

Mark Scheme (Results)

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Pearson Edexcel GCE In Mathematics (9MA0) Paper 32 Mechanics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS General Instructions for Marking

- 1. The total number of marks for the paper is 100.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
- 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol $\sqrt[4]{}$ will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- ***** The answer is printed on the paper
- The second mark is dependent on gaining the first mark
- 4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- Where a candidate has made multiple responses <u>and indicates which response</u> <u>they wish to submit</u>, examiners should mark this response.
 If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most</u> <u>complete</u>.

- 6. Ignore wrong working or incorrect statements following a correct answer.
- 7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- dM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.
 N.B. Over-accuracy or under-accuracy of correct answers should only be penalised once per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

 M(A) Taking moments about A
 N2L Newton's Second Law (Equation of Motion)
 NEL Newton's Experimental Law (Newton's Law of Impact)
 HL Hooke's Law
 SHM Simple harmonic motion
 PCLM Principle of conservation of linear momentum
 RHS, LHS Right hand side, left hand side

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Que	estion	Scheme	Marks	AOs
1	(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $t = 2$: $\mathbf{v} = 4\mathbf{i} + 2(2\mathbf{i} - 3\mathbf{j})$ OR integration: $\mathbf{v} = (2\mathbf{i} - 3\mathbf{j})t + 4\mathbf{i}$, with $t = 2$	M1	3.1a
		$\mathbf{v} = 8\mathbf{i} - 6\mathbf{j}$	A1	1.1b
			(2)	
1	(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ at $t = 3$: $(\mathbf{i} + \mathbf{j}) + \left[3 \times 4\mathbf{i} + \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2 \right]$ OR: find \mathbf{v} at $t = 3$: $4\mathbf{i} + 3(2\mathbf{i} - 3\mathbf{j}) = (10\mathbf{i} - 9\mathbf{j})$ then use $\mathbf{r} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$ $(\mathbf{i} + \mathbf{j}) + \left[\frac{1}{2} \left[4\mathbf{i} + (10\mathbf{i} - 9\mathbf{j}) \right] \times 3 \right]$ or $\mathbf{r} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ $(\mathbf{i} + \mathbf{j}) + \left[3 \times (10\mathbf{i} - 9\mathbf{j}) - \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2 \right]$ OR integration: $\mathbf{r} = (\mathbf{i} + \mathbf{j}) + \left[(2\mathbf{i} - 3\mathbf{j}) \frac{1}{2}t^2 + 4t\mathbf{i} \right]$, with $t = 3$	M1	3.1a
		r = 22i - 12.5j	A1	2.2a
			(2)	
		·	(4 n	narks)
Note	es: A	ccept column vectors throughout		
1a	M1	Complete method to find v , using ruva <i>t</i> or integration (M0 if i and/or j is missing)		
	A1	Apply isw if they also find the speed		
1b	M1	Complete method to find the p.v. but this mark can be scored if they omit $(\mathbf{i} + \mathbf{j})$ i.e. the M1 is for the expression in the square bracket If they integrate, the M1 is earned once the expression in the square bracket is seen with $t = 3$ (M0 if i and/or j is missing)		en
	A1	cao		

Question	Scheme	Marks	AOs
	Mark parts (a) and (b) together		
2(a)	Equation of motion for A	M1	3.3
	$3mg\sin\alpha - F - T = 3ma$	A1	1.1b
		(2)	
2(b)	Resolve perpendicular to the plane	M1	3.4
	$R = 3mg\cos\alpha$	A1	1.1b
	$F = \frac{1}{6}R$	B1	1.2
	Equation of motion for <i>B</i> OR for whole system	M1	3.3
	$T - mg = ma$ OR $3mg\sin\alpha - F - mg = 3ma + ma$	A1	1.1b
	Complete method to solve for <i>a</i>	DM1	3.1b
	$a = \frac{1}{10}g *$	A1*	2.2a
		(7)	
2(c)		B1	1.1b
	e.g. acceleration (of <i>B</i>) is constant; dependent on first B1	DB1	2.4
		(2)	
2(d)	e.g. the tensions in the two equations of motion would be different. Tension on <i>A</i> would be different to tension on <i>B</i>	B1	3.5a
		(1)	
	·	(12 n	narks)
	N.B. If m's are consistently missing treat as a MR, so max (a) M1A0 (b) M1A0B0M1A1M1A1 (c) B1B1 (d) B1		
I	For (a) and (b), allow verification, but must see full equations of motion Equation in T and a with correct no. of terms, condone sign errors and		
2a M1	confusion (If one of the 3's is missing, allow M1) N.B. Treat sin(3/5) etc as an A error but allow recovery	a 5111 005	

