GCE

Mathematics

Advanced GCE **4733/01**

Probability and Statistics 2

Mark Scheme for June 2010

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1	(i)(a)	$1 - P(\le 6) = 1 - 0.8675$ $= \mathbf{0.1325}$	M1 A1 **2**	1 – .9361 or 1 – .8786 or 1 – .8558: M1. .9721: M0 Or 0.132 or 0.133
	(b)	Po(0.42) $e^{-0.42}\dfrac{0.42^2}{2!} = \mathbf{0.05795}$	M1 M1 A1 **3**	Po(0.42) stated or implied Correct formula, any numerical λ Answer, art 0.058. Interpolation in tables: M1B2
	(ii)	E.g. “Contagious so incidences do not occur independently”, or “more cases in winter so not at constant average rate”	B2 **2**	Contextualised reason, referred to conditions: B2. No marks for mere learnt phrases or spurious reasons, e.g. not just “independently, singly and constant average rate”. See notes.
2	(i)	B(10, 0.35) P(< 3) $= \mathbf{0.2616}$	M1 M1 A1 **3**	B(10, 0.35) stated or implied Tables used, e.g. 0.5138 or 0.3373, or formula ± 1 term Answer 0.2616 or better or 0.262 only
	(ii)	Binomial requires being chosen independently, which this is not, but unimportant as population is large	B2 **2**	Focus on “Without replacement” negating independence condition. It doesn’t negate “constant probability” condition but can allow B1 if “selected”. See notes
3	(i)	$\left(\dfrac{32-40}{\sigma}\right) = \Phi^{-1}(0.2) = -0.842$ σ = 9.5[06]	M1 B1 A1 **3**	Standardise and equate to Φ^{-1}, allow “1 –” errors, σ^2, cc 0.842 seen Answer, 9.5 or in range [9.50, 9.51], c.w.o.
	(ii)	B(90, 0.2) ≈ N(18, 14.4) $1-\Phi\left(\dfrac{19.5-18}{\sqrt{14.4}}\right) = 1 - \Phi(0.3953)$ $= 1 - 0.6537 \quad = \mathbf{0.3463}$	B1 M1 A1 M1 A1 A1 **6**	B(90, 0.2) stated or implied N, their *np* … … variance their *npq*, allow √ errors Standardise with *np* and *npq*, allow √, cc errors, e.g. .396, .448, .458, .486, .472; √*npq* and cc correct Answer, a.r.t. 0.346 [NB: 0.3491 from Po: 1/6]
4		$H_0 : p = 0.4$, $H_1 : p > 0.4$ $R \sim B(16, 0.4)$:	B1 B1 M1	Fully correct, B2. Allow *π*. *p* omitted or μ used in both, or > wrong: B1 only. *x* or $\bar{x}$ or 6.4 etc: B0 B(16, 0.4) stated or implied, allow N(6.4, 3.84)
	(α)	$P(R \ge 11) = 0.0191$ > 0.01	A1 A1	Allow for P(≤ 10) = 0.9808, and < 0.99, or *z* = 2.092 or *p* = 0.018, but *not* P(≤ 11) = 0.9951 or P(= 11) = 0.0143 Explicit comp with .01, or *z*<2.326, *not* from ≤11 or =11
	(β)	CR $R \ge 12$ and 11 < 12 Probability 0.0049	A1 A1	Must be clear that it’s ≥ 12 and not ≤ 11 Needs to be seen, allow 0.9951 here, or *p* = .0047 from N
		Do not reject H_0. Insufficient evidence that proportion of commuters who travel by train has increased	M1 A1 FT **7**	Needs like-with-like, P(*R* ≥ 11) or CR *R* ≥ 12 Conclusion correct on their *p* or CR, contextualised, not too assertive, e.g. “evidence that” needed. Normal, *z* = 2.34, “reject” [no cc] can get 6/7
5	(i)	(a) $30 + 1.645 \times \dfrac{5}{\sqrt{10}}$ = 32.6 Therefore critical region is $\bar{t} > 32.6$	M1 B1 A1 A1 FT **4**	30 + 5*z*/√10, allow ± but not just –, allow √ errors *z* = 1.645 seen, allow – Critical value, art 32.6 “ > *c*” or “≥ *c*”, FT on *c* provided > 30, can’t be recovered. Withhold if not clear which is CR
		(b) $P(\bar{t} < 32.6 \mid \mu = 35)$ $\dfrac{32.6-35}{5/\sqrt{10}}$ [= –1.5178] **0.0645**	M1* dep*M1 A1 **3**	Need their *c*, final answer < 0.5 and *μ* = 35 at least, but allow answer > 0.5 if consistent with their (i) Standardise their CV with 35 and √10 or 10 Answer in range [0.064, 0.065], or 0.115 from 1.96 in (a)
	(ii)	$(32.6 - \mu) = 0$ μ = 32.6 20 + 0.6*m* = 32.6 *m* = **21**	M1 A1 FT M1 A1 **4**	Standardise *c* with μ, equate to Φ^{-1}, can be implied by: μ = their *c* Equate and solve for *m*, allow from 30 or 35 Answer, a.r.t. 21, c.a.o. MR: 0.05: M1 A0 M1, 16.7 A1 FT Ignore variance throughout (ii)

