EXPERT TUITION

Maths Questions By Topic:

Algebra & Functions
Mark Scheme

A-Level Edexcel

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Question	Scheme	Marks	AOs
1	$\text{f}(1) = a(1)^3 + 10(1)^2 - 3a(1) - 4 = 0$	M1	3.1a
	$6 - 2a = 0 \Rightarrow a = ...$	M1	1.1b
	$a = 3$	A1	1.1b
		(3)	
			(3 marks)

Notes

Main method seen:

M1: Attempts f (1) = 0 to set up an equation in a It is implied by $a + 10 - 3a - 4 = 0$

Condone a slip but attempting f (−1) = 0 is M0

M1: Solves a linear equation in a .

Using the main method it is dependent upon having set f (±1) = 0

It is implied by a solution of $\pm a \pm 10 \pm 3a \pm 4 = 0$.

Don't be concerned about the mechanics of the solution.

A1: a = 3 (following correct work)

....

Answers without working scores 0 marks. The method must be made clear. Candidates cannot guess. However if a candidate states for example, when $a = 3$, $\text{f}(x) = 3x^3 + 10x^2 - 9x - 4$ and shows that $(x-1)$ is a factor of this $\text{f}(x)$ by an allowable method, they should be awarded M1 M1 A1

E.g. 1: $3x^3 + 10x^2 - 9x - 4 = (x-1)(3x^2 + 13x + 4)$ Hence $a = 3$

E.g. 2: $\text{f}(x) = 3x^3 + 10x^2 - 9x - 4$, $\text{f}(1) = 3 + 10 - 9 - 4 = 0$ Hence $a = 3$

The solutions via this method must end with the value for a to score the A1

....

....

Other methods are available. They are more difficult to determine what the candidate is doing.
Please send to review if you are uncertain
It is important that a correct method is attempted so look at how the two M's are scored

Amongst others are:
Alt (1) by inspection which may be seen in a table/g

	ax^2	$(10+a)x$	4
x	ax^3	$(10+a)x^2$	$4x$
-1	$-ax^2$	$-(10+a)x$	-4

$ax^3+10x^2-3ax-4=(x-1)\left(ax^2+(10+a)x+4\right)$ and sets terms in x equal

$-3a=-(10+a)+4 \Rightarrow 2a=6 \Rightarrow a=3$

M1: This method is implied by a **correct** equation, usually $-3a=-(10+a)+4$

M1: Attempts to find the quadratic factor which must be of the form $ax^2+\mathrm{g}(a)x\pm 4$ and then forms and solves a linear equation formed by linking the coefficients or terms in x

....

Alt (2) By division:

$$\begin{array}{r|l} & ax^2+(\pm 10\pm a)x+(10-2a) \\ x-1 & ax^3 \;+10x^2 \quad -3ax-4 \\ & \underline{ax^3-ax^2} \\ & \quad (10+a)x^2-3ax \\ & \quad \underline{(10+a)x^2-(10+a)x} \\ & \qquad\qquad (-2a+10)x \end{array}$$

M1: This method is implied by a **correct** equation, usually $-10+2a=-4$

M1: Attempts to divide with quotient of $ax^2+(\pm 10\pm a)x+\mathrm{h}(a)$ and then forms and solves a linear equation in a formed by setting the remainder = 0.

Question	Scheme	Marks	AOs
2(a)	$f(x)=(x-2)^2 \pm ...$	M1	1.2
	$f(x)=(x-2)^2+1$	A1	1.1b
		(2)	
(b)(i)	$P=(0, 5)$	B1	1.1b
(b)(ii)	$Q=(2, 1)$	B1ft	1.1b
		(2)	
			(4 marks)
	Notes		

(a)

M1: Achieves $(x-2)^2 \pm ...$ or states $a = -2$

A1: Correct expression $(x-2)^2+1$ ISW after sight of this

Condone $a = -2$ and $b = 1$. Condone $(x-2)^2+1=0$

(b)

(i) B1: Correct coordinates for P. Allow to be expressed $x=0, y=5$

(ii) B1ft: Correct coordinates for Q. Allow to be expressed $x=2, y=1$ (Score for the correct answer or follow through their part (a) so allow $(-a, b)$ where a and b are numeric)

Score in any order if they state $P=(0, 5)$ and $Q=(2, 1)$

....

Allow part (b) to be awarded from a sketch. So award

First B1 from a sketch crossing the y-axis at 5

Second B1 from a sketch with minimum at $(2, 1)$

....

Question	Scheme	Marks	AOs
3	$\mathrm{f}'(x) = 2x + \dfrac{4x-4}{2x^2-4x+5}$	M1 A1	1.1b 1.1b
	$2x + \dfrac{4x-4}{2x^2-4x+5} = 0 \Rightarrow 2x(2x^2-4x+5)+4x-4=0$	dM1	1.1b
	$2x^3-4x^2+7x-2=0$ *	A1*	2.1
		(4)	

M1: Differentiates $\ln(2x^2-4x+5)$ to obtain $\dfrac{g(x)}{2x^2-4x+5}$ where g(x) could be 1

A1: For $\mathrm{f}'(x) = 2x + \dfrac{4x-4}{2x^2-4x+5}$

dM1: Sets their $\mathrm{f}'(x) = ax + \dfrac{g(x)}{2x^2-4x+5}$ = 0 and uses "**correct**" algebra, condoning slips, to obtain a cubic equation. E.g Look for $ax(2x^2-4x+5) \pm g(x) = 0$ o.e. , condoning slips, followed by some attempt to simplify

A1*: Achieves $2x^3-4x^2+7x-2=0$ with no errors. (The dM1 mark must have been awarded)

Question	Scheme	Marks	AOs
4	Finds critical values $x^2 - x > 20 \Rightarrow x^2 - x - 20 > 0 \Rightarrow x = (5, -4)$	M1	1.1b
	Chooses outside region for their values Eg. $x > 5, x < -4$	M1	1.1b
	Presents solution in set notation $\{x : x < -4\} \cup \{x : x > 5\}$ oe	A1	2.5
		(3)	
			(3 marks)

Notes

M1: Attempts to find the critical values using an algebraic method. Condone slips but an allowable method should be used and two critical values should be found

M1: Chooses the outside region for their critical values. This may appear in incorrect inequalities such as $5 < x < -4$

A1: Presents in set notation as required $\{x : x < -4\} \cup \{x : x > 5\}$ Accept $\{x < -4 \cup x > 5\}$.

Do not accept $\{x < -4,\ x > 5\}$

Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.

Question	Scheme	Marks	AOs
5 (a)	$3x^3-17x^2-6x=0 \Rightarrow x(3x^2-17x-6)=0$	M1	1.1a
	$\Rightarrow x(3x+1)(x-6)=0$	dM1	1.1b
	$\Rightarrow x=0,-\frac{1}{3},6$	A1	1.1b
		(3)	
(b)	Attempts to solve $(y-2)^2=n$ where n is any solution $\ldots 0$ to (a)	M1	2.2a
	Two of $2, 2\pm\sqrt{6}$	A1ft	1.1b
	All three of $2, 2\pm\sqrt{6}$	A1	2.1
		(3)	
			(6 marks)

Notes

(a)

M1: Factorises out or cancels by x to form a quadratic equation.

dM1: Scored for an attempt to find x. May be awarded for factorisation of the quadratic or use of the quadratic formula.

A1: $x=0,-\frac{1}{3},6$ and no extras

(b)

M1: Attempts to solve $(y-2)^2=n$ where n is any solution $\ldots 0$ to (a). At least one stage of working must be seen to award this mark. Eg $(y-2)^2=0 \Rightarrow y=2$

A1ft: Two of $2, 2\pm\sqrt{6}$ but follow through on $(y-2)^2=n \Rightarrow y=2\pm\sqrt{n}$ where n is a positive solution to part (a). (Provided M1 has been scored)

A1: All three of $2, 2\pm\sqrt{6}$ and no extra solutions. (Provided M1A1 has been scored)

Question	Scheme	Marks	AOs
6 (a)	$\mathrm{f}(x) = -3x^2 + 12x + 8 = -3(x \pm 2)^2 + ...$	M1	1.1b
	$= -3(x-2)^2 + ...$	A1	1.1b
	$= -3(x-2)^2 + 20$	A1	1.1b
		(3)	
(b)	Coordinates of $M = (2, 20)$	B1ft B1ft	1.1b 2.2a
		(2)	

Notes:

(a)

M1: Attempts to take out a common factor and complete the square. Award for $-3(x \pm 2)^2 + ...$

Alternatively attempt to compare $-3x^2 + 12x + 8$ to $ax^2 + 2abx + ab^2 + c$ to find values of a and b

A1: Proceeds to a form $-3(x-2)^2 + ...$ or via comparison finds $a = -3, b = -2$

A1: $-3(x-2)^2 + 20$

(b)

B1ft: One correct coordinate

B1ft: Correct coordinates. Allow as $x = ..., y = ...$

Follow through on their $(-b, c)$

Question	Scheme	Marks	AOs
7 (a) (i)	Uses $\frac{dy}{dx}=-3$ at $x=2 \Rightarrow 12a+60-39=-3$	M1	1.1b
	Solves a correct equation and shows one correct intermediate step $12a+60-39=-3 \Rightarrow 12a=-24 \Rightarrow a=-2$ *	A1*	2.1
(a) (ii)	Uses the fact that $(2,10)$ lies on C $10=8a+60-78+b$	M1	3.1a
	Subs $a=-2$ into $10=8a+60-78+b \Rightarrow b=44$	A1	1.1b
		(4)	
(b)	$f(x)=-2x^3+15x^2-39x+44 \Rightarrow f'(x)=-6x^2+30x-39$	B1	1.1b
	Attempts to show that $-6x^2+30x-39$ has no roots Eg. calculates $b^2-4ac=30^2-4\times-6\times-39=-36$	M1	3.1a
	States that as $f'(x)\neq 0 \Rightarrow$ hence $f(x)$ has no turning points *	A1*	2.4
		(3)	
(c)	$-2x^3+15x^2-39x+44 \equiv (x-4)(-2x^2+7x-11)$	M1 A1	1.1b 1.1b
		(2)	
(d)	Deduces either intercept. $(0,44)$ or $(20,0)$	B1 ft	1.1b
	Deduces both intercepts $(0,44)$ and $(20,0)$	B1 ft	2.2a
		(2)	
			(11 marks)

Notes

(a)(i)

M1: Attempts to use $\frac{dy}{dx}=-3$ at $x=2$ to form an equation in a. Condone slips but expect to see two of the powers reduced correctly

A1*: Correct differentiation with one correct intermediate step before $a=-2$

(a)(ii)

M1: Attempts to use the fact that $(2,10)$ lies on C by setting up an equation in a and b with $a=-2$ leading to $b=...$

A1: $b=44$

(b)

B1: $f'(x)=-6x^2+30x-39$ oe

M1: Correct attempt to show that "$-6x^2+30x-39$" has no roots.
This could involve an attempt at

- finding the numerical value of b^2-4ac
- finding the roots of $-6x^2+30x-39$ using the quadratic formula (or their calculator)
- completing the square for $-6x^2+30x-39$

A1*: A fully correct method with reason and conclusion. Eg as $b^2-4ac=-36<0, f'(x)\neq 0$ meaning that no stationary points exist

(c)

M1: For an attempt at division (seen or implied) Eg $-2x^3+15x^2-39x+b \equiv (x-4)\left(-2x^2 ... \pm \frac{b}{4}\right)$

A1: $(x-4)\left(-2x^2+7x-11\right)$ Sight of the quadratic with no incorrect working seen can score both marks.

(d)

See scheme. You can follow through on their value for b

Question	Scheme	Marks	AOs
8 (a)	Either attempts $\frac{3x-7}{x-2}=7 \Rightarrow x=...$ Or attempts $f^{-1}(x)$ and substitutes in $x=7$	M1	3.1a
	$\frac{7}{4}$ oe	A1	1.1b
		(2)	
(b)	Attempts $ff(x)=\frac{3\times\left(\frac{3x-7}{x-2}\right)-7}{\left(\frac{3x-7}{x-2}\right)-2}=\frac{3\times(3x-7)-7(x-2)}{3x-7-2(x-2)}$	M1, dM1	1.1b 1.1b
	$=\frac{2x-7}{x-3}$	A1	2.1
		(3)	
		(5 marks)	
Notes:			

(a)

M1: For either attempting to solve $\frac{3x-7}{x-2}=7$. Look for an attempt to multiply by the $(x-2)$ leading to a value for *x*.

Or score for substituting in $x=7$ in $f^{-1}(x)$. FYI $f^{-1}(x)=\frac{2x-7}{x-3}$

The method for finding $f^{-1}(x)$ should be sound, but you can condone slips.

A1: $\frac{7}{4}$

(b)

M1: For an attempt at fully substituting $\frac{3x-7}{x-2}$ into $f(x)$. Condone slips but the expression must have a correct form. E.g. $\frac{3\times\left(\frac{*-*}{*-*}\right)-a}{\left(\frac{*-*}{*-*}\right)-b}$ where *a* and *b* are positive constants.

dM1: Attempts to multiply **all** terms on the numerator and denominator by $(x-2)$ to create a fraction $\frac{P(x)}{Q(x)}$ where both $P(x)$ and $Q(x)$ are linear expressions. Condone $\frac{P(x)}{Q(x)}\times\frac{x-2}{x-2}$

A1: Reaches $\frac{2x-7}{x-3}$ via careful and accurate work. Implied by $a=2, b=-7$ following correct work.

....

Methods involving $\frac{3x-7}{x-2}\equiv a+\frac{b}{x-2}$ may be seen. The scheme can be applied in a similar way

FYI $\frac{3x-7}{x-2}\equiv 3-\frac{1}{x-2}$

Question	Scheme	Marks	AOs
9	Attempts equation of line Eg Substitutes $(-2,13)$ into $y = mx + 25$ and finds m	M1	1.1b
	Equation of l is $y = 6x + 25$	A1	1.1b
	Attempts equation of C Eg Attempts to use the intercept $(0,25)$ within the equation $y = a(x \pm 2)^2 + 13,$ in order to find a	M1	3.1a
	Equation of C is $y = 3(x+2)^2 + 13$ or $y = 3x^2 + 12x + 25$	A1	1.1b
	Region R is defined by $3(x+2)^2 + 13 < y < 6x + 25$ o.e.	B1ft	2.5
		(5)	
			(5 marks)
Notes:			

The first two marks are awarded for finding the equation of the line

M1: Uses the information in an attempt to find an equation for the line l.

E.g. Attempt using two points: Finds $m = \pm\frac{25-13}{2}$ and uses of one of the points in their $y = mx + c$ or equivalent to find c. Alternatively uses the intercept as shown in main scheme.

A1: $y = 6x + 25$ seen or implied. This alone scores the first two marks. Do not accept $l = 6x + 25$

It must be in the form $y =$ ... but the correct equation can be implied from an inequality. E.g. $\ldots < y < 6x + 25$.

The next two marks are awarded for finding the equation of the curve

M1: A complete method to find the constant a in $y = a(x \pm 2)^2 + 13$ or the constants a, b in $y = ax^2 + bx + 25$.

An alternative to the main scheme is deducing equation is of the form $y = ax^2 + bx + 25$ and setting and solving a pair of simultaneous equations in a and b using the point (-2, 13) the gradient being 0 at $x = -2$. Condone slips. Implied by $C = 3x^2 + 12x + 25$ or $3x^2 + 12x + 25$

FYI the correct equations are $13 = 4a - 2b + 25\ (2a - b = -6)$ and $-4a + b = 0$

A1: $y = 3(x+2)^2 + 13$ or equivalent such as $y = 3x^2 + 12x + 25$, $f(x) = 3(x+2)^2 + 13$.

Do not accept $C = 3x^2 + 12x + 25$ or just $3x^2 + 12x + 25$ for the A1 but may be implied from an inequality or from an attempt at the area, E.g. $\int 3x^2 + 12x + 25\,dx$

B1ft: Fully defines the region R. Follow through on their equations for l and C.

Allow strict or non -strict inequalities as long as they are used consistently.

E.g. Allow for example "$3(x+2)^2 + 13 < y < 6x + 25$" "$3(x+2)^2 + 13 \leqslant y \leqslant 6x + 25$"

Allow the inequalities to be given separately, e.g. $y < 6x + 25,\ y > 3(x+2)^2 + 13$. Set notation may be used so $\{(x,y): y > 3(x+2)^2 + 13\} \cap \{(x,y): y < 6x + 25\}$ is fine but condone with or without any of $(x,y) \leftrightarrow y \leftrightarrow x$

Incorrect examples include "$y < 6x + 25$ **or** $y > 3(x+2)^2 + 13$", $\{(x,y): y > 3(x+2)^2 + 13\} \cup \{(x,y): y < 6x + 25\}$

Values of x could be included but they must be correct. So $3(x+2)^2 + 13 < y < 6x + 25,\ x < 0$ is fine

If there are multiple solutions mark the final one.

Question	Scheme	Marks	AOs
10(a)	$\mathrm{f}(x)=4\left(x^2-2\right)\mathrm{e}^{-2x}$		
	Differentiates to $\mathrm{e}^{-2x}\times 8x+4\left(x^2-2\right)\times -2\mathrm{e}^{-2x}$	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x)=8\mathrm{e}^{-2x}\left\{x-\left(x^2-2\right)\right\}=8\left(2+x-x^2\right)\mathrm{e}^{-2x}$ *	A1*	2.1
		(3)	
(b)	States roots of $\mathrm{f}'(x)=0$ $x=-1,2$	B1	1.1b
	Substitutes one x value to find a y value	M1	1.1b
	Stationary points are $\left(-1,-4\mathrm{e}^2\right)$ and $\left(2,8\mathrm{e}^{-4}\right)$	A1	1.1b
		(3)	
(c)	(i) Range $\left[-8\mathrm{e}^2,\infty\right)$ o.e. such as $\mathrm{g}(x)\geqslant -8\mathrm{e}^2$	B1ft	2.5
	(ii) For • Either attempting to find $2\mathrm{f}(0)-3=2\times -8-3=(-19)$ and identifying this as the **lower bound** • Or attempting to find $2\times "8\mathrm{e}^{-4}"-3$ and identifying this as the **upper bound**	M1	3.1a
	Range $\left[-19,16\mathrm{e}^{-4}-3\right]$	A1	1.1b
		(3)	
			(9 marks)
Notes:			

(a)

M1: Attempts the product rule and uses $\mathrm{e}^{-2x}\to k\mathrm{e}^{-2x},\quad k\neq 0$

If candidate states $u=4(x^2-2), v=\mathrm{e}^{-2x}$ with $u'=....,v'=...\mathrm{e}^{-2x}$ it can be implied by their $vu'+uv'$

If they just write down an answer without working award for $\mathrm{f}'(x)=px\mathrm{e}^{-2x}\pm q\left(x^2-2\right)\mathrm{e}^{-2x}$

They may multiply out first $\mathrm{f}(x)=4x^2\mathrm{e}^{-2x}-8\mathrm{e}^{-2x}$. Apply in the same way condoning slips

Alternatively attempts the quotient rule on $\mathrm{f}(x)=\dfrac{u}{v}=\dfrac{4\left(x^2-2\right)}{\mathrm{e}^{2x}}$ with $v'=k\mathrm{e}^{2x}$ and $\mathrm{f}'(x)=\dfrac{vu'-uv'}{v^2}$

A1: A correct $\mathrm{f}'(x)$ which may be unsimplified.

Via the quotient rule you can award for $\mathrm{f}'(x)=\dfrac{8x\mathrm{e}^{2x}-8\left(x^2-2\right)\mathrm{e}^{2x}}{\mathrm{e}^{4x}}$ o.e.

A1*: Proceeds correctly to given answer showing all necessary steps.

The $\mathrm{f}'(x)$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ must be present at some point in the solution

This is a "show that" question and there must not be any errors. All bracketing must be correct.

Allow a candidate to move from the **simplified** unfactorised answer of $\mathrm{f}'(x)=8x\mathrm{e}^{-2x}-8\left(x^2-2\right)\mathrm{e}^{-2x}$

to the given answer in one step.

Do not allow it from an **unsimplified** $f'(x) = 4 \times 2xe^{-2x} + 4(x^2 - 2) \times -2e^{-2x}$

Allow the expression / bracketed expression to be written in a different order.

So, for example, $8(x - x^2 + 2)e^{-2x}$ is OK

(b)

B1: States or implies $x = -1, 2$ (as the roots of $f'(x) = 0$)

M1: Substitutes one *x* value of their solution to $f'(x) = 0$ in f(*x*) to find a *y* value.

Allow decimals here (3sf). FYI, to 3 sf, $-4e^2 = -29.6$ and $8e^{-4} = 0.147$

Some candidates just write down the *x* coordinates but then go on in part (c) to find the ranges using the *y* coordinates. Allow this mark to be scored from work in part (c)

A1: Obtains $(-1, -4e^2)$ and $(2, 8e^{-4})$ as the stationary points. This must be scored in (b). Remember to isw after a correct answer. Allow these to be written separately. E.g. $x = -1,\ y = -4e^2$

Extra solutions, e.g. from $x = 0$ will be penalised on this mark.

(c)(i)

B1ft: For a correct range written using correct notation.

Follow through on 2 × their minimum "*y*" value from part (b), providing it is negative.

Condone a decimal answer if this is consistent with their answer in (b) to 3sf or better.

Examples of correct responses are $[-8e^2, \infty)$, $g \geqslant -8e^2$, $y \geqslant -8e^2$, $\{q \in \mathbb{R}, q \geqslant -8e^2\}$

(c)(ii)

M1: See main scheme. Follow through on $2 \times$ their "$8e^{-4}$" $- 3$ for the upper bound.

A1: Range $[-19, 16e^{-4} - 3]$ o.e. such as $-19 \leqslant y \leqslant 16e^{-4} - 3$ but must be exact

Question	Scheme	Marks	AOs
11	Attempts to differentiate $x^n \to x^{n-1}$ seen once	M1	1.1b
	$y = 2x^3 - 4x + 5 \Rightarrow \frac{dy}{dx} = 6x^2 - 4$	A1	1.1b
	For substituting $x = 2$ into their $\frac{dy}{dx} = 6x^2 - 4$	dM1	1.1b
	For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x-2)$	ddM1	1.1b
	$y = 20x - 27$	A1	1.1b
		(5)	
			(5 marks)

Notes

M1: Attempts to differentiate $x^n \to x^{n-1}$ seen once. Score for $x^3 \to x^2$ or $\pm 4x \to 4$ or $+5 \to 0$

A1: $\left(\frac{dy}{dx} =\right) 6x^2 - 4$ which may be unsimplified $6x^2 - 4 + C$ is A0

dM1: Substitutes $x = 2$ into their $\frac{dy}{dx}$. The first M must have been awarded.

Score for sight of embedded values, or sight of "$\frac{dy}{dx}$ at $x = 2$ is" or a correct follow through.

Note that 20 on its own is not enough as this can be done on a calculator.

ddM1: For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x-2)$

It is dependent upon both previous M's.

If the form $y = mx + c$ is used they must proceed as far as $c = ...$

A1: Completely correct $y = 20x - 27$ (and in this form)

Question	Scheme	Marks	AOs
12 (i)	$x\sqrt{2}-\sqrt{18}=x \Rightarrow x\left(\sqrt{2}-1\right)=\sqrt{18} \Rightarrow x=\dfrac{\sqrt{18}}{\sqrt{2}-1}$	M1	1.1b
	$\Rightarrow x=\dfrac{\sqrt{18}}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}$	dM1	3.1a
	$x=\dfrac{\sqrt{18}\left(\sqrt{2}+1\right)}{1}=6+3\sqrt{2}$	A1	1.1b
		(3)	
(ii)	$4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 2^{6x-4}=2^{-\frac{3}{2}}$	M1	2.5
	$6x-4=-\dfrac{3}{2} \Rightarrow x=\ldots$	dM1	1.1b
	$x=\dfrac{5}{12}$	A1	1.1b
		(3)	
			(6 marks)

Notes

(i)

M1: Combines the terms in *x*, factorises and divides to find *x*. Condone sign slips and ignore any attempts to simplify $\sqrt{18}$

Alternatively squares both sides $x\sqrt{2}-\sqrt{18}=x \Rightarrow 2x^2-12x+18=x^2$

dM1: Scored for a complete method to find *x*. In the main scheme it is for making *x* the subject and then multiplying both numerator and denominator by $\sqrt{2}+1$

In the alternative it is for squaring both sides to produce a 3TQ and then factorising their quadratic equation to find *x*. (usual rules apply for solving quadratics)

A1: $x=6+3\sqrt{2}$ only following a correct intermediate line. Allow $\dfrac{6+3\sqrt{2}}{1}$ as an intermediate line.

In the alternative method the $6-3\sqrt{2}$ must be discarded.

(ii)

M1: Uses correct mathematical notation and attempts to set both sides as powers of 2 or 4.

Eg $2^{ax+b}=2^c$ or $4^{dx+e}=4^f$ is sufficient for this mark.

Alternatively uses logs (base 2 or 4) to get a linear equation in *x*.

$4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow \log_2 4^{3x-2}=\log_2\dfrac{1}{2\sqrt{2}} \Rightarrow 2\left(3x-2\right)=\log_2\dfrac{1}{2\sqrt{2}}$.

Or $4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 3x-2=\log_4\dfrac{1}{2\sqrt{2}}$

Or $4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 4^{3x}=4\sqrt{2} \Rightarrow 3x=\log_4 4\sqrt{2}$

dM1: Scored for a complete method to find x.
Scored for setting the indices of 2 or 4 equal to each other and then solving to find x.
There must be an attempt on both sides.

You can condone slips for this mark Eg bracketing errors $4^{3x-2} = 2^{2\times 3x-2}$ or $\frac{1}{2\sqrt{2}} = 2^{-1+\frac{1}{2}}$

In the alternative method candidates cannot just write down the answer to the rhs.

So expect some justification. E.g. $\log_2 \frac{1}{2\sqrt{2}} = \log_2 2^{-\frac{3}{2}} = -\frac{3}{2}$

or $\log_4 \frac{1}{2\sqrt{2}} = \log_4 2^{-\frac{3}{2}} = -\frac{3}{2}\times\frac{1}{2}$ condoning slips as per main scheme

or $3x = \log_4 4\sqrt{2} \Rightarrow 3x = 1+\frac{1}{4}$

A1: $x = \frac{5}{12}$ with correct intermediate work

Question	Scheme	Marks	AOs
13 (a)	$g(5) = 2\times5^3 + 5^2 - 41\times5 - 70 = ...$	M1	1.1a
	$g(5) = 0 \Rightarrow (x-5)$ is a factor, hence $g(x)$ is divisible by $(x-5)$.	A1	2.4
		(2)	
(b)	$2x^3 + x^2 - 41x - 70 = (x-5)(2x^2 ... x \pm 14)$	M1	1.1b
	$= (x-5)(2x^2 + 11x + 14)$	A1	1.1b
	Attempts to factorise quadratic factor	dM1	1.1b
	$(g(x)) = (x-5)(2x+7)(x+2)$	A1	1.1b
		(4)	
(c)	$\int 2x^3 + x^2 - 41x - 70 \, dx = \frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x$	M1 A1	1.1b 1.1b
	Deduces the need to use $\int_{-2}^{5} g(x)dx$ $-\frac{1525}{3} - \frac{190}{3}$	M1	2.2a
	Area $= 571\frac{2}{3}$	A1	2.1
		(4)	
			(10 marks)

Notes

(a)

M1: Attempts to calculate $g(5)$ Attempted division by $(x-5)$ is M0

Look for evidence of embedded values or two correct terms of
$g(5) = 250 + 25 - 205 - 70 = ...$

A1: Correct calculation, reason and conclusion. It must follow M1. Accept, for example,

$g(5) = 0 \Rightarrow (x-5)$ is a factor, hence divisible by $(x-5)$

$g(5) = 0 \Rightarrow (x-5)$ is a factor ✓

Do not allow if candidate states

$f(5) = 0 \Rightarrow (x-5)$ is a factor, hence divisible by $(x-5)$ **(It is not f)**

$g(x) = 0 \Rightarrow (x-5)$ is a factor **(It is not g(*x*) and there is no conclusion)**

This may be seen in a preamble before finding $g(5) = 0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Attempts to find the quadratic factor by inspection (correct coefficients of first term and ± last term) or by division (correct coefficients of first term and ± second term). Allow this to be scored from division in part (a)

A1: $(2x^2 + 11x + 14)$ You may not see the $(x-5)$ which can be condoned

dM1: Correct attempt to factorise their $(2x^2 + 11x + 14)$

A1: $(g(x)=)\ (x-5)(2x+7)(x+2)$ or $(g(x)=)\ (x-5)(x+3.5)(2x+4)$

It is for the product of factors and not just a statement of the three factors
Attempts with calculators via the three roots are likely to score 0 marks. The question was "Hence" so the two M's must be awarded.

(c)

M1: For $x^n \to x^{n+1}$ for any of the terms in x for $g(x)$ so
$2x^3 \to ...x^4,\ x^2 \to ...x^3,\ -41x \to ...x^2,\ -70 \to ...x$

A1: $\int 2x^3 + x^2 - 41x - 70\,dx = \frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x$ which may be left unsimplified (ignore any reference to $+C$)

M1: Deduces the need to use $\int_{-2}^{5} g(x)dx$.

This may be awarded from the limits on their integral (either way round) or from embedded values which can be subtracted either way round.

