Mark Scheme (Results)

Summer 2013

GCE Further Pure Mathematics 2 (6668/01R)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2013
Publications Code UA035974

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2+bx+c)=(x+p)(x+q), \text{ where } |pq|=|c|, \text{ leading to x =}$$

$$(ax^2+bx+c)=(mx+p)(nx+q), \text{ where } |pq|=|c| \text{ and } |mn|=|a|, \text{ leading to x =}$$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0, \quad$ leading to x =...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers
Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1.	$z = x \qquad w = \dfrac{x+2\mathrm{i}}{\mathrm{i}x}$	M1A1
	$w = \dfrac{1}{\mathrm{i}} + \dfrac{2\mathrm{i}}{\mathrm{i}x}$	
	$u + \mathrm{i}v = -\mathrm{i} + \dfrac{2}{x}$	
	$\left(u = \dfrac{2}{x}\right) \qquad v = -1$	M1
	$\therefore w$ is on the line $v + 1 = 0$	A1
		4 Marks

NOTES

M1 for replacing at least one z with x to obtain (ie show an appreciation that $y = 0$)

A1 $w = \dfrac{x+2\mathrm{i}}{\mathrm{i}x}$

M1 for writing w as $u + \mathrm{i}v$ and equating real or imaginary parts to obtain either u or v in terms of x or just a number

A1 for giving the equation of the line $v + 1 = 0$ oe must be in terms of v

Question Number	Scheme	Marks
	Q1 - ALTERNATIVE 1: $w = \dfrac{x+iy+2i}{i(x+iy)}$ Replacing z with $x+iy$ $w = \dfrac{x+iy+2i}{-y+ix} \times \dfrac{-y-ix}{-y-ix}$ $w = \dfrac{(x+i(y+2))(-y-ix)}{y^2+x^2}$ $w = \dfrac{2x-i(x^2+y^2+2y)}{y^2+x^2}$ $w = \dfrac{2x-ix^2}{x^2} = \dfrac{2}{x} - i$ **Using y = 0**. This is where the first M1 may be awarded. A1 if correct even if expression is unsimplified **but denominator must be real** $v = -1$ M1, A1 as in main scheme	M1A1 M1A1
	Q1 - ALTERNATIVE 2: $z = \dfrac{2i}{iw-1}$ Writing the transformation as a function of w $z = \dfrac{2i}{i(u+iv)-1}$ $z = \dfrac{2i}{(-v-1)+iu} \times \dfrac{(-v-1)-iu}{(-v-1)-iu}$ $z = \dfrac{2u+2i(-v-1)}{(-v-1)^2+u^2} = \dfrac{2u}{(-v-1)^2+u^2} + i\left(\dfrac{2(-v-1)}{(-v-1)^2+u^2}\right)$ $\left(\dfrac{2(-v-1)}{(v-1)^2+u^2}\right) = 0$ or simply $-2(v+1) = 0$ **Using y = 0** . This is where the first M1 may be awarded. A1 if correct even if expression is unsimplified **but denominator must be real** $v = -1$ M1, A1 as in main mark scheme above	M1A1 M1A1

Question Number	Scheme	Marks
2.	**NB** Allow the first 5 marks with = instead of inequality	
	$\frac{6x}{3-x} > \frac{1}{x+1}$	
	$6x(3-x)(x+1)^2 - (3-x)^2(x+1) > 0$	M1
	$(3-x)(x+1)\left(6x^2+6x-3+x\right) > 0$	
	$(3-x)(x+1)(3x-1)(2x+3) > 0$	M1dep
	Critical values $3, -1$	B1
	and $-\frac{3}{2}, \frac{1}{3}$	A1, A1
	Use critical values to obtain both of $-\frac{3}{2} < x < -1$ $\quad \frac{1}{3} < x < 3$	M1A1cso
		7 Marks

NOTES

M1 for multiplying through by $(x+1)^2(3-x)^2$

OR: for collecting one side of the inequality and attempting to form a single fraction (see alternative in mark scheme)

M1dep for collecting on one side of the inequality and factorising the result of the above (usual rules for factorising the quadratic)

OR: for factorising the numerator - must be a three term quadratic - usual rules for factorising a quadratic (see alternative in mark scheme)

<u>Dependent on the first M mark</u>

B1 for the critical values 3, -1

A1 for either $-\frac{3}{2}$ or $\frac{1}{3}$

A1 for the second of these

NB: the critical values need not be shown explicitly - they may be shown on a sketch or just appear in the ranges or in the working for the ranges.

M1 using **their** 4 critical values to obtain appropriate ranges e.g. use a sketch graph of a quartic, (which must be the correct shape and cross the *x*-axis at the cvs) or a table or number line

A1cso for both of $-\frac{3}{2} < x < -1,\quad \frac{1}{3} < x < 3$

Notes for Question 2 Continued

Set notation acceptable i.e. $\left(-\frac{3}{2},-1\right)\cup\left(\frac{1}{3},3\right)$ All brackets must be round; if square brackets appear anywhere then A0.

If both ranges correct, no working is needed for the last 2 marks, but any working shown must be correct.

Purely graphical methods are unacceptable as the question specifies "Use algebra…".

