A-level
MATHEMATICS
7357/1
Paper 1

Mark scheme

June 2020

Version: 1.0 Final Mark Scheme

206A7357/1/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

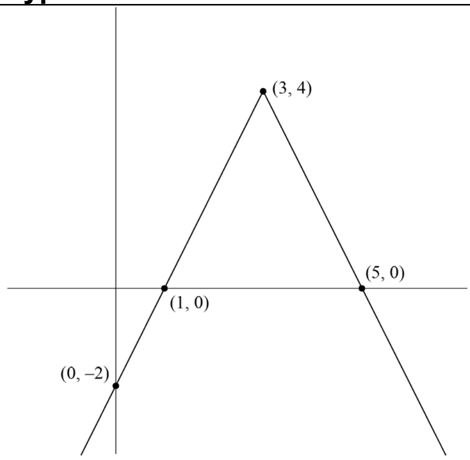
Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1a	Circles the correct answer	1.1b	B1	$\|x\| < \frac{9}{2}$
	Subtotal		**1**	
1b	Circles the correct answer	1.1b	B1	3
	Subtotal		**1**	
	Question Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles the correct answer	2.3	R1	$f(x) = \frac{1}{x}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles the correct answer	2.2a	R1	1
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Sketches an inverted V shape graph Condone lack of symmetry	1.1a	M1	
	Sketches an inverted V shape in the correct quadrants Condone lack of symmetry or absence of curve to the left of (0 , - 2)	1.1b	A1	
	Correctly labels all three intersections with coordinate axis. Accept the coordinates of each point or x values on x axis and y value on y axis Ignore any other values	1.1b	A1	
	Total		**3**	
4(b)	Obtains at least one correct critical value using a correct method. Can be read off graph or calculator Condone use of equals or incorrect inequality sign	1.1a	M1	$2 < x < 4$
	Writes correct solution in a correct form Accept $x > 2$, $x < 4$ or (2 , 4)	1.1b	A1	
	Subtotal		**2**	
	Question Total		**5**	

<table>
<tr><th>Q</th><th>Marking instructions</th><th>AO</th><th>Marks</th><th>Typical solution</th></tr>
<tr><td rowspan="2">5</td><td>Selects and begins to use a suitable method of proof.
Exhaustion:
Must check at least two correct values for n in the range $0 \le n < 4$ and make at least two correct comparisons. Comparisons are implied by ticks/crosses or use of true/false
Direct proof:
Takes logs to any base of both sides and uses a law of logs correctly once
Contradiction:
Must be clear they are attempting contradiction with
“$0 \le n < 4$ and $2^{n+2} \le 3^n$”
assumed explicitly.
Condone strict inequality</td><td>3.1a</td><td>M1</td><td rowspan="2"><table><tr><td>n</td><td>2^{n+2}</td><td>3^n</td><td></td></tr><tr><td>0</td><td>4</td><td>1</td><td>4>1</td></tr><tr><td>1</td><td>8</td><td>3</td><td>8>3</td></tr><tr><td>2</td><td>16</td><td>9</td><td>16>9</td></tr><tr><td>3</td><td>32</td><td>27</td><td>32>27</td></tr></table>
Hence $2^{n+2} > 3^n$ for integer values of n such that $0 \le n < 4$</td></tr>
<tr><td>Completes a reasoned mathematical
argument, proving $2^{n+2} > 3^n$ when n is an integer and
$0 \le n < 4$. Must include a fully correct concluding statement which refers to ‘integer’ or lists the four integers
If using direct proof or contradiction they must use the laws of logs correctly to remove n from the exponent. Condone use of equality if direct proof used</td><td>2.1</td><td>R1</td></tr>
<tr><td></td><td>**Total**</td><td></td><td>**2**</td><td></td></tr>
</table>