A1: For clear work showing all algebraic steps leading to area = $571\frac{2}{3}$ oe

So allow $\int_{-2}^{5} 2x^3 + x^2 - 41x - 70\,dx = \left[\frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x\right]_{-2}^{5} = -\frac{1715}{3} \Rightarrow$ area $= \frac{1715}{3}$ for 4 marks

Condone spurious notation, as long as the algebraic steps are correct. If they find $\int_{-2}^{5} g(x)dx$ then withhold the final mark if they just write a positive value to this integral since $\int_{-2}^{5} g(x)dx = -\frac{1715}{3}$

Note $\int_{-2}^{5} 2x^3 + x^2 - 41x - 70\,dx \Rightarrow \frac{1715}{3}$ with no algebraic integration seen scores M0A0M1A0

Question	Scheme	Marks	AOs
14 (a)	Deduces $g(x) = ax^3 + bx^2 + ax$	B1	2.2a
	Uses $(2,9) \Rightarrow 9 = 8a + 4b + 2a$ $\Rightarrow 10a + 4b = 9$	M1 A1	2.1 1.1b
	Uses $g'(2) = 0 \Rightarrow 0 = 12a + 4b + a$ $\Rightarrow 13a + 4b = 0$	M1 A1	2.1 1.1b
	Solves simultaneously $\Rightarrow a, b$	dM1	1.1b
	$g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$	A1	1.1b
		(7)	
(b)	Attempts $g''(x) = -18x + \frac{39}{2}$ and substitutes $x = 2$	M1	1.1b
	$g''(2) = -\frac{33}{2} < 0$ hence maximum	A1	2.4
		(2)	
			(9 marks)

Notes

(a)

B1: Uses the information given to deduce that $g(x) = ax^3 + bx^2 + ax$. (Seen or implied)

M1: Uses the fact that $(2,9)$ lies on the curve so uses $x = 2, y = 9$ within a cubic function

A1: For a simplified equation in just two variables. E.g. $10a + 4b = 9$

M1: Differentiates their cubic to a quadratic and uses the fact that $g'(2) = 0$ to obtain an equation in *a* and *b*.

A1: For a different simplified equation in two variables E.g. $13a + 4b = 0$

dM1: Solves simultaneously $\Rightarrow a = ..., b = ...$ It is dependent upon the B and both M's

A1: $g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$

(b)

M1: Attempts $g''(x) = -18x + \frac{39}{2}$ and substitutes $x = 2$. Award for second derivatives of the form $g''(x) = Ax + B$ with $x = 2$ substituted in.
Alternatively attempts to find the value of their $g'(x)$ or $g(x)$ either side of $x = 2$ (by substituting a value for *x* within 0.5 either side of 2)

A1: $g''(2) = -\frac{33}{2} < 0$ hence maximum. (allow embedded values but they must refer to the sign or that it is less than zero)
If $g'(x) = -9x^2 + \frac{39}{2}x - 3$ or $g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$ is calculated either side of $x = 2$ then the values must be correct or embedded correctly (you will need to check these) they need to compare $g'(x) > 0$ to the left of $x = 2$ and $g'(x) < 0$ to the right of $x = 2$ or $g(x) <$ 9 to the left and $g(x) >$ 9 to the right of $x = 2$ hence maximum.

Note If they only sketch the cubic function $g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$ then award M1A0

Question	Scheme	Marks	AOs
15	Attempts $f(-3)=3\times(-3)^3+2a\times(-3)^2-4\times-3+5a=0$	M1	3.1a
	Solves linear equation $23a=69\Rightarrow a=...$	M1	1.1b
	$a=3$ cso	A1	1.1b
		(3)	
			(3 marks)

M1: Chooses a suitable method to set up a correct equation in a which may be unsimplified.
This is mainly applying $f(-3)=0$ leading to a correct equation in a.
Missing brackets may be recovered.
Other methods may be seen but they are more demanding
If division is attempted must produce **a correct equation** in a similar way to the $f(-3)=0$ method

$$\begin{array}{r}
3x^2+(2a-9)x+23-6a \\
x+3\overline{)3x^3+2ax^2-4x+5a} \\
\underline{3x^3+9x^2} \\
(2a-9)x^2-4x \\
\underline{(2a-9)x^2+(6a-27)x} \\
(23-6a)x+5a \\
(23-6a)x+69-18a
\end{array}$$

So accept $5a=69-18a$ or equivalent, where it implies that the remainder will be 0
You may also see variations on the table below. In this method the terms in x are equated to -4

	$3x^2$	$(2a-9)x$	$\frac{5a}{3}$
x	$3x^3$	$(2a-9)x^2$	$\frac{5a}{3}x$
3	$9x^2$	$(6a-27)x$	$5a$

$$6a-27+\frac{5a}{3}=-4$$

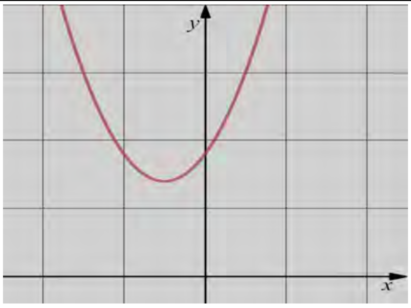
M1: This is scored for an attempt at solving a linear equation in a.
For the main scheme it is dependent upon having attempted $f(\pm3)=0$. Allow for a linear equation in a leading to $a=...$. Don't be too concerned with the mechanics of this.

Via division accept $x+3\overline{)3x^3+2ax^2-4x+5a}$ (quotient $3x^2...$) followed by a remainder in a set $=0\Rightarrow a=...$

or two terms in a are equated so that the remainder = 0
FYI the correct remainder via division is $23a+12-81$ oe

A1: $a=3$ cso

An answer of 3 with no incorrect working can be awarded 3 marks

Question	Scheme	Marks	AOs
16 (a)	$2x^2+4x+9=2(x\pm k)^2\pm....$ $a=2$	B1	1.1b
	Full method $2x^2+4x+9=2(x+1)^2\pm...$ $a=2$ & $b=1$	M1	1.1b
	$2x^2+4x+9=2(x+1)^2+7$	A1	1.1b
		(3)	
(b)	 U shaped curve any position but not through (0,0)	B1	1.2
	y - intercept at $(0,9)$	B1	1.1b
	Minimum at $(-1,7)$	B1ft	2.2a
		(3)	
(c)	(i) Deduces translation with one correct aspect.	M1	3.1a
	Translate $\begin{pmatrix}2\\-4\end{pmatrix}$	A1	2.2a
	(ii) $\text{h}(x)=\dfrac{21}{"2(x+1)^2+7"}\Rightarrow$ (maximum) value $\dfrac{21}{"7"}(=3)$	M1	3.1a
	$0<\text{h}(x)\leqslant 3$	A1ft	1.1b
		(4)	
			(10 marks)

(a)

B1: Achieves $2x^2+4x+9=2(x\pm k)^2\pm....$ or states that $a=2$

M1: Deals correctly with first two terms of $2x^2+4x+9$.

Scored for $2x^2+4x+9=2(x+1)^2\pm...$or stating that $a=2$ and $b=1$

A1: $2x^2+4x+9=2(x+1)^2+7$

Note that this may be done in a variety of ways including equating $2x^2+4x+9$ with the expanded form of $a(x+b)^2+c\equiv ax^2+2abx+ab^2+c$

(b)

B1: For a U-shaped curve in any position not passing through $(0,0)$. Be tolerant of slips of the pen but do not allow if the curve bends back on itself

B1: A curve with a y - intercept on the +ve y axis of 9. The curve cannot just stop at $(0,9)$

Allow the intercept to be marked 9, $(0,9)$ but not $(9,0)$

B1ft: For a minimum at $(-1,7)$ in quadrant 2. This may be implied by -1 and 7 marked on the axes in the correct places. The curve must be a U shape and not a cubic say.

Follow through on a minimum at $(-b,c)$, marked in the correct quadrant, for their $a(x+b)^2+c$

(c)(i)

M1: Deduces translation with one correct aspect or states $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ with no reference to 'translate'.

Allow instead of the word translate, shift or move. $\mathrm{g}(x)=\mathrm{f}(x-2)-4$ can score M1

For example, possible methods of arriving at this deduction are:

- $\mathrm{f}(x)\to\mathrm{g}(x)$ is $2x^2+4x+9\to 2(x-2)^2+4(x-2)+5$ So $\mathrm{g}(x)=\mathrm{f}(x-2)-4$
- $\mathrm{g}(x)=2(x-1)^2+3$ New curve has its minimum at $(1,3)$ so $(-1,7)\to(1,3)$
- Using a graphical calculator to sketch $y=\mathrm{g}(x)$ and compares to the sketch of $y=\mathrm{f}(x)$

In almost all cases you will not allow if the candidate gives two **different types of** transformations. Eg, stretch and .....

A1: Requires both 'translate' and '$\begin{pmatrix} 2 \\ -4 \end{pmatrix}$' Allow 'shift' or move' instead of translate.

So condone " Move shift 2 (units) to the right and move 4 (units) down

However, for M1 A1, it is possible to reflect in $x=0$ and translate $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$, so please consider all responses.

SC: If the candidate writes translate $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ or " move 2 (units) to the left and 4 (units) up" score M1 A0

(c)(ii)

M1: Correct attempt at finding the maximum value (although it may not be stated as a maximum)

- Uses part (a) to write $\mathrm{h}(x)=\dfrac{21}{"2(x+1)^2+7"}$ and attempts to find $\dfrac{21}{\text{their "7"}}$
- Attempts to differentiate, sets $4x+4=0\to x=-1$ and substitutes into $\mathrm{h}(x)=\dfrac{21}{2x^2+4x+9}$
- Uses a graphical calculator to sketch $y=\mathrm{h}(x)$ and establishes the 'maximum' value $(...,3)$

A1ft: $0<\mathrm{h}(x)\leqslant 3$ Allow for $0<\mathrm{h}\leqslant 3$ $(0,3]$ and $0<y\leqslant 3$ but not $0<x\leqslant 3$

Follow through on their $a(x+b)^2+c$ so award for $0<\mathrm{h}(x)\leqslant \dfrac{21}{c}$

Question	Scheme		Marks	AOs
17(i)	$16a^2 = 2\sqrt{a} \Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	$16a^2 - 2\sqrt{a} = 0$ $\Rightarrow 2a^{\frac{1}{2}}\left(8a^{\frac{3}{2}} - 1\right) = 0$ $\Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	M1	1.1b
	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	M1	1.1b
	$\Rightarrow a = \frac{1}{4}$	$\Rightarrow a = \frac{1}{4}$	A1	1.1b
	Deduces that $a = 0$ is a solution		B1	2.2a
			(4)	
(ii)	$b^4 + 7b^2 - 18 = 0 \Rightarrow \left(b^2 + 9\right)\left(b^2 - 2\right) = 0$		M1	1.1b
	$b^2 = -9, 2$		A1	1.1b
	$b^2 = k \Rightarrow b = \sqrt{k}, k > 0$		dM1	2.3
	$b = \sqrt{2}$, $-\sqrt{2}$ only		A1	1.1b
			(4)	
				(8 marks)

Notes

(i)

M1: Combines the two algebraic terms to reach $a^{\pm\frac{3}{2}} = C$ or equivalent such as $\left(\sqrt{a}\right)^3 = C$ $\left(C \neq 0\right)$

An alternative is via squaring and combining the algebraic terms to reach $a^{\pm 3} = k, k > 0$

Eg. $...a^4 = ...a \Rightarrow a^{\pm 3} = k$ or $...a^4 = ...a \Rightarrow ...a^4 - ...a = 0 \Rightarrow ...a\left(a^3 - ...\right) = 0 \Rightarrow a^3 = ...$

Allow for slips on coefficients.

M1: Undoes the indices correctly for their $a^{\frac{m}{n}} = C$ (So M0 M1 A0 is possible)
You may even see logs used.

A1: $a = \frac{1}{4}$ and no other solutions apart from 0 Accept exact equivalents Eg 0.25

B1: Deduces that $a = 0$ is a solution.

(ii)

M1: Attempts to solve as a quadratic equation in b^2
Accept $\left(b^2 + m\right)\left(b^2 + n\right) = 0$ with $mn = \pm 18$ or solutions via the use of the quadratic formula Also allow candidates to substitute in another variable, say $u = b^2$ and solve for u

A1: Correct solution. Allow for $b^2 = 2$ or $u = 2$ with no incorrect solution given.
Candidates can choose to omit the solution $b^2 = -9$ or $u = -9$ and so may not be seen

dM1: Finds at least one solution from their $b^2 = k \Rightarrow b = \sqrt{k}, k > 0$. Allow $b = 1.414$

A1: $b=\sqrt{2}$, $-\sqrt{2}$ only. The solution asks for real values so if $3i$ is given then score A0

Notes on Question 17 continue

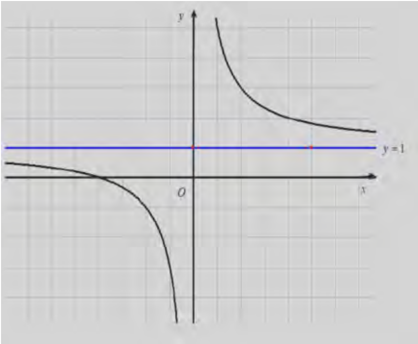
Answers with minimal or no working:

In part **(i)**

- no working, just answer(s) with they can score the B1
- If they square and proceed to the quartic equation $256a^4 = 4a$ oe, and then write down the answers they can have access to all marks.

In part (ii)

- Accept for 4 marks $b^2 = 2 \Rightarrow b = \pm\sqrt{2}$
- No working, no marks.

Question	Scheme	Marks	AOs
18(a)	 $\frac{1}{x}$ shape in 1st quadrant	M1	1.1b
	Correct	A1	1.1b
	Asymptote $y=1$	B1	1.2
		(3)	
(b)	Combines equations $\Rightarrow \frac{k^2}{x}+1=-2x+5$	M1	1.1b
	$(\times x)\Rightarrow k^2+1x=-2x^2+5x \Rightarrow 2x^2-4x+k^2=0$ *	A1*	2.1
		(2)	
(c)	Attempts to set $b^2-4ac=0$	M1	3.1a
	$8k^2=16$	A1	1.1b
	$k=\pm\sqrt{2}$	A1	1.1b
		(3)	
			(8 marks)

Notes

(a)

M1: For the shape of a $\frac{1}{x}$ type curve in Quadrant 1. It must not cross either axis and have acceptable curvature. Look for a negative gradient changing from $-\infty$ to 0 condoning "slips of the pencil". (See Practice and Qualification for clarification)

A1: Correct shape and position for both branches.
It must lie in Quadrants 1, 2 and 3 and have the correct curvature including asymptotic behaviour

B1: Asymptote given as $y=1$. This could appear on the diagram or within the text.
Note that the curve does not need to be asymptotic at $y=1$ but this must be the only horizontal asymptote offered by the candidate.

(b)

M1: Attempts to combine $y=\frac{k^2}{x}+1$ with $y=-2x+5$ to form an equation in just x

A1*: Multiplies by x (the processed line must be seen) and proceeds to given answer with no slips.
Condone if the order of the terms are different $2x^2+k^2-4x=0$

(c)

M1: Deduces that $b^2-4ac=0$ or equivalent for **the given equation.**
If *a*, *b* and *c* are stated only accept $a=2, b=\pm4, c=k^2$ so $4^2-4\times2\times k^2=0$
Alternatively completes the square $x^2-2x+\frac{1}{2}k^2=0 \Rightarrow (x-1)^2=1-\frac{1}{2}k^2 \Rightarrow$ "$1-\frac{1}{2}k^2$" $=0$

A1: $8k^2=16$ or exact simplified equivalent. Eg $8k^2-16=0$

If a, b and c are stated they must be correct. Note that b^2 appearing as 4^2 is correct
Note on Question 18 continue
A1: $k=\pm\sqrt{2}$ and following correct a, b and c if stated

A solution via differentiation would be awarded as follows

M1: Sets the gradient of the curve $=-2 \Rightarrow -\dfrac{k^2}{x^2}=-2 \Rightarrow x=(\pm)\dfrac{k}{\sqrt{2}}$ oe and attempts to substitute into $2x^2-4x+k^2=0$

A1: $2k^2=(\pm)2\sqrt{2}k$ oe

A1: $k=\pm\sqrt{2}$

Question	Scheme	Marks	AOs
19 (a)	Attempts $f(4)=2\times4^3-13\times4^2+8\times4+48$	M1	1.1b
	$f(4)=0\Rightarrow(x-4)$ is a factor	A1	1.1b
		(2)	
(b)	$2x^3-13x^2+8x+48=(x-4)(2x^2...x-12)$	M1	2.1
	$=(x-4)(2x^2-5x-12)$	A1	1.1b
	Attempts to factorise quadratic factor or solve quadratic eqn	dM1	1.1b
	$f(x)=(x-4)^2(2x+3)\Rightarrow f(x)=0$ has only two roots, 4 and -1.5	A1	2.4
		(4)	
(c)	Deduces either three roots or deduces that $f(x)$ is moved down two units	M1	2.2a
	States three roots, as when $f(x)$ is moved down two units there will be three points of intersection (with the x - axis)	A1	2.4
		(2)	
(d)	For sight of $k=\pm4,\pm\frac{3}{2}$	M1	1.1b
	$k=4,-\frac{3}{2}$	A1ft	1.1b
		(2)	
			(10 marks)

Notes

(a)

M1: Attempts to calculate $f(4)$.

Do not accept $f(4)=0$ without sight of embedded values or calculations.

If values are not embedded look for two correct terms from $f(4)=128-208+32+48$

Alternatively attempts to divide by $(x-4)$. Accept via long division or inspection.

See below for awarding these marks.

A1: Correct reason with conclusion. Accept $f(4)=0$, hence factor as long as M1 has been scored.

This should really be stated on one line after having performed a correct calculation. It could appear as a preamble if the candidate states "If $f(4)=0$, then $(x-4)$ is a factor before doing the calculation and then writing hence proven or ✓ oe.

If division/inspection is attempted it must be correct and there must be some attempt to explain why they have shown that $(x-4)$ is a factor. Eg Via division they must state that there is no remainder, hence factor

(b)

M1: Attempts to find the quadratic factor by inspection (correct first and last terms) or by division (correct first two terms)

Notes on Question 19 continue

So for inspection award for $2x^3-13x^2+8x+48=(x-4)(2x^2...x\pm 12)$

$$\begin{array}{r} 2x^2-5x \\ x-4\overline{)\,2x^3-13x^2+8x+48} \end{array}$$

For division look for
$$\begin{array}{r} 2x^3-8x^2 \\ \hline -5x^2 \end{array}$$

A1: Correct quadratic factor $(2x^2-5x-12)$ For division award for sight of this "in the correct place" You don't have to see it paired with the $(x-4)$ for this mark.

If a student has used division in part (a) they can score the M1 A1 in (b) as soon as they start attempting to factorise their $(2x^2-5x-12)$.

dM1: Correct attempt to solve or factorise their $(2x^2-5x-12)$ including use of formula

Apply the usual rules $(2x^2-5x-12)=(ax+b)(cx+d)$ where $ac=\pm 2$ and $bd=\pm 12$

Allow the candidate to move from $(x-4)(2x^2-5x-12)$ to $(x-4)^2(2x+3)$ for this mark.

A1: Via factorisation

Factorises twice to $\text{f}(x)=(x-4)(2x+3)(x-4)$ or $\text{f}(x)=(x-4)^2(2x+3)$ or

$\text{f}(x)=2(x-4)^2\left(x+\frac{3}{2}\right)$ followed by a valid explanation why there are only two roots.

The explanation can be as simple as

- hence $x=4$ and $-\frac{3}{2}$ (only). The roots must be correct
- only two distinct roots as 4 is a repeated root

There must be some understanding between roots and factors.

E.g. $\text{f}(x)=(x-4)^2(2x+3)$

only two distinct roots is insufficient.

This would require two distinct factors, so there are two distinct roots.

Via solving.

Factorsises to $(x-4)(2x^2-5x-12)$ and solves $2x^2-5x-12=0\Rightarrow x=4,-\frac{3}{2}$ followed by an explanation that the roots are $4,4,-\frac{3}{2}$ so only two distinct roots.

Note that this question asks the candidate to use algebra so you cannot accept any attempt to use their calculators to produce the answers.

(c)

M1: For a valid **deduction.**

Accept **either** there are 3 roots **or** states that it is a solution of $\text{f}(x)=2$ or $\text{f}(x)-2=0$

A1: Fully explains:

Eg. States three roots, as $\text{f}(x)$ is moved down by **two** units (giving three points of intersection with the x - axis)

Eg. States three roots, as it is where $\text{f}(x)=2$ (You may see $y=2$ drawn on the diagram)

Notes on Question 19 continue
(d) **M1:** For sight of ± 4 **and** $\pm\frac{3}{2}$ Follow through on ± their roots. **A1ft:** $k = 4, -\frac{3}{2}$ Follow through on their roots. Accept $4, -\frac{3}{2}$ but not $x = 4, -\frac{3}{2}$

Question	Scheme	Marks	AOs
20 (a)	Attempts $P=100-6.25(15-9)^2$	M1	3.4
	$=-125$ $\therefore$ not sensible as the company would make a loss	A1	2.4
		(2)	
(b)	Uses $P>80 \Rightarrow (x-9)^2<3.2$ or $P=80 \Rightarrow (x-9)^2=3.2$	M1	3.1b
	$\Rightarrow 9-\sqrt{3.2}<x<9+\sqrt{3.2}$	dM1	1.1b
	Minimum Price = £7.22	A1	3.2a
		(3)	
(c)	States (i) maximum profit =£ 100 000 and (ii) selling price £9	B1 B1	3.2a 2.2a
		(2)	
			(7 marks)

(a)

M1: Substitutes $x=15$ into $P=100-6.25(x-9)^2$ and attempts to calculate. This is implied by an answer of -125. Some candidates may have attempted to multiply out the brackets before they substitute in the x = 15. This is acceptable as long as the function obtained is quadratic. There must be a calculation seen or implied by the value of -125.

A1: Finds $P=-125$ or states that $P<0$ **and** explains that (this is not sensible as) the company would make a loss.

Condone $P=-125$ followed by an explanation that it is not sensible as the company would make a loss of £125 rather than £125 000. An explanation that it is not sensible as "the profit cannot be negative" , "the profit is negative" or "the company will not make any money" , "they might make a loss" is incomplete/incorrect. You may ignore any misconceptions or reference to the price of the toy being too cheap for this mark.

Alt: **M1:** Sets P = 0 and finds $x=5,13$ A1: States $15>13$ and states makes a loss

(b)

M1: Uses $P...80$ where ... is any inequality or "$=$"in $P=100-6.25(x-9)^2$ and proceeds to $(x-9)^2...k$ where k >0 and ... is any inequality or "$=$"

Eg. Condone $P<80$ in $P=100-6.25(x-9)^2 \Rightarrow (x-9)^2<k$ where k >0 If the candidate attempts to multiply out then allow when they achieve a form $ax^2+bx+c=0$

dM1: Award for solving to find the two positive values for x. Allow decimal answers

FYI correct answers are $\Rightarrow 9-\sqrt{3.2}<x<9+\sqrt{3.2}$ Accept $\Rightarrow x=9\pm\sqrt{3.2}$

Condone incorrect inequality work $100-6.25(x-9)^2>80 \Rightarrow (x-9)^2>3.2 \Rightarrow x>9\pm\sqrt{3.2}$

Alternatively award if the candidate selects the lower of their two positive values $9-\sqrt{3.2}$

A1: Deduces that the minimum Price = £7.22 (£7.21 is not acceptable)

(c)

(i) B1: Maximum Profit = £ 100 000 with units. Accept 100 thousand pound(s).

(ii) B1: Selling price = £9 with units

SC 1: Missing units in (b) and (c) only penalise once in these parts, withhold the final mark.

SC 2: If the answers to (c) are both correct, but in the wrong order score SC B1 B0

If (i) and (ii) are not written out score in the order given.

Question	Scheme	Marks	AOs
21(a)	$(g(-2)) = 4\times -8 - 12\times 4 - 15\times -2 + 50$	M1	1.1b
	$g(-2) = 0 \Rightarrow (x+2)$ is a factor	A1	2.4
		(2)	
(b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 - 20x + 25)$	M1 A1	1.1b 1.1b
	$= (x+2)(2x-5)^2$	M1 A1	1.1b 1.1b
		(4)	
(c)	(i) $x \leqslant -2,\ x = 2.5$	M1 A1ft	1.1b 1.1b
	(ii) $x = -1,\ x = 1.25$	B1ft	2.2a
		(3)	
			(9 marks)

(a)

M1: Attempts $g(-2)$ Some sight of (-2) embedded or calculation is required.

So expect to see $4\times(-2)^3 - 12\times(-2)^2 - 15\times(-2) + 50$ embedded

Or $-32 - 48 + 30 + 50$ condoning slips for the M1

Any attempt to divide or factorise is M0. (See demand in question)

A1: $g(-2) = 0 \Rightarrow (x+2)$ is a factor.

Requires a correct statement and conclusion. Both "$g(-2) = 0$" and "$(x+2)$ is a factor" must be seen in the solution. This may be seen in a preamble before finding $g(-2) = 0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

Also accept, in one coherent line/sentence, explanations such as, 'as g (x) =0 when $x = -2$, $(x+2)$ is a factor.'

(b)

M1: Attempts to divide $g(x)$ by $(x+2)$ May be seen and awarded from part (a)

If inspection is used expect to see $4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 \text{..........} \pm 25)$

If algebraic / long division is used expect to see

$$x+2\overline{)\ 4x^3 - 12x^2 - 15x + 50}\quad\text{with quotient } 4x^2 \pm 20x$$

A1: Correct quadratic factor is $(4x^2 - 20x + 25)$ may be seen and awarded from part (a)

M1: Attempts to factorise their $(4x^2 - 20x + 25)$ usual rule $(ax+b)(cx+d), ac = \pm 4, bd = \pm 25$

A1: $(x+2)(2x-5)^2$ oe seen on a single line. $(x+2)(-2x+5)^2$ is also correct.

Allow recovery for all marks for $g(x) = (x+2)(x-2.5)^2 = (x+2)(2x-5)^2$

(c)(i)

M1: For identifying that the solution will be where the curve is on or below the axis. Award for either $x \leqslant -2$ or $x = 2.5$ Follow through on their $g(x) = (x+2)(ax+b)^2$ only where $ab < 0$ (that is a positive root). Condone $x < -2$ See SC below for $g(x) = (x+2)(2x+5)^2$

A1ft: BOTH $x \leqslant -2$, $x = 2.5$ Follow through on their $-\frac{b}{a}$ of their $g(x) = (x+2)(ax+b)^2$

May see $\{x \leqslant -2 \cup x = 2.5\}$ which is fine.

(c) (ii)

B1ft: For deducing that the solutions of $g(2x) = 0$ will be where $x = -1$ and $x = 1.25$

Condone the coordinates appearing $(-1, 0)$ and $(1.25, 0)$

Follow through on their 1.25 of their $g(x) = (x+2)(ax+b)^2$

……..

SC: If a candidate reaches $g(x) = (x+2)(2x+5)^2$, clearly incorrect because of Figure 2, we will award

In (i) M1 A0 for $x \leqslant -2$ or $x < -2$

In (ii) B1 for $x = -1$ and $x = -1.25$

……

Alt (b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(ax+b)^2$ $= a^2x^3 + (2ba + 2a^2)x^2 + (b^2 + 4ab)x + 2b^2$		
	Compares terms to get either a or b	M1	1.1b
	Either $a = 2$ or $b = -5$	A1	1.1b
	Multiplies out expression $(x+2)(\pm 2x \pm 5)^2$ and compares to $4x^3 - 12x^2 - 15x + 50$	M1	
	All terms must be compared or else expression must be multiplied out and establishes that $4x^3 - 12x^2 - 15x + 50 = (x+2)(2x-5)^2$	A1	1.1b
		(4)	

Question	Scheme	Marks	AOs
22(a)	$(4,-3)$	B1	1.2
		(1)	
(b)	$x=6$	B1	1.1b
		(1)	
(c)	$x \le 4$	B1	1.1b
		(1)	
(d)	$k>1.5$	B1	2.2a
		(1)	
			(4 marks)

Question	Scheme	Marks	AOs
23(a)	$f(-3)=(-3)^3+3\times(-3)^2-4\times(-3)-12$	M1	1.1b
	$f(-3)=0 \Rightarrow (x+3)$ is a factor $\Rightarrow$ Hence $f(x)$ is divisible by $(x+3)$.	A1	2.4
		(2)	
(b)	$x^3+3x^2-4x-12=(x+3)(x^2-4)$	M1	1.1b
	$=(x+3)(x+2)(x-2)$	dM1 A1	1.1b 1.1b
		(3)	
(c)	$\dfrac{x^3+3x^2-4x-12}{x^3+5x^2+6x}=\dfrac{....}{x(x^2+5x+6)}$	M1	3.1a
	$=\dfrac{(x+3)(x+2)(x-2)}{x(x+3)(x+2)}$	dM1	1.1b
	$=\dfrac{(x-2)}{x}=1-\dfrac{2}{x}$	A1	2.1
		(3)	
			(8 marks)

Notes:

(a)

M1: Attempts $f(-3)$

A1: Achieves $f(-3)=0$ and explains that $(x+3)$ is a factor and hence $f(x)$ is divisible by $(x+3)$.

(b)

M1: Attempts to divide by $(x+3)$ to get the quadratic factor.

By division look for the first two terms. ie x^2+0x

$$\begin{array}{r} x^2 \pm 0x.............. \\ x+3 \overline{\big) x^3+3x^2-4x-12} \\ \underline{x^3+3x^2} \end{array}$$

By inspection look for the first and last term $x^3+3x^2-4x-12=(x+3)(x^2+..x\pm4)$

dM1: For an attempt at factorising their (x^2-4). (Need to check first and last terms)

A1: $f(x)=(x+3)(x+2)(x-2)$

(c)

M1: Takes a common factor of x out of the denominator and writes the numerator in factors.

Alternatively rewrites to $x^3+3x^2-4x-12=A(x^3+5x^2+6x)+B(x^2+5x+6)$

dM1: Further factorises the denominator and cancels

Alternatively compares terms or otherwise to find either A or B

A1: Shows that $\dfrac{x^3+3x^2-4x-12}{x^3+5x^2+6x}=1-\dfrac{2}{x}$ with no errors or omissions

In the alternative there must be a reference to

$x^3+3x^2-4x-12\equiv1(x^3+5x^2+6x)-2(x^2+5x+6)$ and hence $\dfrac{x^3+3x^2-4x-12}{x^3+5x^2+6x}=1-\dfrac{2}{x}$

Question	Scheme	Marks	AOs
24(a)	Sets $3x-2\sqrt{x}=8x-16$	B1	1.1a
	$2\sqrt{x}=16-5x$ $4x=(16-5x)^2 \Rightarrow x=..$	$5x+2\sqrt{x}-16=0$ $\Rightarrow\left(5\sqrt{x}\pm 8\right)\left(\sqrt{x}\pm 2\right)=0$	M1	3.1a
	$25x^2-164x+256=0$	$\left(5\sqrt{x}-8\right)\left(\sqrt{x}+2\right)=0$	A1	1.1b
	$(25x-64)(x-4)=0 \Rightarrow x=..$	$\sqrt{x}=\frac{8}{5},(-2)\Rightarrow x=..$	M1	1.1b
	$x=\frac{64}{25}$ only	A1	2.3
		(5)	
(b)	Attempts to solve $3x-2\sqrt{x}=0$	M1	2.1
	Correct solution $x=\frac{4}{9}$	A1	1.1b
	$y\leqslant 3x-2\sqrt{x}$, $y>8x-16$ $x\geqslant\frac{4}{9}$	B1ft	1.1b
		(3)	
			(8 marks)

Notes:

(a)

B1: Sets the equations equal to each other and achieves a correct equation

M1: Awarded for the key step in solving the problem. This can be awarded via two routes. Both routes must lead to a value for *x*.

