## AS Pure Mathematics 8MA0: Specimen Paper 1 Mark Scheme

| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 1 (a) | $y=2 x^{3}-2 x^{2}-2 x+8 \Rightarrow \frac{\mathrm{~d} y}{\mathrm{~d} x}=6 x^{2}-4 x-2$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | $\begin{aligned} & 1.1 \mathrm{~b} \\ & 1.1 \mathrm{~b} \end{aligned}$ |
|  |  | (2) |  |
| (b) | Attempts $6 x^{2}-4 x-2>0 \Rightarrow(6 x+2)(x-1)>0$ | M1 | 1.1b |
|  | $x=-\frac{1}{3}, 1$ | A1 | 1.1b |
|  | Chooses outside region | M1 | 1.1 b |
|  | $\left\{x: x<-\frac{1}{3}\right\} \cup\{x: x>1\}$ | A1 | 2.5 |
|  |  | (4) |  |
| (6 marks) |  |  |  |

## Notes:

(a)

M1: Attempts to differentiate. Allow for two correct terms un-simplified
A1: $\quad \frac{\mathrm{d} y}{\mathrm{~d} x}=6 x^{2}-4 x-2$
(b)

M1: Attempts to find the critical values of their $\frac{\mathrm{d} y}{\mathrm{~d} x}>0$ or their $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$
A1: Correct critical values $x=-\frac{1}{3}, 1$
M1: Chooses the outside region
A1: $\left\{x: x<-\frac{1}{3}\right\} \cup\{x: x>1\}$ or $\left\{x: x \in \mathrm{R} \quad x<-\frac{1}{3}\right.$ or $\left.x>1\right\}$
Accept also $\left\{x: x,-\frac{1}{3}\right\} \cup\{x: x \ldots 1\}$

| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 2 (a) | $\overrightarrow{A B}=\overrightarrow{O B}-\overrightarrow{O A}=6 \mathbf{i}-3 \mathbf{j}-(4 \mathbf{i}+2 \mathbf{j})$ | M1 | 1.1b |
|  | $=2 \mathbf{i}-5 \mathbf{j}$ | A1 | 1.1b |
|  |  | (2) |  |
| 1(b) | Explains that $\overrightarrow{O C}$ is parallel to $\overrightarrow{A B}$ as $8 \mathbf{i}-20 \mathbf{j}=4 \times(2 \mathbf{i}-5 \mathbf{j})$ | M1 | 1.1b |
|  | As $\overrightarrow{O C}=4 \times \overrightarrow{A B}$ it is parallel to it and not the same length Hence $O A B C$ is a trapezium. | A1 | 2.4 |
|  |  | (2) |  |
| (4 marks) |  |  |  |
| Notes: |  |  |  |
| (a) <br> M1: Attempts $\overrightarrow{A B}=\overrightarrow{O B}-\overrightarrow{O A}$ or equivalent. This may be implied by one correct component <br> A1: $2 \mathbf{i}-5 \mathbf{j}$ <br> (b) <br> M1: Attempts to compare vectors $\overrightarrow{O C}$ and $\overrightarrow{A B}$ by considering their directions <br> A1: Fully explains why $O A B C$ is a trapezium. (The candidate is required to state that $O C$ is parallel to $A B$ but not the same length as it.) |  |  |  |


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 3(a) | Uses or implies that $V=a t+b$ | B1 | 3.3 |
|  | Uses both $4=24 a+b$ and $2.8=60 a+b$ to get either $a$ or $b$ | M1 | 3.1b |
|  | Uses both $4=24 a+b$ and $2.8=60 a+b$ to get both $a$ and $b$ | M1 | 1.1b |
|  | $\Rightarrow V=-\frac{1}{30} t+4.8$ | A1 | 1.1b |
|  |  | (4) |  |
| (b) | (i) States that the initial volume is $4.8 \mathrm{~m}^{3}$ | B1 ft | 3.4 |
|  | (ii) Attempts to solve $0=-\frac{1}{30} t+4.8$ | M1 | 3.4 |
|  | States 144 minutes | A1 | 1.1b |
|  |  | (3) |  |
| (c) | States any logical reason <br> - The tank will leak more quickly at the start due to the greater water pressure <br> - The hole will probably get larger and so will start to leak more quickly <br> - Sediment could cause the leak to be plugged and so the tank would not empty. | B1 | 3.5b |
|  |  | (1) |  |