	Q2 – ALTERNATIVE 1:	
	$\frac{6x}{3-x}-\frac{1}{x+1}>0$	
	$\frac{6x(x+1)-(3-x)}{(3-x)(x+1)}>0$	M1
	$\frac{(3x-1)(2x+3)}{(3-x)(x+1)}>0$	M1dep
	Critical values $3,\ -1$	B1
	and $-\frac{3}{2},\ \frac{1}{3}$	A1A1
	Use critical values to obtain both of $-\frac{3}{2}<x<-1$ $\quad \frac{1}{3}<x<3$	M1A1cso
		7 Marks

Question Number	Scheme	Marks
3(a)	$\frac{2}{(r+1)(r+3)}=\frac{A}{r+1}+\frac{B}{r+3}$ $2=A(r+3)+B(r+1)$ $\frac{2}{(r+1)(r+3)}=\frac{1}{r+1}-\frac{1}{r+3}$ **N.B.** for M mark you may see no working. Some will just use the "cover up" method to write the answer directly. This is acceptable.	M1A1 **(2)**
(b)	$\sum\frac{2}{(r+1)(r+3)}=\sum\frac{1}{r+1}-\frac{1}{r+3}$ $=\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+...$ $+\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{1}{n+2}\right)+\left(\frac{1}{n+1}-\frac{1}{n+3}\right)$ $=\frac{1}{2}+\frac{1}{3}-\frac{1}{n+2}-\frac{1}{n+3}$ $=\frac{5(n+2)(n+3)-6(n+3)-6(n+2)}{6(n+2)(n+3)}$ $=\frac{5n^2+25n+30-12n-30}{6(n+2)(n+3)}$ $=\frac{n(5n+13)}{6(n+2)(n+3)}$ $*$	M1A1ft M1 A1 **(4)**
(c)	$\sum_{10}^{100}=\sum_{1}^{100}-\sum_{1}^{9}$ $=\frac{100(500+13)}{6\times102\times103}-\frac{9\times58}{6\times11\times12}=\frac{1425}{1751}-\frac{29}{44}=0.81382\ldots-0.65909\ldots$ = 0.1547... = 0.155	M1 A1 (2)
		8 Marks

Notes for Question 3

Question 3a

M1 for attempting the PFs - any valid method

A1 for correct PFs $\frac{2}{(r+1)(r+3)}=\frac{1}{r+1}-\frac{1}{r+3}$

N.B. for M mark you may see no working. Some will just use the "cover up" method to write the answer directly. This is acceptable. Award M1A1 if correct, M0A0 otherwise.

Question 3b

If all work in *r* instead of *n*, penalise last A mark only.

M1 for using **their** PFs to list at least 3 terms at the start and 2 terms at the end so the cancelling can be seen. Must start at $r=1$ and end at $r=n$

A1ft for correct terms follow through their PFs

M1 for picking out the (4) remaining terms and attempting to form a single fraction (unsimplified numerator with at least 2 terms correct)

A1cso for $\frac{n(5n+13)}{6(n+2)(n+3)}$ * (Check all steps in the working are correct - in particular 3rd line from end in the mark scheme.)

NB: If final answer reached correctly from $\frac{1}{2}+\frac{1}{3}-\frac{1}{n+2}-\frac{1}{n+3}$ (i.e. working shown from this point onwards) give 4/4 (even without individual terms listed)

Notes for Question 3 Continued
Question 3c M1 for attempting $\sum_{1}^{100} - \sum_{1}^{9}$ using the result from (b) (with numbers substituted) Use of $\sum_{1}^{100} - \sum_{1}^{10}$ scores M0 A1cso for sum = 0.155

Question Number	Scheme	Marks
4(a)	$y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5y=0$	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+y\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}+2\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+5\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$	M1M1
	$\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=\dfrac{-5\dfrac{\mathrm{d}y}{\mathrm{d}x}-3\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}}{y}$	A2,1,0 **(4)**
	Q4a – ALTERNATIVE 1:	
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{-5y-\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}{y}=-5-\dfrac{1}{y}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	
	$\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=\dfrac{1}{y^2}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^3-\dfrac{2}{y}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)$	M1M1 A2,1,0
(b)	When $x=0$ $\quad \dfrac{\mathrm{d}y}{\mathrm{d}x}=2$ and $y=2$	
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{1}{2}(-10-4)=-7$	M1A1
	$\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=\dfrac{-10-3\times2\times-7}{2}=16$	A1
	$y=2+2x-\dfrac{7}{2(!)}x^2+\dfrac{16}{3!}x^3+\ldots$	M1
	$y=2+2x-\dfrac{7}{2}x^2+\dfrac{8}{3}x^3$	A1 **(5)** **9 Marks**

Question Number	Scheme	Marks
	Alternative: $y=2+2x+ax^2+bx^3$	M1
	$\left(2+2x+ax^2+bx^3\right)\left(2a+6bx\right)+\left(2+2ax+3bx^2...\right)^2$ $+5\left(2+2x+ax^2+bx^3\right)=0$	M1
	Coeffs x^0 : $4a+4+10=0 \quad a=-\frac{7}{2}$	A1
	Coeffs x: $4a+12b+8a+10=0 \Rightarrow b=\frac{8}{3}$	A1
	$y=2+2x-\frac{7}{2}x^2+\frac{8}{3}x^3$	A1

NOTES

Accept the dash notation in this question

Question 4a

M1 for using the product rule to differentiate $y\frac{d^2y}{dx^2}$.