- Making the $\sqrt{x}$ term the subject and squaring both sides (not each term)
- Recognising that this is a quadratic in $\sqrt{x}$ and attempting to factorise $\Rightarrow\left(5\sqrt{x}\pm 8\right)\left(\sqrt{x}\pm 2\right)=0$

A1: A correct intermediate line $25x^2-164x+256=0$ or $\left(5\sqrt{x}-8\right)\left(\sqrt{x}+2\right)=0$

M1: A correct method to find at least one value for *x*. Way One it is for factorising (usual rules), Way Two it is squaring at least one result of their $\sqrt{x}$

A1: Realises that $x=\frac{64}{25}$ is the only solution $x=\frac{64}{25},4$ is A0

(b) **M1:** Attempts to solve $3x-2\sqrt{x}=0$ For example

Allow $3x=2\sqrt{x}\Rightarrow 9x^2=4x\Rightarrow x=...$

Allow $3x=2\sqrt{x}\Rightarrow x^{\frac{1}{2}}=\frac{2}{3}\Rightarrow x=...$

A1: Correct solution to $3x-2\sqrt{x}=0\Rightarrow x=\frac{4}{9}$

B1: For a **consistent** solution defining *R* using either convention

Either $y\leqslant 3x-2\sqrt{x}$, $y>8x-16$ $x\geqslant\frac{4}{9}$ Or $y<3x-2\sqrt{x}$, $y\geqslant 8x-16$ $x>\frac{4}{9}$

Question	Scheme	Marks	AOs
25 (a)(i)	$\mathrm{f}(x) = x^3 + ax^2 - ax + 48, \ x \in \mathbb{R}$		
	$\mathrm{f}(-6) = (-6)^3 + a(-6)^2 - a(-6) + 48$	M1	1.1b
	$= -216 + 36a + 6a + 48 = 0 \Rightarrow 42a = 168 \Rightarrow a = 4$ *	A1*	1.1b
(a)(ii)	Hence, $\mathrm{f}(x) = (x+6)(x^2 - 2x + 8)$	M1	2.2a
		A1	1.1b
		(4)	
(b)	$2\log_2(x+2) + \log_2 x - \log_2(x-6) = 3$		
	E.g. • $\log_2(x+2)^2 + \log_2 x - \log_2(x-6) = 3$ • $2\log_2(x+2) + \log_2\left(\dfrac{x}{x-6}\right) = 3$	M1	1.2
	$\log_2\left(\dfrac{x(x+2)^2}{(x-6)}\right) = 3 \quad \left[\text{or } \log_2\left(x(x+2)^2\right) = \log_2\left(8(x-6)\right)\right]$	M1	1.1b
	$\left(\dfrac{x(x+2)^2}{(x-6)}\right) = 2^3 \quad \left\{\text{i.e. } \log_2 a = 3 \Rightarrow a = 2^3 \text{ or } 8\right\}$	B1	1.1b
	$x(x+2)^2 = 8(x-6) \Rightarrow x(x^2+4x+4) = 8x - 48$		
	$\Rightarrow x^3 + 4x^3 + 4x = 8x - 48 \Rightarrow x^3 + 4x^3 - 4x + 48 = 0$ *	A1 *	2.1
		(4)	
(c)	$2\log_2(x+2) + \log_2 x - \log_2(x-6) = 3 \Rightarrow x^3 + 4x^3 - 4x + 48 = 0$		
	$\Rightarrow (x+6)(x^2 - 2x + 8) = 0$		
	Reason 1: E.g. • $\log_2 x$ is not defined when $x = -6$ • $\log_2(x-6)$ is not defined when $x = -6$ • $x = -6$, but $\log_2 x$ is only defined for $x > 0$ Reason 2: • $b^2 - 4ac = -28 < 0$, so $(x^2 - 2x + 8) = 0$ has no (real) roots		
	At least one of Reason 1 or Reason 2	B1	2.4
	Both Reason 1 and Reason 2	B1	2.1
		(2)	
			(10 marks)

Question 25 Notes:	
(a)(i)	
M1:	Applies $f(-6)$
A1*:	Applies $f(-6)=0$ to show that $a=4$
(a)(ii)	
M1:	Deduces $(x+6)$ is a factor of $f(x)$ and attempts to find a quadratic factor of $f(x)$ by either equating coefficients or by algebraic long division
A1:	$(x+6)(x^2-2x+8)$
(b)	
M1:	Evidence of applying a correct law of logarithms
M1:	Uses correct laws of logarithms to give either • an expression of the form $\log_2(h(x))=k$, where k is a constant • an expression of the form $\log_2(g(x))=\log_2(h(x))$
B1:	Evidence in their working of $\log_2 a=3 \Rightarrow a=2^3$ or 8
A1*:	Correctly proves $x^3+4x^3-4x+48=0$ with no errors seen
(c)	
B1:	See scheme
B1:	See scheme

Question	Scheme	Marks	AOs
26 (a)	Attempts to use an appropriate model; e.g. $y = A(3 - x)(3 + x)$ or $y = A(9 - x^2)$	M1	3.3
	e.g. $y = A(9 - x^2)$ Substitutes $x = 0, y = 5 \Rightarrow 5 = A(9 - 0) \Rightarrow A = \frac{5}{9}$	M1	3.1b
	$y = \frac{5}{9}(9 - x^2)$ or $y = \frac{5}{9}(3 - x)(3 + x)$, $\{-3 \leqslant x \leqslant 3\}$	A1	1.1b
		(3)	
(b)	Substitutes $x = \frac{2.4}{2}$ into their $y = \frac{5}{9}(9 - x^2)$	M1	3.4
	$y = \frac{5}{9}(9 - x^2) = 4.2 > 4.1 \Rightarrow$ Coach can enter the tunnel	A1	2.2b
		(2)	
(b) **Alt 1**	$4.1 = \frac{5}{9}(9 - x^2) \Rightarrow x = \frac{9\sqrt{2}}{10}$, so maximum width $= 2\left(\frac{9\sqrt{2}}{10}\right)$	M1	3.4
	$= 2.545... > 2.4 \Rightarrow$ Coach can enter the tunnel	A1	2.2b
		(2)	
(c)	E.g. • Coach needs to enter through the centre of the tunnel. This will only be possible if it is a one-way tunnel • In real-life the road may be cambered (and not horizontal) • The quadratic curve *BCA* is modelled for the entrance to the tunnel but we do not know if this curve is valid throughout the entire length of the tunnel • There may be overhead lights in the tunnel which may block the path of the coach	B1	3.5b
		(1)	
			(6 marks)

Question 26 Notes:

(a)	
M1:	Translates the given situation into an appropriate quadratic model – see scheme
M1:	Applies the maximum height constraint in an attempt to find the equation of the model – see scheme
A1:	Finds a suitable equation – see scheme
(b)	
M1:	See scheme
A1:	Applies a fully correct argument to infer {by assuming that curve *BCA* is quadratic and the given measurements are correct}, that is possible for the coach to enter the tunnel
(c)	
B1:	See scheme

Question	Scheme	Marks	AOs
27 (a)	$y=\dfrac{3x-5}{x+1} \Rightarrow y(x+1)=3x-5 \Rightarrow xy+y=3x-5 \Rightarrow y+5=3x-xy$	M1	1.1b
	$\Rightarrow y+5=x(3-y) \Rightarrow \dfrac{y+5}{3-y}=x$	M1	2.1
	Hence $\mathrm{f}^{-1}(x)=\dfrac{x+5}{3-x}, \quad x\in\mathbb{R},\ x\neq 3$	A1	2.5
		(3)	
(b)	$\mathrm{ff}(x)=\dfrac{3\left(\dfrac{3x-5}{x+1}\right)-5}{\left(\dfrac{3x-5}{x+1}\right)+1}$	M1	1.1a
	$=\dfrac{\dfrac{3(3x-5)-5(x+1)}{x+1}}{\dfrac{(3x-5)+(x+1)}{x+1}}$	M1	1.1b
		A1	1.1b
	$=\dfrac{9x-15-5x-5}{3x-5+x+1}=\dfrac{4x-20}{4x-4}=\dfrac{x-5}{x-1}$ (note that $a=-5$)	A1	2.1
		(4)	
(c)	$\mathrm{fg}(2)=\mathrm{f}(4-6)=\mathrm{f}(-2)=\dfrac{3(-2)-5}{-2+1}=11$	M1	1.1b
		A1	1.1b
		(2)	
(d)	$\mathrm{g}(x)=x^2-3x=(x-1.5)^2-2.25$. Hence $\mathrm{g}_{\min}=-2.25$	M1	2.1
	Either $\mathrm{g}_{\min}=-2.25$ or $\mathrm{g}(x)\geqslant -2.25$ or $\mathrm{g}(5)=25-15=10$	B1	1.1b
	$-2.25\leqslant \mathrm{g}(x)\leqslant 10$ or $-2.25\leqslant y\leqslant 10$	A1	1.1b
		(3)	
(e)	E.g. • the function g is many-one • the function g is not one-one • the inverse is one-many • $\mathrm{g}(0)=\mathrm{g}(3)=0$	B1	2.4
		(1)	
		(13 marks)	

Question **27** Notes:	
(a)	
M1:	Attempts to find the inverse by cross-multiplying and an attempt to collect all the *x*-terms (or swapped *y*-terms) onto one side
M1:	A fully correct method to find the inverse
A1:	A correct $f^{-1}(x) = \dfrac{x+5}{3-x},\ x \in \mathbb{R},\ x \neq 3$, expressed fully in function notation (including the domain)
(b)	
M1:	Attempts to substitute $f(x) = \dfrac{3x-5}{x+1}$ into $\dfrac{3f(x)-5}{f(x)+1}$
M1:	Applies a method of "rationalising the denominator" for both their numerator and their denominator.
A1:	$\dfrac{\dfrac{3(3x-5)-5(x+1)}{x+1}}{\dfrac{(3x-5)+(x+1)}{x+1}}$ which can be simplified or un-simplified
A1:	Shows $ff(x) = \dfrac{x+a}{x-1}$ where $a = -5$ or $ff(x) = \dfrac{x-5}{x-1}$, with no errors seen.
(c)	
M1:	Attempts to substitute the result of $g(2)$ into f
A1:	Correctly obtains $fg(2) = 11$
(d)	
M1:	Full method to establish the minimum of g. E.g. • $(x \pm a)^2 + b$ leading to $g_{min} = b$ • Finds the value of *x* for which $g'(x) = 0$ and inserts this value of *x* back into $g(x)$ in order to find to g_{min}
B1:	For either • finding the correct minimum value of g (Can be implied by $g(x) \geqslant -2.25$ or $g(x) > -2.25$) • stating $g(5) = 25 - 15 = 10$
A1:	States the correct range for g. E.g. $-2.25 \leqslant g(x) \leqslant 10$ or $-2.25 \leqslant y \leqslant 10$
(e)	
B1:	See scheme

Question	Scheme	Marks	AOs
28(a)	States or uses $f(+3) = 0$	M1	1.1b
	$4(3)^3 – 12(3)^2 + 2(3) – 6 = 108 – 108 + 6 – 6 = 0$ and so $(x – 3)$ is a factor	A1	1.1b
		(2)	
(b)	Begins division or factorisation so x $4x^3 – 12x^2 + 2x – 6 = (x −3)(4x^2 +\dots)$	M1	2.1
	$4x^3 – 12x^2 + 2x – 6 = (x −3)(4x^2 + 2)$	A1	1.1b
	Considers the roots of their quadratic function using completion of square or discriminant	M1	2.1
	$(4x^2 + 2) = 0$ has no real roots with a reason (e.g. negative number does not have a real square root, or $4x^2 + 2 > 0$ for all x So $x = 3$ is the only real root of $f(x) = 0$ *	A1*	2.4
		(4)	
			(6 marks)

Notes:

(a)

M1: States or uses $f(+3) = 0$

A1: See correct work evaluating and achieving zero, together with correct conclusion

(b)

M1: Needs to have $(x – 3)$ and first term of quadratic correct

A1: Must be correct – may further factorise to $2(x − 3)(2x^2 + 1)$

M1: Considers their quadratic for no real roots by use of completion of the square or consideration of discriminant then

A1*: A correct explanation

Question	Scheme	Marks	AOs
29	Realises that $k = 0$ will give no real roots as equation becomes $3 = 0$ (proof by contradiction)	B1	3.1a
	(For $k \neq 0$) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$	M1	2.4
	$4k(4k - 3) < 0$ with attempt at solution	M1	1.1b
	So $0 < k < \frac{3}{4}$, which together with $k = 0$ gives $0 \leqslant k < \frac{3}{4}$ *	A1*	2.1
			(4 marks)

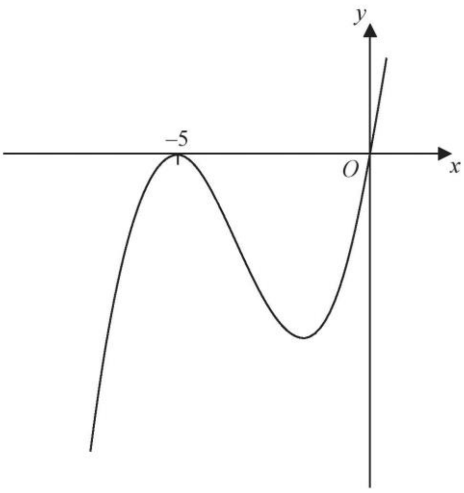
Notes:

B1: Explains why $k = 0$ gives no real roots

M1: Considers discriminant to give quadratic inequality – does not need the $k \neq 0$ for this mark

M1: Attempts solution of quadratic inequality

A1*: Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)

Question	Scheme	Marks	AOs
30(a)	$x^3+10x^2+25x=x(x^2+10x+25)$	M1	1.1b
	$=x(x+5)^2$	A1	1.1b
		(2)	
(b)	 A cubic with correct orientation	M1	1.1b
	Curve passes through the origin (0, 0) and touches at (– 5, 0) (see note below for ft)	A1ft	1.1b
		(2)	
(c)	Curve has been translated a to the left	M1	3.1a
	$a = -2$	A1ft	3.2a
	$a = 3$	A1ft	1.1b
		(3)	
			(7 marks)

Notes:

(a)
M1: Takes out factor x
A1: Correct factorisation – allow $x(x + 5)(x + 5)$

(b)
M1: Correct shape
A1ft: Curve passes through the origin (0, 0) and touches at (– 5, 0) – allow follow through from incorrect factorisation

(c)
M1: May be implied by one of the correct answers for a or by a statement
A1ft: ft from their cubic as long as it meets the x-axis only twice
A1ft: ft from their cubic as long as it meets the x-axis only twice

Question	Scheme	Marks	AOs
31(a)	Sets $2xy+\frac{\pi x^2}{2}=250$	B1	2.1
	Obtain $y=\frac{250-\frac{\pi x^2}{2}}{2x}$ and substitute into P	M1	1.1b
	Use $P=2x+2y+\pi x$ with their y substituted	M1	2.1
	$P=2x+\frac{250}{x}-\frac{\pi x^2}{2x}+\pi x=2x+\frac{250}{x}+\frac{\pi x}{2}$ *	A1*	1.1b
		(4)	
(b)	$x>0$ and $y>0$ (distance) $\Rightarrow \frac{250-\frac{\pi x^2}{2}}{2x}>0$ or $250-\frac{\pi x^2}{2}>0$ o.e.	M1	2.4
	As x and y are distances they are positive so $0<x<\sqrt{\frac{500}{\pi}}$ *	A1*	3.2a
		(2)	
(c)	Differentiates P with negative index correct in $\frac{dP}{dx}$; $x^{-1}\to x^{-2}$	M1	3.4
	$\frac{dP}{dx}=2-\frac{250}{x^2}+\frac{\pi}{2}$	A1	1.1b
	Sets $\frac{dP}{dx}=0$ and proceeds to $x=$	M1	1.1b
	Substitutes their x into $P=2x+\frac{250}{x}+\frac{\pi x}{2}$ to give perimeter = 59.8 M	A1	1.1b
		(4)	
(10 marks)			

Question 31 continued	
Notes:	
(a)	
B1:	Correct area equation
M1:	Rearranges **their** area equation to make y the subject of the formula and attempt to use with an expression for P
M1:	Use correct equation for perimeter with their y substituted
A1*:	Completely correct solution to obtain and state printed answer
(b)	
M1:	States $x > 0$ and $y > 0$ and uses their expression from (a) to form inequality
A1*:	Explains that x and y are positive because they are distances, and uses correct expression for y to give the printed answer correctly
(c)	
M1:	Attempt to differentiate P (deals with negative power of x correctly)
A1:	Correct differentiation
M1:	Sets derived function equal to zero and obtains $x =$
A1:	The value of x may not be seen (it is 8.37 to 3sf or $\sqrt{\left(\frac{500}{4+\pi}\right)}$) Need to see awrt 59.8 M with units included for the perimeter

Question	Scheme	Marks	AOs
32(a)	Sets $H=0\Rightarrow 1.8+0.4d-0.002d^2=0$	M1	3.4
	Solves using an appropriate method, for example $d=\dfrac{-0.4\pm\sqrt{(0.4)^2-4(-0.002)(1.8)}}{2\times -0.002}$	dM1	1.1b
	Distance = awrt 204(m) only	A1	2.2a
		(3)	
(b)	States the initial height of the arrow above the ground.	B1	3.4
		(1)	
(c)	$1.8+0.4d-0.002d^2=-0.002\left(d^2-200d\right)+1.8$	M1	1.1b
	$=-0.002\left((d-100)^2-10000\right)+1.8$	M1	1.1b
	$=21.8-0.002(d-100)^2$	A1	1.1b
		(3)	
(d)	(i) 22.1 metres	B1ft	3.4
	(ii) 100 metres	B1ft	3.4
		(2)	
		(9 marks)	

Notes:

(a)

M1: Sets $H=0\Rightarrow 1.8+0.4d-0.002d^2=0$

M1: Solves using formula, which if stated must be correct, by completing square (look for $(d-100)^2=10900\Rightarrow d=..$) or even allow answers coming from a graphical calculator

A1: Awrt 204 m only

(b)

B1: States it is the initial height of the arrow above the ground. Do not allow " it is the height of the archer"

(c)

M1: Score for taking out a common factor of -0.002 from at least the d^2 and d terms

M1: For completing the square for their $\left(d^2-200d\right)$ term

A1: $=21.8-0.002(d-100)^2$ or exact equivalent

(d)

B1ft: For their '21.8+0.3' =22.1m

B1ft: For their 100m

Question	Scheme	Marks	AOs
33(a)	$y \leqslant 7$	B1	2.5
		(1)	
(b)	$\text{f}(1.8) = 7 - 2\times1.8^2 = 0.52 \Rightarrow \text{gf}(1.8) = \text{g}(0.52) = \dfrac{3\times0.52}{5\times0.52-1} = ...$	M1	1.1b
	$\text{gf}(1.8) = 0.975$ oe e.g. $\dfrac{39}{40}$	A1	1.1b
		(2)	
(c)	$y = \dfrac{3x}{5x-1} \Rightarrow 5xy - y = 3x \Rightarrow x(5y-3) = y$	M1	1.1b
	$\left(\text{g}^{-1}(x) =\right)\dfrac{x}{5x-3}$	A1	2.2a
		(2)	
		(5 marks)	

Notes

(a)

B1: Correct range. Allow $\text{f}(x)$ or f for y. Allow e.g. $\{y \in \mathbb{R} : y \leqslant 7\}$, $-\infty < y \leqslant 7$, $(-\infty, 7]$

(b)

M1: Full method to find f (1.8) and substitutes the result into g to obtain a value.
Also allow for an attempt to substitute x = 1.8 into an attempt at gf (x).

E.g. $\text{gf}(x) = \dfrac{3(7-2x^2)}{5(7-2x^2)-1} = \dfrac{3\left(7-2(1.8)^2\right)}{5\left(7-2\times(1.8)^2\right)-1} = ...$

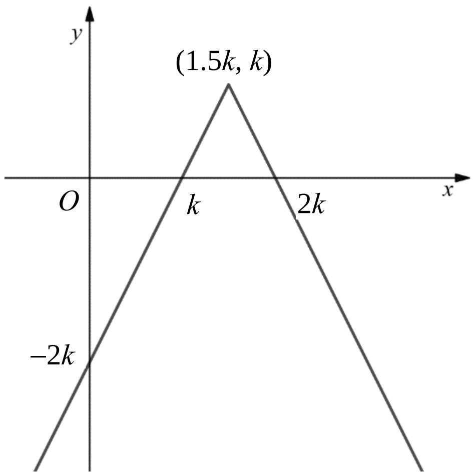
A1: Correct value

(c)

M1: Correct attempt to cross multiply, followed by an attempt to factorise out x from an xy term and an x term.
If they swap x and y at the start then it will be for an attempt to cross multiply followed by an attempt to factorise out y from an xy term and a y term.

A1: Correct expression. Allow equivalent correct expressions e.g. $\dfrac{-x}{3-5x}, \dfrac{1}{5} + \dfrac{3}{25x-15}$

Ignore any domain if given.

Question	Scheme	Marks	AOs
34(a)			
	∧ shape in any position	B1	1.1b
	Correct x-intercepts or coordinates	B1	1.1b
	Correct y-intercept or coordinates	B1	1.1b
	Correct coordinates for the vertex of a ∧ shape	B1	1.1b
		(4)	
(b)	$x = k$	B1	2.2a
	$k-(2x-3k)=x-k \Rightarrow x = ...$	M1	3.1a
	$x=\frac{5k}{3}$	A1	1.1b
	Set notation is required here for this mark $\left\{x : x<\frac{5k}{3}\right\} \cap \{x : x>k\}$	A1	2.5
		(4)	
(c)	$x = 3k$ **or** $y = 3 - 5k$	B1ft	2.2a
	$x = 3k$ **and** $y = 3 - 5k$	B1ft	2.2a
		(2)	
			(10 marks)

Notes

(a) **Note that the sketch may be seen on Figure 4**
B1: See scheme
B1: Correct x-intercepts. Allow as shown or written as (k, 0) and (2k, 0) and condone coordinates written as (0, k) and (0, 2k) as long as they are in the correct places.
B1: Correct y-intercept. Allow as shown or written as (0, –2k) or (–2k, 0) as long as it is in the correct place. Condone k – 3k for – 2k.
B1: Correct coordinates as shown
Note that the marks for the intercepts and the maximum can be seen away from the sketch but the coordinates must be the right way round or e.g. as y = 0, x = k etc. These marks can be awarded without a sketch but if there is a sketch, such points must not contradict the sketch.
(b)
B1: Deduces the correct critical value of $x = k$. May be implied by e.g. $x > k$ or $x < k$
M1: Attempts to solve $k – (2x – 3k) = x – k$ or an equivalent equation/inequality to find the other critical value. Allow this mark for reaching k = … or x = … as long as they are solving the required equation.
A1: Correct value
A1: Correct answer using the correct set notation.

Allow e.g. $\left\{x : x \in \mathbb{R}, k < x < \frac{5k}{3}\right\}$, $\left\{x : \ k < x < \frac{5k}{3}\right\}$, $x \in \left(k, \frac{5k}{3}\right)$ and allow "|" for " :"

But $\left\{x : x < \frac{5k}{3}\right\} \cup \{x : x > k\}$ scores A0 $\left\{x : \ k < x, \ x < \frac{5k}{3}\right\}$ scores A0

(c)

B1ft: Deduces one correct coordinate. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" or $y = 3 - 5\times$"k" but must be in terms of k.

Allow as coordinates or $x = ..$, $y = \ldots$

B1ft: Deduces both correct coordinates. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" and $y = 3 - 5\times$"k" but must be in terms of k.

Allow as coordinates or $x = ..$, $y = \ldots$

If coordinates are given the wrong way round and not seen correctly as $x = \ldots$, $y = \ldots$

e.g. $(3 - 5k, 3k)$ this is B0B0

Question	Scheme	Marks	AOs
35(a)	$x = -4$ **or** $y = -5$	B1	1.1b
	$P(-4, -5)$	B1	2.2a
		(2)	
(b)	$3x + 40 = -2(x+4) - 5 \Rightarrow x = ...$	M1	1.1b
	$x = -10.6$	A1	2.1
		(2)	
(c)	$a > 2$	B1	2.2a
	$y = ax \Rightarrow -5 = -4a \Rightarrow a = \frac{5}{4}$	M1	3.1a
	$\{a : a \leqslant 1.25\} \cup \{a : a > 2\}$	A1	2.5
		(3)	
			(7 marks)

Notes:

(a)

B1: One correct coordinate. Either $x = -4$ or $y = -5$ or $(-4, \ldots)$ or $(\ldots, -5)$ seen.

B1: Deduces that $P(-4, -5)$ Accept written separately e.g. $x = -4$, $y = -5$

(b)

M1: Attempts to solve $3x + 40 = -2(x+4) - 5 \Rightarrow x = ...$ Must reach a value for x.

You may see the attempt crossed out but you can still take this as an attempt to solve the required equation.

A1: $x = -10.6$ oe e.g. $-\frac{53}{5}$ only. If other values are given, e.g. $x = -37$ they must be rejected or the $-\frac{53}{5}$ clearly chosen as their answer. Ignore any attempts to find y.

Alternative by squaring:

$$3x + 40 = 2|x+4| - 5 \Rightarrow 3x + 45 = 2|x+4| \Rightarrow 9x^2 + 270x + 2025 = 4\left(x^2 + 8x + 16\right)$$

$$\Rightarrow 5x^2 + 238x + 1961 = 0 \Rightarrow x = -37, -\frac{53}{5}$$

M1 for isolating the $|x+4|$, squaring both sides and solving the resulting quadratic

A1 for selecting the $-\frac{53}{5}$

Correct answer with no working scores both marks.

(c)

B1: Deduces that $a > 2$

M1: Attempts to find a value for a using their $P(-4, -5)$

Alternatively attempts to solve $ax = 2(x + 4) - 5$ and $ax = 2(x + 4) - 5$ to obtain a value for a.

A1: Correct range in acceptable set notation.