(8 marks)

## Notes:

(a)

B1: Uses or implies that $V=a t+b$
You may award this at their final line but it must be $V=\mathrm{f}(t)$
M1: Awarded for translating the problem in context and starting to solve. It is scored when both $4=24 a+b$ and $2.8=60 a+b$ are written down and the candidate proceeds to find either $a$ or $b$. You may just see a line $\pm \frac{4-2.8}{60-24}$
M1: Uses $4=24 a+b$ and $2.8=60 a+b$ to find both $a$ and $b$
A1: $V=-\frac{1}{30} t+4.8$ or exact equivalent. Eg $30 V+t=144$
(b)(i)

B1ft: Follow through on their ' $b$ '
(b)(ii)

M1: States that $V=0$ and finds $t$ by attempting to solve their $0=-\frac{1}{30} t+4.8$
A1: States 144 minutes
(c)

B1: States any logical reason. There must be a statement and a reason that matches See scheme

| Question |  | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 4(a) | $(4,-3)$ | B1 | 1.2 |
|  |  | (1) |  |
| (b) | $x=6$ | B1 | 1.1b |
|  |  | (1) |  |
| (c) | $x \leq 4$ | B1 | 1.1b |
|  |  | (1) |  |
| (d) | $k>1.5$ | B1 | 2.2a |
|  |  | (1) |  |
| (4 marks) |  |  |  |
| Notes: |  |  |  |
| See m/scheme |  |  |  |


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 5(a) | $\mathrm{f}(-3)=(-3)^{3}+3 \times(-3)^{2}-4 \times(-3)-12$ | M1 | 1.1b |
|  | $\mathrm{f}(-3)=0 \Rightarrow(x+3)$ is a factor $\Rightarrow$ Hence $\mathrm{f}(x)$ is divisible by $(x+3)$. | A1 | 2.4 |
|  |  | (2) |  |
| (b) | $x^{3}+3 x^{2}-4 x-12=(x+3)\left(x^{2}-4\right)$ | M1 | 1.1b |
|  | $=(x+3)(x+2)(x-2)$ | $\begin{gathered} \mathrm{dM} 1 \\ \mathrm{~A} 1 \end{gathered}$ | $\begin{aligned} & 1.1 \mathrm{~b} \\ & 1.1 \mathrm{~b} \end{aligned}$ |
|  |  | (3) |  |
| (c) | $\frac{x^{3}+3 x^{2}-4 x-12}{x^{3}+5 x^{2}+6 x}=\frac{\ldots .}{x\left(x^{2}+5 x+6\right)}$ | M1 | 3.1a |
|  | $=\frac{(x+3)(x+2)(x-2)}{x(x+3)(x+2)}$ | dM1 | 1.1b |
|  | $=\frac{(x-2)}{x}=1-\frac{2}{x}$ | A1 | 2.1 |
|  |  | (3) |  |

(8 marks)

## Notes:

(a)

M1: Attempts $\mathrm{f}(-3)$
A1: Achieves $\mathrm{f}(-3)=0$ and explains that $(x+3)$ is a factor and hence $\mathrm{f}(x)$ is divisible by $(x+3)$.
(b)

M1: Attempts to divide by $(x+3)$ to get the quadratic factor.
By division look for the first two terms. ie $x^{2}+0 x \quad \frac{x^{2} \pm 0 x \ldots \ldots \ldots \ldots . . . . . . . . . . . . ~}{x+3} \begin{aligned} & x^{3}+3 x^{2}-4 x-12\end{aligned}$

$$
x^{3}+3 x^{2}
$$

By inspection look for the first and last term $x^{3}+3 x^{2}-4 x-12=(x+3)\left(x^{2}+. . x \pm 4\right)$
dM1: For an attempt at factorising their $\left(x^{2}-4\right)$. (Need to check first and last terms)
A1: $\mathrm{f}(x)=(x+3)(x+2)(x-2)$
(c)