Q	Marking instructions	AO	Marks	Typical solution
6(a)(i)	Explains that Tom's solution does not include an arbitrary constant Accept Tom forgot the +c There is no constant on the RHS	2.4	E1	Tom's solution has no constant of integration
	Subtotal		**1**	
6(a)(ii)	Explains that the constant is in the wrong place or Explains that the k should not be there or that k = 1 or Shows that differentiating does not give $\frac{1}{x}$ or The constant has been multiplied instead of being added or It should be $\ln kx$ not $k \ln x$	2.4	E1	Although there is a constant, it is in the wrong place
	Subtotal		**1**	
6(b)	Rewrites $ln\, Ax$ as $\ln A + \ln x$ Condone use of any letter for A to demonstrate the log rule used Condone use of log without a specified base	1.1a	M1	ln Ax = $\ln A + \ln x$ This is equivalent as c = ln A
	Deduces explicitly that c = ln A clearly demonstrating equivalence OE	2.2a	R1	
	Subtotal		**2**	
	Question Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)(i)	Substitutes 2 into formula correctly to obtain $u_2 = -1$ PI by correct $u_3 = 2$	1.1a	M1	$u_2 = -1$ $u_3 = 2$
	Obtains correct $u_3 = 2$ and no further working resulting in a contradictory value for u_3	1.1b	A1	
	Subtotal		**2**	
7(a)(ii)	Deduces correct $u_{50} = -1$	2.2a	B1	$u_{50} = -1$
	Subtotal		**1**	
7(b)	Deduces correct $u_1 = -2$ Accept any correct value eg $\sqrt{2}$ or $-\sqrt{2}$ Condone if ± 2 seen	2.2a	B1	$u_1 = -2$
	Subtotal		**1**	
	Question Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Uses sin = -1 in the model to obtain -3.87 + 11.7 If a *t* value is used then the sine must evaluate to -1 or Differentiates, sets the derivative equal to 0 and obtains a value for t which they substitute back into the formula	3.4	M1	$sin\left(\frac{2\pi(t+101.75)}{365}\right)=-1$ $-3.87+11.7=7.83$ $7\text{hours } 50\text{mins}$
	Obtains correct answer Accept 470 minutes, $\frac{47}{6}$ or $7\frac{5}{6}$ hours	3.2a	A1	
	Subtotal		**2**	
8(b)	Uses model to form equation or inequality with H =14 Condone incorrect inequality	3.4	M1	$3.87\,sin\left(\frac{2\pi(t+101.75)}{365}\right)+11.7=14$
	Solves equation to obtain at least two correct values of t Can be rounded or truncated Eg -64.77, 43.779, 300.22, 408.77	1.1b	A1	$t=300.22$ or $t=408.77$ $408-300=108$
	Subtracts an appropriate pair of t values to obtain number of consecutive days Condone any rounding to the nearest whole number or truncation of their pair of values Accept 109 or 107 Alternative method = 43 + (365 – 300) = 108	3.2a	A1	
	Subtotal		**3**	
8(c)	Explains that Sofia's refinement would increase the amplitude of the graph Accept The range of the graph would increase It would increase the fluctuation of the graph	3.3	M1	Sofia's refinement would increase the range of the graph Sofia's graph suggests this is not the case, so the refinement is not appropriate
	Explains that Sofia's refinement is not appropriate as her data/graph suggests a lower amplitude OE	3.5c	A1	
	Subtotal		**2**	
	Question Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(i)	Deduces an appropriate value for x and substitutes into at least one side of the given identity Any value of $x \neq -2, -1$	2.2a	M1	$\frac{2x^2+x}{(x+1)(x+2)^2} \equiv \frac{1}{x+1} - \frac{6}{(x+2)^2}$ Let $x = 0 \Rightarrow LHS = 0$ $RHS = \frac{1}{1} - \frac{6}{4} = -\frac{1}{2} \neq 0$ ∴Chloe's answer must be incorrect
	Shows that LHS≠RHS and concludes that Chloe's answer must be incorrect Accept $\frac{2x^2+x}{(x+1)(x+2)^2} \neq \frac{1}{x+1} - \frac{6}{(x+2)^2}$	2.1	R1	
	Subtotal		**2**	
9(a)(ii)	Explains that Chloe should have included an additional term with $x+2$ in the denominator or Explains that Chloe should have included (Bx + C) as the numerator for $(x+2)^2$	2.3	E1	Chloe should have included $\frac{C}{x+2}$
	Subtotal		**1**	
9(b)	Writes an identity of the correct form Condone use of equals signs	1.1a	M1	$\frac{2x^2+x}{(x+1)(x+2)^2} \equiv \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ $2x^2 + x \equiv A(x+2)^2 + B(x+1)(x+2) + C(x+1)$ $x = -1 \Rightarrow A = 1$ $x = -2 \Rightarrow C = -6$ $x^2: A + B = 2 \Rightarrow B = 1$ $\frac{2x^2+x}{(x+1)(x+2)^2} \equiv \frac{1}{x+1} + \frac{1}{x+2} - \frac{6}{(x+2)^2}$
	Uses a suitable method to obtain all three of '*their*' constants. For example by substituting or comparing coefficients Only award the M1 if the identity used results from correctly removing fractions from '*their*' chosen partial fraction form	3.1a	M1	
	Obtains any two correct constants If Bx + C is used, then B = 1 and C = - 4	1.1b	A1	
	Obtains all three correct values for the constant numerators	1.1b	A1	
	Subtotal		**4**	
	Question Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)(i)	Obtains correct first term	1.1b	B1	21
	Subtotal		**1**	
10(a)(ii)	Obtains correct common difference	1.1b	B1	4
	Subtotal		**1**	
10(a)(iii)	Obtains correct number of terms	1.1b	B1	16
	Subtotal		**1**	
10(b)(i)	Finds or uses at least one of the first term, the common difference, the last term or the number of terms correctly or Expresses given series as a difference of two series using n= 1 to 100 and n = 1 to 9. Either $\sum_{n=1}^{100}(br+c) - \sum_{n=1}^{n=9}(br+c)$ or $b\sum_{n=1}^{100} r + 100c - b\sum_{n=1}^{n=9} r - 9c$	1.1b	B1	$n = 91$ $a = 10b + c$ $d = b$ $\mathrm{L} = 100b + c$ $\frac{91}{2}(2(10b+c) + 90b) = 7735$ $91(55b + c) = 7735$ $55b + c = 85$
	Forms an equation in terms of b and c for the sum of n terms using '*their*' first term, '*their*' number of terms and either '*their*' common difference or '*their*' last term Alternative $\frac{100}{2}[2b + 2c + 99b]$ $- \frac{9}{2}[2b + 2c + 8b]$	3.1a	M1	
	Obtains correct equation ACF Alternative 5050b +100c - 45b - 9c = 7735 or 5005b + 91c = 7735	1.1b	A1	
	Completes rigorous argument to show the required result. This must include at least one single step of correct working between the initial correct formula and the given answer AG	2.1	R1	
	Subtotal		**4**	