Examples:

$\{a : a \leqslant 1.25\} \cup \{a : a > 2\}$

$\{a : a \leqslant 1.25\}, \{a : a > 2\}$

$\{a : a \leqslant 1.25 \text{ or } a > 2\}$

$\{a : a \leqslant 1.25, \ a > 2\}$

$(-\infty, 1.25] \cup (2, \infty)$

$(-\infty, 1.25], (2, \infty)$

Question	Scheme	Marks	AOs
36 (a)	$gg(0) = g((0-2)^2+1) = g(5) = 4(5)-7 = 13$	M1	2.1
		A1	1.1b
		(2)	
(b)	Solves either $(x-2)^2+1=28 \Rightarrow x=...$ or $4x-7=28 \Rightarrow x=...$	M1	1.1b
	At least one critical value $x=2-3\sqrt{3}$ or $x=\frac{35}{4}$ is correct	A1	1.1b
	Solves both $(x-2)^2+1=28 \Rightarrow x=...$ and $4x-7=28 \Rightarrow x=...$	M1	1.1b
	Correct final answer of '$x<2-3\sqrt{3}$, $x>\frac{35}{4}$'	A1	2.1
	Note: Writing awrt -3.20 or a truncated -3.19 or a truncated -3.2 in place of $2-3\sqrt{3}$ is accepted for any of the A marks	**(4)**	
(c)	<u>h is a one-one</u> {function (or mapping) so has an inverse} <u>g is a many-one</u> {function (or mapping) so does not have an inverse}	B1	2.4
		(1)	
(d) Way 1	$\left\{h^{-1}(x) = -\frac{1}{2} \Rightarrow\right\}$ $x = h\left(-\frac{1}{2}\right)$	M1 B1 on epen	1.1b
	$x=\left(-\frac{1}{2}-2\right)^2+1$ **Note:** Condone $x=\left(\frac{1}{2}-2\right)^2+1$	M1	1.1b
	$\Rightarrow x = 7.25$ only **cso**	A1	2.2a
		(3)	
(d) Way 2	{their $h^{-1}(x)$ } $= \pm 2 \pm \sqrt{x \pm 1}$	M1	1.1b
	Attempts to solve $\pm 2 \pm \sqrt{x\pm 1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x \pm 1} = ...$	M1	1.1b
	$\Rightarrow x = 7.25$ only **cso**	A1	2.2a
		(3)	
		(10 marks)	

Notes for Question 36

(a)	
M1:	Uses a complete method to find $gg(0)$. E.g. • Substituting $x=0$ into $(0-2)^2+1$ and the result of this into the relevant part of $g(x)$ • Attempts to substitute $x=0$ into $4((x-2)^2+1)-7$ or $4(x-2)^2-3$
A1:	$gg(0)=13$
(b)	
M1:	See scheme
A1:	See scheme
M1:	See scheme
A1:	Brings all the strands of the problem together to give a correct solution.
Note:	You can ignore inequality symbols for any of the M marks
Note:	If a 3TQ is formed (e.g. $x^2-4x-23=0$) then a correct method for solving a 3TQ is required for the relevant method mark to be given.
Note:	Writing $(x-2)^2+1=28 \Rightarrow (x-2)+1=\sqrt{28} \Rightarrow x=-1+\sqrt{28}$ (i.e. taking the square-root of each term to solve $(x-2)^2+1=28$ is not considered to be an acceptable method)
Note:	Allow set notation. E.g. $\{x \in \mathbb{R}: x<2-3\sqrt{3} \cup x>8.75\}$ is fine for the final A mark

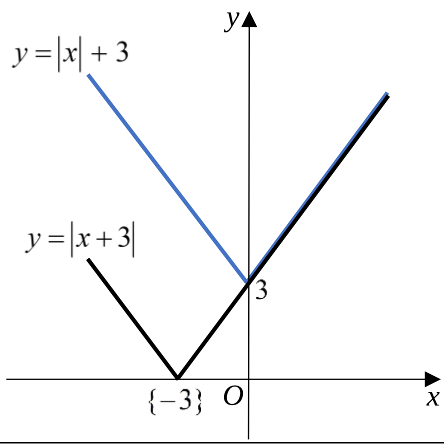
Notes for Question 36 Continued	
(b)	***continued***
Note:	Give final A0 for $\{x \in \mathbb{R}: x < 2 - 3\sqrt{3} \cap x > 8.75\}$
Note:	Give final A0 for $2 - 3\sqrt{3} > x > 8.75$
Note:	Allow final A1 for their writing a final answer of "$x < 2 - 3\sqrt{3}$ and $x > \frac{35}{4}$"
Note:	Allow final A1 for a final answer of $x < 2 - 3\sqrt{3},\ x > \frac{35}{4}$
Note:	Writing $2 - \sqrt{27}$ in place of $2 - 3\sqrt{3}$ is accepted for any of the A marks
Note:	Allow final A1 for a final answer of $x < -3.20,\ x > 8.75$
Note:	Using 29 instead of 28 is M0 A0 M0 A0
(c)	
B1:	A correct explanation that conveys the underlined points
Note:	A minimal acceptable reason is "h is a one-one and g is a many-one"
Note:	Give B1 for "h^{-1} is one-one and g^{-1} is one-many"
Note:	Give B1 for "h is a one-one and g is not"
Note:	Allow B1 for "g is a many-one and h is not"
(d)	**Way 1**
M1:	Writes $x = \mathrm{h}\left(-\frac{1}{2}\right)$
M1:	See scheme
A1:	Uses $x = \mathrm{h}\left(-\frac{1}{2}\right)$ to deduce that $x = 7.25$ only, **cso**
(d)	**Way 2**
M1:	See scheme
M1:	See scheme
A1:	Use a correct $\mathrm{h}^{-1}(x) = 2 - \sqrt{x-1}$ to deduce that $x = 7.25$ only, **cso**
Note:	Give final A0 cso for $2 + \sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A0 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A1 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow -\sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Allow final A1 for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$

Question	Scheme	Marks	AOs
37	$\mathrm{g}(x) = \dfrac{2x+5}{x-3},\ x \geq 5$		
(a) **Way 1**	$\mathrm{g}(5) = \dfrac{2(5)+5}{5-3} = 7.5 \Rightarrow \mathrm{gg}(5) = \dfrac{2(\text{"}7.5\text{"})+5}{\text{"}7.5\text{"}-3}$	M1	1.1b
	$\mathrm{gg}(5) = \dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(a) **Way 2**	$\mathrm{gg}(x) = \dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3} \Rightarrow \mathrm{gg}(5) = \dfrac{2\left(\dfrac{2(5)+5}{(5)-3}\right)+5}{\left(\dfrac{2(5)+5}{(5)-3}\right)-3}$	M1	1.1b
	$\mathrm{gg}(5) = \dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(b)	$\{\text{Range:}\}\ \ 2 < y \leq \dfrac{15}{2}$	B1	1.1b
		(1)	
(c) **Way 1**	$y = \dfrac{2x+5}{x-3} \Rightarrow yx - 3y = 2x + 5 \Rightarrow yx - 2x = 3y + 5$	M1	1.1b
	$x(y-2) = 3y+5 \Rightarrow x = \dfrac{3y+5}{y-2}\ \left\{\text{or } y = \dfrac{3x+5}{x-2}\right\}$	M1	2.1
	$\mathrm{g}^{-1}(x) = \dfrac{3x+5}{x-2},\ \ 2 < x \leq \dfrac{15}{2}$	A1ft	2.5
		(3)	
(c) **Way 2**	$y = \dfrac{2x-6+11}{x-3} \Rightarrow y = 2 + \dfrac{11}{x-3} \Rightarrow y - 2 = \dfrac{11}{x-3}$	M1	1.1b
	$x - 3 = \dfrac{11}{y-2} \Rightarrow x = \dfrac{11}{y-2} + 3\ \left\{\text{or } y = \dfrac{11}{x-2} + 3\right\}$	M1	2.1
	$\mathrm{g}^{-1}(x) = \dfrac{11}{x-2} + 3,\ \ 2 < x \leq \dfrac{15}{2}$	A1ft	2.5
		(3)	
		(6 marks)	

Notes for Question 37

(a)	
M1:	Full method of attempting $\mathrm{g}(5)$ and substituting the result into g
Note:	**Way 2:** Attempts to substitute $x = 5$ into $\dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3}$, o.e. Note that $\mathrm{gg}(x) = \dfrac{9x-5}{14-x}$
A1:	Obtains $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$ or an exact equivalent
Note:	Give A0 for 4.4 or 4.444... without reference to $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$

Notes for Question **37** Continued	
(b)	
B1:	States $2 < y \le \frac{15}{2}$ Accept any of $2 < \text{g} \le \frac{15}{2}$, $2 < \text{g}(x) \le \frac{15}{2}$, $\left(2, \frac{15}{2}\right]$
Note:	Accept $\text{g}(x) > 2$ **and** $\text{g}(x) \le \frac{15}{2}$ o.e.
(c) Way 1	
M1:	Correct method of cross multiplication followed by an attempt to collect terms in *x* or terms in a swapped *y*
M1:	A complete method (i.e. as above and also factorising and dividing) to find the inverse
A1ft:	Uses correct notation to correctly define the inverse function g^{-1}, where the domain of g^{-1} stated correctly or correctly followed through (using correct notation) on the values shown in their range in part (b). Allow $\text{g}^{-1} : x \to$. Condone $\text{g}^{-1} = ...$ Do not accept $y = ...$
Note:	Correct notation is required when stating the domain of $\text{g}^{-1}(x)$. Allow $2 < x \le \frac{15}{2}$ or $\left(2, \frac{15}{2}\right]$ Do not allow any of e.g. $2 < \text{g} \le \frac{15}{2}$, $2 < \text{g}^{-1}(x) \le \frac{15}{2}$
Note:	Do not allow A1ft for following through their range in (b) to give a domain for g^{-1} as $x \in \mathbb{R}$
(c) Way 2	
M1:	Writes $y = \frac{2x+5}{x-3}$ in the form $y = 2 \pm \frac{k}{x-3}$, $k \ne 0$ and rearranges to isolate *y* and 2 on one side of their equation. **Note:** Allow the equivalent method with *x* swapped with *y*
M1:	A complete method to find the inverse
A1ft:	As in Way 1
Note:	If a candidate scores no marks in part (c), but • states the domain of g^{-1} correctly, **or** • states a domain of g^{-1} which is correctly followed through on the values shown in their range in part (b) then give special case (SC) M1 M0 A0

Question	Scheme		Marks	AOs
38(i), (ii) Way 1		V shaped graph {reasonably} symmetrical about the y-axis with vertical interpret (0, 3) or 3 stated or marked on the positive y-axis	B1	1.1b
		Superimposes the graph of $y=\lvert x+3\rvert$ on top of the graph of $y=\lvert x\rvert+3$	M1	3.1a
	the graph of $y=\lvert x\rvert+3$ is either the same or above the graph of $y=\lvert x+3\rvert$ {for corresponding values of x} **or** when $x \ge 0$, both graphs are equal (or the same) when $x<0$, the graph of $y=\lvert x\rvert+3$ is above the graph of $y=\lvert x+3\rvert$		A1	2.4
			(3)	
38(ii) Way 2	**<u>Reason 1</u>** When $x \ge 0$, $\lvert x\rvert+3=\lvert x+3\rvert$ **<u>Reason 2</u>** When $x<0$, $\lvert x\rvert+3>\lvert x+3\rvert$	Any one of Reason 1 or Reason 2	M1	3.1a
		Both Reason 1 and Reason 2	A1	2.4
				(3 marks)

Notes for Question 38	
38(i)	
B1:	See scheme
3(ii)	
M1:	For constructing a method of comparing $\lvert x\rvert+3$ with $\lvert x+3\rvert$. See scheme.
A1:	Explains fully why $\lvert x\rvert+3 \ge \lvert x+3\rvert$. See scheme.
Note:	Do not allow either $x>0$, $\lvert x\rvert+3 \ge \lvert x+3\rvert$ or $x \ge 0$, $\lvert x\rvert+3 \ge \lvert x+3\rvert$ as a valid reason
Note	$x=0$ (or where necessary $x=-3$) need to be considered in their solutions for A1
Note:	Do not allow an incorrect statement such as $x \le 0$, $\lvert x\rvert+3>\lvert x+3\rvert$ for A1

Notes for Question 38 Continued			
38(ii)			
Note:	Allow M1A1 for $x > 0,\ \lvert x\rvert + 3 = \lvert x+3\rvert$ and for $x \le 0,\ \lvert x\rvert + 3 \ge \lvert x+3\rvert \geqslant$		
Note:	Allow M1 for any of • x is positive, $\lvert x\rvert + 3 = \lvert x+3\rvert$ • x is negative, $\lvert x\rvert + 3 > \lvert x+3\rvert$ • $x > 0,\ \lvert x\rvert + 3 = \lvert x+3\rvert$ • $x \le 0,\ \lvert x\rvert + 3 \ge \lvert x+3\rvert$ • $x > 0,\ \lvert x\rvert + 3$ and $\lvert x+3\rvert$ are equal • $x \ge 0,\ \lvert x\rvert + 3$ and $\lvert x+3\rvert$ are equal • when $x \ge 0$, both graphs are equal • for positive values $\lvert x\rvert + 3$ and $\lvert x+3\rvert$ are the same Condone for M1 • $x \le 0,\ \lvert x\rvert + 3 > \lvert x+3\rvert$ • $x < 0,\ \lvert x\rvert + 3 \ge \lvert x+3\rvert$		
38(ii) Way 3	• For $x > 0,\ \lvert x\rvert + 3 = \lvert x+3\rvert$ • For $-3 < x < 0$, as $\lvert x\rvert + 3 > 3$ and $\{0 <\}\ \lvert x+3\rvert < 3$, then $\lvert x\rvert + 3 > \lvert x+3\rvert$ • For $x \le -3$, as $\lvert x\rvert + 3 = -x + 3$ and $\lvert x+3\rvert = -x - 3$, then $\lvert x\rvert + 3 > \lvert x+3\rvert$	M1 A1	3.1a 2.4

Question	Scheme	Marks	AOs
39	(a) $\mathrm{f}(x)=-3x^3+8x^2-9x+10, \quad x\in\mathbb{R}$		
(a)	(i) $\{\mathrm{f}(2)=-24+32-18+10 \Rightarrow\}\ \mathrm{f}(2)=0$	B1	1.1b
	(ii) $\{\mathrm{f}(x)=\}\ (x-2)(-3x^2+2x-5)$ or $(2-x)(3x^2-2x+5)$	M1	2.2a
		A1	1.1b
		(3)	
(b)	$-3y^6+8y^4-9y^2+10=0 \Rightarrow (y^2-2)(-3y^4+2y^2-5)=0$		
	Gives a partial explanation by • explaining that $-3y^4+2y^2-5=0$ has no {real} solutions with a reason, e.g. $b^2-4ac=(2)^2-4(-3)(-5)=-56<0$ • or stating that $y^2=2$ has 2 {real} solutions or $y=\pm\sqrt{2}$ {only}	M1	2.4
	Complete proof that the given equation has exactly two {real} solutions	A1	2.1
		(2)	
(c)	$3\tan^3\theta-8\tan^2\theta+9\tan\theta-10=0;\ 7\pi\le\theta<10\pi$		
	{Deduces that} there are 3 solutions	B1	2.2a
		(1)	
			(6 marks)

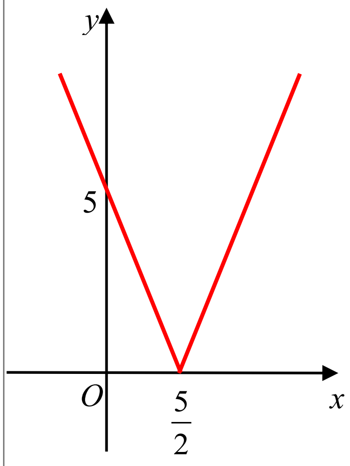
Notes for Question 39

(a)(i)	
B1:	$\mathrm{f}(2)=0$ or 0 stated by itself in part (a)(i)
(a)(ii)	
M1:	Deduces that $(x-2)$ or $(2-x)$ is a factor and attempts to find the other quadratic factor by • using long division to obtain either $\pm3x^2 \pm kx+...,\ k=\text{value}\ne0$ or $\pm3x^2 \pm \alpha x+\beta,\ \beta=\text{value}\ne0,\ \alpha$ can be 0 • factorising to obtain their quadratic factor in the form $(\pm3x^2 \pm kx \pm c),\ k=\text{value}\ne0$, c can be 0, or in the form $(\pm3x^2 \pm \alpha x \pm \beta),\ \beta=\text{value}\ne0,\ \alpha$ can be 0
A1:	$(x-2)(-3x^2+2x-5),\ (2-x)(3x^2-2x+5)$ or $-(x-2)(3x^2-2x+5)$ stated together as a product
(b)	
M1:	See scheme
A1:	See scheme. Proof must be correct ***with no errors***, e.g. giving an incorrect discriminant value
Note:	Correct calculation e.g. $(2)^2-4(-3)(-5),\ 4-60$ or -56 must be given for the first explanation
Note:	Note that M1 can be allowed for • a correct follow through calculation for the discriminant of their "$-3y^4+2y^2-5$" which would lead to a value <0 together with an explanation that $-3y^4+2y^2-5=0$ has no {real} solutions • **or** for the omission of <0
Note:	<0 must also been stated in a discriminant method for A1
Note:	Do not allow A1 for incorrect working, e.g. $(2)^2-4(-3)(-5)=-54<0$
Note:	$y^2=2 \Rightarrow y=\pm2$, so 2 solutions is not allowed for A1, but can be condoned for M1
Note:	Using the formula on $-3y^4+2y^2-5=0$ or $-3x^2+2x-5=0$ gives y^2 or $x=\dfrac{-2\pm\sqrt{-56}}{-6}$ or $\dfrac{-1\pm\sqrt{-14}}{-3}$

Notes for Question **39** Continued	
Note:	Completing the square on $-3x^2+2x-5=0$ gives $x^2-\frac{2}{3}x+\frac{5}{3}=0 \Rightarrow \left(x-\frac{1}{3}\right)^2-\frac{1}{9}+\frac{5}{3}=0 \Rightarrow x=\frac{1}{3}\pm\sqrt{\frac{-14}{9}}$
Note:	Do not recover work for part (b) in part (c)
(c)	
B1:	See scheme
Note:	Give B0 for stating $\theta =$ awrt 23.1, awrt 26.2, awrt 29.4 **without** reference to 3 solutions

Question	Scheme	Marks	AOs
40 (a) **Way 1**	$H = Ax(40 - x)$ {or $H = Ax(x - 40)$ }	M1	3.3
	$x = 20,\ H = 12 \Rightarrow 12 = A(20)(40-20) \Rightarrow A = \frac{3}{100}$	dM1	3.1b
	$H = \frac{3}{100}x(40-x)$ or $H = -\frac{3}{100}x(x-40)$	A1	1.1b
		(3)	
(a) **Way 2**	$H = 12 - \lambda(x-20)^2$ {or $H = 12 + \lambda(x-20)^2$ }	M1	3.3
	$x = 40,\ H = 0 \Rightarrow 0 = 12 - \lambda(40-20)^2 \Rightarrow \lambda = \frac{3}{100}$	dM1	3.1b
	$H = 12 - \frac{3}{100}(x-20)^2$	A1	1.1b
		(3)	
(a) **Way 3**	$H = ax^2 + bx + c$ (or deduces $H = ax^2 + bx$) **Both** $x = 0,\ H = 0 \Rightarrow 0 = 0 + 0 + c \Rightarrow c = 0$ **and either** $x = 40,\ H = 0 \Rightarrow 0 = 1600a + 40b$ **or** $x = 20,\ H = 12 \Rightarrow 12 = 400a + 20b$ **or** $\frac{-b}{2a} = 20$ $\{\Rightarrow b = -40a\}$	M1	3.3
	$b = -40a \Rightarrow 12 = 400a + 20(-40a) \Rightarrow a = -0.03$ so $b = -40(-0.03) = 1.2$	dM1	3.1b
	$H = -0.03x^2 + 1.2x$	A1	1.1b
		(3)	
(b)	$\{H = 3 \Rightarrow\}\ 3 = \frac{3}{100}x(40-x) \Rightarrow x^2 - 40x + 100 = 0$ **or** $\{H = 3 \Rightarrow\}\ 3 = 12 - \frac{3}{100}(x-20)^2 \Rightarrow (x-20)^2 = 300$	M1	3.4
	e.g. $x = \frac{40 \pm \sqrt{1600 - 4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	$\left\{\text{chooses } 20 + \sqrt{300} \Rightarrow\right\}$ greatest distance = awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. • the ground is horizontal • the ball needs to be kicked from the ground • the ball is modelled as a particle • the horizontal bar needs to be modelled as a line • there is no wind or air resistance on the ball • there is no spin on the ball • no obstacles in the trajectory (or path) of the ball • the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
			(7 marks)

Notes for Question **40**	
(a)	
M1:	Translates the situation given into a suitable equation for the model. E.g. **Way 1:** {Uses $(0, 0)$ and $(40, 0)$ to write} $H = Ax(40 - x)$ o.e. {or $H = Ax(x - 40)$ }
	Way 2: {Uses $(20, 12)$ to write} $H = 12 - \lambda(x-20)^2$ or $H = 12 + \lambda(x-20)^2$
	Way 3: Writes $H = ax^2 + bx + c$, **and** uses $(0, 0)$ to deduce $c = 0$ **and** an attempt at using either $(40, 0)$ or $(20, 12)$ **Special Case:** Allow SC M1dM0A0 for not deducing $c = 0$ but attempting to apply both $(40, 0)$ and $(20, 12)$
dM1:	Applies a complete strategy with appropriate constraints to find all constants in their model. **Way 1:** Uses $(20, 12)$ on their model and finds $A = ...$ **Way 2:** Uses either $(40, 0)$ or $(0, 0)$ on their model to find $\lambda = ...$ **Way 3:** Uses $(40, 0)$ and $(20, 12)$ on their model to find $a = ...$ and $b = ...$
A1:	Finds a correct equation linking *H* to *x* E.g. $H = \frac{3}{100}x(40-x)$, $H = 12 - \frac{3}{100}(x-20)^2$ or $H = -0.03x^2 + 1.2x$
Note:	Condone writing *y* in place of *H* for the M1 and dM1 marks.
Note:	Give final A0 for $y = -0.03x^2 + 1.2x$
Note:	Give special case M1dM0A0 for writing down any of $H = 12 - (x-20)^2$ or $H = x(40-x)$ or $H = x(x-40)$
Note:	Give M1 dM1 for finding $-0.03x^2 + 1.2x$ or $a = -0.03, b = 1.2, c = 0$ in an implied $ax^2 + bx$ or $ax^2 + bx + c$ (with no indication of $H = ...$)
(b)	
M1:	Substitutes $H = 3$ into their quadratic equation and proceeds to obtain a 3TQ or a quadratic in the form $(x \pm \alpha)^2 = \beta;\ \alpha, \beta \neq 0$
Note:	E.g. $1.2x - 0.03x^2 = 3$ or $40x - x^2 = 100$ are acceptable for the 1st M mark
Note:	Give M0 dM0 A0 for $(\text{their } A)x^2 = 3 \Rightarrow x = ...$ or their $(\text{their } A)x^2 + (\text{their } k) = 3 \Rightarrow x = ...$
dM1:	Correct method of solving their quadratic equation to give ***at least one solution***
A1:	Interprets their solution in the original context by selecting the larger correct value ***and states correct units for their value***. E.g. Accept awrt 37.3 m or $\left(20 + \sqrt{300}\right)$ m or $\left(20 + 10\sqrt{3}\right)$ m
Note:	Condone the use of inequalities for the method marks in part (b)
(c):	
B1:	See scheme
Note:	Give no credit for the following reasons • *H* (or the height of ball) is negative when $x > 40$ • Bounce of the ball should be considered after hitting the ground • Model will not be true for a different rugby ball • Ball may not be kicked in the same way each time

Question	Scheme	Marks	AOs
41(a)	 Correct graph in quadrant 1 and quadrant 2 with V on the x-axis	B1	1.1b
	States $(0, 5)$ and $\left(\frac{5}{2}, 0\right)$ **or** $\frac{5}{2}$ marked in the correct position on the x-axis **and** 5 marked in the correct position on the y-axis	B1	1.1b
		(2)	
(b)	$\lvert 2x-5 \rvert > 7$		
	$2x-5=7 \Rightarrow x=\ldots$ **and** $-(2x-5)=7 \Rightarrow x=\ldots$	M1	1.1b
	{critical values are $x=6, -1 \Rightarrow$} $x<-1$ or $x>6$	A1	1.1b
		(2)	
(c)	$\lvert 2x-5 \rvert > x-\frac{5}{2}$		
	E.g. • Solves $2x-5=x-\frac{5}{2}$ to give $x=\frac{5}{2}$ and solves $-(2x-5)=x-\frac{5}{2}$ to also give $x=\frac{5}{2}$ • Sketches graphs of $y=\lvert 2x-5 \rvert$ and $y=x-\frac{5}{2}$. Indicates that these graphs meet at the point $\left(\frac{5}{2}, 0\right)$	M1	3.1a
	Hence using set notation, e.g. • $\left\{x: x<\frac{5}{2}\right\} \cup \left\{x: x>\frac{5}{2}\right\}$ • $\left\{x \in \mathbb{R}, x \neq \frac{5}{2}\right\}$ • $\mathbb{R} - \left\{\frac{5}{2}\right\}$	A1	2.5
		(2)	
		(6 marks)	

Question 41 Notes:	
(a)	
B1:	See scheme
B1:	See scheme
(b)	
M1:	See scheme
A1:	Correct answer, e.g. • $x < -1$ or $x > 6$ • $x < -1 \cup x > 6$ • $\{x : x < -1\} \cup \{x : x > 6\}$
(c)	
M1:	A complete process of finding that $y = \lvert 2x - 5 \rvert$ and $y = x - \frac{5}{2}$ meet at ***only*** one point. This can be achieved either algebraically or graphically.
A1:	See scheme. **Note:** Final answer must be expressed using set notation.

Question	Scheme	Marks	AOs
42	$3x - 2y = k$ intersects $y = 2x^2 - 5$ at two distinct points		
	Eliminate y and forms quadratic equation = 0 or quadratic expression $\{= 0\}$	M1	3.1a
	$\{3x - 2(2x^2 - 5) = k \Rightarrow\}\ -4x^2 + 3x + 10 - k = 0$	A1	1.1b
	$\{"b^2 - 4ac" > 0 \Rightarrow\}\ 3^2 - 4(-4)(10 - k) > 0$	dM1	2.1
	$9 + 16(10 - k) > 0 \Rightarrow 169 - 16k > 0$		
	Critical value obtained of $\frac{169}{16}$	B1	1.1b
	$k < \frac{169}{16}$ o.e.	A1	1.1b
		(5)	
42 Alt 1	Eliminate y and forms quadratic equation = 0 or quadratic expression $\{= 0\}$	M1	3.1a
	$y = 2\left(\frac{1}{3}(k + 2y)\right)^2 - 5 \Rightarrow y = \frac{2}{9}(k^2 + 4ky + 4y^2) - 5$		
	$8y^2 + (8k - 9)y + 2k^2 - 45 = 0$	A1	1.1b
	$\{"b^2 - 4ac" > 0 \Rightarrow\}\ (8k - 9)^2 - 4(8)(2k^2 - 45) > 0$	dM1	2.1
	$64k^2 - 144k + 81 - 64k^2 + 1440 > 0 \Rightarrow -144k + 1521 > 0$		
	Critical value obtained of $\frac{169}{16}$	B1	1.1b
	$k < \frac{169}{16}$ o.e.	A1	1.1b
		(5)	
42 Alt 2	$\frac{dy}{dx} = 4x,\ m_l = \frac{3}{2} \Rightarrow 4x = \frac{3}{2} \Rightarrow x = \frac{3}{8}$. So $y = 2\left(\frac{3}{8}\right)^2 - 5 = -\frac{151}{32}$	M1 A1	3.1a 1.1b
	$k = 3\left(\frac{3}{8}\right) - 2\left(-\frac{151}{32}\right) \Rightarrow k = \ldots$	dM1	2.1
	Critical value obtained of $\frac{169}{16}$	B1	1.1b
	$k < \frac{169}{16}$ o.e.	A1	1.1b
		(5)	
			(5 marks)

Question **42** Notes:	
M1:	Complete strategy of eliminating x or y and manipulating the resulting equation to form a quadratic equation = 0 or a quadratic expression $\{= 0\}$
A1:	Correct algebra leading to either • $-4x^2 + 3x + 10 - k = 0$ or $4x^2 - 3x - 10 + k = 0$ or a one-sided quadratic of either $-4x^2 + 3x + 10 - k$ or $4x^2 - 3x - 10 + k$ • $8y^2 + (8k - 9)y + 2k^2 - 45 = 0$ or a one-sided quadratic of e.g. $8y^2 + (8k - 9)y + 2k^2 - 45$
dM1:	Depends on the previous M mark. Interprets $3x - 2y = k$ intersecting $y = 2x^2 - 5$ at two distinct points by applying $"b^2 - 4ac" > 0$ to their quadratic equation or one-sided quadratic.
B1:	See scheme
A1:	Correct answer, e.g. • $k < \frac{169}{16}$ • $\left\{k : k < \frac{169}{16}\right\}$
Alt 2	
M1:	Complete strategy of using differentiation to find the values of x and y where $3x - 2y = k$ is a tangent to $y = 2x^2 - 5$
A1:	Correct algebra leading to $x = \frac{3}{8},\ y = -\frac{151}{32}$
dM1:	Depends on the previous M mark. Full method of substituting their $x = \frac{3}{8},\ y = -\frac{151}{32}$ into l and attempting to find the value for k.
B1:	See scheme
A1:	Deduces correct answer, e.g. • $k < \frac{169}{16}$ • $\left\{k : k < \frac{169}{16}\right\}$

Question	Scheme	Marks	AOs
43	Sets $\mathrm{f}(-2)=0 \Rightarrow 2\times(-2)^3-5\times(-2)^2+a\times-2+a=0$	M1	3.1a
	Solves linear equation $2a-a=-36 \Rightarrow a=$	dM1	1.1b
	$\Rightarrow a=-36$	A1	1.1b
			(3 marks)

Notes:

M1: Selects a suitable method given that $(x+2)$ is a factor of $\mathrm{f}(x)$
Accept either setting $\mathrm{f}(-2)=0$ or attempted division of $\mathrm{f}(x)$ by $(x+2)$

dM1: Solves linear equation in a. Minimum requirement is that there are two terms in 'a' which must be collected to get $..a=.. \Rightarrow a=$

A1: $a=-36$

Question	Scheme	Marks	AOs
44 (a)	$\text{gf}(x) = 3\ln e^x$	M1	1.1b
	$= 3x, \ (x \in \mathbb{R})$	A1	1.1b
		(2)	
(b)	$\text{gf}(x) = \text{fg}(x) \Rightarrow 3x = x^3$	M1	1.1b
	$\Rightarrow x^3 - 3x = 0 \Rightarrow x =$	M1	1.1b
	$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$	M1	2.2a
		(3)	
			(5 marks)

Notes:

(a)

M1: For applying the functions in the correct order

A1: The simplest form is required so it must be $3x$ and not left in the form $3\ln e^x$

An answer of $3x$ with no working would score both marks

(b)

M1: Allow the candidates to score this mark if they have $e^{3\ln x} =$ their $3x$

M1: For solving their cubic in x and obtaining at least one solution.

A1: For either stating that $x = \sqrt{3}$ **only** as $\ln x$ (or $3\ln x$) is not defined at $x = 0$ and $-\sqrt{3}$

or stating that $3x = x^3$ would have three answers, one positive one negative and one zero but $\ln x$ (or $3\ln x$) is not defined for $x \leqslant 0$ so therefore there is only one (real) answer.

Note: Student who mix up fg and gf can score full marks in part (b) as they have already been penalised in part (a)

Question	Scheme	Marks	AOs
45(i)	$x^2-6x+10=(x-3)^2+1$	M1	2.1
	Deduces "always true" as $(x-3)^2 \geqslant 0 \Rightarrow (x-3)^2+1 \geqslant 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true This could be if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a>0$ then $ax>b \Rightarrow x>\frac{b}{a}$ if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $=(n+1)^2-n^2=2n+1$	M1	3.1a
	Deduces "Always true" as $2n+1$ = (even +1) = odd	A1	2.2a
		(2)	
			(6 marks)

Notes:

(i)

M1: Attempts to complete the square or any other valid reason. Allow for a graph of $y=x^2-6x+10$ or an attempt to find the minimum by differentiation

A1: States always true with a valid reason for their method

(ii)

M1: For an explanation that it need not be true (sometimes). This could be if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$ or simply $-3x>6 \Rightarrow x<-2$

A1: Correct statement (sometimes true) and explanation

(iii)

M1: Sets up the proof algebraically.
For example by attempting $(n+1)^2-n^2=2n+1$ or $m^2-n^2=(m-n)(m+n)$ with $m=n+1$

A1: States always true with reason and proof
Accept a proof written in words. For example
If integers are consecutive, one is odd and one is even
When squared $\text{odd}\times\text{odd}=\text{odd}$ and $\text{even}\times\text{even}=\text{even}$
The difference between odd and even is always odd, hence always true
Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.