M1 for differentiating $5y$ and using the product rule or chain rule to differentiate $\left(\frac{dy}{dx}\right)^2$

A2,1,0 for $\frac{d^3y}{dx^3}=\frac{-5\frac{dy}{dx}-3\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}}{y}$ Give A1A1 if fully correct, A1A0 if **one** error and A0A0 if more than one error. If there are two sign errors and no other error then give A1A0.

Do NOT deduct if the two $\frac{d^2y}{dx^2}$ terms are shown separately.

Alternative to Q4a

Can be re-arranged first and then differentiated.

M1M1 for differentiating, product and chain rule both needed (or quotient rule as an alternative to product rule)

A2,1,0 for $\frac{d^3y}{dx^3}=\frac{1}{y^2}\left(\frac{dy}{dx}\right)^3-\frac{2}{y}\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$ Give A1A1 if fully correct, A1A0 if **one** error and A0A0 if more than one error

Notes for Question 4 Continued

Question 4b

M1 for substituting $\frac{\mathrm{d}y}{\mathrm{d}x}=2$ and $y=2$ in **the equation** to obtain a numerical value for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

A1 for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-7$

A1 for obtaining the correct value, 16, for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$

M1 for using the series $y=\mathrm{f}(x)=\mathrm{f}(0)+x\mathrm{f}'(0)+\frac{x^2}{2!}\mathrm{f}''(0)+\frac{x^3}{3!}\mathrm{f}'''(0)+\ldots$ (2! or 2, 3! or 6) (The general series may be shown explicitly or implied by their substitution)

A1 for $y=2+2x-\frac{7}{2}x^2+\frac{8}{3}x^3$ oe Must have y = .. and be in ascending powers of

Alternative to Q4b

M1 for setting $y=2+2x+ax^2+bx^3$

M1 for $\left(2+2x+ax^2+bx^3\right)\left(2a+6bx\right)+\left(2+2ax+3bx^2\ldots\right)^2+5\left(2+2x+ax^2+bx^3\right)=0$

A1 for equating constant terms to get $a=-\frac{7}{2}$

A1 for equating coeffs of x^2 to get $b=\frac{8}{3}$

A1 for $y=2+2x-\frac{7}{2}x^2+\frac{8}{3}x^3$

Question Number	Scheme	Marks
5(a)	I.F. $=\mathrm{e}^{\int 2\tan x\,\mathrm{d}x}=\mathrm{e}^{2\ln\sec x}=\sec^2 x$	M1A1
	$y\sec^2 x=\int\sec^2 x\sin 2x\,\mathrm{d}x$	M1
	$y\sec^2 x=\int\frac{2\sin x\cos x}{\cos^2 x}\mathrm{d}x=2\int\tan x\,\mathrm{d}x$	
	$y\sec^2 x=2\ln\sec x\ (+c)$	M1depA1
	$y=\frac{2\ln\sec x+c}{\sec^2 x}$	A1ft
		(6)
(b)	$y=2,\ \ x=\frac{\pi}{3}$	
	$2=\frac{2\ln\sec\left(\frac{\pi}{3}\right)+c}{\sec^2\left(\frac{\pi}{3}\right)}$	
	$2=\frac{2\ln(2)+c}{4}$	
	$c=8-2\ln 2$	M1A1
	$x=\frac{\pi}{6}\quad y=\frac{2\ln\sec\left(\frac{\pi}{6}\right)+8-2\ln 2}{\sec^2\left(\frac{\pi}{6}\right)}$	
	$y=\frac{2\ln\frac{2}{\sqrt{3}}+8-2\ln 2}{\frac{4}{3}}$	M1
	$y=\frac{3}{4}\left(8+2\ln\frac{1}{\sqrt{3}}\right)=6+\frac{3}{2}\ln\frac{1}{\sqrt{3}}=6-\frac{3}{4}\ln 3$	A1
		(4)
		10 Marks

Question Number	Scheme	Marks
	Alternative: c may not appear explicitly: $y\sec^2\frac{\pi}{6}-2\sec^2\frac{\pi}{3}=2\ln\left(\frac{\sec\frac{\pi}{6}}{\sec\frac{\pi}{3}}\right)$	M1A1
	$\frac{4}{3}y-8=2\ln\frac{1}{\sqrt{3}}$	
	$y=\frac{3}{4}\left(8+2\ln\frac{1}{\sqrt{3}}\right)=6+\frac{3}{2}\ln\frac{1}{\sqrt{3}}=6-\frac{3}{4}\ln 3$	M1A1

NOTES

Question 5a

M1 for the $e^{\int 2\tan x\,dx}$ or $e^{\int \tan x\,dx}$ and attempting the integration - $e^{(2)\ln\sec x}$ should be seen if final result is not $\sec^2 x$

A1 for IF $=\sec^2 x$

M1 for multiplying the equation by **their** IF and attempting to integrate the lhs

M1dep for attempting the integration of the rhs $\sin 2x=2\sin x\cos x$ and $\sec x=\frac{1}{\cos x}$ needed. Dependent on the second M mark

A1cao for all integration correct ie $y\sec^2 x=2\ln\sec x\ (+c)$ constant not needed

A1ft for re-writing **their** answer in the form $y=\ldots$ Accept any equivalent form but the constant must be present. eg $y=\frac{\ln\left(A\sec^2 x\right)}{\sec^2 x}$, $y=\cos^2 x\left[\ln\left(\sec^2 x\right)+c\right]$

