OCR
Oxford Cambridge and RSA

GCE

Mathematics

Unit **4725**: Further Pure Mathematics 1

Advanced Subsidiary GCE

Mark Scheme for June 2017

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the

establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be

the last (complete) attempt and ignore the others.

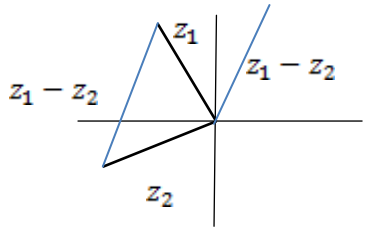
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1.			$\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)-8n$	M1	Use at least 2 correct standard results, must have 3 terms
				A1	Obtain correct unsimplified answer
				M1	Attempt to factorise, must get at least a factor of n and simplify
			$\frac{1}{6}n(2n^2-50)$	A1	Obtain correct answer
			$\frac{1}{3}n(n-5)(n+5)$	A1 **[5]**	Obtain correct final answer
2.			$x^2-y^2=43, 2xyi=-6\sqrt{10}\,i$	M1	Attempt to equate real and imaginary parts of $(x+iy)^2$ and $43-6\sqrt{10}i$
				A1	Obtain both results a.e.f.
				M1	Obtain and solve a quadratic in x^2 or y^2 and attempt to square root [$x^4-43x^2-90=0, y^4+43y^2-90=0$]
			$3\sqrt{5}-i\sqrt{2}, \quad -3\sqrt{5}+i\sqrt{2}$ Or $\pm(3\sqrt{5}-i\sqrt{2})$ or $\pm 3\sqrt{5}\mp i\sqrt{2}$	A1A1 **[5]**	Obtain correct answers as complex numbers a.e.e.f. **N.B.** $\pm(3\sqrt{5}\pm i\sqrt{2})$ gets A1, $\pm(3\sqrt{5}\mp i\sqrt{2})$ gets A1, $\pm 3\sqrt{5}\pm i\sqrt{2}$ gets A0
3.	(i)		$7\mathbf{A}-\mathbf{I}=\begin{pmatrix}6 & 28\\ -14 & 7a-1\end{pmatrix}$	B2 **[2]**	B1 for 3 elements correct or B1 for 4 elements correct but brackets omitted
	(ii)		*Either* $(\mathbf{A}^{-1}\mathbf{B}^{-1})^{-1}=\mathbf{BA}$ $\begin{pmatrix}1 & 28+3a\\ -9 & 4+5a\end{pmatrix}$	B1 M1 A1 **[3]**	Stated or used Attempt at multiplication of **BA** or **AB**, 2 elements correct Obtain correct answer
			Or $\mathbf{A}^{-1}=\frac{1}{a+8}\begin{pmatrix}a & -4\\ 2 & 1\end{pmatrix}$, $\mathbf{B}^{-1}=\frac{1}{82}\begin{pmatrix}5 & -3\\ -1 & 7\end{pmatrix}$	B1	Both correct
			$\begin{pmatrix}1 & 28+3a\\ -9 & 4+5a\end{pmatrix}$	M1 A1	Attempt at multiplication of their $\mathbf{A}^{-1}\mathbf{B}^{-1}$ only, ignore dets, 2 elements correct Obtain correct answer

Question			Answer	Marks	Guidance
4.				B1	Show clearly that result is true when $n = 1$
			$\frac{n}{2n+1}+\frac{1}{(2n+1)(2n+3)}$	M1*	Add correct $(n + 1)$th term to given result
			$\frac{n(2n+3)+1}{(2n+1)(2n+3)}$	DM1	Express as a single fraction with a correct denominator
			$\frac{n+1}{2n+3}$	A1	Show correct factorisation and obtain correct simplified answer
				B1 **[5]**	Clear statement of induction conclusion, previous 4 marks must be earned. Must include somewhere "true for $n = 1$", "true for n implies true for $n + 1$", "true for all n "
5.	**(i)**			B1 B1 B1 **[3]**	Shear, must be shear (only) otherwise 0/3 x-axis invariant, allow parallel to or along x-axis, in x direction (not in x-axis) Image of e.g. (0, 1) is (5,1) or column vectors allow 0.197^c, 11.3^o, $\tan^{-1}$ (1/5) or the complement, ignore scale factor if all OK otherwise
	(ii)		$\begin{pmatrix}1 & 0\\0 & 4\end{pmatrix}$	B1 B1 **[2]**	Each column correct
	(iii)		$\begin{pmatrix}1 & 5\\0 & 4\end{pmatrix}$	M1 A1ft **[2]**	Multiply matrices in correct order, or consider image of columns of P under Q Obtain correct answer, ft their (ii)
	iv)		(Area =) 4	M1 A1ft **[2]**	Find the determinant of a relevant matrix Obtain correct answer, ft their (iii) N.B. it is possible to consider scale factor for each transformation or draw a diagram

