OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4733:** Probability and Statistics 2

Mark Scheme for June 2013

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (OCR) Statistics strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

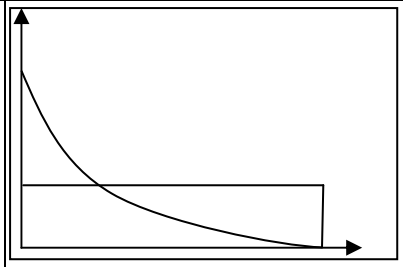
On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		89, 90, 91, 91, 92	B2 **2**	All correct; B2; one error (e.g. all –1), B1	Allow 088, etc
	(ii)		Not all equally likely (91 more than 90 etc) Multiply by 1000 and ignore if > 853	B1 B1 **2**	Imply different likelihood/probability Or equivalent method. Not "ignore repeats". Ignore extras.	*Not* "same pupil *is* selected twice" Number students, use random numbers *and ignore outside range*: B1
2			$\text{Po}(2 \times 10^6)$ $\approx \text{N}(2 \times 10^6, 2 \times 10^6)$ $\Phi\left(\frac{1998999.5 - 2\times10^6}{\sqrt{2\times10^6}}\right) = \Phi(-0.70746)$ **= 0.2396**	M1 A1 A1 A1 **4**	N(their 40λ) Both parameters correct, allow √ here Standardise, mean 40λ, sd $\sqrt{40\lambda}$ (*not* 40λ) Answer, a.r.t. 0.240 (no cc: M1A1A0A1)	*Correct cc must be seen for this A1* NB: no cc gives Φ(–0.7071), 0.23975, wrong cc gives Φ(–0.70675), 0.23986
3	(i)		$\frac{\mu - 157.18}{\sigma/\sqrt{80}} = 1.282$; $\frac{\mu - 164.76}{\sigma/\sqrt{80}} = 0.5244$ Solve simultaneously: μ **= 170** σ **= 89.44**	M1 A1 B1 B1 A1 A1 **6**	Standardise once with √80 or 80 and z, signs may be wrong, allow "1–" errors Both correct ***including signs***, no cc 1.28(155) seen anywhere, correct to 3 SF [0.524, 0.525] seen anywhere μ, a.r.t. 170 to 3 SF (169.98) σ, in range [89, 90], *not* isw *Don't* allow surds, e.g. $40\sqrt{5}$	Allow cc, but *not* 0.1, 0.7, 0.9, 0.3 or Φ(these) [= .5398, .758, .8159, .6179] z may be wrong (provided it *is* z) Ignore signs Ignore signs CWO×2 but allow from inaccurate z if answer(s) within limits. Look out for –89.44: A0A0
	(ii)	(a)	In using normal tables	B1	Or equiv, e.g. "standardising", "dist of $\overline{Y}$"	Any reference to $\sigma/\sqrt{80}$: B0
		(b)	Parent distribution not known	B1	Allow "it is not normal", etc	No extras
		(c)	n large, nothing wrong seen [must be in correct order, no repeats]	B1 **3**	If numerical, must be of the form "$n > n_0$" or "$n \geq n_0$" with $30 \leq n_0 \leq 60$	*Not* "≥ 80".

Question			Answer	Marks	Guidance	
4			$H_0: \lambda = 3.2$ (or 0.32) [Allow μ] $H_1: \lambda > 3.2$ (or 0.32) [Allow μ] $R \sim$ Po(3.2) α: P($R \geq 6$) = 0.1054 > 0.01 β: CR ≥ 9 and 6 < 9, with probability 0.0057 Do not reject H_0. Insufficient evidence of an increase in the number of floods.	B2 M1 A1 A1 A1 A1 M1 A1 ft **7**	Both correct, B2. One error, e.g. wrong/ no/different symbols, or two-tail, B1 Stated or implied, e.g. N(3.2, 3.2) [0.105, 0.106] before rounding Explicit comparison with 0.01 CR ≥ 9 stated; allow CV = 9 if comparison ft 0.0057 or 0.9943 seen, and 6 compared Consistent first conclusion Conclusion, mentions "floods", "evidence" *Not* "evidence of no increase" P($R \leq 6$) = 0.9554; P($R > 6$) = 0.0446; P($R = 6$) = 0.0608: max B2 M1 P($R < 6$) = 0.8946 *and* compare 0.99 etc: can get full marks. Else A0A0M0A0	But x, $\bar{x}$, r, t etc: B0. E(X), words: B1 E.g. H_0: $\lambda_0 = 3.2$, H_1: $\lambda_1 > 3.2$: B1 P(= 6) or (≤ 6) or > 6 or normal: no more marks, maximum B2M1. needs correct method and like-with-like comparison, but 0.01 needn't be explicit
5	(i)			M1 A1 B1 **3**	Upwards parabola, not below x-axis Correct place, not extending beyond limits, ignore pointed at a Horizontal straight line, not beyond limits, y-intercept below curve (unless curve makes this meaningless)	*[scales/annotations not needed]* Touching axes (not asymptotic) Don't need vertical lines i.e., 3/3 only if wholly right
	(ii)		$\int_0^a \frac{3}{a^3} x(x-a)^2 \,\mathrm{d}x$ $= \int_0^a \frac{3}{a^3}(x^3 - 2ax^2 + a^2x)\,\mathrm{d}x$ $= \left[\frac{3}{a^3}\left(\frac{x^4}{4} - \frac{2ax^3}{3} + \frac{a^2x^2}{2}\right)\right]_0^a$ $= \frac{a}{4}$	M1 M1 A1 B1 A1 **5**	Attempt this integral, correct limits seen somewhere Method for $\int xf(x)$, e.g. multiply out or parts, independent of first M1 Correct form for integration, e.g. multiplied out correctly, or correct first stage of parts Correct indefinite integral $\frac{a}{4}$ or exact equivalent (e.g. 0.25a) only	Multiplication: needs 3 terms E.g. $\frac{3}{a^3}x\frac{(x-a)^3}{3} - \int \frac{3}{a^3}\frac{(x-a)^3}{3}\mathrm{d}x$ E.g. $\frac{3}{a^3}x\frac{(x-a)^3}{3} - \frac{3}{a^3}\frac{(x-a)^4}{12}$ Limits not seen anywhere: can get M0M1A0B1A0