Notes for Question 5 Continued
Question 5b M1 for using the given values $y = 2,\ x = \frac{\pi}{3}$ in **their** general solution to obtain a value for the constant of integration A1 for eg $c = 8 - 2\ln 2$ or $A = \frac{1}{4}e^8$ (Check the constant is correct for their correct answer for (a)). Answers to 3 significant figures acceptable here and can include $\cos\frac{\pi}{3}$ or $\sec\frac{\pi}{3}$ M1 for using **their** constant and $x = \frac{\pi}{6}$ in **their** general solution and attempting the simplification to the required form. A1cao for $y = 6 - \frac{3}{4}\ln 3$ $\left(\frac{3}{4} \text{ or } 0.75\right)$ **Alternative to 5b** M1 for finding the difference between $y\sec^2\frac{\pi}{6}$ and $2\sec^2\frac{\pi}{3}$ (or equivalent with their general solution) A1 for $y\sec^2\frac{\pi}{6} - 2\sec^2\frac{\pi}{3} = 2\ln\left(\frac{\sec\frac{\pi}{6}}{\sec\frac{\pi}{3}}\right)$ M1 for re-arranging to $y = \ldots$ and attempting the simplification to the required form A1cao for $y = 6 - \frac{3}{4}\ln 3$ $\left(\frac{3}{4} \text{ or } 0.75\right)$

Question Number	Scheme	Marks
6(a)	$z^n + z^{-n} = e^{in\theta} + e^{-in\theta}$	
	$= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$	
	$= 2\cos n\theta$ *	M1A1
		(2)
(b)	$\left(z+z^{-1}\right)^5 = 32\cos^5\theta$	B1
	$\left(z+z^{-1}\right)^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$	M1A1
	$32\cos^5\theta = \left(z^5+z^{-5}\right) + 5\left(z^3+z^{-3}\right) + 10\left(z+z^{-1}\right)$	
	$= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	M1
	$\cos^5\theta = \frac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$ *	A1
		(5)
(c)	$\cos 5\theta + 5\cos 3\theta + 10\cos\theta = -2\cos\theta$	M1
	$16\cos^5\theta = -2\cos\theta$	A1
	$2\cos\theta\left(8\cos^4\theta + 1\right) = 0$	
	$8\cos^4\theta + 1 = 0$ no solution	B1
	$\cos\theta = 0$	
	$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$	A1
		(4)
		11 Marks

Notes for Question 6

Question 6a

M1 for using de Moivre's theorem to show that either $z^n = \cos n\theta + \mathrm{i}\sin n\theta$ or $z^{-n} = \cos n\theta - \mathrm{i}\sin n\theta$

A1 for completing to the given result $z^n + z^n = 2\cos n\theta$ *

Question 6b

B1 for using the result in (a) to obtain $\left(z+z^{-1}\right)^5 = 32\cos^5\theta$ Need not be shown explicitly.

M1 for attempting to expand $\left(z+z^{-1}\right)^5$ by binomial, Pascal's triangle or multiplying out the brackets. If ${}^nC_r$ is used do not award marks until changed to numbers

A1 for a correct expansion $\left(z+z^{-1}\right)^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$

M1 for replacing $\left(z^5+z^{-5}\right), \left(z^3+z^{-3}\right), \left(z+z^{-1}\right)$ with $2\cos 5\theta,\ 2\cos 3\theta,\ 2\cos\theta$ and equating their revised expression to their result for $\left(z+z^{-1}\right)^5 = 32\cos^5\theta$

A1cso for $\cos^5\theta = \dfrac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$ *

Question 6c

M1 for attempting re-arrange the equation with one side matching the bracket in the result in (b) Question states "hence", so no other method is allowed.

A1 for using the result in (b) to obtain $16\cos^5\theta = -2\cos\theta$ oe

B1 for stating that there is no solution for $8\cos^4\theta + 1 = 0$ oe eg $8\cos^4\theta + 1 \neq 0$ $8\cos^4\theta + 1 > 0$ or "ignore" but $\cos\theta = \sqrt[4]{-\dfrac{1}{8}}$ without comment gets B0

A1 for $\theta = \dfrac{\pi}{2}$ **and** $\dfrac{3\pi}{2}$ and no more in the range. **Must** be in radians, can be in decimals (1.57..., 4.71... 3 sf or better)

Question Number	Scheme	Marks
7(a)	$y = \lambda t^2 e^{3t}$	
	$\frac{dy}{dt} = 2\lambda t e^{3t} + 3\lambda t^2 e^{3t}$	M1A1
	$\frac{d^2y}{dt^2} = 2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t}$	A1
	$2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t} - 12\lambda t e^{3t} - 18\lambda t^2 e^{3t} + 9\lambda t^2 e^{3t} = 6e^{3t}$	M1dep
	$\lambda = 3$	A1cso
		(5)
	NB. Candidates who give $\lambda = 3$ without all this working get 5/5 provided no erroneous working is seen.	
(b)	$m^2 - 6m + 9 = 0$	
	$(m-3)^2 = 0$	
	C.F. $(y =)\ (A + Bt)e^{3t}$	M1A1
	G.S. $y = (A + Bt)e^{3t} + 3t^2 e^{3t}$	A1ft
		(3)
(c)	$t = 0\ \ y = 5\ \ \Rightarrow A = 5$	B1
	$\frac{dy}{dt} = Be^{3t} + 3(A + Bt)e^{3t} + 6te^{3t} + 9t^2 e^{3t}$	M1
	$\frac{dy}{dt} = 4 \qquad 4 = B + 15$	M1dep
	$B = -11$	A1
	Solution: $y = (5 - 11t)e^{3t} + 3t^2 e^{3t}$	A1ft
		(5)
		13 Marks

Notes for Question 7

Question 7a

M1 for differentiating $y = \lambda t^2 e^{3t}$ wrt t. Product rule must be used.