2b	M1	Correct no. of terms, condone sign errors and sin/cos confusion Allow if appears in (a)
	A1	Correct equation
	B1	Seen anywhere in (a) or (b), including on a diagram
	M1	Equation (for <i>B</i>) in <i>T</i> and <i>a</i> with correct no. of terms, condone sign errors and sin/cos confusionOR Whole system equation with correct no. of terms, condone sign errors and sin/cos confusion
	A1	Correct equation
	DM1	Complete method (trig may not be substituted), dependent on M1 in (a) and second M1 in (b) if they use two equations, or second M1 in (b) if they use one equation.
	A1*	Correct answer correctly obtained.
2c	B1	Straight line starting at the origin (could be reflected in the <i>t</i> -axis). B0 if continuous vertical line at the end.
	DB1	Dependent on first B1, for any equivalent statement
2d	B1	B0 if incorrect extras

Scheme	Marks	AOs
Part (a) is a 'Show that' so equations need to be given in full to earn A marks		
$\begin{array}{c} C \\ S \\ B \\ G \\ R \\ Mg \\ D \\ A \\ F \end{array}$		
Moments equation: (M1A0 for a moments inequality)	M1	3.3
$\begin{split} & M(A), \ mga\cos\theta = 2Sa\sin\theta \\ & M(B), \ mga\cos\theta + 2Fa\sin\theta = 2Ra\cos\theta \\ & M(C), \ F \times 2a\sin\theta = mga\cos\theta \\ & M(D), \ 2Ra\cos\theta = mga\cos\theta + 2Sa\sin\theta \\ & M(G), \ Ra\cos\theta = Fa\sin\theta + Sa\sin\theta . \end{split}$	A1	1.1b
$(\diamondsuit) \ R = mg \ \mathbf{OR} \ (\leftrightarrow) \ F = S$	B1	3.4
Use their equations (they must have enough) and $F \le \mu R$ to give an inequality in μ and θ only (allow DM1 for use of $F = \mu R$ to give an <i>equation</i> in μ and θ only)	DM1	2.1
$\mu \ge \frac{1}{2} \cot \theta *$	A1*	2.2a
	(5)	
$\begin{array}{c} C \\ R \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$		
Moments equation:	M1	3.4
$\begin{split} \mathbf{M}(A), \ mga\cos\theta &= 2Na\sin\theta\\ \mathbf{M}(B), \ mga\cos\theta + 2kmga\sin\theta &= 2Ra\cos\theta + \frac{1}{2}mg2a\sin\theta\\ \mathbf{M}(D), \ 2Ra\cos\theta &= mga\cos\theta + N2a\sin\theta\\ \mathbf{M}(G), \ kmga\sin\theta + Na\sin\theta &= \frac{1}{2}mga\sin\theta + Ra\cos\theta \end{split}$	A1	1.1b
	Part (a) is a 'Show that' so equations need to be given in full to earn A marks $C \qquad S \qquad B$ $Moments equation: (M1A0 for a moments inequality)$ M(A), mga cos $\theta = 2Sa \sin \theta$ M(B), mga cos $\theta = 2Sa \sin \theta$ M(C), $F \times 2a \sin \theta = mga \cos \theta$ M(C), $F \times 2a \sin \theta = mga \cos \theta$ M(C), $F \times 2a \sin \theta = mga \cos \theta$ M(C), $Ra \cos \theta = Fa \sin \theta + Sa \sin \theta$. (\updownarrow) $R = mg$ OR (\leftrightarrow) $F = S$ Use their equations (they must have enough) and $F \le \mu R$ to give an inequality in μ and θ only (allow DM1 for use of $F = \mu R$ to give an inequality in μ and θ only $\mu \ge \frac{1}{2} \cot \theta^*$ Moments equation: M(A), mga cos $\theta = 2Na \sin \theta$ M(B), mga cos $\theta + 2kmga \sin \theta = 2Ra \cos \theta + \frac{1}{2}mg 2a \sin \theta$ M(D), $2Ra \cos \theta = mga \cos \theta + N2a \sin \theta$	Part (a) is a 'Show that' so equations need to be given in full to earn A marksC B C B Moments equation: (M1A0 for a moments inequality)M1M(A), $mga \cos \theta = 2Sa \sin \theta$ M(B), $mga \cos \theta = 2Sa \sin \theta = 2Ra \cos \theta$ M(C), $F \times 2a \sin \theta = mga \cos \theta$ M(D), $2Ra \cos \theta = mga \cos \theta + 2Sa \sin \theta$ M(G), $Ra \cos \theta = Fa \sin \theta + Sa \sin \theta$.(\updownarrow) $R = mg$ OR (\leftrightarrow) $F = S$ Use their equations (they must have enough) and $F \le \mu R$ to give an inequality in μ and θ only (allow DM1 for use of $F = \mu R$ to give an inequality in μ and θ only $\mu \ge \frac{1}{2} \cot \theta^*$ A1*(5)Moments equation:M1M(A), $mga \cos \theta = 2Na \sin \theta$ M(B), $mga \cos \theta + 2kmga \sin \theta = 2Ra \cos \theta + \frac{1}{2}mg2a \sin \theta$ M(D), $2Ra \cos \theta = mga \cos \theta + N2a \sin \theta$ A1

