions	**11**
8	**(i)**	<u>Long division method</u>		<u>Identity method</u>	
		Evidence of division process as far as 1st stage incl sub	M1	$\equiv Q(x-1)+R$	
		(Quotient =) $x-4$	A1	$Q = x-4$	
		(Remainder =) 2 ISW	A1 **3**	$R = 2$; N.B. might be B1	
	(ii)	**(a)** Separate variables; $\int \frac{1}{y-5}\,\mathrm{d}y = \int \frac{x^2-5x+6}{x-1}\,\mathrm{d}x$	M1	'$\int$' may be implied later	
		Change $\frac{x^2-5x+6}{x-1}$ into their (Quotient + $\frac{\text{Rem}}{x-1}$)	M1		
		$\ln(y-5)$ =√ (integration of their previous result) (+c)ISW	√A1 **3**	f.t. if using Quot + $\frac{\text{Rem}}{x-1}$	
	(ii)	**(b)** Substitute y = 7, x = 8 into their eqn containing 'c'	M1	& attempt 'c' (-3.2 , $\ln\frac{2}{49}$)	
		Substitute x = 6 and their value of 'c'	M1	& attempt to find y	
		<u>y = 5.00 (5.002529)</u> Also $5 + \frac{50}{49}e^{-6}$	A2 **4**	Accept 5, 5.0,	
		Beware: <u>any</u> wrong working anywhere → A0 even if answer is one of the acceptable ones.			
			10		

9 (i)	Attempt to multiply out $(x+\cos 2x)^2$	M1	Min of 2 correct terms
	<u>Finding $\int 2x\cos 2x\,\mathrm{d}x$</u>		
	Use $u=2x, \mathrm{d}v=\cos 2x$	M1	1st stage $\mathrm{f}(x)+/-\int \mathrm{g}(x)\,\mathrm{d}x$
	1st stage $x\sin 2x-\int \sin 2x\,\mathrm{d}x$	A1	
	$\therefore \int 2x\cos 2x\,\mathrm{d}x = x\sin 2x+\frac{1}{2}\cos 2x$	A1	
	<u>Finding $\int \cos^2 2x\,\mathrm{d}x$</u>		
	Change to $k\int +/-1+/-\cos 4x\,\mathrm{d}x$	M1	where $k=\frac{1}{2}$, 2 or 1
	Correct version $\frac{1}{2}\int 1+\cos 4x\,\mathrm{d}x$	A1	
	$\int \cos 4x\,\mathrm{d}x = \frac{1}{4}\sin 4x$	B1	seen anywhere in this part
	Result = $\frac{1}{2}x+\frac{1}{8}\sin 4x$	A1	
	(i) ans = $\frac{1}{3}x^3+x\sin 2x+\frac{1}{2}\cos 2x+\frac{1}{2}x+\frac{1}{8}\sin 4x$ (+ c)	A1 **9**	Fully correct
(ii)	V = $\pi\int_0^{\frac{1}{2}\pi}(x+\cos 2x)^2\ (\mathrm{d}x)$	M1	
	Use limits 0 & $\frac{1}{2}\pi$ correctly on their **(i)** answer	M1	
	(**i**) correct value = $\frac{1}{24}\pi^3-\frac{1}{2}+\frac{1}{4}\pi-\frac{1}{2}$	A1	
	Final answer = $\pi\left(\frac{1}{24}\pi^3+\frac{1}{4}\pi-1\right)$	A1 **4**	**c.a.o.** No follow-through
		$\boxed{13}$	

<u>Alternative methods</u>

2 If $y=\frac{\cos x}{1-\sin x}$ is changed into $y(1-\sin x)=\cos x$, award

M1 for clear use of the product rule (though possibly trig differentiation inaccurate)

A1 for $-y\cos x+(1-\sin x)\frac{\mathrm{d}y}{\mathrm{d}x}=-\sin x$ AEF

B1 for reducing to a fraction with $1-\sin x$ or $-\sin x+\sin^2 x+\cos^2 x$ in the numerator

A1 for correct final answer of $\frac{1}{1-\sin x}$ or $(1-\sin x)^{-1}$

If $y=\frac{\cos x}{1-\sin x}$ is changed into $y=\cos x(1-\sin x)^{-1}$, award

M1 for clear use of the product rule (though possibly trig differentiation inaccurate)

A1 for $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\cos^2 x(1-\sin x)^{-2}+(1-\sin x)^{-1}.-\sin x$ AEF

B1 for reducing to a fraction with $1-\sin x$ or $-\sin x+\sin^2 x+\cos^2 x$ in the numerator

A1 for correct final answer of $\dfrac{1}{1-\sin x}$ or $(1-\sin x)^{-1}$

6(ii)(a) If candidates use some long drawn-out method to find 'a' instead of the direct route, allow

M1 as before, for producing the 3 equations

M1 for any satisfactory method which will/does produce 'a', however involved

A2 for $a=-2$

7(ii) Marks for obtaining this Cartesian equation are not available in part (i).

<u>If part (ii) is done first and then part (i) is attempted using the Cartesian equation, award marks as follow:</u>

<u>Method 1 where candidates differentiate implicitly</u>

M1 for attempt at implicit differentiation

A1 for $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2y-2}{1-2x}$ AEF

M1 for substituting parametric values of x and y

A2 for simplifying to $\frac{2(t+1)^2}{(t+3)^2}$

A1 for finish as in original method

<u>Method 2 where candidates manipulate the Cartesian equation to find x = or y=</u>

M1 for attempt to re-arrange so that either $y = \mathrm{f}(x)$ or $x = \mathrm{g}(y)$

A1 for correct $y=\frac{2-2x}{1-2x}$ AEF or $x=\frac{2-y}{2-2y}$ AEF

M1 for differentiating as a quotient

A2 for obtaining $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{(1-2x)^2}$ or $\frac{(2-2y)^2}{2}$

A1 for finish as in original method

8(ii)(b) If definite integrals are used , then

M2 for $[\ ]_y^7=[\ ]_6^8$ or equivalent or M1 for $[\ ]_7^y=[\ ]_6^8$ or equivalent

A2 for 5, 5.0, 5.00 (5.002529) <u>with caveat as in main scheme</u> dep M2

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