A1 for correct differentiation ie $\frac{dy}{dt} = 2\lambda t e^{3t} + 3\lambda t^2 e^{3t}$

A1 for a correct second differential $\frac{d^2y}{dt^2} = 2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t}$

M1dep for substituting their differentials in the equation and obtaining a numerical value for λ
Dependent on the first M mark.

A1cso for $\lambda = 3$ (no incorrect working seen)

NB. Candidates who give $\lambda = 3$ without all this working get 5/5 provided no erroneous working is seen. Candidates who attempt the differentiation should be marked on that. If they then go straight to $\lambda = 3$ without showing the substitution, give M1A1 if differentiation correct and M1A0 otherwise, as the solution is incorrect. If $\lambda \neq 3$ then the M mark is only available if the substitution is shown.

Question 7b

M1 for solving the 3 term quadratic auxiliary equation to obtain a value or values for *m* (usual rules for solving a quadratic equation)

A1 for the CF $(y =)$ $(A + Bt)e^{3t}$

A1ft for using **their** CF and **their numerical** value of λ in the particular integral to obtain the general solution $y = (A + Bt)e^{3t} + 3t^2 e^{3t}$ Must have *y* = ... and rhs must be a function of *t*.

Question 7c

B1 for deducing that $A = 5$

M1 for differentiating **their** GS to obtain $\frac{dy}{dt} = ...$ The product rule must be used.

M1dep for using $\frac{dy}{dt} = 4$ and **their** value for *A* in **their** $\frac{dy}{dt}$ to obtain an equation for *B* Dependent on the previous M mark (of (c))

A1cao and cso for $B = -11$

A1ft for using **their** numerical values *A* and *B* in **their** GS from (b) to obtain the particular solution. Must have *y* = ... and rhs must be a function of *t*.

Question Number	Scheme	Marks
8 (a)	$A = (4\times)\int_0^{\frac{\pi}{4}} \frac{9}{2}\cos 2\theta \,\mathrm{d}\theta$	M1A1(limits for A mark only)
	$= 18\left[\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$	M1
	$9\left[\sin\frac{\pi}{2} - 0\right] = 9$	A1
		(4)
(b)	$r = 3(\cos 2\theta)^{\frac{1}{2}}$	
	$r\sin\theta = 3(\cos 2\theta)^{\frac{1}{2}}\sin\theta$	M1
	$\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta) = \left\{-3\times\frac{1}{2}(\cos 2\theta)^{-\frac{1}{2}}\times 2\sin 2\theta\sin\theta + 3(\cos 2\theta)^{\frac{1}{2}}\cos\theta\right\}$	M1depA1
	At max/min: $\frac{-3\sin 2\theta\sin\theta}{(\cos 2\theta)^{\frac{1}{2}}} + 3(\cos 2\theta)^{\frac{1}{2}}\cos\theta = 0$	M1
	$\sin 2\theta\sin\theta = \cos 2\theta\cos\theta$	
	$2\sin^2\theta\cos\theta = (1-2\sin^2\theta)\cos\theta$	
	$\cos\theta(1-4\sin^2\theta) = 0$	
	$(\cos\theta = 0)\quad \sin^2\theta = \frac{1}{4}$	
	$\sin\theta = \pm\frac{1}{2}\qquad \theta = \pm\frac{\pi}{6}$	M1A1
	$r\sin\frac{\pi}{6} = 3\left(\cos\frac{\pi}{3}\right)^{\frac{1}{2}}\times\frac{1}{2} = \frac{3\sqrt{2}}{4}$	B1
	$\therefore$ length $PS = \frac{3\sqrt{2}}{2}$, (length PQ = 6)	

Question Number	Scheme	Marks
	Shaded area = $6\times\dfrac{3\sqrt{2}}{2}-9, =9\sqrt{2}-9$ oe	M1,A1 **(9)** **13 Marks**

NOTES

Question 8a

M1 for $A=\dfrac{1}{2}\int r^2\,\mathrm{d}\theta=\dfrac{1}{2}\int\alpha\cos 2\theta\,\mathrm{d}\theta$ with $\alpha=3$ or 9 4 or 2 and limits not needed for this mark - ignore any shown.

A1 for $A=(4\times)\int_0^{\frac{\pi}{4}}\dfrac{9}{2}\cos 2\theta\,\mathrm{d}\theta$ Correct limits $\left(0,\dfrac{\pi}{4}\right)$ with multiple 4 or $\left(-\dfrac{\pi}{4},\dfrac{\pi}{4}\right)$ with multiple 2 needed. 4 or 2 may be omitted here, provided it is used later.