Question			Answer	Marks	Guidance	
5	(iii)		S is concentrated more towards 0 Therefore T has bigger variance	M1 A1 **2**	Reason that shows understanding of PDF Correct conclusion	*Not*, e.g., "T is constant"
6			$H_0 : \mu = 38.4$ [Allow E(X) both times] $H_1 : \mu \neq 38.4$	B2	Both correct: B2. One error e.g. no or different symbols, one-tail etc, B1	But $\bar{x}$, x, t etc B0. E.g. H_0: $\mu_0 = 38.4$, H_1: $\mu_1 \neq 38.4$: B1
			$\hat{\mu} = \bar{x} = 36.68$	B1	36.68 seen anywhere	H_0: $\mu = 36.68$, H_1: $\mu \neq 36.68$: B0B0B1 *See below and exemplars*
			$\hat{\sigma}^2 = \frac{50}{49}\left(\frac{70027.37}{50} - 36.68^2\right) = 56.25$	M1 M1 A1	Use biased variance formula [55.125] Multiply by 50/49 56.25	Single formula: M2 or M0. If M0, a divisor of 49 seen anywhere gets M1 Allow rounded if clearly correct
			α: $z = \frac{36.68 - 38.4}{\sqrt{56.25/50}} = -1.62$ > -2.576 [*or* $0.0525 > .005$]	M1 A1 A1ft	Standardise using $\sqrt{50}$ or 50 z, a.r.t. –1.62 or $p = 0.0525$ Compare $-z$ with -2.576 or $+z$ with 2.576	If 50 missing, no more marks p in range [0.052, 0.053] Ft on z. Or p explicitly with 0.005
			β: CV is $38.4 - 2.576\sqrt{\frac{56.25}{50}} = 35.6677$ $36.68 > 35.6677$	M1 A1 A1ft	CV $38.4 - z\sigma/\sqrt{50}$, ignore 38.4 + anything A.r.t 35.7 CV ft and correct comparison	$36.68 + z\sigma/\sqrt{50}$: M1A0A0, M0A0 Ft on wrong z or on $\sqrt{}$ only
			Do not reject H_0. Insufficient evidence of a change in crop yield	M1 A1ft	Correct first conclusion, needs correct method & comparison if seen Contextualised, "evidence" somewhere *Not* "evidence of no change"	Like-with-like, needs μ and $\bar{x}$ right way round, needs 50 Ft on wrong TS and/or CV
					Biased variance [55.125; –1.638 or 0.0508] can get B2B1 M1M0A0 M1A0A1M1A1 (max 8) σ^2 used [–1.529 or 0.0632, or –0.l2162 or 0.4144]: B2B1 M1M1A1 M1A0A1M1A1 (max 10) No $\sqrt{50}$ [–0.2293 or 0.4092]: B2B1 M1M1A1M0 (max 6) H_0/H_1 in terms of 36.68: can get last 4 marks *only* if (36.68 – 38.4) seen, and not (38.4 – 36.68)	
				11		