Question	Scheme	Marks	AOs
46(a)	$f(x) \geqslant 5$	B1	1.1b
		(1)	
(b)	Uses $-2(3-x)+5=\frac{1}{2}x+30$	M1	3.1a
	Attempts to solve by multiplying out bracket, collect terms etc $\frac{3}{2}x=31$	M1	1.1b
	$x=\frac{62}{3}$ only	A1	1.1b
		(3)	
(c)	Makes the connection that there must be two intersections. Implied by either end point $k>5$ or $k \leqslant 11$	M1	2.2a
	$\{k : k \in \mathbb{R}, 5 < k \leqslant 11\}$	A1	2.5
		(2)	
		(6 marks)	

Notes:

(a)

B1: $f(x) \geqslant 5$ Also allow $f(x) \in [5, \infty)$

(b)

M1: Deduces that the solution to $f(x)=\frac{1}{2}x+30$ can be found by solving

$-2(3-x)+5=\frac{1}{2}x+30$

M1: Correct method used to solve their equation. Multiplies out bracket/ collects like terms

A1: $x=\frac{62}{3}$ only. Do not allow 20.6

(c)

M1: Deduces that two distinct roots occurs when $y=k$ intersects $y=f(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k>5$ or $k \leqslant 11$

A1: Correct solution only $\{k : k \in \mathbb{R}, 5 < k \leqslant 11\}$

Question Number	Scheme		Marks
47.(i) Way 1	$\sqrt{48}=\sqrt{16}\sqrt{3}$ or $\dfrac{6}{\sqrt{3}}=6\dfrac{\sqrt{3}}{3}$	Writes one of the terms of the given expression correctly in terms of $\sqrt{3}$	M1
	$\Rightarrow \sqrt{48}-\dfrac{6}{\sqrt{3}}=2\sqrt{3}$	A correct answer of $2\sqrt{3}$. A correct answer with **no** working implies both marks.	A1
			(2)
(i) Way 2	$\sqrt{48}=2\sqrt{12}$ or $\dfrac{6}{\sqrt{3}}=\sqrt{12}$	Writes one of the terms of the given expression correctly in terms of $\sqrt{12}$	M1
	$2\sqrt{12}-\sqrt{12}=\sqrt{12}=2\sqrt{3}$	A correct answer of $2\sqrt{3}$. A correct answer with **no** working implies both marks.	A1
			(2)
(i) Way 3	$\sqrt{48}=\dfrac{12}{\sqrt{3}}$ or $\sqrt{48}=\dfrac{\sqrt{144}}{\sqrt{3}}$	Writes $\sqrt{48}$ correctly as $\dfrac{12}{\sqrt{3}}$ or $\dfrac{\sqrt{144}}{\sqrt{3}}$	M1
	$\dfrac{12}{\sqrt{3}}-\dfrac{6}{\sqrt{3}}=\dfrac{6}{\sqrt{3}}=2\sqrt{3}$	A correct answer of $2\sqrt{3}$. A correct answer with **no** working implies both marks.	A1
			(2)
(i) Way 4	$\sqrt{48}-\dfrac{6}{\sqrt{3}}=\dfrac{\sqrt{3}\sqrt{48}-\dots}{\sqrt{3}}=\dfrac{12-\dots}{\sqrt{3}}$ or $\sqrt{48}-\dfrac{6}{\sqrt{3}}=\dfrac{\sqrt{3}\sqrt{48}-\dots}{\sqrt{3}}=\dfrac{\sqrt{144}-\dots}{\sqrt{3}}$	Writes $\sqrt{48}$ correctly as $\dfrac{12}{\sqrt{3}}$ or $\dfrac{\sqrt{144}}{\sqrt{3}}$	M1
	$\dfrac{12-6}{\sqrt{3}}=\dfrac{6}{\sqrt{3}}=2\sqrt{3}$	A correct answer of $2\sqrt{3}$. A correct answer with **no** working implies both marks.	A1
			(2)

(ii) **Way 1**	$81=3^4$ or $\log_3 81=6x-3$	For $81=3^4$ or $\log_3 81=6x-3$. This may be implied by subsequent work.	B1
	$3^{6x-3}=3^4$ or $\log_3 81=6x-3$ $\Rightarrow 4=6x-3\Rightarrow x=...$	Solves an equation of the form $6x-3=k$ where k is their power of 3.	M1
	$\Rightarrow x=\frac{4+3}{6}=\frac{7}{6}$	$\frac{7}{6}$ or $1\frac{1}{6}$ or $1.1\dot{6}$ with a dot over the 6	A1
			(3)
Way 2	$3=81^{\frac{1}{4}}$	For $3=81^{\frac{1}{4}}$. This may be implied by subsequent work.	B1
	$81^{\frac{6x-3}{4}}=81\Rightarrow\frac{6x-3}{4}=1\Rightarrow x=...$	Solves an equation of the form $k(6x-3)=1$ where k is their power of 81.	M1
	$\Rightarrow x=\frac{4+3}{6}=\frac{7}{6}$	$\frac{7}{6}$ or $1\frac{1}{6}$ or $1.1\dot{6}$ with a dot over the 6	A1
			(3)
Way 3	$81=9^2$ **and** $3=9^{\frac{1}{2}}$	For $81=9^2$ **and** $3=9^{\frac{1}{2}}$. This may be implied by subsequent work.	B1
	$9^{\frac{6x-3}{2}}=9^2\Rightarrow\frac{6x-3}{2}=2\Rightarrow x=...$	Solves an equation of the form $p(6x-3)=q$ where p is their power of 9 for the 3 and q is their power of 9 for the 81.	M1
	$\Rightarrow x=\frac{4+3}{6}=\frac{7}{6}$	$\frac{7}{6}$ or $1\frac{1}{6}$ or $1.1\dot{6}$ with a dot over the 6	A1
			(3)
Way 4	$3^{6x-3}=3^{6x}\times3^{-3}$	For writing 3^{6x-3} correctly in terms of 3^{6x}	B1
	$3^{6x}=81\times3^3=3^7$ $\Rightarrow 6x=7\Rightarrow x=...$	Solves an equation of the form $6x=k$ where k is their $3^3\times81$ written as a power of 3.	M1
	$\Rightarrow x=\frac{7}{6}$	$\frac{7}{6}$ or $1\frac{1}{6}$ or $1.1\dot{6}$ with a dot over the 6	A1
			(3)
Way 5	$\log 3^{6x-3}=\log 81$	Takes logs of both sides	B1
	$6x-3=\frac{\log 81}{\log 3}$ $6x-3=4\Rightarrow x=...$	Solves an equation of the form $6x-3=k$ where k is their $\frac{\log 81}{\log 3}$	M1
	$\Rightarrow x=\frac{7}{6}$	$\frac{7}{6}$ or $1\frac{1}{6}$ or $1.1\dot{6}$ with a dot over the 6	A1
			(3)
			(5 marks)

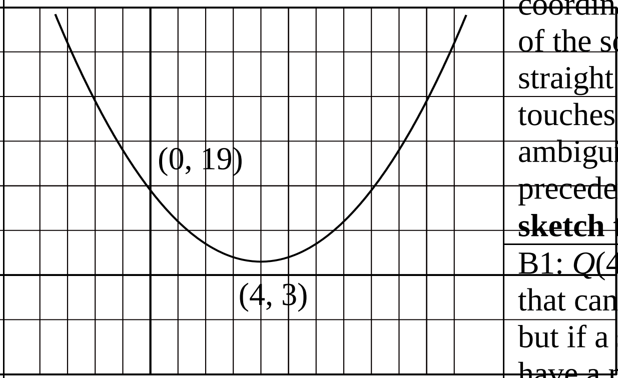
Note:

The question does not specify the form of the final answer in (b) and so if answers are left un-simplified as e.g. $\frac{\log_3 81+3}{6}$, $\frac{\log_3 2187}{6}$ **then allow full marks if correct.**

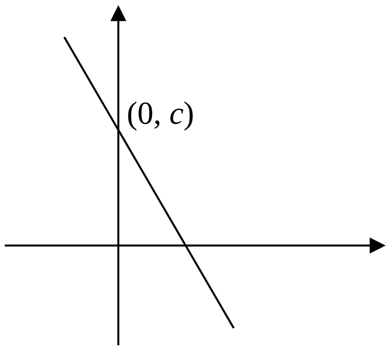
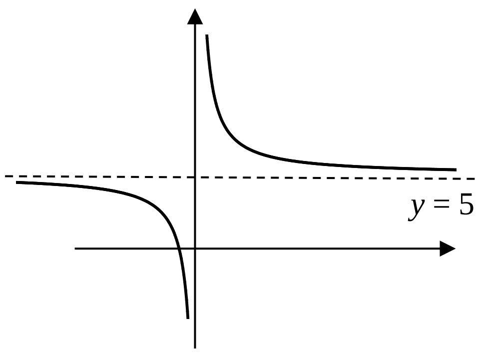
Question Number	Scheme		Marks
48.(a)	$x^2-10x+23=(x\pm5)^2\pm A$	For an attempt to complete the square. Note that if their $A = 23$ then this is M0 by the General Principles.	M1
	$(x-5)^2-2$	Correct expression. Ignore "= 0".	A1
			(2)
(b)	$(x\pm5)^2-A\Rightarrow x=\dots$ or $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}\Rightarrow x=\dots$ $\left(x=\dfrac{10\pm\sqrt{10^2-4(1)(23)}}{2}\right)$	Uses their completion of the square for **positive** A or uses the correct quadratic formula to obtain at least one value for x	M1
	$x=5\pm\sqrt{2}$	Correct exact values. If using the quadratic formula must reach as far as $\dfrac{10\pm\sqrt{8}}{2}$	A1
			(2)
(c)	$\left(5\pm\sqrt{2}\right)^2=27+10\sqrt{2}$	Attempts to square **any** solution from part (b). Allow poor squaring e.g. $\left(5+\sqrt{2}\right)^2=25+2=27$. **Do not allow** for substituting e.g. $5+\sqrt{2}$ into $x^2-10x+23$.	M1
	$=27+10\sqrt{2}$	Accept equivalent forms such as $27+\sqrt{200}$. If any extra answers are given, this mark should be withheld.	A1
			(2)
	Allow candidates to start again:		
	$y-10y^{0.5}+23=0\Rightarrow y^{0.5}=\dfrac{10\pm\sqrt{10^2-4\times23}}{2}=5\pm\sqrt{2}$ $y=\left(5\pm\sqrt{2}\right)^2=\dots$		M1
	$=27+10\sqrt{2}$	Accept equivalent forms such as $27+\sqrt{200}$. If any extra answers are given, this mark should be withheld.	A1
			(6 marks)

Question Number	Scheme		Marks
49(a)	(4, 7)	Accept (4, 7) or $x = 4$, $y = 7$ or a sketch of $y = \mathrm{f}(x-2)$ with a maximum point marked at (4, 7). (Condone missing brackets) There should be no other coordinates.	B1
			(1)
(b)	$(x =)$ 2.5	Allow (2.5, 0) (condone missing brackets) but no other values or points. Allow a sketch of f(2x) with the only x-intercept marked at $x = 2.5$ (Allow (0, 2.5) marked in the correct place.	B1
			(1)
(c)	$y = 1$ (oe e.g. $y - 1 = 0$)	Must be an equation and not just '1' and no other asymptotes stated.	B1
			(1)
(d)	$k \le 1$ **or** $k = 7$	Either of $k \le 1$ or $k = 7$ Accept either of $y \le 1$ or $y = 7$ Note that $k = 7$ may sometimes be seen embedded in e.g. $k = 0, 1, 7$ and can score B1 here.	B1
	$k \le 1 \quad k = 7$	Both correct and in terms of k with no other solutions.	B1
			(2)
			(5 marks)

Question Number	Scheme		Marks
50(a)	$b^2-4ac=(4k)^2-4(-2)(20+13k)$	Attempts to use b^2-4ac with $a=\pm(20\pm13k)$, $b=\pm4k$, $c=\pm2$. This could be as part of the quadratic formula or as $b^2<4ac$ or as $b^2>4ac$ or as $\sqrt{b^2-4ac}$ etc. If it is part of the quadratic formula only look for use of b^2-4ac. There must be no x's. If they gather to the lhs, condone the omission of the "– " on the "$4k$".	M1
	$(4k)^2-4(-2)(20+13k)$	For a correct un-simplified expression.	A1
	$b^2-4ac<0$ $\Rightarrow (4k)^2-4(-2)(20+13k)<0$	Uses $b^2-4ac<0$ or e.g. $b^2<4ac$ with their values of a, b and c in terms of k. The " < 0" must appear before the final printed answer but can appear as $b^2-4ac<0$ at the start.	M1
	$16k^2+160+104k<0$ $\Rightarrow 2k^2+13k+20<0$*	Reaches the printed answer with no errors, including bracketing errors, or contradictory statements and sufficient working shown. Note that the statement $(20+13k)x^2-4kx-2<0$ or starting with e.g. $20x^2<4kx-13kx^2+2$ would be an error.	A1*
			(4)
(b)	$2k^2+13k+20=0\Rightarrow k=...$ e.g. $(2k+5)(k+4)=0\Rightarrow k=...$	Attempt to solve the **<u>given</u>** quadratic to find 2 values for k. See general guidance.	M1
	$\Rightarrow k=-\frac{5}{2},-4$	Both correct. May be implied by e.g. $k<-\frac{5}{2}$, $k<-4$ or seen on a sketch. If they use the quadratic formula allow $\frac{-13\pm3}{4}$ for this mark but not $\sqrt{9}$ for 3 and allow e.g. $-\frac{13}{4}\pm\frac{3}{4}$ if they complete the square.	A1
	$-4<k<-\frac{5}{2}$ Allow equivalent values e.g. $-\frac{10}{4}$ i.e. the critical values must be in the form $\frac{a}{b}$ where a and b are integers	M1: Chooses 'inside' region for **their critical values** i.e. Lower Limit $<k<$ Upper Limit or e.g. Lower Limit $\le k\le$ Upper Limit A1: Allow $k\in(-4,-\frac{5}{2})$ or just $(-4,-\frac{5}{2})$ and allow $k>-4$ **and** $k<-2.5$ and $-\frac{5}{2}>k>-4$ but $k>-4$, $k<-\frac{5}{2}$ scores M1A0. $-\frac{5}{2}<k<-4$ is M0A0	M1A1
	Allow working in terms of x in (b) but the answer must be in terms of k for the final mark.		
			(4)
			(8 marks)

Question Number	Scheme		Marks
51.(a)	$\mathrm{f}(x) = (x-4)^2 + 3$	M1: $\mathrm{f}(x) = (x \pm 4)^2 \pm \alpha,\ \alpha \neq 0$ (where α is a single number or a numerical expression $\neq 0$)	M1A1
		A1: Allow $(x + {}^-4)^2 + 3$ and ignore any spurious "= 0"	
	Allow *a* = -4, *b* = 3 to score both marks		
			(2)
(b)		B1: U shape anywhere even with no axes. Do not allow a "V" shape i.e. with an obvious vertex.	B1
		B1: *P*(0, 19). Allow (0, 19) or just 19 marked in the correct place as long as the curve (or straight line) passes through or touches here and allow (19, 0) as long as it is marked in the correct place. Correct coordinates may be seen in the body of the script as long as the curve (or straight line) passes through or touches here. If there is any ambiguity, the sketch has precedence. **(There must be a sketch to score this mark)**	B1
		B1: *Q*(4, 3). Correct coordinates that can be scored without a sketch but if a sketch is drawn then it must have a minimum in the first quadrant and no other turning points. May be seen in the body of the script. If there is any ambiguity, the sketch has precedence. Allow this mark if 4 is clearly marked on the *x*-axis below the minimum and 3 is marked clearly on the *y*-axis and corresponds to the minimum,	B1
			(3)

(c)	$PQ^2 = (0-4)^2 + (19-3)^2$	**Correct** use of Pythagoras' Theorem on 2 points of the form (0, p) and (q, r) where $q \neq 0$ and $p \neq r$ with p, q and r numeric.	M1
	$PQ = \sqrt{4^2 + 16^2}$	Correct un-simplified numerical expression for PQ including the square root. **<u>This must come from a correct *P* and *Q*</u>**. Allow e.g $PQ = \sqrt{(0-4)^2 + (19-3)^2}$. Allow $\pm\sqrt{(0-4)^2 + (19-3)^2}$	A1
	$PQ = 4\sqrt{17}$	Cao and cso i.e. **<u>This must come from a correct *P* and *Q*</u>**.	A1
	Note that it is possible to obtain the correct value for PQ from (-4,3) and (0, 19) and e.g. (0, 13) and (4, -3) but the A marks in (c) can only be awarded for the correct P and Q.		
			(3)
			(8 marks)

Question Number	Scheme		Marks
52(a)(i)		B1: Straight line with negative gradient anywhere even with no axes.	B1
		B1: Straight line with an intercept at (0, c) or just c marked on the positive y-axis provided the line passes through the positive y-axis. Allow (c, 0) as long as it is marked in the correct place. Allow (0, c) in the body of the script but in any ambiguity, the sketch has precedence. **Ignore any intercepts with the *x*-axis.**	B1
(a)(ii)		**<u>Either:</u>** For the shape of a $y=\frac{1}{x}$ curve in any position. It must have two branches and be asymptotic horizontally and vertically with no obvious "overlap" with the asymptotes, but otherwise be generous. The curve may bend away from the asymptote a little at the end. Sufficient curve must be seen to suggest the asymptotic behaviour, both vertically and horizontally and the branches must approach the same asymptote **<u>Or</u>** the **equation** $y = 5$ seen independently i.e. whether the sketch has an asymptote here or not. Do not allow $y \neq 5$ or $x = 5$.	B1
		B1: Fully correct graph **and** with a horizontal asymptote on the positive y-axis. The asymptote does not have to be drawn but the **equation** $y = 5$ must be seen. The shape needs to be reasonably accurate with the "ends" not bending away significantly from the asymptotes and the branches must approach the same asymptote. Ignore $x = 0$ given as an asymptote.	B1
	Allow sketches to be on the same axes.		
			(4)

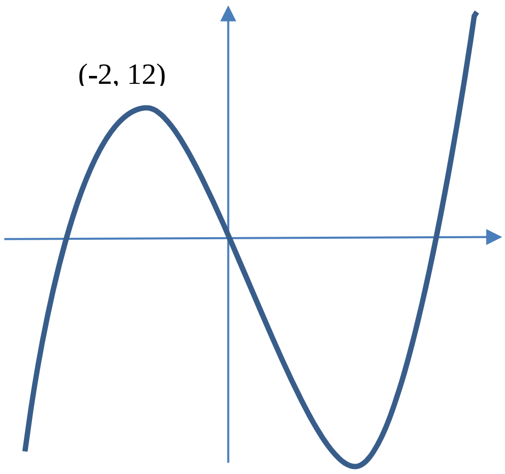
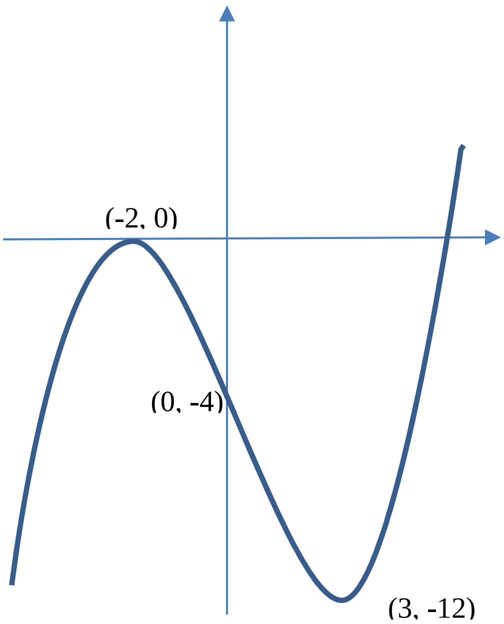
EXPERT TUITION

(b)	$\frac{1}{x}+5=-3x+c \Rightarrow 1+5x=-3x^2+cx$ $\Rightarrow 3x^2+5x-cx+1=0$	Sets $\frac{1}{x}+5=-3x+c$, attempts to multiply by *x* and collects terms (to one side). Allow e.g. ">" or "<" for "=" . At least 3 of the terms must be multiplied by *x*, e.g. allow one slip. The ' = 0' may be implied by subsequent work and provided correct work follows, full marks are still possible in (b).	M1
	$b^2-4ac=(5-c)^2-4\times1\times3$	Attempts to use $b^2 - 4ac$ with **their** *a*, *b* and *c* from **their equation** where $a=\pm3,\ b=\pm5\pm c$ and $c=\pm1$. This could be as part of the quadratic formula or as $b^2 < 4ac$ or as $b^2 > 4ac$ or as $\sqrt{b^2-4ac}$ etc. If it is part of the quadratic formula only look for use of $b^2 - 4ac$. There must be no *x*'s.	M1
	$(5-c)^2>12$*	Completes proof with **no errors or incorrect statements** and with the ">" appearing correctly before the final answer, which could be from $b^2 - 4ac > 0$. Note that the statement $3x^2+5x-cx+1>0$ or starting with e.g. $\frac{1}{x}+5>-3x+c$ would be an error.	A1*
	Note: A minimum for (b) could be, $\frac{1}{x}+5=-3x+c \Rightarrow 3x^2+5x-cx+1(=0)$ (M1) $b^2>4ac \Rightarrow (5-c)^2>12$ (M1A1) If $b^2>4ac$ is not seen then $4\times3\times1$ needs to be seen explicitly.		
			(3)

EXPERT TUITION

(c)	$(5-c)^2=12 \Rightarrow (c=)5\pm\sqrt{12}$ or $(5-c)^2=12 \Rightarrow c^2-10c+13=0$ $\Rightarrow (c=)\dfrac{-^-10\pm\sqrt{(-10)^2-4\times13}}{2}$	M1: Attempts to find at least one critical value using the result in (b) or by expanding and solving a 3TQ (See General Principles) (the "= 0" may be implied) A1: Correct critical values in any form. Note that $\sqrt{12}$ may be seen as $2\sqrt{3}$.	M1A1
	$c<"5-\sqrt{12}",\ c>"5+\sqrt{12}"$	Chooses outside region. The '0 <' can be ignored for **this** mark. So look for c < their $5-\sqrt{12}$, c > their $5+\sqrt{12}$. This could be scored from $5+\sqrt{12}<c<5-\sqrt{12}$ or $5-\sqrt{12}>c>5+\sqrt{12}$. Evidence is to be taken from their answers not from a diagram.	M1
	$0<c<5-\sqrt{12},\ c>5+\sqrt{12}$	Correct ranges including the '0 <' e.g. answer as shown or each region written separately or e.g. $\left(0,5-\sqrt{12}\right)$, $\left(5+\sqrt{12},\infty\right)$. The critical values may be un-simplified but must be at least $\dfrac{10+\sqrt{48}}{2}$, $\dfrac{10-\sqrt{48}}{2}$. Note that $0<c<5-\sqrt{12}$ **and** $c>5+\sqrt{12}$ would score M1A0.	A1
	Allow the use of *x* rather than *c* in (c) but the final answer must be in terms of *c*.		
			(4)
			(11 marks)

Question Number	Scheme	Notes	Marks
53.(a)	$\sqrt{50}-\sqrt{18}=5\sqrt{2}-3\sqrt{2}$	$\sqrt{50}=5\sqrt{2}$ or $\sqrt{18}=3\sqrt{2}$ **and** the other term in the form $k\sqrt{2}$. This mark may be implied by the correct answer $2\sqrt{2}$	M1
	$=2\sqrt{2}$	Or $a=2$	A1
			[2]
(b) **WAY 1**	$\dfrac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}=\dfrac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where a is numeric. This is all that is required for this mark.	M1
	$=\dfrac{12\sqrt{3}}{"2"\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{12\sqrt{6}}{4}$	Rationalises the denominator by a correct method e.g. multiplies numerator and denominator by $k\sqrt{2}$ to obtain a multiple of $\sqrt{6}$. Note that multiplying numerator and denominator by $2\sqrt{2}$ or $-2\sqrt{2}$ is quite common and is acceptable for this mark. May be implied by a correct answer. **This is dependent on the first M1.**	dM1
	$=3\sqrt{6}$ or $b=3$, $c=6$	Cao and cso	A1
			[3]
(b) WAY 2	$\dfrac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}\times\dfrac{\sqrt{50}+\sqrt{18}}{\sqrt{50}+\sqrt{18}}$ **or** $\dfrac{12\sqrt{3}}{5\sqrt{2}-3\sqrt{2}}\times\dfrac{5\sqrt{2}+3\sqrt{2}}{5\sqrt{2}+3\sqrt{2}}$	For rationalising the denominator by a correct method i.e. multiplying numerator and denominator by $k\left(\sqrt{50}+\sqrt{18}\right)$	M1
	$\dfrac{60\sqrt{6}+36\sqrt{6}}{50-18}$	For replacing numerator by $\alpha\sqrt{6}+\beta\sqrt{6}$. **This is dependent on the first M1 and there is no need to consider the denominator for this mark.**	dM1
	$=3\sqrt{6}$ or $b=3$, $c=6$	Cao and cso	A1
			[3]
(b) WAY 3	$\dfrac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}=\dfrac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where a is numeric. This is all that is required for this mark.	M1
	$=\dfrac{12\sqrt{3}}{2\sqrt{2}}=\dfrac{6\sqrt{3}}{\sqrt{2}}=\dfrac{\sqrt{108}}{\sqrt{2}}=\sqrt{54}=\sqrt{9}\sqrt{6}$	Cancels to obtain a multiple of $\sqrt{6}$. **This is dependent on the first M1.**	dM1
	$=3\sqrt{6}$ Or $b=3$, $c=6$	Cao and cso	A1
			[3]
(b) WAY 4	$\dfrac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}=\dfrac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where a is numeric. This is all that is required for this mark.	M1
	$\left(\dfrac{12\sqrt{3}}{"2"\sqrt{2}}\right)^2=\dfrac{432}{8}$		
	$\sqrt{54}=\sqrt{9}\sqrt{6}$	Obtains a multiple of $\sqrt{6}$. **This is dependent on the first M1.**	dM1
	$=3\sqrt{6}$ Or $b=3$, $c=6$	Cao and cso (do not allow $\pm 3\sqrt{6}$)	A1
			5 marks

Question Number	Scheme	Notes	Marks
	Note original points are A(-2, 4) and B(3, -8)		
54.(a)		Similar shape to given figure passing through the origin. A cubic shape with a maximum in the second quadrant and a minimum in the 4th quadrant. There must be evidence of a change in at least one of the y-coordinates (inconsistent changes in the y-coordinates are acceptable) but **not the x-coordinates**.	B1
		Maximum at (-2, 12) and minimum at (3, -24) with coordinates written the right way round. Condone missing brackets. The coordinates may appear on the sketch, or separately in the text (not necessarily referenced as A and B). If they are on the sketch, the x and y coordinates can be positioned correctly on the axes rather than given as coordinate pairs. In cases of ambiguity, the sketch has precedence. The origin does not need to be labelled. Nor do the x and y axes.	B1
			[2]
(b)		A positive cubic which does not pass through the origin with a maximum to the left of the y-axis and a minimum to the right of the y-axis.	M1
		Maximum at (-2, 0) and minimum at (3, -12). Condone missing brackets. For the max allow just -2 or (0, -2) if marked in the correct place. If the coordinates are in the text, they must appear as (-2, 0) and must not contradict the sketch. The curve must **touch** the x-axis at (-2, 0). For the min allow coordinates as shown or 3 and -12 to be marked in the correct places on the axes. In cases of ambiguity, the sketch has precedence.	A1
		Crosses y-axis at (0, -4). Allow just -4 (not +4) and allow (-4, 0) if marked in the correct place. If the coordinates are in the text, they must appear as (0, -4) and must not contradict the sketch. In cases of ambiguity, the sketch has precedence.	A1
			[3]
			5 marks

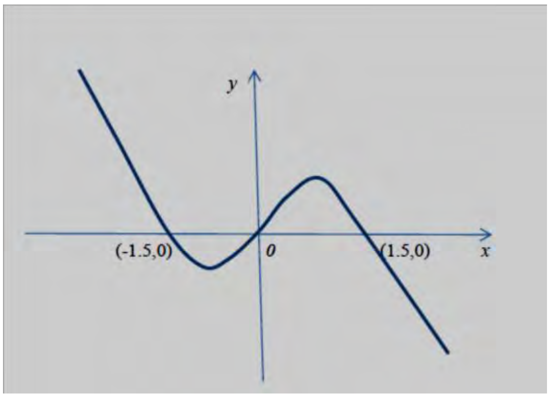
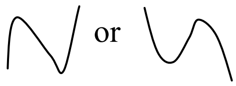
Question Number	Scheme	Notes	Marks
	WAY 1		
55.	$y = -4x - 1$ $\Rightarrow (-4x-1)^2 + 5x^2 + 2x = 0$	Attempts to makes y the subject of the linear equation and substitutes into the other equation. Allow slips e.g. substituting $y = -4x + 1$ etc.	M1
	$21x^2 + 10x + 1 = 0$	Correct 3 term quadratic (terms do not need to be all on the same side). The "= 0" may be implied by subsequent work.	A1
	$(7x+1)(3x+1)=0 \Rightarrow (x=) -\frac{1}{7}, -\frac{1}{3}$	dM1: Solves a **3 term** quadratic by the usual rules (see general guidance) to give at least one value for x. **Dependent on the first method mark.** A1: $(x=) -\frac{1}{7}, -\frac{1}{3}$ (two separate correct exact answers). Allow exact equivalents e.g. $(x=) -\frac{6}{42}, -\frac{14}{42}$	dM1 A1
	$y = -\frac{3}{7}, \frac{1}{3}$	M1: Substitutes to find at least one y value (Allow substitution into their rearranged equation above but not into an equation that has not been seen earlier). You may need to check here if there is no working and x values are incorrect. A1: $y = -\frac{3}{7}, \frac{1}{3}$ (two correct exact answers) Allow exact equivalents e.g. $y = -\frac{18}{42}, \frac{14}{42}$	M1 A1
	Coordinates do not need to be paired		
	Note that if the linear equation is explicitly rearranged to $y = 4x + 1$, this gives the correct answers for x and possibly for y. In these cases, if it is not already lost, deduct the final A1.		
			[6]
	WAY 2		
	$x = -\frac{1}{4}y - \frac{1}{4}$ $\Rightarrow y^2 + 5(-\frac{1}{4}y - \frac{1}{4})^2 + 2(-\frac{1}{4}y - \frac{1}{4}) = 0$	Attempts to makes x the subject of the linear equation and substitutes into the other equation. Allow slips in the rearrangement as above.	M1
	$\frac{21}{16}y^2 + \frac{1}{8}y - \frac{3}{16} = 0 \quad (21y^2 + 2y - 3 = 0)$	Correct 3 term quadratic (terms do not need to be all on the same side). The "= 0" may be implied by subsequent work.	A1
	$(7y+3)(3y-1)=0 \Rightarrow (y=) -\frac{3}{7}, \frac{1}{3}$	dM1: Solves a **3 term** quadratic by the usual rules (see general guidance) to give at least one value for y. **Dependent on the first method mark.** A1: $(y=) -\frac{3}{7}, \frac{1}{3}$ (two separate correct exact answers). Allow exact equivalents e.g. $(y=) -\frac{18}{42}, \frac{14}{42}$	dM1 A1
	$x = -\frac{1}{7}, -\frac{1}{3}$	M1: Substitutes to find at least one x value (Allow substitution into their rearranged equation above but not into an equation that has not been seen earlier). You may need to check here if there is no working and y values are incorrect. A1: $x = -\frac{1}{7}, -\frac{1}{3}$ (two correct exact answers) Allow exact equivalents e.g. $x = -\frac{6}{42}, -\frac{14}{42}$	M1 A1
	Coordinates do not need to be paired		
	Note that if the linear equation is explicitly rearranged to $x = (y + 1)/4$, this gives the correct answers for y and possibly for x. In these cases, if it is not already lost, deduct the final A1.		
			[6]
			6 marks