M1: Takes a common factor of $x$ out of the denominator and writes the numerator in factors.
Alternatively rewrites to $x^{3}+3 x^{2}-4 x-12=A\left(x^{3}+5 x^{2}+6 x\right)+B\left(x^{2}+5 x+6\right)$
dM1: Further factorises the denominator and cancels
Alternatively compares terms or otherwise to find either $A$ or $B$
A1: Shows that $\frac{x^{3}+3 x^{2}-4 x-12}{x^{3}+5 x^{2}+6 x}=1-\frac{2}{x}$ with no errors or omissions
In the alternative there must be a reference to

$$
x^{3}+3 x^{2}-4 x-12 \equiv 1\left(x^{3}+5 x^{2}+6 x\right)-2\left(x^{2}+5 x+6\right) \text { and hence } \frac{x^{3}+3 x^{2}-4 x-12}{x^{3}+5 x^{2}+6 x}=1-\frac{2}{x}
$$

| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 6(i) | Tries at least one value in the interval $\operatorname{Eg} 4^{2}-4-1=11$ | M1 | 1.1b |
|  | States that when $n=8$ it is FALSE and provides evidence $8^{2}-8-1=55=(11 \times 5) \quad$ Hence NOT PRIME | A1 | 2.4 |
|  |  | (2) |  |
| (ii) | Knows that an odd number is of the form $2 n+1$ | B1 | 3.1a |
|  | Attempts to simplify $(2 n+1)^{3}-(2 n+1)^{2}$ | M1 | 2.1 |
|  | $\cdots \ldots \ldots$. and factorise $8 n^{3}+8 n^{2}+2 n=2\left(4 n^{3}+4 n^{2}+1 n\right)=$ | dM1 | 1.1b |
|  | with statement $2 \times .$. is always even | A1 | 2.4 |
|  |  | (4) |  |
| Alt (ii) | Let the odd number be ' $n$ ' and attempts $n^{3}-n^{2}$ | B1 | 3.1a |
|  | Attempts to factorise $n^{3}-n^{2}=n^{2}(n-1)$ | M1 | 2.1 |
|  | States that $n^{2}$ is odd (odd $\times$ odd) and ( $n-1$ ) is even (odd -1$)$ | dM1 | 1.1 b |
|  | States that the product is even ( odd $\times$ even) | A1 | 2.4 |
| (6 marks) |  |  |  |

## Notes: See above

(i)

M1: Attempts any $n^{2}-n-1$ for $n$ in the interval. It is acceptable just to show $8^{2}-8-1=55$
A1: States that when $n=8$ it is FALSE and provides evidence. A comment that $55=11 \times 5$ and hence not prime is required
(ii)

## See scheme for two examples of proof

Note that Alt (i) works equally well with an odd number of the form $2 n-1$
For example $(2 n-1)^{3}-(2 n-1)^{2}=(2 n-1)^{2}\{2 n-1-1\}=(2 n-1)^{2}\{2 n-2\}=2 \times(2 n-1)^{2}\{n-1\}$

| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 7 (a) | $\left(1+\frac{3}{x}\right)^{2}=1+\frac{6}{x}+\frac{9}{x^{2}}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | $\begin{aligned} & 1.1 \mathrm{~b} \\ & 1.1 \mathrm{~b} \end{aligned}$ |
|  |  | (2) |  |
| (b) | $\left(1+\frac{3}{4} x\right)^{6}=1+6 \times\left(\frac{3}{4} x\right)+\ldots$ | B1 | 1.1b |
|  | $\left(1+\frac{3}{4} x\right)^{6}=1+6 \times\left(\frac{3}{4} x\right)+\frac{6 \times 5}{2} \times\left(\frac{3}{4} x\right)^{2}+\frac{6 \times 5 \times 4}{3 \times 2} \times\left(\frac{3}{4} x\right)^{3}+\ldots$ | $\begin{gathered} \text { M1 } \\ \text { A1 } \end{gathered}$ | $\begin{aligned} & 1.1 \mathrm{~b} \\ & 1.1 \mathrm{~b} \end{aligned}$ |
|  | $=1+\frac{9}{2} x+\frac{135}{16} x^{2}+\frac{135}{16} x^{3}+\ldots$ | A1 | 1.1b |
|  |  | (4) |  |
| (c) | $\left(1+\frac{3}{x}\right)^{2}\left(1+\frac{3}{4} x\right)^{6}=\left(1+\frac{6}{x}+\frac{9}{x^{2}}\right)\left(1+\frac{9}{2} x+\frac{135}{16} x^{2}+\frac{135}{16} x^{3}+\ldots\right)$ |  |  |
|  | Coefficient of $x=\frac{9}{2}+6 \times \frac{135}{16}+9 \times \frac{135}{16}=\frac{2097}{16}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | $\begin{aligned} & 2.1 \\ & 1.1 \mathrm{~b} \end{aligned}$ |
|  |  | (2) |  |
| (8 marks) |  |  |  |