10(b)(ii)	Uses or writes down a + 39d or a + d with '*their*' expressions for a and d Must be in terms of b and c	3.1a	B1	$4(11b + c) = 49b + c$ $5b - 3c = 0$ $b = 1.5$ $c = 2.5$
	Uses '*their*' a + 39d and a + d consistently to form 'their' equation $u_{40} = 4u_2$ in terms of b and c. Condone use of 50b + c for the fortieth term Condone $11b + c = 4(49b + c)$ OE with '*their*' a and d in terms of b and c	1.1a	M1	
	Solves $55b + c = 85$ with '*their*' other equation involving b and c PI by obtaining correct values of b and c or Obtains b = -12.75 and c = 786.25 from using $11b + c = 4(49b + c)$	1.1a	M1	
	Obtains correct values of b and c	1.1b	A1	
	Subtotal		**4**	
	Question Total		**11**	

Q	Marking Instructions	AO	Marks	Typical Solution
11(a)	Evaluates f(1) and f(6) using exact logs or decimals Award if seen embedded in calculations using more than one trapezium	1.1a	M1	f(1) = 1.945910149.. f(6)= 0.69314718.. A = $\frac{5}{2}$ (1.9459+0.6931) = 6.5976… =6.60 cm^2
	Evaluates an approximate value of the area of R AWRT 6.60 Condone omission of units	1.1b	A1	
	Subtotal		**2**	
11(b)	Writes or uses the six ordinates as ln 7, ln 6, ln 5, ln 4, ln 3 ln 2 or Obtains the values of the correct six ordinates in decimal form	1.1b	B1	*x* — f(*x*) 1 — 1.9459 2 — 1.7918 3 — 1.6094 4 — 1.3863 5 — 1.0986 6 — 0.6931 Area = $\frac{1}{2}$x1x(1.9459+0.6931+2 (1.7918+1.6094+1.3863+1.0986)) Area= 7.205633 cm^2 Volume of Shape B = 4 x 7.205633 x 0.2 =5.7645… cm^3 Mass of Shape B = 5.7645cm^3 x 10.5 g/cm^3 =60.52731 g =61 g
	Uses the correct formula for the trapezium rule with their six ordinates and h = 1 Award this mark if seven ordinates used with $h = \frac{5}{6}$ Answer for seven = 7.2145648..	1.1a	M1	
	Evaluates an approximate value for the area of R. Must have used six ordinates AWRT 7.2 PI by correct final answer	1.1b	A1	
	Forms an expression for the mass of either one section or all four sections using '*their'* area and consistent units PI by correct final answer	3.1b	M1	
	Obtains an approximate value for the correct mass of Shape B Must state units If seven ordinates used this mark can be awarded as answer would be 61g CAO	3.2a	A1	
	Subtotal		**5**	
11(c)(i)	Explains that the trapezia are all below the curve or Explains that the curve is concave or Draws a diagram and indicates the gaps	3.5a	E1	The trapezia are all below the curve
	Subtotal		**1**	
11(c)(ii)	Explains that numbers have been rounded	3.5a	E1	Numbers in the calculation have been rounded
	Subtotal		**1**	
	Question Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
12(a)	Substitutes $x = \sqrt{3}$ and $y = \frac{\pi}{6}$ to obtain an equation or an expression for A	1.1a	M1	$\left(\sqrt{3}\right)^3 sin\frac{\pi}{6} + cos\frac{\pi}{6} = A\sqrt{3}$ $\frac{3\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = A\sqrt{3}$ $\frac{3}{2} + \frac{1}{2} = A$ $A = 2$
	Completes argument to show A = 2 Must clearly show use of $cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ and $sin\frac{\pi}{6} = \frac{1}{2}$ AG	2.1	R1	
	Subtotal		**2**	
12(b)(i)	Uses implicit differentiation correctly at least once with sight of $sin\, y\frac{dy}{dx}$ or $cos\, y\frac{dy}{dx}$ Condone sign error	3.1a	M1	$3x^2 sin\, y + x^3 cos\, y\frac{dy}{dx} - sin\, y\frac{dy}{dx} = 2$ $\frac{dy}{dx}(x^3 cos\, y - sin\, y) = 2 - 3x^2 sin\, y$ $\frac{dy}{dx} = \frac{2 - 3x^2 sin\, y}{x^3 cos\, y - sin\, y}$
	Uses product rule with sight of $Px^2 sin\, y \pm x^3 cos\, y\frac{dy}{dx}$ Condone omission of $\frac{dy}{dx}$	3.1a	M1	
	Obtains equation of the form $Px^2 sin\, y \pm x^3 cos\, y\frac{dy}{dx} \pm sin\, y\frac{dy}{dx} = 2$	1.1b	A1	
	Obtains completely correct equation	1.1b	A1	
	Isolates $\frac{dy}{dx}$ terms and factorises to complete rigorous argument with no slips to show the given result AG	2.1	R1	
	Subtotal		**5**	
12(b)(ii)	Substitutes $x = \sqrt{3}$ and $y = \frac{\pi}{6}$ to obtain an expression for the gradient	1.1a	M1	$\frac{dy}{dx} = \frac{2 - 3(\sqrt{3})^2 sin\frac{\pi}{6}}{(\sqrt{3})^3\ cos\frac{\pi}{6} - sin\frac{\pi}{6}}$ $= -\frac{5}{8}$
	Obtains correct gradient of $-\frac{5}{8}$ OE	1.1b	A1	
	Subtotal		**2**	