Question Number	Scheme	Notes	Marks
56(a)	$2px^2-6px+4p"="3x-7$ or $y=2p\left(\frac{y+7}{3}\right)^2-6p\left(\frac{y+7}{3}\right)+4p$	**Either:** Compares the given quadratic expression with the given linear expression using <, >, = , ≠ (May be implied) **or** Rearranges $y = 3x - 7$ to make x the subject and substitutes into the given quadratic	M1
	Examples $2px^2-6px+4p-3x+7(=0)$, $\quad -2px^2+6px-4p+3x-7(=0)$ $2p\left(\frac{y+7}{3}\right)^2-6p\left(\frac{y+7}{3}\right)+4p-y(=0)$, $\quad 2py^2+(10p-9)y+8p(=0)$ $y=2px^2-6px+4p-3x+7$		dM1
	Moves all the terms to one side allowing sign errors only. Ignore > 0, < 0, = 0 etc. **The terms do not need to be collected. Dependent on the first method mark.**		
	E.g. $b^2-4ac=(-6p-3)^2-4(2p)(4p+7)$ $b^2-4ac=(10p-9)^2-4(2p)(8p)$	Attempts to use b^2-4ac with their a, b and c where $a=\pm 2p$, $b=\pm(-6p\pm 3)$ and $c=\pm(4p\pm 7)$ or for the quadratic in y, $a=\pm 2p$, $b=\pm(10p\pm 9)$ and $c=\pm 8p$. This could be as part of the quadratic formula or as $b^2<4ac$ or as $b^2>4ac$ or as $\sqrt{b^2-4ac}$ etc. If it is part of the quadratic formula only look for use of b^2-4ac. There must be no x's or y's. **Dependent on both method marks.**	ddM1
	$4p^2-20p+9<0$ *	Obtains printed answer with **no errors** seen (Allow $0>4p^2-20p+9$) **but this** <0 must been seen at some stage before the last line.	A1*
			[4]
(b)	$(2p-9)(2p-1)=0\Rightarrow p=...$ to obtain $p=$	Attempt to solve the **given** quadratic to find 2 values for p. See general guidance.	M1
	$p=\frac{9}{2}, \quad \frac{1}{2}$	Both correct. May be implied by e.g. $p<\frac{9}{2}$, $p<\frac{1}{2}$. Allow equivalent values e.g. 4.5, $\frac{36}{8}$, 0.5 etc. If they use the quadratic formula allow $\frac{20\pm 16}{8}$ for this mark but not $\sqrt{256}$ for 16 and allow e.g. $\frac{5}{2}\pm 2$ if they complete the square.	A1
	$\frac{1}{2}<p<4\frac{1}{2}$ Allow equivalent values e.g. $\frac{36}{8}$ for $4\frac{1}{2}$	M1: Chooses 'inside' region i.e. Lower Limit < p < Upper Limit or e.g. Lower Limit ≤ p ≤ Upper Limit A1: Allow $p\in(\frac{1}{2},4\frac{1}{2})$ or just $(\frac{1}{2},4\frac{1}{2})$ and allow $p>\frac{1}{2}$ **and** $p<4\frac{1}{2}$ and $4\frac{1}{2}>p>\frac{1}{2}$ but $p>\frac{1}{2}$, $p<4\frac{1}{2}$ scores M1A0 $\frac{1}{2}>p>4\frac{1}{2}$ scores M0A0	M1A1
	Allow working in terms of x in (b) but the answer must be in terms of p for the final A mark.		**[4]**
			8 marks

Question Number	Scheme		Marks
57(a)	20	Sight of 20. (4×5 is not sufficient)	B1
			(1)
(b)	$\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$	Multiplies top and bottom by a **correct** expression. This statement is sufficient. NB $2\sqrt{5}+3\sqrt{2} \equiv \sqrt{20}+\sqrt{18}$	M1
	(Allow to multiply top and bottom by $k(2\sqrt{5}+3\sqrt{2})$)		
	$= \frac{...}{2}$	Obtains a denominator of 2 or sight of $\left(2\sqrt{5}-3\sqrt{2}\right)\left(2\sqrt{5}+3\sqrt{2}\right)=2$ with no errors seen in this expansion. May be implied by $\frac{...}{2k}$	A1
	Note that M0A1 is not possible. The 2 must come from a correct method.		
	Note that if M1 is scored there is no need to consider the numerator. **e.g.** $\frac{2(MR?)}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} = \frac{...}{2}$ scores M1A1		
	Numerator = $\sqrt{2}(2\sqrt{5} \pm 3\sqrt{2}) = 2\sqrt{10} \pm 6$	An attempt to multiply the numerator by $\pm\left(2\sqrt{5} \pm 3\sqrt{2}\right)$ and obtain an expression of the form $p+q\sqrt{10}$ where p and q are integers. This may be implied by e.g. $2\sqrt{10}+3\sqrt{4}$ or by their final answer.	M1
	(Allow attempt to multiply the numerator by $k(2\sqrt{5} \pm 3\sqrt{2})$)		
	$\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} = \frac{2\sqrt{10}+6}{2} = 3+\sqrt{10}$	Cso. For the answer as written or $\sqrt{10}+3$ or a statement that a = 3 and b = 10. Score when first seen and ignore any subsequent attempt to 'simplify'. Allow $1\sqrt{10}$ for $\sqrt{10}$	A1
			(4)
			(5 marks)
	Alternative for (b)		
	$\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} = \frac{1}{\sqrt{10}-3}$ or $\frac{2}{2\sqrt{10}-6}$	M1: Divides or multiplies top and bottom by $\sqrt{2}$ A1: $\frac{k}{k\left(\sqrt{10}-3\right)}$	M1A1
	$= \frac{1}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3}$	M1: Multiplies top and bottom by $\sqrt{10}+3$	M1
	$= 3+\sqrt{10}$		A1

Question Number	Scheme		Marks
58.	$y-2x-4=0,\ \ 4x^2+y^2+20x=0$		
	$y=2x+4 \Rightarrow 4x^2+(2x+4)^2+20x=0$ **or** $2x=y-4$ or $x=\dfrac{y-4}{2}$ $\Rightarrow (y-4)^2+y^2+10(y-4)=0$	Attempts to rearrange the linear equation to $y=$... or $x=$... or $2x=$... and attempts to **fully** substitute into the second equation.	M1
	$8x^2+36x+16=0$ or $2y^2+2y-24=0$	M1: Collects terms together to produce quadratic expression = 0. The '= 0' may be implied by later work. A1: Correct three term quadratic equation in x or y. The '= 0' may be implied by later work.	M1 A1
	$(4)(2x+1)(x+4)=0 \Rightarrow x=...$ or $(2)(y+4)(y-3)=0 \Rightarrow y=...$	Attempt to factorise and solve or complete the square and solve or uses a correct quadratic formula **for a 3 term quadratic.**	M1
	$x=-0.5, x=-4$ or $y=-4, y=3$	Correct answers for either both values of x or both values of y (possibly un-simplified)	A1 cso
	Sub into $y=2x+4$ or Sub into $x=\dfrac{y-4}{2}$	Substitutes at least one of their values of x into a **correct** equation as far as $y=$... or substitutes at least one of their values of y into a **correct** equation as far as $y=$...	M1
	$y=3,\ y=-4$ and $x=-4,\ x=-0.5$	Fully correct solutions and simplified. **Pairing not required.** If there are any extra values of x or y, score A0.	A1
			(7 marks)
	Special Case: Uses $y = -2x - 4$		
	$y=2x+4 \Rightarrow 4x^2+(-2x-4)^2+20x=0$		M1
	$8x^2+36x+16=0$		M1A1
	$(4)(2x+1)(x+4)=0 \Rightarrow x=...$		M1
	$x=-0.5, x=-4$		A0
	Sub into $y=2x+4$	Sub into $y=-2x-4$ is M0	M1
	$y=3,\ y=-4$ and $x=-4,\ x=-0.5$		A0

Question Number	Scheme		Marks
59(a)	$b^2-4ac<0 \Rightarrow$ e.g. $4^2-4(p-1)(p-5)<0$ or $0>4^2-4(p-1)(p-5)$ or $4^2<4(p-1)(p-5)$ or $4(p-1)(p-5)>4^2$	M1: Attempts to use $b^2 - 4ac$ with at least two of a, b or c correct. May be in the quadratic formula. Could also be, for example, comparing or equating b^2 and $4ac$. Must be considering the given quadratic equation. Inequality sign not needed for this M1.There must be no x terms. A1: For a correct un-simplified **inequality** that is not the given answer	M1A1
	$4<p^2-6p+5$		
	$p^2-6p+1>0$	Correct solution with **no** errors that includes an expansion of $(p-1)(p-5)$	A1*
			(3)
(b)	$p^2-6p+1=0 \Rightarrow p=...$	For an attempt to solve $p^2-6p+1=0$ (**not <u>their</u> quadratic**) leading to 2 solutions for p (do not allow attempts to **factorise** – must be using the quadratic formula or completing the square)	M1
	$p=3\pm\sqrt{8}$	$p=3\pm2\sqrt{2}$ or any equivalent correct expressions e.g. $p=\dfrac{6\pm\sqrt{32}}{2}$ (May be implied by their inequalities) Discriminant must be a single number not e.g. 36 - 4	A1
	Allow the M1A1 to score anywhere for solving the given quadratic		
	$p<3-\sqrt{8}$ or $p>3+\sqrt{8}$	M1: Chooses outside region – **not dependent on the previous method mark** A1: $p<3-\sqrt{8},\ p>3+\sqrt{8}$ or equivalent e.g. $p<\dfrac{6-\sqrt{32}}{2},\ p>\dfrac{6+\sqrt{32}}{2}$ $\left(-\infty,\ 3-\sqrt{8}\right)\cup\left(3+\sqrt{8},\ \infty\right)$ Allow “,”, “or” or a space between the answers but do **not** allow $p<3-\sqrt{8}$ **and** $p>3+\sqrt{8}$ (this scores M1A0) **Apply ISW if necessary.**	M1A1
A correct solution to the quadratic followed by $p>3\pm\sqrt{8}$ scores M1A1M0A0			
$3+\sqrt{8}<p<3-\sqrt{8}$ scores M1A0			
Allow candidates to use *x* rather than *p* but must be in terms of *p* for the final A1			
			(4)
			(7 marks)

Question Number	Scheme		Marks
60(a)	$9x-4x^3=x(9-4x^2)$ or $-x(4x^2-9)$	Takes out a common factor of x or $-x$ **correctly.**	B1
	$9-4x^2=(3+2x)(3-2x)$ or $4x^2-9=(2x-3)(2x+3)$	$9-4x^2=(\pm3\pm2x)(\pm3\pm2x)$ or $4x^2-9=(\pm2x\pm3)(\pm2x\pm3)$	M1
	$9x-4x^3=x(3+2x)(3-2x)$	Cao but allow equivalents e.g. $x(-3-2x)(-3+2x)$ or $-x(2x+3)(2x-3)$	A1
Note: $4x^3-9x=x(4x^2-9)=x\left(2x-3\right)\left(2x+3\right)$ so $9x-4x^3=x\left(3-2x\right)\left(2x+3\right)$ **would score full marks**			
	Note: Correct work leading to $9x\left(1-\frac{2}{3}x\right)\left(1+\frac{2}{3}x\right)$ **would score full marks**		
	Allow (x ± 0) or (-x ± 0) instead of x and -x		
			(3)
(b)		 A cubic shape with one maximum and one minimum	M1
		Any line or curve drawn passing through (not touching) the origin	B1
		Must be the correct shape and in all four quadrants and pass through (-1.5, 0) and (1.5, 0) (Allow (0, -1.5) and (0, 1.5) or just -1.5 and 1.5 provided they are positioned correctly). **Must be on the diagram** (Allow $\sqrt{\frac{9}{4}}$ for 1.5)	A1
			(3)
(c)	$A=(-2,\ 14)$, $B=(1,\ 5)$	B1: $y=14$ **or** $y=5$ B1: $y=14$ **and** $y=5$	B1 B1
	These must be seen or used in (c)		
	$(AB=)\sqrt{(-2-1)^2+(14-5)^2}\left(=\sqrt{90}\right)$	**Correct** use of Pythagoras <u>including the square root.</u> Must be a correct expression for their A and B if a correct formula is not quoted	M1
	E.g. $AB=\sqrt{(-2+1)^2+(14-5)^2}$ **scores M0.** **However** $AB=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}=\sqrt{(-2+1)^2+(14-5)^2}$ **scores M1**		
	$(AB=)3\sqrt{10}$	cao	A1
			(4)
			(10 marks)
Special case: Use of $4x^3-9x$ for the curve gives (-2, -14) and (1, -5) in part (c). Allow this to score a maximum of B0B0M1A1 as a special case in part (c) as the length AB comes from equivalent work.			

Question Number	Scheme	Marks
61.	(a) $3x-7>3-x$	
	$4x>10$	M1
	$x>2.5,\quad x>\frac{5}{2},\quad \frac{5}{2}<x$ o.e.	A1
		(2)
	(b) Obtain $x^2-9x-36$ and attempt to solve $x^2-9x-36=0$	
	e.g. $(x-12)(x+3)=0$ so $x=\quad,$ or $x=\frac{9\pm\sqrt{81+144}}{2}$	M1
	$12,\ -3$	A1
	$-3\le x\le 12$	M1A1
		(4)
	(c) $2.5<x\le 12$	A1cso
		(1)
		(7 marks)

Notes

(a) M1 Reaching $px > q$ with one or both of p or q correct. Also give for $-4x < -10$

A1 Cao $x > 2.5$ o.e. Accept alternatives to 2.5 like $2\frac{1}{2}$ and $\frac{5}{2}$ even allow $\frac{10}{4}$ and allow $\frac{5}{2}<x$ o.e. This answer must occur and be credited as part (a) A correct answer implies M1A1

Mark parts (b) and (c) together.

(b) M1 Rearrange 3TQ ≤ 0 or 3TQ $= 0$ or even 3TQ > 0 Do not worry about the inequality at this stage AND attempt to solve by factorising, formula or completion of the square with the usual rules (see notes)

A1 12 and -3 seen as critical values

M1 Inside region for their critical values – must be stated – not just a table or a graph

A1 $-3\le x\le 12$ Accept $x\ge -3$ **and** $x\le 12$ or [-3, 12]

For the A mark: Do not accept $x\ge -3$ **or** $x\le 12$ nor $-3 < x < 12$ nor (-3, 12) nor $x\ge -3, x\le 12$

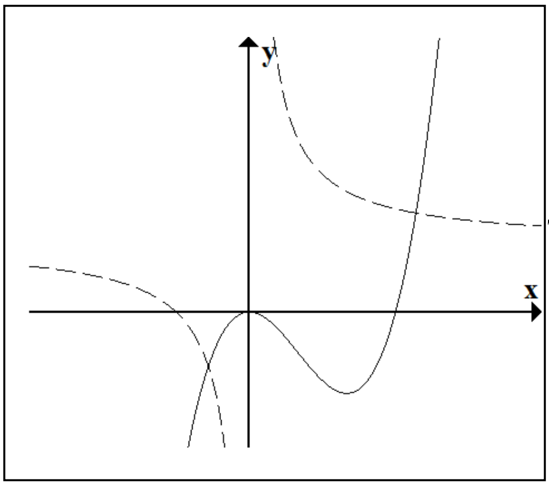
However allow recovery if they follow these statements by a correct statement, either in (b) or as they start part (c)

N.B. $-3\le 0\le 12$ and $x\ge -3, x\le 12$ are poor notation and get M1A0 here.

(c) A1 cso $2.5<x\le 12$ Accept $x>2.5$ and $x\le 12$ Allow $\frac{10}{4}$ Do not accept $x>2.5$ **or** $x\le 12$

Accept (2.5, 12] A graph or table is not sufficient. **Must follow correct earlier work** – except for special case

Special case (c) $x>2.5, x\le 12$; $2.5<0\le 12$ are poor notation – but if this poor notation has been penalised in (b) then allow A1 here. Any other errors are penalised in both (b) and (c).

Question Number	Scheme	Marks
62.	(a) - 1 accept (–1, 0)	B1 (1)
	(b) 	
	Shape	B1
	Touches at (0,0)	B1
	Crosses at (2,0) **only**	B1 (3)
	(c) 2 solutions as **curves** cross twice	B1 ft (1)
		(5 marks)

Notes

N.B. Check original diagram as answer may appear there.

(a) B1 The *x* coordinate of *A* is –1. Accept -1 or (–1,0) on the diagram or stated with or without diagram
Allow (0, -1) on the diagram if it is on the correct axis.

(b) *If no graph is drawn then no marks are available in part (b)*

B1 Correct shape. The position is not important for this mark but the curve must have two clear turning points and be a +ve x^3 curve (with a maximum and minimum)

B1 The graph touches the origin. Accept touching as a maximum or minimum. There must be a sketch for this mark but sketch may be wrong and this mark is independent of previous mark. Origin is where axes cross and may not be labelled. This may be a quadratic or quartic curve for this mark.

B1 The graph crosses the *x*-axis at the point (2,0) **only**. If it crosses at (2,0) and (0,0) this is B0. Accept (0,2) or 2 marked on the correct axis. Accept (2, 0) in the text of the answer provided that the curve crosses the positive *x* axis. There must be a sketch for this mark. Do not give credit if (2,0) appears only in a table with no indication that this is the intersection point. (If in doubt send to review) Graph takes precedence over text for third B mark.

(c) B1ft Two (solutions) **as there are two intersections** (**of the curves)** N.B. Just states 2 with no reason is B0
If the answer states 2 roots and two intersections – or crosses twice this is enough for B1
BUT B0 If there is any wrong **reason** given – e.g. crosses *x* axis twice, or crosses asymptote twice
Isw – is not used for this mark so any wrong statement listed to follow a correct statement will result in B0
Allow ft – so if their graph crosses the hyperbola once – allow "one solution as there is one intersection"
And if it crosses three times – allow "three solutions as there are three intersections" or four etc..
If it does not cross at all (e.g.negative cubic) – allow "no solutions as there are no intersections"
However in (c) if they have sketched a curve (even a fully correct one) but not extended it to intersect the hyperbola and they put "no points of intersection so no solutions" then this scores B0.
Accept "lines or curves cross over twice, or touch twice, or meet twice…etc as explanation, but need some form of words)

Question Number	Scheme	Marks
63.	(a) $80 = 5\times16$ $\sqrt{80} = 4\sqrt{5}$	B1 (1)
	Method 1 (b) $\frac{\sqrt{80}}{\sqrt{5}+1}$ or $\frac{c\sqrt{5}}{\sqrt{5}+1}$ — **Method 2** $(p+q\sqrt{5})(\sqrt{5}+1)=\sqrt{80}$	B1ft
	$= \frac{\sqrt{80}}{\sqrt{5}+1}\times\frac{\sqrt{5}-1}{\sqrt{5}-1}$ or $\frac{\sqrt{80}}{1+\sqrt{5}}\times\frac{1-\sqrt{5}}{1-\sqrt{5}}$ — $p\sqrt{5}+q\sqrt{5}+p+5q = 4\sqrt{5}$	M1
	$= \frac{20-4\sqrt{5}}{4}$ or $\frac{4\sqrt{5}-20}{-4}$ — $p+5q=0$, $p+q=4$	A1
	$= 5-\sqrt{5}$ — $p=5,\ q=-1$	A1cao (4) **(5 marks)**

Notes

(a) B1 Accept $4\sqrt{5}$ or $c = 4$ – no working necessary

(b)

(Method 1)

B1ft Only ft on c See $\frac{\sqrt{80}}{\sqrt{5}+1}$ or $\frac{c\sqrt{5}}{\sqrt{5}+1}$

M1 State intention to multiply by $\sqrt{5}-1$ or $1-\sqrt{5}$ in the numerator **and** the denominator

A1 Obtain denominator of 4 (for $\sqrt{5}-1$) **or** -4 (for $1-\sqrt{5}$) **or** correct simplified numerator of $20-4\sqrt{5}$ or $4(5-\sqrt{5})$ **or** $4\sqrt{5}-20$ or $4(\sqrt{5}-5)$ **So either numerator or denominator must be correct**

A1 Correct answer only. Both **numerator and denominator must have been correct and d**ivision of numerator and denominator by 4 has been performed.

Accept $p=5,\ q=-1$ or accept $5-\sqrt{5}$ or $-\sqrt{5}+5$ Also accept $5-1\sqrt{5}$

(Method 2)

B1ft Only ft on c $(p+q\sqrt{5})(\sqrt{5}+1)=\sqrt{80}$ or $c\sqrt{5}$

M1 Multiply out the lhs and replace $\sqrt{80}$ by $c\sqrt{5}$

A1 Compare rational and irrational parts to give $p+q=4$, **and** $p+5q=0$

A1 Solve equations to give $p=5,\ q=-1$

Common error:

$\frac{\sqrt{80}}{1+\sqrt{5}}\times\frac{1-\sqrt{5}}{1-\sqrt{5}} = \frac{4\sqrt{5}-20}{4} = \sqrt{5}-5$ gets B1 M1 A1 (for correct numerator – denominator is wrong for their product) then A0

Correct answer with no working – send to review – have they used a calculator?

Correct answer after trial and improvement with evidence that $(5-\sqrt{5})(\sqrt{5}+1)=\sqrt{80}$ could earn all four marks

Question Number	Scheme	Marks
64.	(a) Discriminant = $b^2 - 4ac = 8^2 - 4\times 2\times 3, = 40$	M1, A1 (2)
	(b) $2x^2 + 8x + 3 = 2(x^2 + ...)$ or p=2	B1
	$= 2((x+2)^2 \pm ...)$ or $q = 2$	M1
	$= 2(x+2)^2 - 5$ or $p = 2,\ q = 2$ and r = -5	A1 (3)
	(c) Method 1A: Sets derivative "$4x+8$" $= 4 \Rightarrow x =,\quad x = -1$	M1, A1
	Substitute $x = -1$ in $y = 2x^2 + 8x + 3 \quad (\Rightarrow y = -3)$	dM1
	Substitute $x = -1$ and y= -3 in $y = 4x + c$ or into $(y + 3)=4(x + 1)$ and expand	dM1
	$c = 1$ or writing $y = 4x + 1$	A1cso (5)
	Method 1B: Sets derivative "$4x+8$" $= 4 \Rightarrow x =,\quad x = -1$	M1, A1
	Substitute $x = -1$ in $2x^2 + 8x + 3 = 4x + c$	dM1
	Attempts to find value of c	dM1
	$c = 1$ or writing $y = 4x + 1$	A1cso (5)
	Method 2: Sets $2x^2 + 8x + 3 = 4x + \text{c}$ and collects x terms together	M1
	Obtains $2x^2 + 4x + 3 - c = 0$ or equivalent	A1
	States that $b^2 - 4ac = 0$	dM1
	$4^2 - 4\times 2\times(3-c) = 0$ and so c =	dM1
	$c = 1$	A1cso (5)
	Method 3: Sets $2x^2 + 8x + 3 = 4x + \text{c}$ and collects x terms together	M1
	Obtains $2x^2 + 4x + 3 - c = 0$ or equivalent	A1
	Uses $2(x+1)^2 - 2 + 3 - c = 0$ or equivalent	dM1
	Writes -2 +3 –c = 0	dM1
	So c = 1	A1cso (5)
	Also see special case for using a perpendicular gradient (overleaf)	**(10 marks)**

Notes

(a) M1 Attempts to calculate $b^2 - 4ac$ using $8^2 - 4\times 2\times 3$ - must be correct – not just part of a quadratic formula

A1 Cao 40

(b) B1 See 2(….) or p = 2

M1 $..((x+2)^2 \pm ...)$ is sufficient evidence or obtaining q = 2

A1 Fully correct values. $2(x+2)^2 - 5$ or $p = 2$, $q = 2$, $r = –5$ cso.

Ignore inclusion of "=0".

[In many respects these marks are similar to three B marks.
p = 2 is B1; q = 2 is B1 and p = 2, q = 2 and r = -5 is final B1 but they must be entered on epen as **B1 M1 A1]**

Special case: Obtains $2x^2 + 8x + 3 = 2(x+2) - 1$ This may have first B1, for p = 2 then M0A0 (c) Method 1A (Differentiates and puts gradient equal to 4. Needs both x and y to find c)

M1 Attempts to solve their $\frac{dy}{dx} = 4$. They must reach $x = ...$ (Just differentiating is M0 A0)

A1 $x = -1$ (If this follows $\frac{dy}{dx} = 4x + 8$, then give M1 A1 by implication)

dM1 (Depends on previous M mark) Substitutes **their** $x = -1$ into f(x) or into "their f(x) from (b)" to find y

dM1 (Depends on both previous M marks) Substitutes **their** $x = -1$ and **their** $y = -3$ values into $y = 4x + c$ to find c or uses equation of line is (y + "3")=4(x + "1") and rearranges to $y = mx + c$

A1 $c = 1$ or allow for $y = 4x + 1$ cso

(c) Method 1B (Differentiates and puts gradient equal to 4. Also equates equations and uses x to find c)

M1A1 Exactly as in Method 1A above

dM1 (Depends on previous M mark) Substitutes **their** $x = -1$ into $2x^2 + 8x + 3 = 4x + c$

dM1 Attempts to find value of c then A1 as before

(c) Method 2 (uses repeated root to find c by discriminant)

M1 Sets $2x^2 + 8x + 3 = 4x + c$ and tries to collect x terms together

A1 Collects terms e.g. $2x^2 + 4x + 3 - c = 0$ or $-2x^2 - 4x - 3 + c = 0$ or $2x^2 + 4x + 3 = c$ or even $2x^2 + 4x = c - 3$ Allow "=0" to be missing on RHS.

dM1 (If the line is a tangent it meets the curve at just one point so repeated root and $b^2 - 4ac = 0$) Stating that $b^2 - 4ac = 0$ is enough

dM1 Using $b^2 - 4ac = 0$ to obtain equation in terms of c (Eg. $4^2 - 4 \times 2 \times (3 - c) = 0$) AND leading to a solution for c

A1 $c = 1$ or allow for $y = 4x + 1$ cso

(c) Method 3 (Similar to method 2 but uses completion of the square on the quadratic to find repeated root)

M1 Sets $2x^2 + 8x + 3 = 4x + c$ and tries to collect x terms together. May be implied by $2x^2 + 8x + 3 - 4x \pm c$ on one side

A1 Collects terms e.g. $2x^2 + 4x + 3 - c = 0$ or $-2x^2 - 4x - 3 + c = 0$ or $2x^2 + 4x + 3 = c$ or even $2x^2 + 4x = c - 3$ Allow "=0" to be missing on RHS.

dM1 Then use completion of square $2(x+1)^2 - 2 + 3 - c = 0$ (Allow $2(x+1)^2 - k + 3 - c = 0$) where k is non zero. It is enough to give the correct or almost correct (with k) completion of the square

dM1 $-2 + 3 - c = 0$ AND leading to a solution for c (Allow $-1 + 3 - c = 0$) ($x = -1$ has been used)

A1 $c = 1$ cso

In Method 1 they may use part (b) and differentiate their f(x) and put it equal to 4
They can earn M1, but do not follow through errors.

In Methods 2 and 3 they may use part (b) to write
their $2(x+2)^2 - 5 = 4x + c$. They need to expand and collect x terms together for M1
Then expanding gives $2x^2 + 4x + 3 - c = 0$ for A1 – do not follow through errors
Then the scheme is as before

If they just state $c = 1$ with little or no working – please send to review,

PTO for special case

Special case uses perpendicular gradient (maximum of 2/5)

Sets $4x+8=-\frac{1}{4} \Rightarrow x=,\quad x=-\frac{33}{16}$ M1 A0

Substitute $x=-\frac{33}{16}$ in $y=2x^2+8x+3$ $(\Rightarrow y=-\frac{639}{128})$ M0

Substitute $x=-\frac{33}{16}$ and $y=-\frac{639}{128}$ into $y = 4x + c$ or into $(y+\frac{639}{128})=4(x+\frac{33}{16})$ and expand M1 A0

Question Number	Scheme	Marks
65.	$25x-9x^3=x(25-9x^2)$ $(25-9x^2)$=(5+3x)(5-3x) $25x-9x^3=x(5+3x)(5-3x)$	B1 M1 A1 (3)

B1 Take out a common factor, usually x, to produce $x(25-9x^2)$. Accept $(x\pm 0)(25-9x^2)$ or $-x(9x^2-25)$ Must be correct.
Other possible options are $(5+3x)(5x-3x^2)$ or $(5-3x)(5x+3x^2)$

M1 For factorising their quadratic term, usually $(25-9x^2)=$ (5+3x)(5-3x) Accept sign errors
If $(5\pm 3x)$ has been taken out as a factor first, this is for an attempt to factorise $(5x\mp 3x^2)$

A1 cao $x(5+3x)(5-3x)$ or any equivalent with three factors
e.g. $x(5+3x)(-3x+5)$ or x(3x-5)(-3x-5) etc including –x(3x+5)(3x-5)
isw if they go on to show that $x = 0,\ \pm\frac{5}{3}$

Question Number	Scheme	Marks
66 **Method 1**	$x\sqrt{8}+10=\dfrac{6x}{\sqrt{2}}$ $\times\sqrt{2}\Rightarrow x\sqrt{16}+10\sqrt{2}=6x$ $4x+10\sqrt{2}=6x\Rightarrow 2x=10\sqrt{2}$ $x=5\sqrt{2}$ or a = 5 and b = 2	M1,A1 M1A1 (4)
66 **Method 2**	$x\sqrt{8}+10=\dfrac{6x}{\sqrt{2}}$ $2\sqrt{2}x+10=3\sqrt{2}x$ $\sqrt{2}x=10\Rightarrow x=\dfrac{10}{\sqrt{2}}=\dfrac{10\sqrt{2}}{2},=5\sqrt{2}$ oe	M1A1 M1,A1 (4)

Method 1

M1 For multiplying both sides by √2 – allow a slip e.g. $\sqrt{2}x\sqrt{8}+10=\dfrac{6x}{\sqrt{2}}\times\sqrt{2}$ or

$\sqrt{2}\times 10+x\sqrt{8}=\dfrac{6x}{\sqrt{2}}\times\sqrt{2}$, where one term has an error or the correct $\sqrt{2}(10+x\sqrt{8})=\dfrac{6x}{\sqrt{2}}\times\sqrt{2}$

NB $x\sqrt{8}+10=6x\sqrt{2}$ is M0

A1 A correct equation in x with no fractional terms. Eg $x\sqrt{16}+10\sqrt{2}=6x$ oe.