## Notes:

(a)

M1: Attempts $\left(1+\frac{3}{x}\right)^{2}=A+\frac{B}{x}+\frac{C}{x^{2}}$
A1: $\left(1+\frac{3}{x}\right)^{2}=1+\frac{6}{x}+\frac{9}{x^{2}}$
(b)

B1: First two terms correct, may be un-simplified
M1: Attempts the binomial expansion. Implied by the correct coefficient and power of $x$ seen at least once in term 3 or 4
A1: Binomial expansion correct and un-simplified
A1: Binomial expansion correct and simplified.
(c)

M1: Combines all relevant terms for their $\left(1+\frac{A}{x}+\frac{B}{x^{2}}\right)\left(1+C x+D x^{2}+E x^{3}+\ldots\right)$ to find the coefficient of $x$.
A1: Fully correct

| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 8(a) | (i) $\int_{1}^{a} \sqrt{8 x} \mathrm{~d} x=\sqrt{8} \times \int_{1}^{a} \sqrt{x} \mathrm{~d} x=10 \sqrt{8}=20 \sqrt{2}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | $\begin{aligned} & 2.2 \mathrm{a} \\ & 1.1 \mathrm{~b} \end{aligned}$ |
|  | (ii) $\int_{0}^{a} \sqrt{x} \mathrm{~d} x=\int_{0}^{1} \sqrt{x} \mathrm{~d} x+\int_{1}^{a} \sqrt{x} \mathrm{~d} x=\left[\frac{2}{3} x^{\frac{3}{2}}\right]_{0}^{1}+10=\frac{32}{3}$ | $\begin{gathered} \text { M1 } \\ \text { A1 } \end{gathered}$ | $\begin{gathered} 2.1 \\ 1.1 \mathrm{~b} \end{gathered}$ |
|  |  | (4) |  |
| (b) | $R=\int_{1}^{a} \sqrt{x} \mathrm{~d} x=\left[\frac{2}{3} x^{\frac{3}{2}}\right]_{1}^{a}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | $\begin{aligned} & 1.1 b \\ & 1.1 \mathrm{~b} \end{aligned}$ |
|  | $\frac{2}{3} a^{\frac{3}{2}}-\frac{2}{3}=10 \Rightarrow a^{\frac{3}{2}}=16 \Rightarrow a=16^{\frac{2}{3}}$ | dM1 | 3.1a |
|  | $\Rightarrow a=2^{4 \times \frac{2}{3}}=2^{\frac{8}{3}}$ | A1 | 2.1 |
|  |  | (4) |  |

(8 marks)

## Notes:

(a)(i)

M1: For deducing that $\int_{1}^{a} \sqrt{8 x} \mathrm{~d} x=\sqrt{8} \times \int_{1}^{a} \sqrt{x} \mathrm{~d} x$ attempting to multiply $\int_{1}^{a} \sqrt{x} \mathrm{~d} x$ by $\sqrt{8}$
A1: $20 \sqrt{2}$ or exact equivalent
(a)(ii)

M1: For identifying and attempting to use $\int_{0}^{a} \sqrt{x} \mathrm{~d} x=\int_{0}^{1} \sqrt{x} \mathrm{~d} x+\int_{1}^{a} \sqrt{x} \mathrm{~d} x$
A1: For $\frac{32}{3}$ or exact equivalent
(b)