12(b)(iii)	Forms equation for the tangent (condone normal) at P using '*their'* gradient and $\left(\sqrt{3}, \frac{\pi}{6}\right)$ ACF or Writes the equation as $y = mx + \mathrm{c}$ using '*their*' gradient of tangent (condone normal) and substitutes $\left(\sqrt{3}, \frac{\pi}{6}\right)$ to obtain an equation in c PI by correct exact value for x	3.1a	M1	$y - \frac{\pi}{6} = -\frac{5}{8}(x - \sqrt{3})$ $0 - \frac{\pi}{6} = -\frac{5}{8}(x - \sqrt{3})$ $x = \sqrt{3} + \frac{4\pi}{15}$
	Obtains fully correct equation for the '*their*' **tangent** at P ACF Note $c = \frac{5\sqrt{3}}{8} + \frac{\pi}{6}$ or c = 1.606.. Follow through 'their' gradient of tangent from 12(b)(ii) must be to at least 3 dp	1.1b	A1F	
	Substitutes y = 0 into '*their*' tangent (condone normal) equation and solves to find the x coordinate of Q Accept decimals	3.1a	M1	
	Obtains $x = \sqrt{3} + \frac{4\pi}{15}$ OE must be exact form Eg $x = \frac{8}{5}(\frac{5\sqrt{3}}{8} + \frac{\pi}{6})$	1.1b	A1	
	Subtotal		**4**	
	Question Total		**13**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)(i)	Rearranges to make x the subject by isolating x terms or Swaps x and y and isolates y terms	1.1a	M1	$y = \frac{2x+3}{x-2}$ $xy - 2y = 2x + 3$ $xy - 2x = 2y + 3$ $x(y-2) = 2y + 3$ $x = \frac{2y+3}{y-2}$ $f^{-1}(x) = \frac{2x+3}{x-2}\ x \neq 2$
	Obtains correct rearrangement and factorises ACF PI by final correct answer	1.1b	A1	
	Obtains $f^{-1}(x)$ and states domain Must use fully correct notation	2.5	R1	
	Subtotal		**3**	
13(a)(ii)	Obtains any valid expression in x for $ff(x)$ Can be left unsimplified ISW	1.1b	B1	$ff(x) = x$
	Subtotal		**1**	
13(b)(i)	Deduces the greatest value of g by evaluating g (4)	2.2a	B1	$g(4)$ = 6 Vertex at (1.25 , -1.5625) $\{y : -1.5625 \leq y \leq 6\}$
	Obtains the minimum value of g	3.1a	B1	
	States the range using their finite greatest value and finite minimum value using set notation or interval notation Accept $[-1.5625, 6]$ in interval notation For set notation - use of none curly brackets or commas scores R0	2.5	R1F	
	Subtotal		**3**	
13(b)(ii)	Demonstrates that g is a many to one function by using an appropriate method eg Sketches the function Or Evaluates $g(x)$ at two points that give the same answer.	2.4	E1	$g(0) = 0 = g(2.5)$ g is many to one so it does not have an inverse.
	Deduces that g is many to one and states that g has no inverse Or Explains that g is not one to one and states that g has no inverse	2.2a	E1	
	Subtotal		**2**	