M1 An attempt to solve their linear equation in x to produce an answer of the form $a\sqrt{2}$ or $a\sqrt{50}$

A1 $5\sqrt{2}$ oe (accept $1\sqrt{50}$)

Method 2

M1 For writing √8 as 2√2 or $\dfrac{6}{\sqrt{2}}$ as 3√2

A1 A correct equation in x with no fractional terms. Eg $2\sqrt{2}x+10=3\sqrt{2}x$ *or* $x\sqrt{8}+10=3\sqrt{2}x$ oe.

M1 An attempt to solve their linear equation in x to produce an answer of the form $a\sqrt{2}$ or $a\sqrt{50}$

$\sqrt{2}x=10\Rightarrow x=\dfrac{10}{\sqrt{2}}=\dfrac{10\sqrt{2}}{2},=5\sqrt{2}$

or $\sqrt{2}x=10\Rightarrow 2x^2=100\Rightarrow x^2=50\Rightarrow x=\sqrt{50}$ or $5\sqrt{2}$

A1 $5\sqrt{2}$ oe Accept $1\sqrt{50}$

Question Number	Scheme	Marks
67(a).	$P = 20x + 6$ o.e	B1
	$20x + 6 > 40 \Rightarrow x >$	M1
	$x > 1.7$	A1*
		(3)
	Mark parts (b) and (c) together	
(b)	$A = 2x(2x+1) + 2x(6x+3) = 16x^2 + 8x$	B1
	$16x^2 + 8x - 120 < 0$	M1
	Try to solve their $2x^2 + x - 15 = 0$ e.g. $(2x-5)(x+3) = 0$ so $x =$	M1
	Choose inside region	M1
	$-3 < x < \frac{5}{2}$ or $0 < x < \frac{5}{2}$ (as x is a length)	A1
(c)		(5)
	$1.7 < x < \frac{5}{2}$	B1cao
		(1)
		(9 marks)

(a) B1 Correct expression for perimeter but may not be simplified so accept
$2x + 1 + 2x + 4x + 2 + 2x + 6x + 3 + 4x$ or $2(10x + 3)$ or any equivalent

M1: Set $P > 40$ with their linear expression for P (this may not be correct but should be a sum of sides) and manipulate to get $x >$…

A1* cao $x > 1.7$. This is a given answer, there must not be any errors, but accept $1.7 < x$

(b) Marks parts (b) and (c) together

B1 Writes a correct statement in x for the area. It need not be simplified. You may isw
Amongst numerous possibilities are.
$2x(2x+1) + 2x(6x+3)$, $16x^2 + 8x$, $4x(6x+3) - 2x(4x+2)$, $4x(2x+1) + 2x(4x+2)$

M1 Sets their quadratic expression < 120 and collects on one side of the inequality

M1 For an attempt to solve a 3 Term quadratic equation producing two solutions by factorising, formula or completion of the square with the usual rules (see notes)

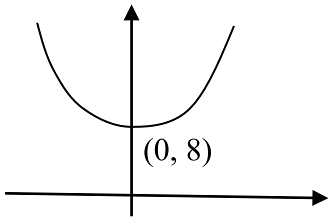
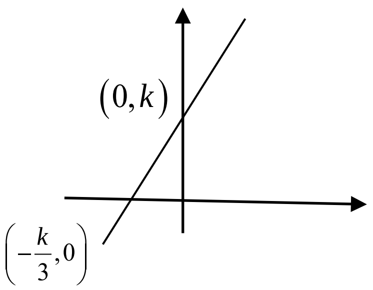
M1 For choosing the 'inside' region. Can follow through from their critical values – must be stated – not just a table or a graph. Can also be implied by $0 < x <$ upper value

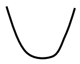
A1 $-3 < x < \frac{5}{2}$. Accept $x > -3$ **and** $x < 2.5$ or (-3, 2.5)

As x is a width, accept $0 < x < \frac{5}{2}$ Also accept $\frac{10}{4}$ or 2.5 instead of $\frac{5}{2}$. $\leq$ would be M1A0

Allow this final answer to part (b) to appear as answer to part (c) This would score final M1A1 in (b) then B0 in (c)

(c) B1cao $1.7 < x < \frac{5}{2}$. Must be correct. [This does not imply final M1 in (b)]

Question Number	Scheme	Marks
68(a)	 U shaped parabola – symmetric about y axis	B1
	Graph passes through (0, 8)	B1
	 Shape and position for L	M1
	Both $\left(-\frac{k}{3},0\right)$ and $(0,k)$	A1
		(4)
	Allow marks **even if** on the **same** diagram	
68(b)	Method 1:	
	Equate curves $\frac{1}{3}x^2+8=3x+k$ and proceed to collect terms on one side	M1
	$\frac{1}{3}x^2-3x+(8-k)$	A1
	Method 1a: Uses "$b^2=4ac$" | Method 1b: Attempt $\frac{1}{3}(x-\frac{9}{2})^2-\lambda+8-k$	dM1
	Method 1a: $9=4\times\frac{1}{3}\times(8-k)\Rightarrow k=..$ | Method 1b: Deduce that $k=8-\lambda$	dM1
	$k=\frac{5}{4}$ o.e.	A1
		(5)
	Method 2 :	
	Attempts to set $\frac{dy}{dx}=3$	M1
	$\frac{2}{3}x=3\Rightarrow x=4.5$	A1
	Method 2a: Substitutes $x=$"4.5" into $y=\frac{1}{3}x^2+8\Rightarrow y=....(14.75)$ | Method 2b: Substitutes $x=$"4.5" into $\frac{1}{3}x^2+8=3x+k$	dM1
	Method 2a: Substitutes both their x and y into $y=3x+k$ to find k | Method 2b: Finds $k=$	dM1
	$k=1.25$ o.e.	A1
		(5)
		(9 marks)

(a) B1 Shape for C.  Approximately Symmetrical about the y axis

B1 Coordinates of (0, 8) There must be a graph.

Accept graph crossing positive y axis with only 8 marked. Accept (8,0) if given on y axis.

M1 Shape for L. A straight line with positive gradient and positive intercept

A1 Coordinates of (0, k) and (-k/3, 0) or k marked on y axis, and –k/3 marked on x axis or even

Accept (k, 0) on y axis and (0, -k/3) on x axis

(b) Either

Methods 1

M1 Equate curves $\frac{1}{3}x^2+8=3x+k$ and proceed to collect x terms on one side and $(8-k)$ terms together on the same side or on the other side

A1 Achieves an expression that leads to the point of intersection e.g $\frac{1}{3}x^2-3x+(8-k)$

Method 1a

dM1 (depends on previous M mark) Uses the fact that $b^2=4ac$ or 'b^2-4ac' $=0$ is true

dM1 (depends on previous M mark) Solves their $b^2=4ac$, leading to k=..

A1 cso $k=\frac{5}{4}$ Accept equivalents like 1.25 etc.

Method 1b

dM1 (depends on previous M mark) Uses completion of the square as shown in scheme

dM1 (depends on previous M mark) Uses $k=8-\lambda$

A1 cso $k=\frac{5}{4}$ Accept equivalents like 1.25 etc.

Methods 2

M1 Equate $\frac{\mathrm{d}y}{\mathrm{d}x}=3$ Not given just for derivative

A1 Solves to get $x=4.5$

Method 2a

dM1 Substitutes their 4.5 into equation for C to give y coordinate

dM1 Substitutes both their x and y into $y=3x+k$ to find k

A1 cso $k=\frac{5}{4}$ Accept equivalents like 1.25 etc.

Method 2b

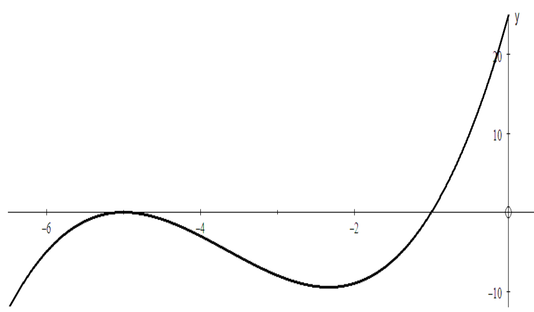
dM1 Substitutes their 4.5 into $\frac{1}{3}x^2+8=3x+k$

dM1 Finds k

A1 cso $k=\frac{5}{4}$ Accept equivalents like 1.25 etc.

Question Number	Scheme		Marks
69	$\frac{7+\sqrt{5}}{\sqrt{5}-1}\times\frac{(\sqrt{5}+1)}{(\sqrt{5}+1)}$	Multiplies top and bottom by a correct expression. This statement is sufficient.	M1
	(Allow to multiply top and bottom by $k(\sqrt{5}+1)$)		
	$=\frac{...}{4}$	Obtains a denominator of 4 or sight of $(\sqrt{5}-1)(\sqrt{5}+1)=4$	A1cso
	Note that M0A1 is not possible. The 4 must come from a correct method.		
	$(7+\sqrt{5})(\sqrt{5}+1)=7\sqrt{5}+5+7+\sqrt{5}$	An attempt to multiply the numerator by $(\pm\sqrt{5}\pm1)$ and get 4 terms with at least 2 correct for their $(\pm\sqrt{5}\pm1)$. (May be implied)	M1
	$3+2\sqrt{5}$	Answer as written or a = 3 and b = 2. (Allow $2\sqrt{5}+3$)	A1cso
	Correct answer with no working scores full marks		
			[4]
Way 2	$\frac{7+\sqrt{5}}{\sqrt{5}-1}\times\frac{(-\sqrt{5}-1)}{(-\sqrt{5}-1)}$	Multiplies top and bottom by a correct expression. This statement is sufficient.	M1
	(Allow to multiply top and bottom by $k(-\sqrt{5}-1)$)		
	$=\frac{...}{-4}$	Obtains a denominator of -4	A1cso
	$(7+\sqrt{5})(-\sqrt{5}-1)=-7\sqrt{5}-5-7-\sqrt{5}$	An attempt to multiply the numerator by $(\pm\sqrt{5}\pm1)$ and get 4 terms with at least 2 correct for their $(\pm\sqrt{5}\pm1)$. (May be implied)	M1
	$3+2\sqrt{5}$	Answer as written or a = 3 and b = 2	A1cso
	Correct answer with no working scores full marks		
			[4]
	Alternative using Simultaneous Equations: $\frac{(7+\sqrt{5})}{\sqrt{5}-1}=a+b\sqrt{5}\Rightarrow 7+\sqrt{5}=(a-b)\sqrt{5}+5b-a$ M1 Multiplies and collects rational and irrational parts $a-b=1,\ \ 5b-a=7$ A1 Correct equations $a=3,\ \ b=2$ M1 for attempt to solve simultaneous equations A1 both answers correct		

Question Number	Scheme		Marks
70(a)	$6x + x > 1 - 8$	Attempts to expand the bracket and collect x terms on one side and constant terms on the other. Condone sign errors and allow one error in expanding the bracket. Allow $<, \le, \ge, =$ instead of $>$.	M1
	$x > -1$	Cao	A1
	Do not isw here, mark their final answer.		
			(2)
(b)	$(x+3)(3x-1)[=0]$ $\Rightarrow x = -3$ and $\frac{1}{3}$	M1: Attempt to solve the quadratic to obtain two critical values A1: $x = -3$ **and** $\frac{1}{3}$ (may be implied by their inequality). Allow all equivalent fractions for -3 and 1/3. (Allow 0.333 for 1/3)	M1A1
	$-3 < x < \frac{1}{3}$	M1: Chooses "inside" region (The letter x does not need to be used here) A1ft: Allow $x < \frac{1}{3}$ **and** $x > -3$ or $\left(-3, \frac{1}{3}\right)$ or $x < \frac{1}{3} \cap x > -3$. Follow through their critical values. (must be in terms of x here) Allow all equivalent fractions for -3 and 1/3. Both ($x < \frac{1}{3}$ **or** $x > -3$) and ($x < \frac{1}{3}$**,** $x > -3$) as a final answer score A0.	M1A1ft
			(4)
			[6]
	Note that use of $\le$ or $\ge$ appearing in an otherwise correct answer in (a) or (b) should only be penalised once, the first time it occurs.		

Question Number	Scheme		Marks
71 (a)		Horizontal translation – does **not** have to cross the *y*-axis on the right but must at least reach the *x*-axis.	B1
		Touching at (-5, 0). This could be stated anywhere or -5 could be marked on the *x*-axis. Or (0, -5) **marked in the correct place.** Be fairly generous with 'touching' if the intention is clear.	B1
		The right hand tail of their cubic shape crossing at (-1, 0). This could be stated anywhere or -1 could be marked on the *x*-axis. Or (0, -1) **marked in the correct place.** The curve must **cross** the *x*-axis and not stop at -1.	B1
			(3)
(b)	$(x+5)^2(x+1)$	Allow $(x+3+2)^2(x-1+2)$	B1
			(1)
(c)	When $x = 0$, $y = 25$	M1: Substitutes $x = 0$ into their expression in **part (b)** which is not f(x). This may be implied by their answer. Note that the question asks them to use part (b) but allow independent methods.	M1 A1
		A1: $y = 25$ (Coordinates not needed)	
	If they expand <u>incorrectly</u> prior to substituting $x = 0$, score M1 A0 **NB f(x + 2) =** $x^3 + 11x^2 + 35x + 25$		
			(2)
			[6]

EXPERT TUITION

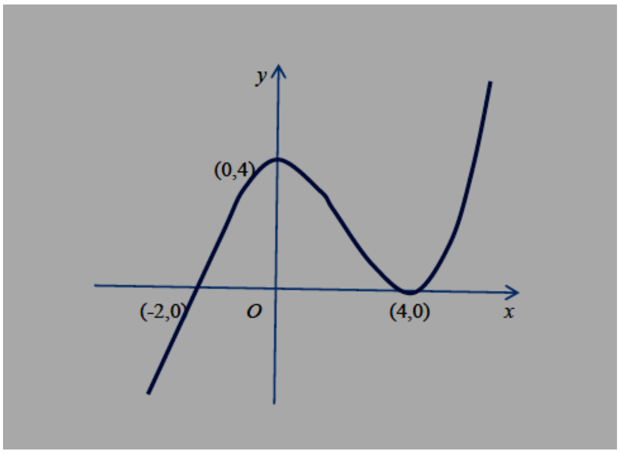
Question Number	Scheme		Marks
72	$(3-x^2)^2 = 9-6x^2+x^4$	An attempt to expand the numerator obtaining an expression of the form $9 \pm px^2 \pm qx^4, \quad p,q \neq 0$	M1
	$9x^{-2}+x^2$	Must come from $\dfrac{9+x^4}{x^2}$	A1
	-6	Must come from $\dfrac{-6x^2}{x^2}$	A1
	Alternative 1: Writes $\dfrac{(3-x^2)^2}{x^2}$ as $(3x^{-1}-x)^2$ and attempts to expand = M1 then A1A1 as in the scheme.		
	Alternative 2: Sets $(3-x^2)^2 = 9+Ax^2+Bx^4$, expands $(3-x^2)^2$ and compares coefficients = M1 then A1A1 as in the scheme.		
			(3)

Question Number	Scheme	Marks	
73(a)	$x^2-4k(1-2x)+5k(=0)$	Makes *y* the subject from the first equation and substitutes into the second equation (= 0 not needed here) or eliminates *y* by a correct method.	M1
	So $x^2+8kx+k=0$ *	Correct completion to printed answer. There must be no incorrect statements.	A1cso
			(2)
(b)	$(8k)^2-4k$	M1: Use of b^2-4ac (Could be in the quadratic formula or an inequality, = 0 not needed yet). There must be some correct substitution but there must be no *x*'s. No formula quoted followed by e.g. $8k^2-4k=0$ is M0. A1: Correct expression. Do not condone missing brackets unless they are implied by later work but can be implied by $(8k)^2>4k$ etc.	M1 A1
	$k=\frac{1}{16}$ (oe)	Cso (Ignore any reference to *k* = 0) but there must be no contradictory earlier statements. A fully correct solution with no errors.	A1
			(3)
(b) Way 2 Equal roots	$\Rightarrow x^2+8kx+k=(x+\sqrt{k})^2$ $\Rightarrow 8k=2\sqrt{k}$	M1: Correct strategy for equal roots A1: Correct equation	M1A1
	$k=\frac{1}{16}$ (oe)	Cso (Ignore any reference to *k* = 0)	A1
(b) Way 3	Completes the Square $x^2+8kx+k=(x+4k)^2-16k^2+k$ $\Rightarrow 16k^2-k=0$	M1: $(x\pm 4k)^2\pm p\pm k,\ p\neq 0$ A1: Correct equation	M1A1
	$k=\frac{1}{16}$ (oe)	Cso (Ignore any reference to *k* = 0)	A1
			(3)
(c)	$x^2+\frac{1}{2}x+\frac{1}{16}=0$ so $(x+\frac{1}{4})^2=0\Rightarrow x=$	Substitutes their value of *k* into the given quadratic and attempt to solve their 2 or 3 term quadratic as far as *x* = (may be implied by substitution into the quadratic formula) or starts again and substitutes their value of *k* into the second equation and solves simultaneously to obtain a value for *x*.	M1
	$x=-\frac{1}{4},\ y=1\frac{1}{2}$	First A1 one answer correct, second A1 both answers correct.	A1A1
	Special Case: $x^2+\frac{1}{2}x+\frac{1}{16}=0\Rightarrow x=-\frac{1}{4},\ \frac{1}{4}\Rightarrow y=1\frac{1}{2},\ \frac{1}{2}$ allow M1A1A0		
			(3)
			[8]

Question Number	Scheme		Marks
74 (a)	$\left(-\frac{3}{4}, 0\right)$. Accept $x = -\frac{3}{4}$		B1
			(1)
(b)	$y = 4$	B1: One correct asymptote	B1B1
	$x = 0$ or 'y-axis'	B1: Both correct asymptotes and no extra ones.	
	Special case $x \neq 0$ **and** $y \neq 4$ scores B1B0		
			(2)

Question Number	Scheme	Notes	Marks
75.	$\frac{15}{\sqrt{3}} = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 5\sqrt{3}$	M1: Attempts to multiply numerator and denominator by √3. This may be implied by a correct answer. A1: $5\sqrt{3}$	M1A1
	$\sqrt{27} = 3\sqrt{3}$		B1
	$\frac{15}{\sqrt{3}} - \sqrt{27} = 2\sqrt{3}$		A1
	Correct answer only scores full marks		
			[4]
Way 2	$\frac{15}{\sqrt{3}} - \sqrt{27} = \frac{15 - \sqrt{81}}{\sqrt{3}} \left(= \frac{6}{\sqrt{3}} \right)$	Terms combined correctly with a common denominator (Need not be simplified)	B1
	$\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3}$	M1: Attempts to multiply numerator and denominator by √3. This may be implied by a correct answer. A1: $\frac{6\sqrt{3}}{3}$	M1A1
	$\frac{15}{\sqrt{3}} - \sqrt{27} = 2\sqrt{3}$		A1
			[4]
	Note that $\frac{15}{\sqrt{3}} - \sqrt{27} = \frac{15\sqrt{3}}{3} - 3\sqrt{3} = 15\sqrt{3} - 9\sqrt{3} = 6\sqrt{3}$ is quite common and scores M1A0B1A0 (i.e. $5\sqrt{3}$ is never seen)		

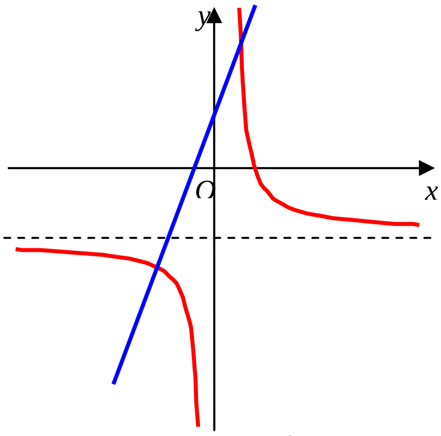
Question Number	Scheme	Notes	Marks
	Ignore any references to the units in this question		
76.(a)	length is '$x + 4$'	May be implied	B1
	$x+x+x+4+x+4>19.2 \Rightarrow x>..$	$2x+2(x\pm4)>19.2$ and proceeds to $x>$ ..... (Accept 'invisible' brackets) Attempts 2 widths + 2 lengths > 19.2 leading to $x>$	M1
	E.g. $x+x+4x+4x>19.2 \Rightarrow x>1.92$ scores B0M1A0		
	$x > 2.8$ *	Achieves $x > 2.8$ with no errors	A1(*)
			(3)
	Mark parts (b) and (c) together		
(b)(i)	$x(x+4)<21$	Cao	B1
b(ii)	$x^2+4x-21<0$ $(x+7)(x-3)<0 \Rightarrow x=...$	Multiply out lhs, produce 3TQ = 0 and attempt to solve leading to x =... according to general guidelines	M1
	Either -7 < x < 3 **or** 0 < x < 3	M1: Attempts the 'inside' for their critical values (may be from a 2TQ here) A1: Accept either -7 < x < 3 or 0 < x < 3 or (x > -7 **and** x < 3) or (x > 0 **and** x < 3) but not e.g. (x > -7, x < 3) or (x > -7 **or** x < 3) (There is no specific need for them to realise x > 0)	M1A1
	Note that <u>many</u> candidates stop here		
			(4)
(c)	$2.8 < x < 3$	Follow through their answers to (a) and (b) Provided "their 3" > 2.8	B1ft
			(1)
			[8]
	Examples		
	$x(x-4)<21 \Rightarrow x^2-4x-21<0$ $(x-7)(x+3)<0,\ x=7, x=-3$ $-3<x<7\ or\ 0<x<7$ $2.8<x<7$ Scores B0M1M1A0B1ft	$x\times4x<21 \Rightarrow 4x^2-21<0$ $(2x-\sqrt{21})(2x+\sqrt{21})<0,\ x=\pm\frac{\sqrt{21}}{2}$ $-\frac{\sqrt{21}}{2}<x<\frac{\sqrt{21}}{2}\ or\ 0<x<\frac{\sqrt{21}}{2}$ $2.8<x<\frac{\sqrt{21}}{2}$ Scores B0M0M1A0B0	

Question Number	Scheme	Notes	Marks
77.(a)	$f(x)=(x+1)(x-2)^2$	M1: Either stating or writing down that ($x \pm 1$) **or** ($x \pm 2$) is a factor – may be implied by their f(x)	M1A1B1
		A1: **Both** (x + 1) **and** (x - 2) are factors - may be implied by their f(x)	
		B1: y or f(x)= (x + 1)(x - 2)^2	
	$=(x+1)(x^2-4x+4)=x^3-3x^2+4$	M1: Multiplying out a quadratic to get 3 terms and then multiplying by the linear term to form a cubic.	M1A1
		A1: x^3-3x^2+4 or a = -3, b = 0, c = 4	
			(5)
(b)			
		Same shape and position (ignore any coordinates) with the maximum on the y-axis	B1
		y intercept = 4 or their 'c'	B1ft
		x coordinates at -2 and 4 or marked as coordinates. Allow (0, -2) and (0, 4) if they are marked in the correct position. The curve must cross or at least stop at x = -2	B1
			(3)
			[8]
(a) Way 2	$x=0, y=4 \Rightarrow c=4$	Uses (0, 4) to obtain c = 4 (can be just stated)	B1
	$x=-1, y=0 \Rightarrow -1+a-b+c=0$ $x=2, y=0 \Rightarrow 8+4a+2b+c=0$	Uses both (-1, 0) and (2, 0) in $y=x^3+ax^2+bx+c$ to form 2 simultaneous equations. Allow the equations to contain c here.	M1
	$a-b=-3$ $4a+2b=-12$ $\Rightarrow a = ..\ or\ b = ...$	Solves simultaneously with a value for c to obtain a value for a or a value for b	M1
	Either a = -3 **or** b = 0		A1
	Both a = -3 **and** b = 0		A1

Question Number	Scheme	Notes	Marks
77(a) Way 3	$\frac{dy}{dx} = 3x^2 + 2ax + b$	M1: $x^n \to x^{n-1}$ at least once including c → 0	M1
	$x = 0 \Rightarrow \frac{dy}{dx} = 0 \Rightarrow b = 0$	Correct value for b	A1
	$x = 0, y = 4 \Rightarrow c = 4$	Uses (0, 4) to obtain c = 4 (can be just stated)	B1
	$3(2)^2 + 2a(2) + b = 0$ or $(-1)^3 + a(-1)^2 + b(-1) + 4 = 0$	Obtains an equation in a	M1
	$a = -3$	Correct value for a	A1
			(5)
	Special case: A common incorrect approach is to assume the cubic is of the form e.g. $f(x) = x(x \pm 1)(x \pm 2) + 4$ This scores B1 only for c = 4		
			[8]

Question Number	Scheme	Marks
78.	$x(1-4x^2)$ Accept $x(-4x^2+1)$ or $-x(4x^2-1)$ or $-x(-1+4x^2)$ or even $4x(\frac{1}{4}-x^2)$ or equivalent	B1
	quadratic (or initial cubic) into two brackets	M1
	$x(1-2x)(1+2x)$ or $-x(2x-1)(2x+1)$ or $x(2x-1)(-2x-1)$	A1
		[3]
		3 marks
	Notes	
	B1: Takes out a factor of x or $-x$ or even $4x$. This line may be implied by correct final answer, but if this stage is shown it **must be correct**. So **B0** for $x(1+4x^2)$ **M1:** Factorises the quadratic resulting from their first factorisation using usual rules (see note 1 in General Principles). e.g. $x(1-4x)(x-1)$. Also allow attempts to factorise cubic such as $(x-2x^2)(1+2x)$ etc N.B. Should not be completing the square here. **A1:** Accept either $x(1-2x)(1+2x)$ or $-x(2x-1)(2x+1)$ or $x(2x-1)(-2x-1)$. (No fractions for this final answer)	
	Specific situations	
	Note: $x(1-4x^2)$ followed by $x(1-2x)^2$ scores B1M1A0 as factors follow quadratic factorisation criteria **And** $x(1-4x^2)$ followed by $x(1-4x)(1+4x)$ **B1M0A0.**	
	Answers with no working: Correct answer gets all three marks B1M1A1	
	: $x(2x-1)(2x+1)$ gets B0M1A0 if **no working** as $x(4x^2-1)$ would earn B0	
	Poor bracketing: e.g. $(-1+4x^2)-x$ gets B0 unless subsequent work implies bracket round the $-x$ in which case candidate may recover the mark by the following correct work.	

Question Number	Scheme	Marks
79 (i)	$\left(5-\sqrt{8}\right)\left(1+\sqrt{2}\right)$ $=5+5\sqrt{2}-\sqrt{8}-4$ $=5+5\sqrt{2}-2\sqrt{2}-4$ — $\sqrt{8}=2\sqrt{2}$, seen or implied at any point. $=1+3\sqrt{2}$ — $1+3\sqrt{2}$ or $a=1$ and $b=3$.	 M1 B1 A1 **[3]**
(ii)	Method 1: **Either** $\sqrt{80}+\frac{30}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$; $=4\sqrt{5}+\ldots$; $=4\sqrt{5}+6\sqrt{5}$ Method 2: **Or** $\left(\frac{\sqrt{400}+30}{\sqrt{5}}\right)\frac{\sqrt{5}}{\sqrt{5}}$; $=\left(\frac{20+..}{..}\right)\frac{..}{..}$; $=\left(\frac{50\sqrt{5}}{5}\right)$ Method 3: $\sqrt{80}+\frac{\sqrt{900}}{\sqrt{5}}=\sqrt{80}+\sqrt{180}$; $=4\sqrt{5}+\ldots$; $=4\sqrt{5}+6\sqrt{5}$ $=10\sqrt{5}$	M1 B1 A1 **[3]**
Alternative for (i)	$\left(5-2\sqrt{2}\right)\left(1+\sqrt{2}\right)$ — This earns the B1 mark. $=5+5\sqrt{2}-2\sqrt{2}-2\sqrt{2}\sqrt{2}$ — Multiplies out correctly with $2\sqrt{2}$. This may be seen or implied and may be simplified e.g. $=5+3\sqrt{2}\;-2\sqrt{4}$ o.e. For earlier use of $2\sqrt{2}$ $=1+3\sqrt{2}$ — $1+3\sqrt{2}$ or $a=1$ and $b=3$.	 M1 B1 A1 **[3]** **6 marks**
	Notes	
(i) (ii)	**M1:** Multiplies out brackets correctly giving four correct terms or simplifying to correct expansion. (This may be implied by correct answer) – can appear as table **B1:** $\sqrt{8}=2\sqrt{2}$, seen or implied at any point **A1:** Fully and correctly simplified to $1+3\sqrt{2}$ or $a=1$ and $b=3$. **M1:** Rationalises denominator i.e. Multiplies $\left(\frac{k}{\sqrt{5}}\right)$ by $\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$ or $\left(\frac{-\sqrt{5}}{-\sqrt{5}}\right)$, seen or implied or uses Method 3 or similar e.g. $\left(\frac{30}{\sqrt{5}}\right)=\frac{6\times5}{\sqrt{5}}=6\sqrt{5}$ **B1**: (Independent mark) States $\sqrt{80}=4\sqrt{5}$ Or either $\sqrt{400}=20$ *or* $\sqrt{80}\sqrt{5}=20$ at any point if they use Method 2. **A1:** $10\sqrt{5}$ or $c=10$.	
	N.B There are other methods e.g. $\sqrt{80}=\frac{20}{\sqrt{5}}$ (B1) then add $\frac{20}{\sqrt{5}}+\frac{30}{\sqrt{5}}=\frac{50}{\sqrt{5}}$ then M1 A1as before Those who multiply initial expression by $\sqrt{5}$ to obtain $\sqrt{400}+30=20+30=50$ earn M0 B1 A0	

Question Number	Scheme	Marks
80. (a)	 $y = \frac{2}{x}$ is translated up or down.	M1
	$y = \frac{2}{x} - 5$ is in the correct position.	A1
	Intersection with x-axis at $\left(\frac{2}{5}, \{0\}\right)$ **only** Independent mark.	B1
	$y = 4x + 2$: attempt at straight line, with positive gradient with positive y intercept.	B1
	Check graph in question for possible answers and space below graph for answers to part (b) Intersection with x-axis at $\left(-\frac{1}{2}, \{0\}\right)$ **and** y-axis at $\left(\{0\}, 2\right)$.	B1 **[5]**
(b)	Asymptotes : $x = 0$ (or y-axis) and $y = -5$. An asymptote stated correctly. Independent of (a)	B1
	(Lose second B mark for extra asymptotes) These two lines only. Not ft their graph.	B1 **[2]**
(c)	Method 1: $\frac{2}{x} - 5 = 4x + 2$ Method 2: $\frac{y-2}{4} = \frac{2}{y+5}$	M1
	$4x^2 + 7x - 2 = 0 \Rightarrow x =$ $y^2 + 3y - 18 = 0 \rightarrow y =$	dM1
	$x = -2, \frac{1}{4}$ $y = -6, 3$	A1
	When $x = -2$, $y = -6$,When $x = \frac{1}{4}$, $y = 3$ When $y = -6$, $x = -2$ When $y = 3$, $x = \frac{1}{4}$.	M1A1 **[5]**
		12 marks

Notes

(a) **M1:** Curve implies y axis as asymptote and does not change shape significantly. Changed curve needs horizontal asymptote (roughly) Asymptote(s) need not be **shown** but shape of curve should be implying asymptote(s) parallel to x axis. Curve should not remain where it was in the given figure. Both sections move in the same direction. There should be no reflection

A1: Crosses positive x axis. Hyperbola has moved down. Both sections move by **almost** same amount. See sheet on page 19 for guidance.