		S.C. M(C), $mga\cos\theta + \frac{1}{2}mg2a\sin\theta = kmg2a\sin\theta$ M1A1B1				
		$1 + \frac{5}{4} = \frac{5k}{2} \qquad \mathbf{M1}$				
		k = 0.9 A1				
		$N = kmg - F \mathbf{OR} R = mg$	B1	3.3		
		Use their equations (they must have enough) to solve for k (numerical)	DM1	3.1b		
		k = 0.9 oe	A1	1.1b		
			(5)			
			(10 n	narks)		
Not	es:					
3a	M1	Any moments equation with correct terms, condone sign errors and sin/cos confusion				
	A1	Correct equation				
	B1	Correct equation				
	DM1	Dependent on M1, for using their equations (they must have enough) give an inequality in μ and θ only (allow M1 for use of $F = \mu R$ to give an equation in μ and θ only)) and $F \leq r$	uR to		
	A1*	Given answer correctly obtained with no wrong working seen (e.g. if $F = \mu R$ anywhere, A0)	f they use			
3b	M1	Any moments equation with correct terms, condone sign errors				
	A1	Correct equation				
	B1	Correct equation				
	DM1	Dependent on M1, for using their equations (they must have enough) with trig substituted, to solve for k , which must be numerical.				
	A1	cao				