Question			Answer	Marks	Guidance
6	(i)			B1 B1 B1ft **[3]**	z_1 shown in 2nd quadrant, as a point or line segment z_2 shown in 3rd quadrant, as a point or line segment $z_1 - z_2$ clearly shown as line segment (either position) or point in 1st quadrant, $\lvert z_1 - z_2 \rvert$ must be > 3 and arg($z_1 - z_2$) reasonably accurate, ft from incorrect z_1 or z_2, ignore scales, penalise wrong "vector" arrows if shown
	(ii)		$3\sqrt{2}$ $7\pi/20$	B1 M1 A1 M1 A1 **[5]**	Find $z_1Oz_2 = 90^0$, or equivalent right angle Use Pythagoras' or other trig method Obtain correct answer a.e.e.f. Find relevant angles Obtain correct answer **N.B.** if they give decimal values in z_1 and z_2, max of M1 M1 only
	(iii)		Perpendicular bisector of Z_1Z_2	B2 **[2]**	Clear statement of required straight line Allow B1 for (straight) line (not circle)
7.	(i)		$\frac{2r+5-(2r-1)}{(2r-1)(2r+5)}$ or $\frac{2r+5-2r+1}{(2r-1)(2r+5)}$ must be seen	B1 **[1]**	Derive **given** result correctly (or use complete partial fractions method) **N.B.** 1st expression could be 2 separate fractions
	(ii)		*Either* 0.629 *Or* $\sum_{2}^{30}\frac{1}{2r-1}=1.68237..$ $\sum_{2}^{30}\frac{1}{2r+5}=1.05383..$ 0.629	M1* A1 A1 DM1 A1 **[5]** M1 A1 A1 M1 A1	Express at least 2 terms as differences using (i), could start at $r = 1$ Obtain $\frac{1}{3},\frac{1}{5},\frac{1}{7}$,obtain $-\frac{1}{61},-\frac{1}{63},-\frac{1}{65}$ these may be unsimplified Show correct cancelling (and subtraction of 1st term if appropriate) Obtain correct answer Attempt to find sum of 2 separate series, could start at $r = 1$ Obtain correct answers, must be at least 4 decimal places Subtract their values Obtain correct answer
	(iii)		71/105	B1 **[1]**	Obtain correct answer

Question	Answer	Marks	Guidance
8.	*Either* $1-\frac{5}{2}i$ is a root	B1	State or use other complex root
	4 is a root	B1	State or use real root
	$\sum\alpha = -\frac{a}{4}$	M1	Use correct symmetric function, must include -ve
	$a = -24$	A1	Obtain correct answer
	$\sum\alpha\beta = \frac{b}{4}$ or $\alpha\beta\gamma = -\frac{c}{4}$	M1	Use another correct symmetric function
	$b = 45, c = -52$	A1 A1	Obtain correct answers
		[7]	
	Or $1-\frac{5}{2}i$ is a root	B1	State or use other complex root
	4 is a root	B1	State or use real root
		M1	Express as product of 3 linear factors
		M1	Expand to obtain a cubic expression $4x^3$ ……
	$a = -24, b = 45, c = -52$	A1A1A1	Obtain correct answers
	Or		
	$\sum\alpha = -\frac{a}{4}$	M1	Use correct symmetric function, must include -ve
	$a = -24$	A1	Obtain correct answer
		M1	Substitute given complex root into the cubic equation and attempt to simplify, use their *a*
		M1	
	$b = 45$	A1	Use imaginary part
		M1	Obtain correct answer
	$c = -52$	A1	Use real part
			Obtain correct answer
	Or		
		B1	
	$256 + 16a + 4b + c = 0$	B1	State of use real root
		M1	Use this root in cubic equation
		M1	Substitute given complex root into the cubic equation and attempt to simplify
	$a = -24, b = 45, c = -52$	A1A1A1	Use real and imaginary parts to obtain 2 more equations for *a* and *b*
			Obtain correct answers

Question			Answer	Marks	Guidance
9	(i)		$2a^2 - 8a + 8$ $a = 2$	M1 M1 A1 M1 A1 **[5]**	Show correct process for det of a 3 × 3, condone sign errors (Cramer's rule is M2) Show correct processes for a 2 × 2 Obtain correct answer Attempt to solve det C = 0, must be a quadratic Obtain correct answer
	(ii)		Any $a \neq 2$, so **C** non-singular or C has an inverse or det **C** $\neq 0$ or det C > 0	B1ft **[1]**	Must be consistent with their (i)
	(iii)		e.g. $x + y = -1, x + y = p$ or $2p - 1 = p - 2$ $p = -1$	M1 A1 A1 **[3]**	Put $a = 2$ (their a from (i)) and attempt to eliminate x, y or z or use eqn1 +eqn2 = eqn3 Obtain a correct pair of equations e.g. in x and y or correct equation Obtain correct answer
10.	(i)		$2ab = ad - bc$	B1 M1 M1 A1 **[4]**	State or use $z^* = a - ib$ Attempt to expand both expressions Equate imaginary parts Obtain **given** answer **N.B.** ignore errors in real parts
	(ii)		*Either* $a^2 - b^2 = bd, ad = 2ab$ $b = \pm\frac{a}{\sqrt{3}}$ *Or* $a^2 - b^2 + 2ab\text{i} = ad\text{i} + \text{bd}$ $d = \frac{(a^2 - b^2 + 2ab\text{i})(b - \text{i}a)}{a^2 + b^2}$ $b = \pm\frac{a}{\sqrt{3}}$	B1 M1 A1 M1 A2 **[6]** B1 M1 A1 M1 A2	State or use $c = 0$ Equate real parts, c need not = 0 at this stage Obtain 2 correct equations a.e.f. Eliminate d Obtain correct answers a.e.f., A1 for 1 correct answer **S.C.** $a = \pm\sqrt{3}b$ gets A1 State or use $c = 0$ in given expression Rearrange to make d the subject and rationalise Obtain correct answer Equate imaginary part to 0 Obtain correct answers a.e.f., A1 for 1 correct answer **S.C.** $a = \pm\sqrt{3}b$ gets A1

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