M1: Attempts to integrate, $x^{\frac{1}{2}} \rightarrow x^{\frac{3}{2}}$
A1: $\quad \int_{1}^{a} \sqrt{x} \mathrm{~d} x=\left[\frac{2}{3} x^{\frac{3}{2}}\right]_{1}^{a}$
dM1: For a whole strategy to find $a$. In the scheme it is awarded for setting $\left[\ldots x^{\frac{3}{2}}\right]_{1}^{a}=10$, using both limits and proceeding using correct index work to find $a$. Alternatively a candidate could assume $a=2^{k}$. In such a case it is awarded for setting $\left[\ldots x^{\frac{3}{2}}\right]_{1}^{2^{k}}=10$, using both limits and proceeding using correct index work to $k=$. .
A1: $a=2^{4 \times \frac{2}{3}}=2^{\frac{8}{3}}$
In the alternative case, a further statement must be seen following $k=\frac{8}{3}$ Hence True

| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 9 | $2 \log _{4}(2-x)-\log _{4}(x+5)=1$ |  |  |
|  | Uses the power law $\log _{4}(2-x)^{2}-\log _{4}(x+5)=1$ | M1 | 1.1 b |
|  | Uses the subtraction law $\log _{4} \frac{(2-x)^{2}}{(x+5)}=1$ | M1 | 1.1b |
|  | $\frac{(2-x)^{2}}{(x+5)}=4 \rightarrow 3 \mathrm{TQ}$ in $x$ | dM1 | 3.1a |
|  | $x^{2}-8 x-16=0$ | A1 | 1.1b |
|  | $(x-4)^{2}=32 \Rightarrow x=$ | M1 | 1.1b |
|  | $x=4-4 \sqrt{2}$ oe only | A1 | 2.3 |
|  |  | (6) |  |
| (6 marks) |  |  |  |

## Notes:

M1: Uses the power law of $\operatorname{logs} 2 \log _{4}(2-x)=\log _{4}(2-x)^{2}$
M1: Uses the subtraction law of logs following the above $\log _{4}(2-x)^{2}-\log _{4}(x+5)=\log _{4} \frac{(2-x)^{2}}{(x+5)}$
Alternatively uses the addition law following use of $1=\log _{4} 4$ That is $1+\log _{4}(x+5)=\log _{4} 4(x+5)$
dM1: This can be awarded for the overall strategy leading to a 3TQ in $x$. It is dependent upon the correct use of both previous M's and for undoing the logs to reach a 3TQ equation in $x$
A1: For a correct equation in $x$
M1: For the correct method of solving their $3 \mathrm{TQ}=0$
A1: $x=4-4 \sqrt{2}$ or exact equivalent only. (For example accept $x=4-\sqrt{32}$ )

| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 10(a) | Attempts to find the radius $\sqrt{(2--2)^{2}+(5-3)^{2}}$ or radius ${ }^{2}$ | M1 | 1.1b |
|  | Attempts $(x-2)^{2}+(y-5)^{2}=r^{\prime 2}$ | M1 | 1.1b |
|  | Correct equation $(x-2)^{2}+(y-5)^{2}=20$ | A1 | 1.1b |
|  |  | (3) |  |
| (b) | Gradient of radius $O P$ where $O$ is the centre of $C=\frac{5-3}{2--2}=\left(\frac{1}{2}\right)$ | M1 | 1.1b |
|  | Equation of $l$ is $-2=\frac{y-3}{x+2}$ | dM1 | 3.1a |
|  | Any correct form $y=-2 x-1$ | A1 | 1.1b |
|  | Method of finding $k$ Substitute $x=2$ into their $y=-2 x-1$ | M1 | 2.1 |
|  | $k=-5$ | A1 | 1.1b |
|  |  | (5) |  |
| (8 marks) |  |  |  |

## Notes:

(a)