6	(a)	N(24, 24) $1-\Phi\left(\frac{30.5-24}{\sqrt{24}}\right) = 1-\Phi(1.327)$ = **0.0923**	B1 B1 M1 A1 A1 **5**	Normal, mean 24 stated or implied Variance or SD equal to mean Standardise 30 with λ and √λ, allow cc or √ errors, e.g. .131 or .1103 ; 30.5 and √λ correct Answer in range [0.092, 0.0925]
	(b)(i)	*p* or *np* [= 196] is too large	B1 **1**	Correct reason, no wrong reason, don't worry about 5 or 15
	(ii)	Consider (200 – *E*) (200 – *E*) ~ Po(4) P(≥ 6) [= 1 – 0.7851] = **0.2149**	M1 M1 M1 A1 **4**	Consider complement Po(200×0.02) Poisson tables used, correct tail, e.g. 0.3712 or 0.1107 Answer a.r.t. 0.215 only
7		$H_0 : \mu = 56.8$ $H_1 : \mu \neq 56.8$ $\bar{x}$ = 17085/300 = 56.95 $\frac{300}{299}\left(\frac{973847}{300} - 56.95^2\right)$ = 2.8637…	B2 B1 M1 M1 A1	Both correct One error: B1, but *not* $\bar{x}$, etc 56.95 or 57.0 seen or implied Biased [2.8541] : M1M0A0 Unbiased estimate method, allow if ÷ 299 seen anywhere Estimate, a.r.t. 2.86 [not 2.85]
	(α)	$z = \frac{56.95-56.8}{\sqrt{2.8637/300}} = 1.535$ 1.535 < 1.645 or 0.0624 > 0.05	M1 A1 A1	Standardise with √300, allow √ errors, cc $z \in [1.53, 1.54]$ or $p \in [0.062, 0.063]$, *not* – 1.535 Compare explicitly *z* with 1.645 or *p* with 0.05, or 2*p* > 0.1, not from μ = 56.95
	(β)	CV $56.8 \pm 1.645 \times \sqrt{\frac{2.8637}{300}}$ 56.96 > 56.95	M1 A1 A1 FT	56.8 + *z*σ/√300, needn't have ±, allow √ errors *z* = 1.645 *c* = 56.96, FT on *z*, and compare 56.95 [c_L = 56.64]
		Do not reject H_0; insufficient evidence that mean thickness is wrong	M1 A1 FT **11**	Consistent first conclusion, needs 300, correct method and comparison Conclusion stated in context, not too assertive, e.g. "evidence that" needed
8	(i)	$\int_1^\infty kx^{-a}dx = \left[k\frac{x^{-a+1}}{-a+1}\right]_1^\infty$ Correctly obtain *k* = *a* – 1 **AG**	M1 B1 A1 **3**	Integrate f(*x*), limits 1 and ∞ (at some stage) Correct indefinite integral Correctly obtain given answer, don't need to see treatment of ∞ but mustn't be wrong. Not k^{-a+1}
	(ii)	$\int_1^\infty 3x^{-3}dx = \left[3\frac{x^{-2}}{-2}\right]_1^\infty = 1\frac{1}{2}$ $\int_1^\infty 3x^{-2}dx = \left[3\frac{x^{-1}}{-1}\right]_1^\infty - (1\frac{1}{2})^2$ Answer ¾	M1 M1 A1 M1 A1 **5**	Integrate *x*f(*x*), limits 1 and ∞ (at some stage) [x^4 is *not* MR] Integrate x^2f(*x*), correct limits Either μ = 1½ or E(X^2) = 3 stated or implied, allow *k*, *k*/2 Subtract their numerical μ^2, allow letter if subs later Final answer ¾ or 0.75 only, cwo, e.g. not from μ =–1½. [SR: Limits 0, 1: can get (i) B1, (ii) M1M1M1]
	(iii)	$\int_1^2 (a-1)x^{-a}dx = \left[-x^{-a+1}\right]_1^2 = 0.9$ $1 - \frac{1}{2^{a-1}} = 0.9,\ 2^{a-1} = 10$ *a* = 4.322	M1* dep*M1 M1 indept A1 **4**	Equate ∫f(*x*)d*x*, one limit 2, to 0.9 or 0.1. [Normal: 0 ex 4] Solve equation of this form to get 2^{a-1} = number Use logs or equivalent to solve 2^{a-1} = number Answer, a.r.t. 4.32. T&I: (M1M1) B2 or B0