Qı	uestion	Scheme	Marks	AOs
		Note that $g = 10$; penalise once for whole question if $g = 9.8$		
	4 (a)	Use $s = ut + \frac{1}{2}at^2$ vertically or any complete method to give an equation in <i>t</i> only	M1	3.4
		$1 - 2^2$	A1	1.1b
		$-70 = 65\sin\alpha \times t - \frac{1}{2} \times g \times t^2$	M (A)1	1.1b
		t = 7 (s)	A1	1.1b
			(4)	
	4(b)	Horizontal velocity component at $A = 65 \cos \alpha$ (60)	B1	3.4
		Complete method to find vertical velocity component at A	M1	3.4
		65 sin $\alpha - g \times 7$ OR $\sqrt{(-25)^2 + 2g \times 70}$ (45)	A1 ft	1.1b
		Sub for trig and square, add and square root : $\sqrt{60^2 + (-45)^2}$	M1	3.1b
		75 Accept 80 (m s ⁻¹)	A1	1.1b
			(5)	
	4(c)	e.g. an approximate value of g has been used, the dimensions of the stone could affect its motion, spin of the stone, $g = 10$ instead of 9.8 has been used, g has been assumed to be constant, wind effect, shape of the stone	B1	3.5b
			(1)	
			(10 n	narks)
Not	es:			
4a	M1	Complete method, correct no. of terms, condone sign errors and sin	cos confus	sion
	A1	Correct equation in <i>t</i> only with at most one error		
	M(A)1	Correct equation in <i>t</i> only		
		N.B. For 'up and down' methods etc, the two A marks are for all the they use, lose a mark for each error.	e equations	s that
	A1	Cao $(g = 9.8, 7.1 \text{ or } 7.11)$ $(g = 9.81, 7.1 \text{ or } 7.12)$		
4b	B1	Seen, including on a diagram.		
	M1	Condone sign errors and sin/cos confusion		
	A1 ft	Correct expression; accept negative of this, follow their <i>t</i>		
	M1	Sub for trig and use Pythagoras		
	A1	Cao $(g = 9.8 \text{ or } 9.81, 75 \text{ or } 74.8)$		

4c B1 B0 if incorrect extras

Que	estion	Scheme	Marks	AOs
		Allow column vectors throughout this question		
5	5(a)	Differentiate v wrt t	M1	3.1a
		$\frac{3}{2}t^{-\frac{1}{2}}\mathbf{i} - 2\mathbf{j} \text{ isw}$	A1	1.1b
			(2)	
5	5(b)	$3t^{\frac{1}{2}} = 2t$	M1	2.1
		Solve for <i>t</i>	DM1	1.1b
		$t = \frac{9}{4}$	A1	1.1b
			(3)	
5	5(c)	Integrate v wrt <i>t</i>	M1	3.1a
		$\mathbf{r} = 2t^{\frac{3}{2}}\mathbf{i} - t^2\mathbf{j}(+\mathbf{C})$	A1	1.1b
		$t = 1, \mathbf{r} = -\mathbf{j} => \mathbf{C} = -2\mathbf{i}$ so $\mathbf{r} = 2t^{\frac{3}{2}}\mathbf{i} - t^{2}\mathbf{j} - 2\mathbf{i}$	A1	2.2a
			(3)	
5	5(d)	$\sqrt{(3t^{\frac{1}{2}})^2 + (2t)^2} = 10$ or $(3t^{\frac{1}{2}})^2 + (2t)^2 = 10^2$	M1	2.1
		$9t + 4t^2 = 100$	M(A)1	1.1b
		t = 4	A1	1.1b
		r = 14i - 16j	M1	1.1b
		$\sqrt{14^2 + (-16)^2}$	M1	3.1a
		$\sqrt{452} (2\sqrt{113}) (m)$	A1	1.1b
			(6)	
			(14 n	narks)
Notes	s:			
5a	M1	Both powers decreasing by 1 (M0 if vector(s) disappear but allo	ow recovery)	
	A1	cao		
5b	M1	Complete method, using \mathbf{v} , to obtain an equation in t only, allow	a sign error	
	DM1	Dependent on M1, solve for <i>t</i>		

	A1	cao
5c	M1	Both powers increasing by 1 (M0 if vectors disappear but allow recovery)
	A1	Correct expression without C
	A1	cao
5d	M1	Use of Pythagoras on \mathbf{v} and 10 to set up equation in t
	M(A)1	Correct 3 term quadratic in t
	A1	cao
	M1	Substitute their numerical t value into their r
	M1	Use of Pythagoras to find the magnitude of their r
	A1	cso