

Mathematics

Advanced GCE 4724/01

Core Mathematics 4

Mark Scheme for June 2010

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- 1 First 2 terms in expansion = $1 - 5x$ B1 (simp to this, now or later)
- 3rd term shown as $\frac{-\frac{5}{3} \cdot -\frac{8}{3}}{2} (3x)^2$ M1 $-\frac{8}{3}$ can be $-\frac{5}{3} - 1$
- $(3x)^2$ can be $9x^2$ or $3x^2$
- = + $20x^2$ A1
- 4th term shown as $\frac{-\frac{5}{3} \cdot -\frac{8}{3} \cdot -\frac{11}{3}}{2 \cdot 3} (3x)^3$ M1 $-\frac{11}{3}$ can be $-\frac{5}{3} - 2$
- $(3x)^3$ can be $27x^3$ or $3x^3$
- = $-\frac{220}{3}x^3$ ISW A1 Accept $-\frac{440}{6}x^3$ ISW

N.B. If 0, SR B2 to be awarded for $1 - \frac{5}{3}x + \frac{20}{9}x^2 - \frac{220}{81}x^3$. Do not mark $(1+x)^{-5/3}$ as a MR.

5

- 2 Attempt quotient rule M1
- [Show fraction with denom $(1 - \sin x)^2$ & num $+/- (1 - \sin x) +/- \sin x +/- \cos x +/- \cos x$]
- Numerator = $(1 - \sin x) \cdot -\sin x - \cos x \cdot -\cos x$ A1 terms in any order
{ Product symbols must be clear or implied by further work }
- Reduce correct numerator to $1 - \sin x$ B1 or $-\sin x + \sin^2 x + \cos^2 x$
- Simplify to $\frac{1}{1 - \sin x}$ ISW A1 Accept $-\frac{1}{\sin x - 1}$

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- 3 $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$ M1 For correct format
- $A(x-1)(x-2) + B(x-2) + C(x-1)^2 \equiv x^2$ M1
- $A = -3$ A1
- $B = -1$ A1 (B1 if cover-up rule used)
- $C = 4$ A1 (B1 if cover-up rule used)

[NB1: Partial fractions need not be written out; correct format + correct values sufficient.

NB2: Having obtained B & C by cover-up rule, candidates may substitute into general expression & algebraically manipulate; the M1 & A1 are then available if deserved.]

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These special cases using different formats are the only other ones to be considered Max

$\frac{A}{x-1} + \frac{Bx+C}{(x-1)^2} + \frac{D}{x-2}$; M1 M1; A0 for any values of A, B & C , A1 or B1 for $D = 4$ 3

$\frac{Ax+B}{(x-1)^2} + \frac{C}{x-2}$; M0 M1; A1 for $A = -3$ and $B = 2$, A1 or B1 for $C = 4$ 3

4 Att by diff to connect dx & du or find $\frac{dx}{du}$ or $\frac{du}{dx}$ (not dx=du) M1 no accuracy; not 'by parts'
 $dx = 2u du$ or $\frac{du}{dx} = \frac{1}{2}(x+2)^{-\frac{1}{2}}$ AEF A1
 Indefinite integral $\rightarrow \int 2(u^2 - 2)^2 \left(\frac{u}{u}\right)(du)$ A1 May be implied later
 {If relevant, cancel u/u and} attempt to square out M1
 {dep $\int kI(du)$ where $k = 2$ or $\frac{1}{2}$ or 1 and $I = (u^2 - 2)^2$ or $(2 - u^2)^2$ or $(u^2 + 2)^2$ }
 Att to change limits if working with f(u) after integration M1 or re-subst into integral attempt and use
 -1 & 7
 Indef integ = $\frac{2}{5}u^5 + /-\frac{8}{3}u^3 + 8u$ or $\frac{1}{10}u^5 + /-\frac{2}{3}u^3 + 2u$ A1 or $\frac{1}{5}u^5 + /-\frac{4}{3}u^3 + 4u$
 $\frac{652}{15}$ or $43\frac{7}{15}$ ISW but no '+c' A1