13(c)	Substitutes $f(x)$ into $g(x)$ correctly	1.1a	M1	$gf(x) = \dfrac{2\left(\frac{2x+3}{x-2}\right)^2 - 5\left(\frac{2x+3}{x-2}\right)}{2}$
	Obtains common denominator of $2(x-2)^2$ or $(x-2)^2$ correctly The fraction(s) must have the fully correct structure	1.1b	A1	$= \dfrac{2(2x+3)^2 - 5(2x+3)(x-2)}{2(x-2)^2}$
	Expands at least two quadratics correctly	1.1a	M1	$= \dfrac{2(4x^2+12x+9)-5(2x^2-x-6)}{2(x^2-4x+4)}$
	Completes rigorous argument to show the required result Must have expanded all three quadratics correctly Terms in the numerator and denominator can be in any order AG	2.1	R1	$= \dfrac{48+29x-2x^2}{2x^2-8x+8}$
	Subtotal		**4**	
13(d)	States $g(x) = 2$ or States $2x^2 - 5x - 4 = 0$ PI by solving correct quadratic PI by sight of $\frac{5+\sqrt{57}}{4}$ or $\frac{5-\sqrt{57}}{4}$	3.1a	M1	$2x^2 - 5x - 4 = 0$ $x = \dfrac{5 \pm \sqrt{57}}{4}$ $a > 0$ since $0 \le x \le 4$
	Determines the exact value of a giving a clear reason for the rejection of the negative root	2.4	R1	$a = \dfrac{5+\sqrt{57}}{4}$
	Subtotal		**2**	
	Question Total		**15**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)	Evaluates $f(0) = -1$ and $f(1) = 2$ or Evaluates two other suitable appropriate values correct to 1 sig fig	1.1a	M1	$f(0) = -1 < 0$ $f(1) = 3 - 1 = 2 > 0$ Change of sign implies root therefore α is between 0 and 1
	Completes argument correctly stating $f(0) < 0$ and $f(1) > 0$ and concludes that $0 < \alpha < 1$	2.1	R1	
	Subtotal		**2**	
14(b)(i)	Uses product rule to obtain an expression of the form $Ax^{\frac{1}{2}}(3^x) + Bx^{-\frac{1}{2}}(3^x)$ A and /or B can be positive or negative	3.1a	M1	$f'(x) = x^{\frac{1}{2}}(3^x)\ln 3 + \frac{1}{2}x^{-\frac{1}{2}}(3^x)$ $= 3^x\left(\ln 3\sqrt{x} + \frac{1}{2\sqrt{x}}\right)$
	Obtains fully correct $f'(x)$	1.1b	A1	$= 3^x\left(\frac{2x\ln 3}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}\right)$
	Completes convincing argument with no slips to show the required result. AG	2.1	R1	$= 3^x\left(\frac{x\ln 9}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}\right)$ $= 3^x\left(\frac{1 + x\ln 9}{2\sqrt{x}}\right)$
	Subtotal		**3**	
14(b)(ii)	Forms correct Newton-Raphson expression PI by correct value of x_2 or x_3 stated to at least 3 decimal places	1.1a	M1	$x_{n+1} = x_n - \dfrac{(3^{x_n}\sqrt{x_n} - 1)}{\dfrac{3^{x_n}(1 + x_n\ln 9)}{2\sqrt{x_n}}}$
	Obtains the correct value of x_3 Must be stated to five decimal places	1.1b	A1	$x_{n+1} = x_n - \dfrac{2\sqrt{x_n}\,(3^{x_n}\sqrt{x_n} - 1)}{3^{x_n}(1 + x_n\ln 9)}$ x_2= 0.5829716.. x_3= 0.4246536.. $x_3 \approx 0.42465$
	Subtotal		**2**	