Specimen Verbal Answers

1	α	"Cases of infection must occur randomly, independently, singly and at constant average rate"	B0
	β	Above + "but it is contagious"	B1
	γ	Above + "but not independent as it is contagious"	B2
	δ	"Not independent as it is contagious"	B2
	ε	"Not constant average rate", or "not independent"	B0
	λ	"Not constant average rate because contagious" *[needs more]*	B1
	ζ	"Not constant average rate because more likely at certain times of year"	B2
	μ	Probabilities changes because of different susceptibilities	B0
	ν	Not constant average rate because of different susceptibilities	B2
	η	Correct but with unjustified or wrong extra assertion *[scattergun]*	B1
	θ	More than one correct assertion, all justified	B2
	π	Valid reason (e.g. "contagious") but not referred to conditions	B1

[Focus is on explaining why the required assumptions might not apply. No credit for regurgitating learnt phrases, such as "events must occur randomly, independently, singly and at constant average rate, even if contextualised.]

2 Don't need either "yes" or "no".

α	"No it doesn't invalidate the calculation"	*[no reason]*	B0
β	"Binomial requires not chosen twice" *[false]*		B0
γ	"Probability has to be constant but here the probabilities change"		B0
δ	Same but "probability <u>of being chosen</u>"	*[false, but allow B1]*	B1
ε	"Needs to be independently chosen but probabilities change" *[confusion]*		B0
ζ	"Needs to be independent but one choice affects another" *[correct]*		B2
η	"The sample is large so it makes little difference"	*[false]*	B0
θ	"The population is large so it makes little difference" *[true]*		B2
λ	Both correct and wrong reasons (scattergun approach)		B1

[Focus is on modelling conditions for binomial: On every choice of a member of the sample, each member of the population is equally likely to be chosen; and each choice is independent of all other choices.
Recall that in fact even without replacement the probability that any one person is chosen is the same for each choice. Also, the binomial "independence" condition <u>does</u> require the possibility of the same person being chosen twice.]

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