M1 for the integration $\cos 2\theta$ to become $\pm\left(\dfrac{1}{2}\right)\sin 2\theta$ Give M0 for $\pm 2\sin 2\theta$. Limits and 4 or 2 not needed

A1cso for using the limits and 4 or 2 as appropriate to obtain 9

ALTERNATIVES ON FOLLOWING PAGES

Notes for Question 8 Continued
Question 8b M1 for $r\sin\theta = 3(\cos 2\theta)^{\frac{1}{2}}\sin\theta$ or $r^2\sin^2\theta = 9\cos 2\theta\sin^2\theta$ 3 or 9 allowed M1dep for differentiating the rhs of the above wrt θ. Product and chain rule must be used. A1 for $\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta) = \left\{-3\times\frac{1}{2}(\cos 2\theta)^{-\frac{1}{2}}\times 2\sin 2\theta\sin\theta + 3(\cos 2\theta)^{\frac{1}{2}}\cos\theta\right\}$ or correct differentiation of $r^2\sin^2\theta = 9\cos 2\theta\sin^2\theta$ M1 for equating their expression for $\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta)$ to 0 M1dep for solving the resulting equation to $\sin k\theta = ...$ or $\cos k\theta = ...$ incuding the use of the appropriate trig formulae (must be correct formulae) A1 for $\sin\theta = \frac{1}{2}$ or $\cos\theta = \frac{\sqrt{3}}{2}$ or $\theta = (\pm)\frac{\pi}{6}$ oe ignore extra answers B1 for the length of $\frac{1}{2}PS = \frac{3\sqrt{2}}{4}$ (1.0606...) or of PS May not be shown explicitly. Give this mark if the correct area of the rectangle is shown. Length of PQ is not needed for this mark. M1 for attempting the shaded area by their $PS \times 6 -$ **their** answer to (a). There must be evidence of PS being obtained using their θ A1 for $9\sqrt{2} - 9$ oe 3.7279....or awrt 3.73 ALTERNATIVES ON FOLLOWING PAGES

Option 1 – using $r\sin\theta$ with/without manipulation of $\cos 2\theta$ before differentiation

Use of $3(\cos 2\theta)^{\frac{1}{2}}\sin\theta$	First M mark
$3(\cos 2\theta)^{\frac{1}{2}}\cos\theta - 3\left(\frac{1}{2}\right)(\cos 2\theta)^{-\frac{1}{2}}(2)\sin 2\theta\sin\theta = 0$ $3(\cos 2\theta)^{\frac{1}{2}}\cos\theta - 3(\cos 2\theta)^{-\frac{1}{2}}\sin 2\theta\sin\theta = 0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$3(\cos 2\theta)^{\frac{1}{2}} - 6(\cos 2\theta)^{-\frac{1}{2}}\sin^2\theta = 0$ $\cos 2\theta - 2\sin^2\theta = 0$	Use of $\sin 2\theta = 2\sin\theta\cos\theta$, division by $3\cos\theta$ and multiplication by $(\cos 2\theta)^{\frac{1}{2}}$ simplify the equation but do not provide specific M marks
$(1-2\sin^2\theta) - 2\sin^2\theta = 0$ $4\sin^2\theta = 1$	Use of $\cos 2\theta = 1-2\sin^2\theta$ gives next M mark provided a value of $\sin\theta$ or alt is reached with no errors seen
$\sin\theta = \pm\frac{1}{2}$ $\left(\theta = \frac{\pi}{6}\right)$	Value of $\sin\theta$ reached with use of $\cos 2\theta = ...$ and no method errors seen (arithmetic slips would be condoned) gives final M mark Second accuracy mark given here.

Use of $3\left(\cos^2\theta-\sin^2\theta\right)^{\frac{1}{2}}\sin\theta$	First M mark Use of $\cos 2\theta=\cos^2\theta-\sin^2\theta$ gives 4th M mark provided a value of $\sin\theta$ or alt is reached with no errors seen after the differentiation
$3\left(\frac{1}{2}\right)\left(\cos^2\theta-\sin^2\theta\right)^{-\frac{1}{2}}\left(-2\cos\theta\sin\theta-2\sin\theta\cos\theta\right)\sin\theta$ $+3\left(\cos^2\theta-\sin^2\theta\right)^{\frac{1}{2}}\cos\theta=0$ $-6\left(\cos^2\theta-\sin^2\theta\right)^{-\frac{1}{2}}\cos\theta\sin^2\theta+3\left(\cos^2\theta-\sin^2\theta\right)^{\frac{1}{2}}\cos\theta=0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$-6\sin^2\theta+3\left(\cos^2\theta-\sin^2\theta\right)=0$ $4\sin^2\theta=1$	Multiplication by $\left(\cos^2\theta-\sin^2\theta\right)^{\frac{1}{2}}$, division by $3\cos\theta$ and use of $\cos^2\theta=1-\sin^2\theta$ simplify the equation but do not provide specific M marks
$\sin\theta=\pm\frac{1}{2}$ $\left(\theta=\frac{\pi}{6}\right)$	Value of $\sin\theta$ reached with use of $\cos 2\theta=\ldots$ and no method errors seen (arithmetic slips would be condoned) gives final M mark Second accuracy mark given here.

