

# OCR

Oxford Cambridge and RSA

## Monday 14 May 2018 – Afternoon

### AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

#### QUESTION PAPER

Candidates answer on the Printed Answer Book.

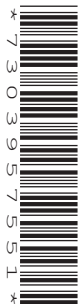
**OCR supplied materials:**

- Printed Answer Book 4725/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



#### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

- 1 The matrices **A**, **B** and **C** are given by  $\mathbf{A} = \begin{pmatrix} 5a \\ 2 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} 7b \\ -3 \end{pmatrix}$  and  $\mathbf{C} = \begin{pmatrix} 3 & 6 \end{pmatrix}$ .

Find

(i)  $5\mathbf{A} - 4\mathbf{B}$ , [2]

(ii)  $\mathbf{BC}$ . [2]

- 2 The complex number  $w$  has modulus 6 and argument  $\frac{2\pi}{3}$ . Find  $\frac{\sqrt{3} + 2i}{w}$ , giving your answer in the form  $x + iy$ , where  $x$  and  $y$  are exact real numbers. [5]

- 3 The matrix **D** is given by  $\mathbf{D} = \begin{pmatrix} d & 0 \\ 0 & 1 \end{pmatrix}$ , where  $d \neq 0$ .

(i) Find  $\mathbf{D}^{-1}$ . [2]

Matrix **D** represents the transformation P.

(ii) Describe fully the transformation P. [2]

The transformation T is represented by the matrix  $\begin{pmatrix} 0 & 1 \\ -d & 0 \end{pmatrix}$  and is equivalent to the transformation P followed by the transformation Q.

(iii) Find the matrix that represents the transformation Q and describe fully the transformation Q. [4]

- 4 The loci  $L_1$ ,  $L_2$  and  $L_3$  are given by  $|z - 3 - 4i| = 2$ ,  $\arg(z - 3 - 4i) = \frac{\pi}{3}$  and  $|z| = |z - 12|$  respectively.

(i) Sketch on a single Argand diagram the loci  $L_1$ ,  $L_2$  and  $L_3$ . [6]

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 - 4i| \geq 2, 0 \leq \arg(z - 3 - 4i) \leq \frac{\pi}{3} \text{ and } |z| \leq |z - 12|. \quad [3]$$

- 5 The cubic equation  $x^3 + 2x^2 + 3x + 4 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

(i) Use the substitution  $x = \frac{1}{u+1}$  to obtain a cubic equation in  $u$  with integer coefficients. [4]

(ii) Hence, or otherwise, find the value of  $\left(\frac{1}{\alpha} - 1\right)\left(\frac{1}{\beta} - 1\right)\left(\frac{1}{\gamma} - 1\right)$ . [3]

- 6 (i) Find  $\sum_{r=1}^n r(r^2 + r - 7)$ , giving your answer in a fully factorised form. [5]

A sequence  $u_0, u_1, u_2, \dots$  is defined by

$$u_0 = 5, u_n = u_{n-1} + n^3 + n^2 - 7n \text{ for } n \geq 1.$$

(ii) By considering  $\sum_{r=1}^n (u_r - u_{r-1})$ , find a formula for  $u_n$  in terms of  $n$ . [3]

[You do not need to factorise your answer.]

- 7 The complex number  $a + 3i$  is a root of the quadratic equation

$$z^2 - (7 + i)z + 16 + ki = 0,$$

where  $a$  and  $k$  are positive real numbers.

(i) Find the value of  $a$  and the value of  $k$ . [7]

(ii) Hence find the other root of the quadratic equation. [2]

- 8 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} a & 1 & -2 \\ -1 & a & 0 \\ 2a & 3 & 1 \end{pmatrix}$ , where  $a$  is a real constant.

(i) Show that  $\mathbf{A}$  is non-singular. [4]

(ii) Find  $\mathbf{A}^{-1}$ . [4]

(iii) Hence solve the three simultaneous equations given below.

$$\begin{aligned} ax + y - 2z &= 2 \\ -x + ay &= 1 \\ 2ax + 3y + z &= 0 \end{aligned}$$

[3]

(iv) Explain briefly why these equations have a unique solution. [1]

- 9 The matrix  $\mathbf{M}$  is given by  $\mathbf{M} = \begin{pmatrix} m & m \\ 0 & 1 \end{pmatrix}$ , where  $m$  is a positive constant.

(i) Find  $\mathbf{M}^2$  and  $\mathbf{M}^3$  in terms of  $m$ . [4]

(ii) Hence suggest a suitable form for the matrix  $\mathbf{M}^n$ , where  $n$  is a positive integer,  $n \geq 2$ . [2]

(iii) Use induction to prove that your answer to part (ii) is correct. [4]

**END OF QUESTION PAPER**

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.