14(b)(iii)	Explains that convergence is impossible Must use the word convergence or convergent	2.4	E1	Convergence is impossible as all values of x_n would equal 0
	Explains that the tangent at x = 0 is vertical or Explains all values of x_n would equal 0 or Demonstrates that several values of x_n would be 0	2.4	E1	
	Subtotal		**2**	
	Question Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
15	Forms a single equation eliminating x or y	3.1a	M1	$6 - e^{\frac{x}{2}} = e^x$
	Obtains a correct rearranged quadratic equation. Either $e^x + e^{\frac{x}{2}} - 6 = 0$ or $\left(e^{\frac{x}{2}} + 3\right)\left(e^{\frac{x}{2}} - 2\right) = 0$ or $e^x + e^{\frac{x}{2}} + \frac{1}{4} = \frac{25}{4}$ OE	1.1b	A1	$e^x + e^{\frac{x}{2}} - 6 = 0$ $\left(e^{\frac{x}{2}} + 3\right)\left(e^{\frac{x}{2}} - 2\right) = 0$ $e^{\frac{x}{2}} = -3$ or 2
	Solves 'their' quadratic Must be a quadratic in $e^{\frac{x}{2}}$ or If squaring is used then it must be a quadratic in e^x or Obtains x = 1.386	1.1a	M1	$e^{\frac{x}{2}} > 0$ so -3 is not a valid solution $\frac{x}{2}$ = ln 2 $x = 2\ln 2 = \ln 4$
	Explains that $e^{\frac{x}{2}} = -3$ is not valid as $e^{\frac{x}{2}} > 0$ or If squaring is used they must clearly check both solutions by substituting and conclude that $\ln 9$ is not valid OE	2.4	E1F	$\int_0^{\ln 4} (6 - e^{\frac{x}{2}} - e^x)\, dx$ = $\left[6x - 2e^{\frac{x}{2}} - e^x\right]_0^{\ln 4}$ = $\left(6\ln 4 - 2e^{\frac{\ln 4}{2}} - e^{\ln 4}\right)$ – $(-2-1)$
	Obtains $x = 2\ln 2$ or $x = \ln 4$	1.1b	A1	
	Forms any definite integral which would contribute to finding the required area This could be $\int_0^{\ln 4} (6 - e^{\frac{x}{2}} - e^x)\, dx$ or $\int_0^{\ln 4} (6 - e^{\frac{x}{2}})\, dx$ or $\int_0^{\ln 4} e^x dx$ or $\int_0^{\ln 4} (e^x + e^{\frac{x}{2}} - 6)\, dx$ Follow through '*their*' value of x for the upper limit	1.1a	M1	= 6 ln 4 – 4 – 4 + 3 = 6 ln 4 – 5
	Forms a fully correct definite integral (or integrals) which would lead to evaluating the correct area Follow through '*their*' incorrect upper limit	3.1a	A1F	

	Integrates '*their'* expressions involving exponentials fully correctly Follow through their exponential expressions – but must have integrated both e^x and $e^{\frac{x}{2}}$ terms Condone missing/incorrect limits	1.1b	B1F	
	Substitutes 0 and '*their'* upper limits into '*their'* integrated expression Must correctly use F (their upper limit) – F (0) for each integral	1.1a	M1	
	Completes rigorous argument by showing explicit evaluation of exponential terms before obtaining final answer AG This mark can be achieved without achieving the E1 mark	2.1	R1	
	Total		**10**	