Use of $3\left(2\cos^2\theta-1\right)^{\frac{1}{2}}\sin\theta$	First M mark Use of $\cos 2\theta = 2\cos^2\theta - 1$ gives 4th M mark provided a value of $\sin\theta$ or alt is reached with no errors seen after the differentiation
$3\left(\frac{1}{2}\right)\left(2\cos^2\theta-1\right)^{-\frac{1}{2}}\left(-4\cos\theta\sin\theta\right)\sin\theta+3\left(2\cos^2\theta-1\right)^{\frac{1}{2}}\cos\theta=0$ $-6\left(2\cos^2\theta-1\right)^{-\frac{1}{2}}\cos\theta\sin^2\theta+3\left(2\cos^2\theta-1\right)^{\frac{1}{2}}\cos\theta=0$	Second (dep) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$-6\sin^2\theta+3\left(2\cos^2\theta-1\right)=0$ $4\sin^2\theta=1$ or $4\cos^2\theta=3$	Multiplication by $\left(\cos^2\theta-\sin^2\theta\right)^{\frac{1}{2}}$, division by $3\cos\theta$ and use of $\sin^2\theta=1-\cos^2\theta$ or vice versa simplify the equation but do not provide specific M marks
$\sin\theta=\pm\frac{1}{2}$ or $\cos\theta=\pm\frac{\sqrt{3}}{2}$ $\left(\theta=\frac{\pi}{6}\right)$	Value of $\sin\theta$ or $\cos\theta$ reached with use of $\cos 2\theta = \ldots$ and no method errors seen (arithmetic slips would be condoned) gives final M mark. Second accuracy mark given here.

Use of $3\left(1-2\sin^2\theta\right)^{\frac{1}{2}}\sin\theta$	First M mark Use of $\cos 2\theta = 2\cos^2\theta - 1$ gives 4th M mark provided a value of $\sin\theta$ or alt is reached with no errors seen after the differentiation
$3\left(\frac{1}{2}\right)\left(1-2\sin^2\theta\right)^{-\frac{1}{2}}\left(-4\cos\theta\sin\theta\right)\sin\theta + 3\left(1-2\sin^2\theta\right)^{\frac{1}{2}}\cos\theta = 0$ $-6\left(1-2\sin^2\theta\right)^{-\frac{1}{2}}\cos\theta\sin^2\theta + 3\left(1-2\sin^2\theta\right)^{\frac{1}{2}}\cos\theta = 0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$-6\sin^2\theta + 3\left(1-2\cos^2\theta\right) = 0$ $4\sin^2\theta = 1$ or $4\cos^2\theta = 3$	Multiplication by $\left(\cos^2\theta - \sin^2\theta\right)^{\frac{1}{2}}$, division by $3\cos\theta$ and use of $\sin^2\theta = 1-\cos^2\theta$ or vice versa simplify the equation but do not provide specific M marks
$\sin\theta = \pm\frac{1}{2}$ or $\cos\theta = \pm\frac{\sqrt{3}}{2}$ $\left(\theta = \frac{\pi}{6}\right)$	Value of $\sin\theta$ or $\cos\theta$ reached with use of $\cos 2\theta = \ldots$ and no method errors seen (arithmetic slips would be condoned) gives final M mark. Second A mark given here.

Option 2 – using $r^2\sin^2\theta$ with/without manipulation of $\cos 2\theta$ before differentiation

Use of $9\cos 2\theta\sin^2\theta$	First M mark even if they have a slip on the 9 and use 3 but must be $\sin^2\theta$
$-9(2)\sin 2\theta\sin^2\theta+9(2)\cos 2\theta\sin\theta\cos\theta=0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$-2\sin^2\theta+\cos 2\theta=0$ or $-\sin 2\theta\sin\theta+\cos 2\theta\cos\theta=0$ leading to $-2\sin^2\theta+\cos 2\theta=0$ or $\cos 3\theta=1$ (compound angle formula)	Division by $9\sin 2\theta$ or $18\sin\theta$ and use of $\sin 2\theta=2\sin\theta\cos\theta$ followed by division by $\cos\theta$ will simplify the equation but not provide specific M marks
$-2\sin^2\theta+1-2\sin^2\theta=0$ $4\sin^2\theta=1$	Use of $\cos 2\theta=1-2\sin^2\theta$ gives next M mark provided a value of $\sin\theta$ or alt is reached with no errors seen
$\sin\theta=\pm\frac{1}{2}$ or $3\theta=2\pi$ (from $\cos 3\theta=1$) $\left(\theta=\frac{\pi}{6}\right)$	Value of $\sin\theta$ or alt reached with use of $\cos 2\theta=\ldots$ and no method errors seen (arithmetic slips would be condoned) gives final M mark Second accuracy mark given here.

Use of $9\left(\cos^2\theta-\sin^2\theta\right)\sin^2\theta$ Could be expanded out to $9\cos^2\theta\sin^2\theta-9\sin^4\theta$ before differentiation in which case the derivative is immediately given by $-18\cos\theta\sin^3\theta+18\cos^3\theta\sin\theta-36\sin^3\theta\cos\theta$	First M mark even if they have a slip on the 9 and use 3 but must be $\sin^2\theta$ Use of $\cos 2\theta = 2\cos^2\theta - 1$ gives 4th M mark provided a value of $\sin\theta$ or alt is reached with no errors seen after the differentiation
$9\left(-2\cos\theta\sin\theta-2\sin\theta\cos\theta\right)\sin^2\theta+9\left(\cos^2\theta-\sin^2\theta\right)2\sin\theta$ $-36\sin^3\theta\cos\theta+18\left(\cos^2\theta-\sin^2\theta\right)\sin\theta\cos\theta=0$ $-36\cos\theta\sin^3\theta+18\cos^3\theta\sin\theta-18\sin^3\theta\cos\theta=0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$18\cos^3\theta\sin\theta-54\sin^3\theta\cos\theta=0$ $\cos^2\theta-3\sin^2\theta=0$ $1-4\sin^2\theta=0$ or $4\cos^2\theta-3=0$	Division by $18\cos\theta\sin\theta$ and use of $\sin^2\theta=1-\cos^2\theta$ or vice versa will simplify the equation but not provide specific M marks
$\sin\theta=\pm\frac{1}{2}$ or $\cos\theta=\pm\frac{\sqrt{3}}{2}$ $\left(\theta=\frac{\pi}{6}\right)$	Value of $\sin\theta$ or $\cos\theta$ reached with use of $\cos 2\theta = \ldots$ and no method errors seen (arithmetic slips would be condoned) gives final M mark Second accuracy mark given here.

