OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4727

Further Pure Mathematics 3

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Monday 13 June 2011
Morning

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 A line l has equation $\frac{x-1}{5} = \frac{y-6}{6} = \frac{z+3}{-7}$ and a plane p has equation $x + 2y - z = 40$.

(i) Find the acute angle between l and p. **[4]**

(ii) Find the perpendicular distance from the point $(1, 6, -3)$ to p. **[2]**

2 It is given that $z = \mathrm{e}^{\mathrm{i}\theta}$, where $0 < \theta < 2\pi$, and $w = \frac{1+z}{1-z}$.

(i) Prove that $w = \mathrm{i}\cot\frac{1}{2}\theta$. **[3]**

(ii) Sketch separate Argand diagrams to show the locus of z and the locus of w. You should show the direction in which each locus is described when θ increases in the interval $0 < \theta < 2\pi$. **[3]**

3 The variables x and y satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = 5\cos 3x.$$

(i) Find the complementary function. **[2]**

(ii) Hence, or otherwise, find the general solution. **[7]**

(iii) Find the approximate range of values of y when x is large and positive. **[2]**

4 A group G, of order 8, is generated by the elements a, b, c. G has the properties

$$a^2 = b^2 = c^2 = e, \quad ab = ba, \quad bc = cb, \quad ca = ac,$$

where e is the identity.

(i) Using these properties and basic group properties as necessary, prove that $abc = cba$. **[2]**

The operation table for G is shown below.

	e	a	b	c	bc	ca	ab	abc
e	e	a	b	c	bc	ca	ab	abc
a	a	e	ab	ca	abc	c	b	bc
b	b	ab	e	bc	c	abc	a	ca
c	c	ca	bc	e	b	a	abc	ab
bc	bc	abc	c	b	e	ab	ca	a
ca	ca	c	abc	a	ab	e	bc	b
ab	ab	b	a	abc	ca	bc	e	c
abc	abc	bc	ca	ab	a	b	c	e

(ii) List all the subgroups of order 2. **[2]**

(iii) List five subgroups of order 4. **[3]**

(iv) Determine whether all the subgroups of G which are of order 4 are isomorphic. **[2]**

5 The substitution $y = u^k$, where k is an integer, is to be used to solve the differential equation

$$x\frac{dy}{dx} + 3y = x^2y^2 \qquad \text{(A)}$$

by changing it into an equation (B) in the variables u and x.

(i) Show that equation (B) may be written in the form

$$\frac{du}{dx} + \frac{3}{kx}u = \frac{1}{k}xu^{k+1}.$$ **[4]**

(ii) Write down the value of k for which the integrating factor method may be used to solve equation (B). **[1]**

(iii) Using this value of k, solve equation (B) and hence find the general solution of equation (A), giving your answer in the form $y = \mathrm{f}(x)$. **[4]**

6 **(a)** The set of polynomials $\{ax + b\}$, where $a, b \in \mathbb{R}$, is denoted by P. Assuming that the associativity property holds, prove that P, under addition, is a group. **[4]**

(b) The set of polynomials $\{ax + b\}$, where $a, b \in \{0, 1, 2\}$, is denoted by Q. It is given that Q, under addition modulo 3, is a group, denoted by $\big(Q, +(\text{mod}3)\big)$.

(i) State the order of the group. **[1]**

(ii) Write down the inverse of the element $2x + 1$. **[1]**

(iii) $\mathrm{q}(x) = ax + b$ is any element of Q other than the identity. Find the order of $\mathrm{q}(x)$ and hence determine whether $\big(Q, +(\text{mod}3)\big)$ is a cyclic group. **[4]**

7 (In this question, the notation ΔABC denotes the area of the triangle ABC.)

The points P, Q and R have position vectors $p\mathbf{i}$, $q\mathbf{j}$ and $r\mathbf{k}$ respectively, relative to the origin O, where p, q and r are positive. The points O, P, Q and R are joined to form a tetrahedron.

(i) Draw a sketch of the tetrahedron and write down the values of ΔOPQ, ΔOQR and ΔORP. **[3]**

(ii) Use the definition of the vector product to show that $\frac{1}{2}\left|\overrightarrow{RP} \times \overrightarrow{RQ}\right| = \Delta PQR$. **[1]**

(iii) Show that $(\Delta OPQ)^2 + (\Delta OQR)^2 + (\Delta ORP)^2 = (\Delta PQR)^2$. **[6]**

8 **(i)** Use de Moivre's theorem to express $\cos 4\theta$ as a polynomial in $\cos\theta$. **[4]**

(ii) Hence prove that $\cos 4\theta \cos 2\theta \equiv 16\cos^6\theta - 24\cos^4\theta + 10\cos^2\theta - 1$. **[1]**

(iii) Use part **(ii)** to show that the only roots of the equation $\cos 4\theta \cos 2\theta = 1$ are $\theta = n\pi$, where n is an integer. **[3]**

(iv) Show that $\cos 4\theta \cos 2\theta = -1$ only when $\cos\theta = 0$. **[3]**