7

5 $\frac{d}{dx}(xy) = x \frac{dy}{dx} + y$ s.o.i. B1 Implied by e.g., $4x \frac{dy}{dx} + y$
 $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$ B1
 Diff eqn(=0 can be implied)(solve for $\frac{dy}{dx}$ and) put $\frac{dy}{dx} = 0$ M1
 Produce only $2x + 4y = 0$ (though AEF acceptable) *A1 without any error seen
 Eliminate x or y from curve eqn & eqn(s) just produced M1
 Produce either $x^2 = 36$ or $y^2 = 9$ dep* A1 Disregard other solutions
($\pm 6, \mp 3$) AEF, as the only answer ISW dep* A1 Sign aspect must be clear

7

6 (i) State/imply scalar product of any two vectors = 0 M1
 Scalar product of correct two vectors = $4 + 2a - 6$ A1 ($4 + 2a - 6 = 0 \rightarrow$ M1A1)
 $a = 1$ A1 **3**
 (ii) (a) Attempt to produce at least two relevant equations M1 e.g. $2t = 3 + 2s \dots$
 Solve two not containing 'a' for s and t M1
 Obtain at least one of $s = -\frac{1}{2}, t = 1$ A1
 Substitute in third equation & produce $a = -2$ A1 **4**
 (b) Method for finding magnitude of any vector M1 possibly involving 'a'
 Using $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$ for the pair of direction vectors M1 possibly involving 'a'
107, 108 (107.548) or 72, 73, 72.4, 72.5 (72.4516) c.a.o. A1 **3** 1.87, 1.88 (1.87707) or 1.26

10

7 (i) Differentiate x as a quotient, $\frac{v \, du - u \, dv}{v^2}$ or $\frac{u \, dv - v \, du}{v^2}$ M1 or product clearly defined

$\frac{dx}{dt} = -\frac{1}{(t+1)^2}$ or $\frac{-1}{(t+1)^2}$ or $-(t+1)^{-2}$ A1 WWW \rightarrow 2

$\frac{dy}{dt} = -\frac{2}{(t+3)^2}$ or $\frac{-2}{(t+3)^2}$ or $-2(t+3)^{-2}$ B1

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ M1 quoted/implied and used

$\frac{dy}{dx} = \frac{2(t+1)^2}{(t+3)^2}$ or $\frac{2(t+3)^{-2}}{(t+1)^{-2}}$ (dep 1st 4 marks) *A1 ignore ref $t = -1, t = -3$

State squares +ve or $(t+1)^2$ & $(t+3)^2$ +ve $\therefore \frac{dy}{dx}$ +ve dep*A1 6 or $(\frac{t+1}{t+3})^2$ +ve. Ignore ≥ 0

(ii) Attempt to obtain t from either the x or y equation M1 No accuracy required

$t = \frac{2-x}{x-1}$ AEF or $t = \frac{2}{y} - 3$ AEF A1

Substitute in the equation not yet used in this part M1 or equate the 2 values of t

Use correct meth to eliminate ('double-decker') fractions M1

Obtain $2x + y = 2xy + 2$ ISW AEF A1 5 but not involving fractions 11

8 (i) Long division method Identity method

Evidence of division process as far as 1st stage incl sub M1 $\equiv Q(x-1) + R$

(Quotient =) $x - 4$ A1 $Q = x - 4$

(Remainder =) 2 ISW A1 3 $R = 2$; N.B. might be B1

(ii) (a) Separate variables; $\int \frac{1}{y-5} \, dy = \int \frac{x^2 - 5x + 6}{x-1} \, dx$ M1 ' \int ' may be implied later

Change $\frac{x^2 - 5x + 6}{x-1}$ into their (Quotient + $\frac{\text{Rem}}{x-1}$) M1

$\ln(y-5) = \sqrt{\text{(integration of their previous result)} (+c)}$ ISW $\sqrt{\text{A1 3}}$ f.t. if using Quot + $\frac{\text{Rem}}{x-1}$

(ii) (b) Substitute $y = 7, x = 8$ into their eqn containing 'c' M1 & attempt 'c' ($-3.2, \ln \frac{2}{49}$)

Substitute $x = 6$ and their value of 'c' M1 & attempt to find y

$y = 5.00$ (5.002529) Also $5 + \frac{50}{49} e^{-6}$ A2 4 Accept 5, 5.0,

Beware: any wrong working anywhere \rightarrow A0 even if answer is one of the acceptable ones.