M1: As scheme or states form of circle is $(x-2)^{2}+(y-5)^{2}=r^{\prime 2}$
M1: As scheme or substitutes $(-2,3)$ into $(x-2)^{2}+(y-5)^{2}=' r^{\prime 2}$
A1: For a correct equation
If students use $x^{2}+y^{2}+2 f x+2 g y+c=0 \mathbf{M 1}: f=2, g=5 \mathbf{M 1}$ : substitutes $(2,5)$ to find value of $c$
A1: $x^{2}+y^{2}-4 x-10 y+9=0$
(b)

M1: Attempts to find the gradient of $O P$ where $O$ is the centre of $C$
dM1: For a complete strategy of finding the equation of $l$ using the perpendicular gradient to $O P$ and the point $(-2,3)$..
A1: Any correct form of $l$ Eg $y=-2 x-1$
M1: Scored for the key step of finding $k$. In this method they are required to substitute $(2, k)$ in their $y=-2 x-1$ and solve for $k$.
A1: $k=-5$
Alt using Pythagoras' theorem
M1: Attempts Pythagoras to find both $P Q$ and $O Q$ in terms of $k$ (where $O$ is centre of $C$ )
dM1: For the complete strategy of using Pythagoras theorem on triangle $P O Q$ to form an equation in $k$
A1: A correct equation in $k$ Eg. $20+(k-3)^{2}+16=(k-5)^{2}$
M1: Scored for a correct attempt to solve their quadratic to find $k$.
A1: $k=-5$

| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 11(i) | $\left(2 \theta+10^{\circ}\right)=\arcsin (-0.6)$ | M1 | 1.1b |
|  | $\left(2 \theta+10^{\circ}\right)=-143.13^{\circ},-36.87^{\circ}, 216.87^{\circ}, 323.13^{\circ}$ (Any two) | A1 | 1.1b |
|  | Correct order to find $\theta=\ldots$ | dM1 | 1.1b |
|  | Two of $\theta=-76.6^{\circ},-23.4^{\circ}, 103.4^{\circ}, 156.6^{\circ}$. | A1 | 1.1b |
|  | $\theta=-76.6^{\circ},-23.4^{\circ}, 103.4^{\circ}, 156.6^{\circ}$, only | A1 | 2.1 |
|  |  | (5) |  |
| (ii) | (a) Explains that the student has not considered the negative value of $x\left(=-29.0^{\circ}\right)$ when solving $\cos x=\frac{7}{8}$ | B1 | 2.3 |
|  | Explains that the student should check if any solutions of $\sin x=0$ (the cancelled term) are solutions of the given equation. $x=0^{\circ}$ should have been included as a solution | B1 | 2.3 |
|  | (b) Attempts to solve $4 \alpha+199^{\circ}=(360-29.0)^{\circ}$ | M1 | 2.2a |
|  | $\alpha=33.0^{\circ}$ | A1 | 1.1b |
|  |  | (4) |  |
| (9 marks) |  |  |  |

## Notes:

(i)

M1: Attempts arcsin $(-0.6)$ implied by any correct answer
A1: Any two of $-143.13^{\circ},-36.87^{\circ}, 216.87^{\circ}, 323.13^{\circ}$
dM1: Correct method to find any value of $\theta$
A1: Any two of $\theta=-76.6^{\circ},-23.4^{\circ}, 103.4^{\circ}, 156.6^{\circ}$.
A1: A full solution leading to all four answers and no extras

$$
\theta=-76.6^{\circ},-23.4^{\circ}, 103.4^{\circ}, 156.6^{\circ}, \text { only }
$$

(ii)(a)

B1: See scheme
B1: See scheme
(ii)(b)

M1: For deducing the smallest positive solution occurs when $4 \alpha+199^{\circ}=(360-29.0)^{\circ}$
A1: $\alpha=33^{\circ}$

| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 12(a) | Sets $3 x-2 \sqrt{x}=8 x-16$ | B1 | 1.1a |
|  | $\begin{array}{c\|c} 2 \sqrt{x}=16-5 x & 5 x+2 \sqrt{x}-16=0 \\ 4 x=(16-5 x)^{2} \Rightarrow x=. . & \Rightarrow(5 \sqrt{x} \pm 8)(\sqrt{x} \pm 2)=0 \end{array}$ | M1 | 3.1a |
|  | $25 x^{2}-164 x+256=0$ $(5 \sqrt{x}-8)(\sqrt{x}+2)=0$ | A1 | 1.1b |
|  | $(25 x-64)(x-4)=0 \Rightarrow x=. . \quad \sqrt{x}=\frac{8}{5},(-2) \Rightarrow x=.$. | M1 | 1.1b |
|  | $x=\frac{64}{25}$ only | A1 | 2.3 |
|  |  | (5) |  |
| (b) | Attempts to solve $3 x-2 \sqrt{x}=0$ | M1 | 2.1 |
|  | Correct solution $\quad x=\frac{4}{9}$ | A1 | 1.1b |
|  | $y, 3 x-2 \sqrt{x}, y>8 x-16 \quad x \ldots \frac{4}{9}$ | B1ft | 1.1b |
|  |  | (3) |  |
| (8 marks) |  |  |  |

## Notes:

## (a)

B1: Sets the equations equal to each other and achieves a correct equation
M1: Awarded for the key step in solving the problem. This can be awarded via two routes. Both routes must lead to a value for $x$.

- Making the $\sqrt{x}$ term the subject and squaring both sides (not each term)
- Recognising that this is a quadratic in $\sqrt{x}$ and attempting to factorise

$$
\Rightarrow(5 \sqrt{x} \pm 8)(\sqrt{x} \pm 2)=0
$$

A1: A correct intermediate line $25 x^{2}-164 x+256=0$ or $(5 \sqrt{x}-8)(\sqrt{x}+2)=0$
M1: A correct method to find at least one value for $x$. Way One it is for factorising (usual rules), Way Two it is squaring at least one result of their $\sqrt{x}$
A1: Realises that $x=\frac{64}{25}$ is the only solution $\quad x=\frac{64}{25}, 4$ is A0
(b) M1: Attempts to solve $3 x-2 \sqrt{x}=0$ For example

Allow $3 x=2 \sqrt{x} \Rightarrow 9 x^{2}=4 x \Rightarrow x=\ldots$
Allow $3 x=2 \sqrt{x} \Rightarrow x^{\frac{1}{2}}=\frac{2}{3} \Rightarrow x=\ldots$
A1: Correct solution to $3 x-2 \sqrt{x}=0 \Rightarrow x=\frac{4}{9}$
B1: For a consistent solution defining $R$ using either convention
Either $y, 3 x-2 \sqrt{x}, y>8 x-16 x \ldots \frac{4}{9}$ Or $y<3 x-2 \sqrt{x}, y \ldots 8 x-16 x>\frac{4}{9}$

| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 13(a) | $0.2 \mathrm{~m}^{2}$ | B1 | 3.4 |
|  |  | (1) |  |
| (b) | $A=0.2 \mathrm{e}^{0.3 t}$ Rate of change $=$ gradient $=\frac{\mathrm{d} A}{\mathrm{~d} t}=0.06 \mathrm{e}^{0.3 t}$ | M1 | 3.1b |
|  | At $t=5 \Rightarrow$ Rate of Growth is $0.06 \mathrm{e}^{1.5}=0.269 \mathrm{~m}^{2} /$ day | A1 | 1.1 b |
|  |  | (2) |  |
| (c) | $100=0.2 \mathrm{e}^{0.3 t} \Rightarrow \mathrm{e}^{0.3 t}=500$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | $\begin{aligned} & 3.1 \mathrm{a} \\ & 1.1 \mathrm{~b} \end{aligned}$ |
|  | $\Rightarrow t=\frac{\ln (500)}{0.3}=20.7$ days $\quad 20$ days 17 hours | $\begin{gathered} \text { M1 } \\ \text { A1 } \end{gathered}$ | $\begin{aligned} & 1.1 \mathrm{~b} \\ & 3.2 \mathrm{a} \end{aligned}$ |
|  |  | (4) |  |
|  | At $t=5 \Rightarrow$ Rate of Growth is $0.06 \mathrm{e}^{1.5}=0.269 \mathrm{~m}^{2} /$ day | A1 | 1.1 b |
|  |  | (2) |  |
| (d) | The model given suggests that the pond is fully covered after 20 days 17 hours. Observed data is inconsistent with this as the pond is only $90 \%$ covered by the end of one month (28/29/30/31 days). <br> Hence the model is not accurate | B1 | 3.5a |
|  |  | (1) |  |
| (8 marks) |  |  |  |

