

# OCR

Oxford Cambridge and RSA

## Friday 19 May 2017 – Morning

### AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

#### QUESTION PAPER

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4725/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



#### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer **all** the questions.

- 1 Find  $\sum_{r=1}^n (r^2 - r - 8)$ , giving your answer in a fully factorised form. [5]
- 2 Use an algebraic method to find the square roots of the complex number  $43 - (6\sqrt{10})i$ . Give your answers in the form  $x + iy$ , where  $x$  and  $y$  are exact real numbers. [5]
- 3 The matrices **A** and **B** are given by  $\mathbf{A} = \begin{pmatrix} 1 & 4 \\ -2 & a \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 7 & 3 \\ 1 & 5 \end{pmatrix}$ , where  $a \neq -8$  and **I** is the  $2 \times 2$  identity matrix. Find
- (i)  $7\mathbf{A} - \mathbf{I}$ , [2]
- (ii)  $(\mathbf{A}^{-1}\mathbf{B}^{-1})^{-1}$ . [3]
- 4 Prove by induction that, for  $n \geq 1$ ,  $\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$ . [5]
- 5 The matrix  $\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$  represents the transformation P.
- (i) Describe fully the transformation P. [3]
- Transformation Q is a stretch, parallel to the y-axis with scale factor 4.
- (ii) Find the matrix that represents transformation Q. [2]
- Transformation T is equivalent to transformation P followed by transformation Q.
- (iii) Find the matrix that represents transformation T. [2]
- (iv) Find the area of the image of the unit square under transformation T. [2]
- 6 The complex number  $z_1$  has modulus 3 and argument  $\frac{3}{5}\pi$ . The complex number  $z_2$  has modulus 3 and argument  $-\frac{9}{10}\pi$ .
- (i) Sketch on a single argand diagram  $z_1$ ,  $z_2$  and  $z_1 - z_2$ . [3]
- (ii) Find the exact value of  $|z_1 - z_2|$  and the exact value of  $\arg(z_1 - z_2)$ . [5]
- (iii) Give a geometrical description of the locus given by  $|z - z_1| = |z - z_2|$ . [2]

7 (i) Show that  $\frac{1}{2r-1} - \frac{1}{2r+5} \equiv \frac{6}{(2r-1)(2r+5)}$ . [1]

Hence find

(ii)  $\sum_{r=2}^{30} \frac{6}{(2r-1)(2r+5)}$ , giving your answer correct to 3 decimal places, [5]

(iii)  $\sum_{r=2}^{\infty} \frac{6}{(2r-1)(2r+5)}$ , giving your answer as a single fraction. [1]

8 In the cubic equation  $4z^3 + az^2 + bz + c = 0$ ,  $a$ ,  $b$  and  $c$  are real numbers. One root is  $1 + \frac{3}{2}i$  and the sum of the roots is 6. Find the values of  $a$ ,  $b$  and  $c$ . [7]

9 The matrix  $C$  is given by  $C = \begin{pmatrix} a & 1 & 1 \\ 3 & a & 1 \\ 5 & 3 & 2 \end{pmatrix}$ .

(i) Find the value of  $a$  for which  $C$  is singular. [5]

In the three simultaneous equations given below,  $p$  is a constant.

$$\begin{aligned} ax + y + z &= p \\ 3x + ay + z &= p - 1 \\ 5x + 3y + 2z &= p - 2 \end{aligned}$$

(ii) Write down one value of  $a$  for which these equations have a unique solution, giving a brief reason. [1]

(iii) Using the value of  $a$  found in (i), find the value of  $p$  for which these equations are consistent. [3]

10 The complex number  $a + ib$  is denoted by  $z$  and the complex number  $c + id$  is denoted by  $w$ .

It is given that  $z^2 = z^*w$ .

(i) Show that  $2ab = ad - bc$ . [4]

(ii) Given that the real part of  $w = 0$ , find the values of  $b$  in terms of  $a$ . [6]

**END OF QUESTION PAPER**

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