10

- 9(i) Attempt to multiply out $(x + \cos 2x)^2$ M1 Min of 2 correct terms
- Finding $\int 2x \cos 2x \, dx$
- Use $u = 2x, dv = \cos 2x$ M1 1st stage $f(x)+/- \int g(x)dx$
- 1st stage $x \sin 2x - \int \sin 2x \, dx$ A1
- $\therefore \int 2x \cos 2x \, dx = x \sin 2x + \frac{1}{2} \cos 2x$ A1
- Finding $\int \cos^2 2x \, dx$
- Change to $k \int +/- 1 +/- \cos 4x \, dx$ M1 where $k = \frac{1}{2}, 2$ or 1
- Correct version $\frac{1}{2} \int 1 + \cos 4x \, dx$ A1
- $\int \cos 4x \, dx = \frac{1}{4} \sin 4x$ B1 seen anywhere in this part
- Result = $\frac{1}{2} x + \frac{1}{8} \sin 4x$ A1
- (i) ans = $\frac{1}{3} x^3 + x \sin 2x + \frac{1}{2} \cos 2x + \frac{1}{2} x + \frac{1}{8} \sin 4x (+ c)$ A1 9 Fully correct
- (ii) $V = \pi \int_0^{\frac{1}{2}\pi} (x + \cos 2x)^2 \, (dx)$ M1
- Use limits 0 & $\frac{1}{2}\pi$ correctly on their (i) answer M1
- (i) correct value = $\frac{1}{24}\pi^3 - \frac{1}{2} + \frac{1}{4}\pi - \frac{1}{2}$ A1
- Final answer = $\pi \left(\frac{1}{24}\pi^3 + \frac{1}{4}\pi - 1 \right)$ A1 4 c.a.o. No follow-through

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Alternative methods

- 2 If $y = \frac{\cos x}{1 - \sin x}$ is changed into $y(1 - \sin x) = \cos x$, award
- M1 for clear use of the product rule (though possibly trig differentiation inaccurate)
- A1 for $-y \cos x + (1 - \sin x) \frac{dy}{dx} = -\sin x$ AEF
- B1 for reducing to a fraction with $1 - \sin x$ or $-\sin x + \sin^2 x + \cos^2 x$ in the numerator
- A1 for correct final answer of $\frac{1}{1 - \sin x}$ or $(1 - \sin x)^{-1}$
- If $y = \frac{\cos x}{1 - \sin x}$ is changed into $y = \cos x(1 - \sin x)^{-1}$, award
- M1 for clear use of the product rule (though possibly trig differentiation inaccurate)
- A1 for $\left(\frac{dy}{dx}\right) = \cos^2 x(1 - \sin x)^{-2} + (1 - \sin x)^{-1} \cdot -\sin x$ AEF

- B1 for reducing to a fraction with $1 - \sin x$ or $-\sin x + \sin^2 x + \cos^2 x$ in the numerator
 A1 for correct final answer of $\frac{1}{1 - \sin x}$ or $(1 - \sin x)^{-1}$

6(ii)(a) If candidates use some long drawn-out method to find 'a' instead of the direct route, allow

- M1 as before, for producing the 3 equations
 M1 for any satisfactory method which will/does produce 'a', however involved
 A2 for $a = -2$

7(ii) Marks for obtaining this Cartesian equation are not available in part (i).

If part (ii) is done first and then part (i) is attempted using the Cartesian equation, award marks as follow:

Method 1 where candidates differentiate implicitly

- M1 for attempt at implicit differentiation
 A1 for $\frac{dy}{dx} = \frac{2y-2}{1-2x}$ AEF
 M1 for substituting parametric values of x and y
 A2 for simplifying to $\frac{2(t+1)^2}{(t+3)^2}$
 A1 for finish as in original method

Method 2 where candidates manipulate the Cartesian equation to find $x =$ or $y =$

- M1 for attempt to re-arrange so that either $y = f(x)$ or $x = g(y)$
 A1 for correct $y = \frac{2-2x}{1-2x}$ AEF or $x = \frac{2-y}{2-2y}$ AEF
 M1 for differentiating as a quotient
 A2 for obtaining $\frac{dy}{dx} = \frac{2}{(1-2x)^2}$ or $\frac{(2-2y)^2}{2}$
 A1 for finish as in original method

8(ii)(b) If definite integrals are used, then

- M2 for $\int_y^7 = \int_6^8$ or equivalent or M1 for $\int_7^y = \int_6^8$ or equivalent
 A2 for 5, 5.0, 5.00 (5.002529) with caveat as in main scheme dep M2

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