Question			Answer	Marks	Guidance	
7			H_0: $p = 0.35$	B2	One error (e.g. μ, no symbol, 2-tailed) B1, but $\bar{x}$, t etc: B0. Allow π	H_0: $\mu = 42$, H_1: $\mu > 42$: B1 only
			H_1: $p > 0.35$			
			B(120, 0.35)	M1	B(120, 0.35) stated or implied	
			≈ N(42, 27.3)	M1	N(*np*, *npq*), their attempt at 120 × 0.35	120 × 0.35 × 0.65 *Not* N(*np*, *nq*).
			α: $z = \frac{49.5 - 42}{\sqrt{27.3}}$	A1ft	Standardise, with their *np* and √*npq*, right cc Allow both 49.5 and 50.5 and both in CR	√50 or √120: M1M1A0A0A1M0A0
			= 1.435	A1	*z* in range [1.43, 1.44] before rounding	Or *p* in range [0.075, 0.0764]
			> 1.282 [or 0.0757 < 0.1]	A1ft	Comparison with 1.282, ft on *z*/*p* or √120	Or *p* explicit comparison with 0.1
			β: CV = 42.5 + 1.282 × √27.3 [= 49.198]	A1ft	CV 42.5 + *z*×√27.3, ignore LH, ft on *np*, *npq*	No cc: 48.618, can get A0A1A0
			z = 1.282 and compare 50	A1	*z* = 1.282 used in RH CV and compare 50	
			CR ≥ 50 or ≥ 49.2	A1ft	CV correct ft on *z*, but don't worry about ≥	Must round up. 49 from 49.2: A1A1A0
			Reject H_0.	M1	Consistent first conclusion, needs correct method and comparison	Can give M1A1 even if comparison not explicit. Allow from exact binomial
			Significant evidence that proportion who know regulations has increased	A1ft	Contextualised, needs "who know regulations" or "pupils", and "evidence"	Ft on TS & CV Or exact equivalent somewhere
			np > 5 [= 42] from normal attempted	M1	From *p* = 0.35 or 5/12, don't need 42	*or n* large or *p* close to 0.5 asserted
			nq = 78 > 5 and no others apart from *n* large	A1	Need 78, or 70 from 5/12, *not npq*	*and* the other qualitative reason asserted
			SC: If B0, B(120, 5/12):		Wrong or no cc [1.627, 0.0519 or 1.5311, 0.0629]: loses (α) first two A1A1 only	
			N(50, 29.17) M1M1		Exact B(120, 0.35): P(≥ 50) = 0.076824, CR ≥ 50. B2M1, M0A0A0A0, M1A1M0A0	
			np > 5, *nq* = 70 > 5: M1A1 Max 4		*NB: If S3 difference of proportions test used, consult PE*	
			SC: P(≥ 42): B2 M1M1A0A0A1M0A0			
				11		
8	**(i)**		B(14, 0.25): Critical region ≥ 7	M1	Use B(14, 0.25) and find *r* for an upper tail *All marks need upper tail*	e.g. CV 5 or 6 or 7, or .1117, .0383, .0103, 0.8883, 0.9617, .9897
				A1	CR ≥ 7 or AR ≤ 6 stated or clearly implied	Not just "CV = 7"
			B(14, 0.4): P(≤ 6)	M1	Find P(in AR when *p* = 0.4) *[indept of M1]*	*Not* P(≥ *r*), e.g. final answer 0.3075
			= 0.6925	A1	Answer 0.692 or 0.693 or a.r.t. 0.6925 or 0.6924 only, *not* isw [0.692452]	NB: expect CV 8 or 9 and answer 0.9825 or 0.9417: M0M0
				4		

Question			Answer	Marks	Guidance	
	(ii)		(a) Decreases	B1	One correct answer & one correct reason *or* two correct answers	Allow from numerical calculation
			(b) Decreases; increased prob (Type I) ⇔ decreased prob (Type II)	B1 **2**	Two correct answers and one correct reason, e.g. "CR becomes larger", etc	Allow equivalent or similar reason Allow from numerical calculation
9	**(i)**		Constant *average* rate; *or* [*] same statement *plus* "breakdowns independent"	B1	State "average" or equiv, "random" or "uniform".	No extras apart from independence (ignore "singly")
			Otherwise it means that they occur at exactly regular intervals	B1 **2**	Correct explanation	Can't get from [*]
	(ii)		No because breakdowns more likely in rush hours, etc	B1 **1**	Any plausible reason for either "yes" or "no" that shows understanding of what the *statistical* concept means	Not "equally likely". *Not* reason for (in)dependence, unless [*], which needs *both* conditions if affirmed
	(iii)		13	B1		
			0.0739	B1 **2**	0.074 or a.r.t. 0.0739. Marks independent	
	(iv)		$e^{-\lambda}\frac{\lambda^2}{2!} = 0.0072$	M1*	Correct formula = their 0.0072 seen	
			$\lambda = \sqrt{(0.0144e^{\lambda})}$	M1dep	Rearrange $e^{-\lambda}$ and square root, to get $\lambda = f(\lambda)$	Allow even if left with e^{λ} or $e^{-\lambda}$ or exact equivalent
			$= 0.12e^{\lambda/2}$	A1	Correctly obtain AG, with $k = 0.5$	
			$8.5 \rightarrow 8.4126$; $8.6 \rightarrow 8.8440$	A1	Two correct evaluations to 4 dp at least	4 dp explicitly required
			Therefore solution between 8.5 and 8.6	A1 **5**	All completely correct and deduction stated	CWO, except allow if only 3 SF

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