Use of $9(2\cos^2\theta-1)\sin^2\theta$ Could be expanded out to $18\cos^2\theta\sin^2\theta-9\sin^2\theta$ before differentiation in which case the derivative is immediately given by $-36\cos\theta\sin^3\theta+36\cos^3\theta\sin\theta-18\sin\theta\cos\theta$	First M mark even if they have a slip on the 9 and use 3 but must be $\sin^2\theta$ Use of $\cos 2\theta=2\cos^2\theta-1$ gives 4th M mark provided a value of $\sin\theta$ or alt is reached with no errors seen after the differentiation
$9(-4\cos\theta\sin\theta)\sin^2\theta+9(2\cos^2\theta-1)2\sin\theta\cos\theta=0$ $-36\sin^3\theta\cos\theta+36\cos^3\theta\sin\theta-18\sin\theta\cos\theta=0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$-2\sin^2\theta+2\cos^2\theta-1=0$ $2\cos 2\theta=1$ or $1-4\sin^2\theta=0$ or $4\cos^2\theta-3=0$	Division by $18\cos\theta\sin\theta$ and use of $\sin^2\theta=1-\cos^2\theta$ or vice versa will simplify the equation but not provide specific M marks It is also possible to use $\cos^2\theta-\sin^2\theta=\cos 2\theta$ here
$\sin\theta=\pm\frac{1}{2}$ or $\cos\theta=\pm\frac{\sqrt{3}}{2}$ or $\cos 2\theta=\frac{1}{2}$ $\left(\theta=\frac{\pi}{6}\right)$	Value of $\sin\theta$ or alt reached with use of $\cos 2\theta=\ldots$ and no method errors seen (arithmetic slips would be condoned) gives final M mark Second accuracy mark given here.

Use of $9\left(1-2\sin^2\theta\right)\sin^2\theta$ Could be expanded out to $9\sin^2\theta - 18\sin^4\theta$ before differentiation in which case the derivative is immediately given by $18\sin\theta\cos\theta - 72\cos\theta\sin^3\theta$	First M mark even if they have a slip on the 9 and use 3 but must be $\sin^2\theta$ Use of $\cos 2\theta = 2\cos^2\theta - 1$ gives 4th M mark provided a value of $\sin\theta$ or alt is reached with no errors seen after the differentiation
$9\left(-4\cos\theta\sin\theta\right)\sin^2\theta + 9\left(1-2\sin^2\theta\right)2\sin\theta\cos\theta = 0$ $-36\sin^3\theta\cos\theta - 36\sin^3\theta\cos\theta + 18\sin\theta\cos\theta = 0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$1 - 4\sin^2\theta = 0$	Division by $18\cos\theta\sin\theta$ will simplify the equation but not provide specific M marks
$\sin\theta = \pm\frac{1}{2}$ or $\cos\theta = \pm\frac{\sqrt{3}}{2}$ or $\cos 2\theta = \frac{1}{2}$ $\left(\theta = \frac{\pi}{6}\right)$	Value of $\sin\theta$ or alt reached with use of $\cos 2\theta = \ldots$ and no method errors seen (arithmetic slips would be condoned) gives final M mark Second accuracy mark given here.

Using the factor formulae after differentiating $3(\cos 2\theta)^{\frac{1}{2}}\sin\theta$ **:**

M1 awarded for using $3(\cos 2\theta)^{\frac{1}{2}}\sin\theta$

$$3\left(\frac{1}{2}\right)(\cos 2\theta)^{-\frac{1}{2}}(-2\sin 2\theta)\sin\theta + 3(\cos 2\theta)^{\frac{1}{2}}\cos\theta = 0$$

M1A1 awarded for correct differentiation using product and chain rule

M1 for setting derivative equal to zero

Multiplication by $(\cos 2\theta)^{\frac{1}{2}}$ and division by 3 gives

$$\cos 2\theta\cos\theta - \sin 2\theta\sin\theta = 0$$
$$\cos 3\theta = 0$$

dM1 mark can now be awarded for using correct trigonometric formulae to reduce the equation to $\cos k\theta = \ldots$ but the A mark requires $\cos\theta = \ldots$ or $\theta = \dfrac{\pi}{6}$

$$3\theta = \frac{\pi}{2}$$
$$\theta = \frac{\pi}{6}$$

The A1 mark can now be awarded

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481
Email publication.orders@edexcel.com

Order Code UA035974 Summer 2013

For more information on Edexcel qualifications, please visit our website www.edexcel.com

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Llywodraeth Cynulliad Cymru
Welsh Assembly Government

Rewarding Learning