OCR
Oxford Cambridge and RSA

Friday 19 May 2017 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 Find $\sum_{r=1}^{n}(r^2-r-8)$, giving your answer in a fully factorised form. **[5]**

2 Use an algebraic method to find the square roots of the complex number $43-(6\sqrt{10})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[5]**

3 The matrices **A** and **B** are given by $\mathbf{A}=\begin{pmatrix}1 & 4\\ -2 & a\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}7 & 3\\ 1 & 5\end{pmatrix}$, where $a \neq -8$ and **I** is the 2×2 identity matrix. Find

(i) $7\mathbf{A}-\mathbf{I}$, **[2]**

(ii) $(\mathbf{A}^{-1}\mathbf{B}^{-1})^{-1}$. **[3]**

4 Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)}=\frac{n}{2n+1}$. **[5]**

5 The matrix $\begin{pmatrix}1 & 5\\ 0 & 1\end{pmatrix}$ represents the transformation P.

(i) Describe fully the transformation P. **[3]**

Transformation Q is a stretch, parallel to the y-axis with scale factor 4.

(ii) Find the matrix that represents transformation Q. **[2]**

Transformation T is equivalent to transformation P followed by transformation Q.

(iii) Find the matrix that represents transformation T. **[2]**

(iv) Find the area of the image of the unit square under transformation T. **[2]**

6 The complex number z_1 has modulus 3 and argument $\frac{3}{5}\pi$. The complex number z_2 has modulus 3 and argument $-\frac{9}{10}\pi$.

(i) Sketch on a single argand diagram z_1, z_2 and z_1-z_2. **[3]**

(ii) Find the exact value of $|z_1-z_2|$ and the exact value of $\arg(z_1-z_2)$. **[5]**

(iii) Give a geometrical description of the locus given by $|z-z_1|=|z-z_2|$. **[2]**

7 **(i)** Show that $\frac{1}{2r-1}-\frac{1}{2r+5}\equiv\frac{6}{(2r-1)(2r+5)}$. **[1]**

Hence find

(ii) $\sum_{r=2}^{30}\frac{6}{(2r-1)(2r+5)}$, giving your answer correct to 3 decimal places, **[5]**

(iii) $\sum_{r=2}^{\infty}\frac{6}{(2r-1)(2r+5)}$, giving your answer as a single fraction. **[1]**

8 In the cubic equation $4z^3+az^2+bz+c=0$, a, b and c are real numbers. One root is $1+\frac{3}{2}\mathrm{i}$ and the sum of the roots is 6. Find the values of a, b and c. **[7]**

9 The matrix **C** is given by $\mathbf{C}=\begin{pmatrix}a & 1 & 1\\ 3 & a & 1\\ 5 & 3 & 2\end{pmatrix}$.

(i) Find the value of a for which **C** is singular. **[5]**

In the three simultaneous equations given below, p is a constant.

$$\begin{aligned} ax+\ y+\ z&=p\\ 3x+ay+\ z&=p-1\\ 5x+3y+2z&=p-2\end{aligned}$$

(ii) Write down one value of a for which these equations have a unique solution, giving a brief reason. **[1]**

(iii) Using the value of a found in **(i)**, find the value of p for which these equations are consistent. **[3]**

10 The complex number $a+\mathrm{i}b$ is denoted by z and the complex number $c+\mathrm{i}d$ is denoted by w.

It is given that $z^2=z^*w$.

(i) Show that $2ab=ad-bc$. **[4]**

(ii) Given that the real part of $w=0$, find the values of b in terms of a. **[6]**

END OF QUESTION PAPER