## Notes:

(a)

B1: $0.2 \mathrm{~m}^{2}$ oe
(b)

M1: Links rate of change to gradient and differentiates $0.2 \mathrm{e}^{0.3 t} \rightarrow k \mathrm{e}^{0.3 t}$
A1: Correct answer $0.269 \mathrm{~m}^{2} /$ day
(c)

M1: Substitutes $A=100$ and proceeds to $\mathrm{e}^{0.3 t}=k$
A1: $\mathrm{e}^{0.3 t}=500$
M1: Correct method when proceeding from $\mathrm{e}^{0.3 t}=k \Rightarrow t=$..
A1: 20 days 17 hours
(d)

B1: Valid conclusion following through on their answer to (c).

| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 14 | $y=(x-2)^{2}(x+3)=\left(x^{2}-4 x+4\right)(x+3)=x^{3}-1 x^{2}-8 x+12$ | B1 | 1.1b |
|  | An attempt to find $x$ coordinate of the maximum point. To score this you must see either <br> - an attempt to expand $(x-2)^{2}(x+3)$, an attempt to differentiate the result, followed by an attempt at solving $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ <br> - an attempt to differentiate $(x-2)^{2}(x+3)$ by the product rule followed by an attempt at solving $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ | M1 | 3.1a |
|  | $y=x^{3}-1 x^{2}-8 x+12 \Rightarrow \frac{\mathrm{~d} y}{\mathrm{~d} x}=3 x^{2}-2 x-8$ | M1 | 1.1b |
|  | Maximum point occurs when $\frac{\mathrm{d} y}{\mathrm{~d} x}=0 \Rightarrow(x-2)(3 x+4)=0$ $\Rightarrow x=-\frac{4}{3}$ | M1 <br> A1 | $\begin{aligned} & 1.1 \mathrm{~b} \\ & 1.1 \mathrm{~b} \end{aligned}$ |
|  | An attempt to find the area under $y=(x-2)^{2}(x+3)$ between two values. To score this you must see an attempt to expand $(x-2)^{2}(x+3)$ followed by an attempt at using two limits | M1 | 3.1a |
|  | Area $=\int\left(x^{3}-1 x^{2}-8 x+12\right) \mathrm{d} x=\left[\frac{x^{4}}{4}-\frac{x^{3}}{3}-4 x^{2}+12 x\right]$ | M1 | 1.1 b |
|  | Uses a top limit of 2 and a bottom limit of their $x=-\frac{4}{3} R=\left[\frac{x^{4}}{4}-\frac{x^{3}}{3}-4 x^{2}+12 x\right]_{-\frac{4}{3}}^{2}$ | M1 | 2.2a |
|  | Uses $=\frac{28}{3}--\frac{1744}{81}=\frac{2500}{81}$ | A1 | 2.1 |
|  |  | (9) |  |
| (9 marks) |  |  |  |
| Notes: |  |  |  |
| B1: Expands $(x-2)^{2}(x+3)$ to $x^{3}-1 x^{2}-8 x+12$ seen at some point in their solution. It may appear just on their integral for example. <br> M1: This is a problem solving mark for knowing the method of finding the maximum point. You should expect to see the key points used (i) differentiation (ii) solution of their $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ |  |  |  |

M1: For correctly differentiating their cubic with at least two terms correct (for their cubic).
M1: For setting their $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ and solves using a correct method (including calculator methods)
A1: $\Rightarrow x=-\frac{4}{3}$
M1: This is a problem solving mark for knowing how integration is used to find the area underneath a curve between two points.
M1: For correctly integrating their cubic with at least two correct terms (for their cubic).
M1: For deducing the top limit is 2 , the bottom limit is their $x=-\frac{4}{3}$ and applying these correctly within their integration.
A1: Shows above steps clearly and proceeds to $R=\frac{2500}{81}$