B1: Check diagram and text of answer. Accept 2/5 or 0.4 shown on x -axis or x = 2/5, or (2/5 , 0) stated clearly in text or on graph. This is **independent** of the graph. Accept (0, 2/5) if clearly on x axis. Ignore any intersection points with y axis. Do not credit work in table of values for this mark.

B1: Must be attempt at a straight line, with positive gradient & with positive y intercept (need not cross x axis)

B1: Accept x = -1/2, or –0.5 shown on x -axis or (-1/2 , 0) or (-0.5,0) in text or on graph and similarly accept 2 on y axis or y = 2 or (0, 2) in text or on graph. **Need not cross curve and allow on separate axes.**

(b) **B1:** For either correct asymptote equation. Second **B1**: For both correct (lose this if extras e.g. $x = \pm 1$ are given also). These asymptotes may follow correctly from equation after wrong graph in (a)

Just y = -5 **is B1 B0** **This may be awarded if given on the graph.** However for other B mark it must be clear that $x = 0$ (or the y-axis) is an asymptote. NB $x \neq 0$, $y \neq -5$ is B1B0

(c) **M1**: Either of these equations is enough for the method mark (May appear labelled as part (b))

dM1: Attempt to solve a 3 term quadratic by factorising, formula, completion of square or implied by correct answers. (see note 1) This mark depends on previous mark.

A1: Need both correct x answers (Accept equivalents e.g. 0.25) or both correct y values (Method 2)

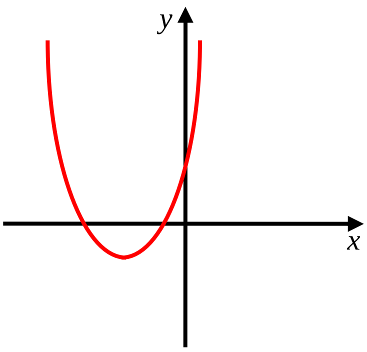
M1: At least one attempt to find *second variable* (usually y) using their *first variable* (usually x) related to line meeting curve. Should not be substituting x or y values from part (a) or (b). This mark is **independent** of previous marks. Candidate may substitute in equation of line or equation of curve.

A1: Need both correct *second variable* answers Need not be written as co-ordinates (allow as in the scheme)

Note: Special case: Answer only with no working in part (c) can have 5 marks if completely correct, with **both** points found. If co-ordinates of just one of the points is correct – with no working – this earns M0 M0 A0 M1 A0 (i.e. 1 / 5)

Question Number	Scheme	Marks
81. (a)	**Method 1:** Attempts $b^2 - 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ $b^2 - 4ac = 6^2 - 4(k+3)(k-5)$ $(b^2 - 4ac =)$ $-4k^2 + 8k + 96$ or $-(b^2 - 4ac =)$ $4k^2 - 8k - 96$ (with no prior algebraic errors) As $b^2 - 4ac > 0$, then $-4k^2 + 8k + 96 > 0$ and so, $k^2 - 2k - 24 < 0$	M1 A1 B1 A1 *
	Method 2: Considers $b^2 > 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ $6^2 > 4(k+3)(k-5)$ $4k^2 - 8k - 96 < 0$ or $-4k^2 + 8k + 96 > 0$ or $9 > (k+3)(k-5)$ (with no prior algebraic errors) and so, $k^2 - 2k - 24 < 0$ following correct work	M1 A1 B1 A1 * **[4]**
(b)	Attempts to solve $k^2 - 2k - 24 = 0$ to give $k =$ ($\Rightarrow$ Critical values, $k = 6, -4$.) $k^2 - 2k - 24 < 0$ gives $-4 < k < 6$	M1 M1 A1 **[3]** **7 marks**
	Notes	
(a)	**Method 1: M1:** Attempts $b^2 - 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ or uses quadratic formula and has this expression under square root. (ignore > 0, < 0 or = 0 for first 3 marks) **A1:** Correct expression for $b^2 - 4ac$ - need not be simplified (may be under root sign) **B1:** Uses algebra to manipulate result **without error** into **one of these** three term quadratics. Again may be under root sign in quadratic formula. If inequality is used early in "proof" may see $4k^2 - 8k - 96 < 0$ and B1 would be given for $4k^2 - 8k - 96$ correctly stated. **A1:** Applies $b^2 - 4ac > 0$ correctly (**or writes** $b^2 - 4ac > 0$ **)** to achieve the **result given** in the question. No errors should be seen. Any incorrect line of argument should be penalised here. There are several ways of reaching the answer; either multiplication of both sides of inequality by –1, or taking every term to other side of inequality. **Need conclusion i.e. printed answer.** **Method 2: M1:** Allow $b^2 > 4ac$, $b^2 < 4ac$ or $b^2 = 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ **A1:** Correct expressions on either side (ignore >, < or =). **B1:** Uses algebra to manipulate result into one of the two three term quadratics or divides both sides by 4 **again without error** **A1:** Produces result with no errors seen from initial consideration of $b^2 > 4ac$ **.**	
(b)	**M1:** Uses factorisation, formula, completion of square method to find two values for k , or finds two **correct** answers with no obvious method **M1:** Their Lower Limit < k < Their Upper Limit . Allow the M mark mark for $\leq$. (Allow k < upper and k > lower) **A1:** $-4 < k < 6$ Lose this mark for $\leq$ Allow (-4, 6) [not square brackets] or k > -4 **and** k < 6 (must be **and** not **or**) Can also use intersection symbol $\cap$ **NOT** k > -4, k < 6 (M1A0)	
	Special case : **In part (a)** uses $c = k$ instead of k - 5 - scores 0 . Allow k + 5 for method marks	
	Special Case: In part (b) Obtaining $-6 < k < 4$ This is a common wrong answer. Give M1 M1 A0 special case.	
	Special Case: In part (b) Use of x instead of k – M1M1A0	
	Special Case: $-4 < k < 6$ and k < -4, k > 6 both given is M0A0 for last two marks. Do not treat as isw.	

EXPERT TUITION

Question Number	Scheme	Marks
82. (a)	This may be done by completion of square or by expansion and comparing coefficients	
	$a = 4$	B1
	$b = 1$	B1
	All three of $a = 4$, $b = 1$ and $c = -1$	B1
		[3]
(b)		
	U shaped quadratic graph.	M1
	The curve is correctly positioned with the minimum in the third quadrant. . It crosses x axis twice on negative x axis and y axis once on positive y axis.	A1
	Curve cuts y-axis at $(\{0\}, 3)$.only	B1
	Curve cuts x-axis at $(-\frac{3}{2}, \{0\})$ and $(-\frac{1}{2}, \{0\})$.	B1
		[4]
		7 marks
	Notes	
(a)	**B1:** States a = 4 or obtains $4(x+b)^2 + c$,	
	B1: States b = 1 or obtains $a(x+1)^2 + c$,	
	B1: States a = 4, b = 1 **and** c = -1 or $4(x+1)^2 - 1$ (Needs all 3 correct for final mark)	
	Special cases: If answer is left as $(2x+2)^2 - 1$ treat as misread B1B0B0	
	or as $2(x+1)^2 - 1$ then the mark is B0B1B0 from scheme	
(b)	**M1:** Any position provided U shaped (be generous in interpretation of U shape but V shape is M0) **A1 :** The curve is correctly positioned with the minimum in the third quadrant. It crosses x axis twice on negative x axis and y axis once on positive y axis. **B1:** Allow 3 on y axis and allow either y = 3 or (0 , 3) if given in text Curve does not need to pass through this point and this mark may be given even if there is no curve at all or if it is drawn as a line. **B1:** Allow –3/2 and –1/2 if given on x axis – need co-ordinates if given in text or x = -3/2, x = -1/2 . Accept decimal equivalents. Curve does not need to pass through these points and this mark may be given even if there is no curve. Ignore third point of intersection and allow touching instead of cutting. So even a cubic curve *might* get M0A0 B1 B1. A V shape with two ruled lines for example might get M0A0B1B1	

Question Number	Scheme	Marks
83.	$\left\{\dfrac{2}{\sqrt{12}-\sqrt{8}}\right\} = \dfrac{2}{\left(\sqrt{12}-\sqrt{8}\right)} \times \dfrac{\left(\sqrt{12}+\sqrt{8}\right)}{\left(\sqrt{12}+\sqrt{8}\right)}$ Writing this is sufficient for M1.	M1
	$= \dfrac{\left\{2\left(\sqrt{12}+\sqrt{8}\right)\right\}}{12-8}$ For $12-8$. This mark can be implied.	A1
	$= \dfrac{2\left(2\sqrt{3}+2\sqrt{2}\right)}{12-8}$	B1 B1
	$= \sqrt{3}+\sqrt{2}$	A1 **cso**
		5
	Notes	

M1: for a correct method to rationalise the denominator.

1st A1: $\left(\sqrt{12}-\sqrt{8}\right)\left(\sqrt{12}+\sqrt{8}\right) \rightarrow 12-8$ or $\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right) \rightarrow 3-2$

1st B1: for $\sqrt{12} = 2\sqrt{3}$ or $\sqrt{48} = 4\sqrt{3}$ seen or implied in candidate's working.

2nd B1: for $\sqrt{8} = 2\sqrt{2}$ or $\sqrt{32} = 4\sqrt{2}$ seen or implied in candidate's working.

2nd A1: for $\sqrt{3}+\sqrt{2}$. Note: $\dfrac{\sqrt{3}+\sqrt{2}}{1}$ as a final answer is A0.

Note: The first accuracy mark is dependent on the first method mark being awarded.

Note: $\frac{1}{2}\sqrt{12} + \frac{1}{2}\sqrt{8} = \sqrt{3}+\sqrt{2}$ with no intermediate working implies the B1B1 marks.

Note: $\sqrt{12} = \sqrt{4}\sqrt{3}$ or $\sqrt{8} = \sqrt{4}\sqrt{2}$ are not sufficient for the B1 marks.

Note: A candidate who writes down (by misread) $\sqrt{18}$ for $\sqrt{8}$ can potentially obtain M1A0B1B1A0, where the 2nd B1 will be awarded for $\sqrt{18} = 3\sqrt{2}$ or $\sqrt{72} = 6\sqrt{2}$

Note: The final accuracy mark is for a correct solution only.

Alternative 1 solution

$\left\{\dfrac{2}{\sqrt{12}-\sqrt{8}}\right\} = \dfrac{2}{\left(2\sqrt{3}-2\sqrt{2}\right)}$ B1 B1

$= \dfrac{1}{\left(\sqrt{3}-\sqrt{2}\right)} \times \dfrac{\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)}$ M1

$= \dfrac{\left\{\left(\sqrt{3}+\sqrt{2}\right)\right\}}{3-2}$ A1 for $3-2$

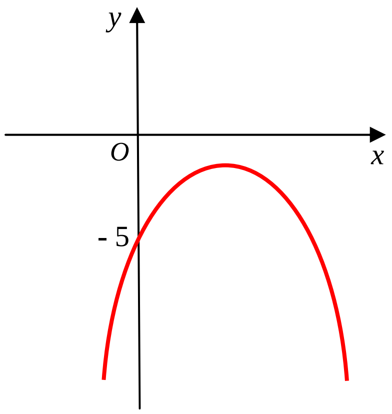
$= \sqrt{3}+\sqrt{2}$ A1

Please record the marks in the relevant places on the mark grid.

Alternative 2 solution

$\left\{\dfrac{2}{\sqrt{12}-\sqrt{8}}\right\} = \dfrac{2}{\left(2\sqrt{3}-2\sqrt{2}\right)} = \dfrac{1}{\left(\sqrt{3}-\sqrt{2}\right)} = \sqrt{3}+\sqrt{2}$, or $\dfrac{2}{\left(2\sqrt{3}-2\sqrt{2}\right)} = \sqrt{3}+\sqrt{2}$

with no incorrect working seen is awarded M1A1B1B1A1.

Question Number	Scheme	Marks
	$4x - 5 - x^2 = q - (x - p)^2$, p, q are integers.	
84. (a)	$\{4x - 5 - x^2 =\} \; -\left[x^2 - 4x + 5\right] = -\left[(x-2)^2 - 4 + 5\right] = -\left[(x-2)^2 + 1\right]$	M1
	$= -1 - (x-2)^2$	A1 A1
		[3]
(b)	$\{"b^2 - 4ac" = \}\; 4^2 - 4(-1)(-5) \quad \{= 16 - 20\}$	M1
	$= -4$	A1
		[2]
(c)		
	Correct ∩ shape	M1
	Maximum **within** the 4th quadrant	A1
	Curve cuts through -5 or $(0, -5)$ marked on the y-axis	B1
		[3]
		8

Notes

(a)

M1: for an attempt to complete the square eg: $\pm(\pm x \pm 2)^2 \pm k - 5,\; k \neq 0$ or $\pm(\pm x \pm 2)^2 \pm \lambda,\; \lambda \neq -5$ seen or implied in working.

1st A1: for $p = -2$ or for $\pm\alpha - (x-2)^2$, α can be 0.

2nd A1: for $q = -1$

Note: Allow M1A1A1 for a correct written down expression of $-1 - (x-2)^2$ Ignore $-1 - (x-2)^2 = 0$.

Note: If a candidate states either $p = -2$ or $q = -1$ or writes $\pm k - (x-2)^2$ then imply the M1 mark.

Note: A candidate who writes down with no working $p = 2$, $q = (\text{a value which is not } -1)$ gets M0A0A0.

Note: Writing $(x-2)^2 - 1$, followed by $p = -2$, $q = -1$ is M1A1A0.

Alternative 1 to (a)

$\{4x - 5 - x^2 =\} \; -\left[x^2 - 4x\right] - 5 = -\left[(x-2)^2 - 4\right] - 5 = -(x-2)^2 + 4 - 5 = -1 - (x-2)^2$

Alternative2 to (a)

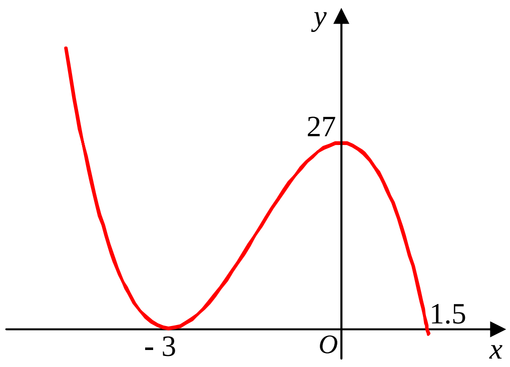
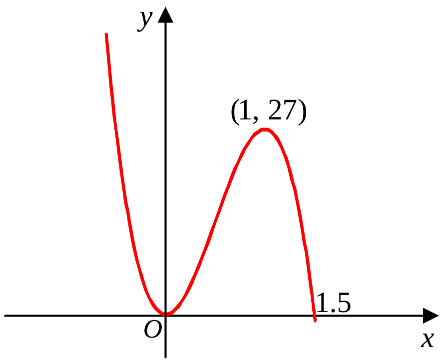
$q - (x+p)^2 = q - (x^2 + 2px + p^2) = -x^2 - 2px + q - p^2$

Compare x terms: $-2p = 4 \Rightarrow \underline{p = -2}$

Compare constant terms: $q - p^2 = -5 \Rightarrow q - 4 = -5 \Rightarrow \underline{q = -1}$

M1: Either $\pm 2p = 4$ or $q \pm p^2 = -5$; **1st A1:** for $p = -2$; **2nd A1:** for $q = -1$

	Alternative 3 to (a) Negating $4x-5-x^2$ gives x^2-4x+5 So, $x^2-4x+5=(x-2)^2-4+5$ $\{=(x-2)^2+1\}$ **M1** for $\pm(\pm x\pm 2)^2\pm k+5$ then stating $p=-2$ is **1st A1** and/or $q=-1$ is **2nd A1**. or writing $-1-(x-2)^2$ is A1A1. **Special Case for part (a):** $q-(x+p)^2=q-(x^2+2px+p^2)=-x^2-2px+q-p^2=4x-5-x^2$ $\Rightarrow -2px+q-p^2=4x-5 \Rightarrow q-p^2+5=4x+2px \Rightarrow q-p^2+5=x(4+2p)$ $\Rightarrow x=\dfrac{q-p^2+5}{4+2p} \Rightarrow p\neq -2$ scores Special Case M1A1A1 **only once** $p\neq -2$ achieved.
(b)	**M1:** for correctly substituting any two of $a=-1, b=4, c=-5$ into b^2-4ac if this is quoted. If b^2-4ac is not quoted then the substitution must be correct. Substitution into $b^2<4ac$ or $b^2=4ac$ or $b^2>4ac$ is M0. **A1:** for -4 only. If they write $-4<0$ treat the <0 as ISW and award A1. If they write $-4\geq 0$ then score A0. So substituting into $b^2-4ac<0$ leading to $-4<0$ can score M1A1 **Note:** Only award marks for use of the discriminant in part (b). **Note:** Award M0 if the candidate uses the quadratic formula UNLESS they later go on to identify that the discriminant is the result of b^2-4ac. **Beware: A number of candidates are writing up their solution to part (b) at the bottom of the second page. So please look!**
(c)	**M1:** Correct $\cap$ shape in any quadrant. **A1:** The maximum must be ***within*** the fourth quadrant to award this mark. **B1:** Curve ***(and not line!)*** cuts through -5 or $(0,-5)$ marked on the y-axis Allow $(-5, 0)$ rather than $(0,-5)$ if marked in the "correct" place on the y-axis. If the curve cuts through the negative y-axis and this is not marked, then you can recover $(0,-5)$ from the candidate's working in part (c). You are not allowed to recover this point, though, from a table of values. **Note:** Do not recover work for part (a) in part (c).

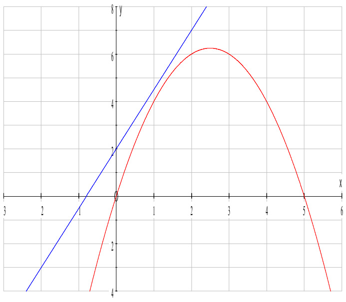
Question Number	Scheme		Marks
85. (a)	{Coordinates of A are} $(4.5, 0)$	See notes below	B1 **[1]**
(b)(i)		Horizontal translation	M1
		-3 and their ft 1.5 on postitive x-axis	A1 ft
		Maximum at 27 marked on the y-axis	B1 **[3]**
(ii)		Correct shape, minimum at $(0, 0)$ and a maximum within the first quadrant.	M1
		1.5 on x-axis	A1 ft
		Maximum at $(1, 27)$	B1 **[3]**
(c)	$\{k =\}\ -17$		B1 **[1]**
			8

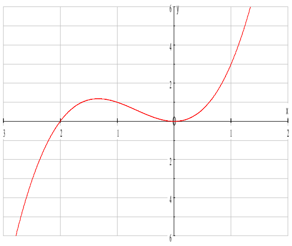
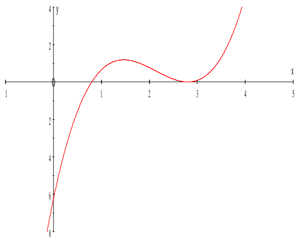
	Notes
(a)	**B1:** For stating either $x = 4.5$ or $\frac{9}{2}$ or $\frac{18}{4}$ etc. or $A = 4.5$ or $\frac{9}{2}$ or $(4.5, 0)$. Can be written on graph. Allow $(0, 4.5)$ marked on curve for B1. Otherwise $(0, 4.5)$ without reference to any of the above is B0.
(b)(i)	**M1:** for any horizontal (left-right) translation where minimum is still **on** x-axis not at $(0, 0)$. Ignore any values.
	A1ft: for -3 **(NOT 3)** and 1.5 (or their x in part (a) – 3) ***evaluated*** and marked on the positive x-axis. Allow $(0, -3)$ and/or $(0, \text{ft } 1.5)$ rather than $(-3, 0)$ and $(\text{ft } 1.5, 0)$ if marked in the "correct" place on the x-axis. **Note:** Candidate ***cannot*** gain this mark if their x in part (a) is less than 3.
	B1: Maximum at 27 marked on the y-axis. **Note**: the maximum must be on the y-axis for this mark.
(ii)	**M1:** for correct shape with minimum still at $(0, 0)$ and a maximum within the first quadrant. Ignore values.
	A1ft: for $\dfrac{\text{their } x \text{ in part } (a)}{3}$; as intercept on x-axis eg: $\dfrac{4.5}{3}$ or 1.5 or $\dfrac{3}{2}$ or $\dfrac{9}{6}$ **Note:** a generalised $\dfrac{A}{3}$ is A0. Allow $(0, \text{ft } 1.5)$ rather than $(\text{ft } 1.5, 0)$ if marked in the "correct" place on the x-axis.
	B1: Maximum at $(1, 27)$ or allow 1 marked on the x-axis and the corresponding 27 marked on the y-axis.
	Note: Be careful to look at the correct graph. The candidate may draw another graph to help them to answer part (c).
	Note: You can recover (b)(i) $(-3, 0)$ and $(\text{ft } 1.5, 0)$ or in (b)(ii) $(\text{ft } 1.5, 0)$ as ***correct coordinates only*** in candidate's working if these are not marked on their sketch(es).
(c)	**B1:** for $(k =)\ -17$ only. **BEWARE**: This could be written in the middle or at the bottom of a page.

Question	Scheme	Marks
86(a)	$\sqrt{32} = 4\sqrt{2}$ or $\sqrt{18} = 3\sqrt{2}$	B1
	$\left(\sqrt{32}+\sqrt{18}=\right)$ $\underline{7\sqrt{2}}$	B1 (2)
(b)	$\times\dfrac{3-\sqrt{2}}{3-\sqrt{2}}$ or $\times\dfrac{-3+\sqrt{2}}{-3+\sqrt{2}}$ seen	M1
	$\left[\dfrac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}\times\dfrac{3-\sqrt{2}}{3-\sqrt{2}}=\right]$ $\dfrac{a\sqrt{2}\left(3-\sqrt{2}\right)}{[9-2]}$ $\rightarrow$ $\dfrac{3a\sqrt{2}-2a}{[9-2]}$ (or better)	dM1
	$=$ $\underline{3\sqrt{2}, -2}$	A1, A1 (4)
ALT	$(b\sqrt{2}+c)(3+\sqrt{2}) = 7\sqrt{2}$ leading to: $3b+c=7$, $3c+2b=0$ e.g. $3(7-3b)+2b=0$ (o.e.)	M1 dM1
		6 marks
	Notes	
(a)	1st B1 for either surd simplified 2nd B1 for $7\sqrt{2}$ or accept $a = 7$. Answer only scores B1B1 NB Common error is $\sqrt{32}+\sqrt{18}=\sqrt{50}=5\sqrt{2}$ this scores B0B0 but can use their "5" in (b) to get M1M1	
(b)	1st M1 for an attempt to multiply by $\dfrac{3-\sqrt{2}}{3-\sqrt{2}}$ (o.e.) Allow poor use of brackets 2nd dM1 for using $a\sqrt{2}$ to correctly obtain a numerator of the form $p + q\sqrt{2}$ where p and q are non-zero integers. Allow arithmetic slips e.g. $21\sqrt{2}-28$ or $3\sqrt{2}\times\sqrt{2}=3$ Follow through their $a = 7$ or a new value found in (b). Ignore denominator. Allow use of letter a. Dependent on 1st M1 So $3\sqrt{32}-\sqrt{64}+3\sqrt{8}-\sqrt{36}$ is M0 until they reduce $p + q\sqrt{2}$ 1st A1 for $3\sqrt{2}$ or accept $b = 3$ from correct working 2nd A1 for -2 or accept $c = -2$ from correct working	
ALT	**Simultaneous Equations** 1st M1 for $(b\sqrt{2}+c)(3+\sqrt{2}) = 7\sqrt{2}$ and forming 2 simultaneous equations. Ft their $a = 7$ 2nd dM1 for solving their simultaneous equations: reducing to a linear equation in one variable	

Question	Scheme	Marks
87. **(a)**	$5x > 20$	M1
	$\underline{\boldsymbol{x > 4}}$	A1 (2)
(b)	$x^2 - 4x - 12 = 0$	
	$(x+2)(x-6)[=0]$	M1
	$x = 6,\ -2$	A1
	$x < -2\ ,\ x > 6$	M1, A1ft
		(4)
		6 marks
	Notes	

(a)	M1 for reducing to the form $px > q$ with one of p or q correct Using $px = q$ is M0 unless > appears later on A1 $x > 4$ only
(b)	1st M1 for multiplying out and attempting to solve a 3TQ with at least $\pm 4x$ or ± 12 See **General Principles** for definitions of "attempt to solve" 1st A1 for 6 and -2 seen. Allow $x > 6$, $x > -2$ etc to score this mark. Values may be on a sketch. 2nd M1 for choosing the "outside region" for their critical values. Do not award simply for a diagram or table – they must have chosen their "outside" regions 2nd A1ft follow through their 2 distinct critical values. Allow "," "or" or a "blank" between answers. Use of "and" is M1A0 i.e. loses the final A1 $-2 > x > 6$ scores M1A0 i.e. loses the final A1 but apply ISW if $x > 6$, $x < -2$ has been seen Accept $(-\infty,-2)\cup(6,\infty)$ (o.e) Use of ≤ instead of < (or ≥ instead of >) loses the final A mark in (b) unless A mark was lost in (a) for $x \geq 4$ in which case allow it here.

Question	Scheme	Marks
88. (a)	$x(5-x)=\frac{1}{2}(5x+4)$ (o.e.)	M1
	$2x^2-5x+4(=0)$ (o.e.) e.g. $x^2-2.5x+2(=0)$	A1
	$b^2-4ac=(-5)^2-4\times2\times4$	M1
	$=25-32$ <0, so no roots <u>or</u> no intersections <u>or</u> no solutions	A1 (4)
(b)	 Curve: $\cap$ shape and passing through (0, 0)	B1
	$\cap$ shape and passing through (5, 0)	B1
	Line : +ve gradient and no intersections with *C*. If no *C* drawn score B0	B1
	Line passing through (0, 2) and $(-0.8, 0)$ marked on axes	B1 (4)
		8 marks
	Notes	
(a)	1st M1 for forming a suitable equation in one variable 1st A1 for a correct 3TQ equation. Allow missing "= 0" Accept $2x^2+4=5x$ etc 2nd M1 for an attempt to evaluate discriminant for their 3TQ. Allow for $b^2>4ac$ or $b^2<4ac$ Allow if it is part of a solution using the formula e.g. $(x=)\frac{5\pm\sqrt{25-32}}{4}$ Correct formula quoted and some correct substitution or a correct expression False factorising is M0 2nd A1 for correct evaluation of discriminant for a correct 3TQ e.g. 25 – 32 (or better) <u>and</u> a comment indicating no roots or equivalent. For <u>contradictory</u> statements score A0	
ALT	2nd M1 for attempt at completing the square $a\left[\left(x\pm\frac{b}{2a}\right)^2-q\right]+c$ 2nd A1 for $\left(x-\frac{5}{4}\right)^2=-\frac{7}{16}$ and a suitable comment	
(b)	**Coordinates must be seen <u>on</u> the diagram. Do not award if only in the body of the script.** **"Passing through" means <u>not</u> stopping at and <u>not</u> touching.** **Allow (0, *x*) and (*y*, 0) if marked on the correct places on the correct axis.** 1st B1 for correct shape and passing through origin. Can be assumed if it passes through the intersection of axes 2nd B1 for correct shape and 5 marked on *x*-axis	
SC	for $\cap$ shape stopping at <u>both</u> (5, 0) <u>and</u> (0, 0) award B0B1 3rd B1 for a line of positive gradient that (if extended) has no intersection with their *C* (possibly extended). Must have both graphs on same axes for this mark. If no *C* given score B0 4th B1 for straight line passing through -0.8 on *x*-axis and 2 on *y*-axis Accept exact fraction equivalents to -0.8 or 2(e.g. $\frac{4}{2}$)	

Question	Scheme	Marks
89. (a)	$[\,y = x^3 + 2x^2\,]$ so $\dfrac{dy}{dx} = 3x^2 + 4x$	M1A1 (2)
(b)	 Shape 	B1
	Touching x-axis at origin	B1
	Through and not touching or stopping at -2 on x –axis. Ignore extra intersections.	B1 (3)
(c)	At $x = -2$: $\dfrac{dy}{dx} = 3(-2)^2 + 4(-2) = 4$	M1
	At $x = 0$: $\dfrac{dy}{dx} = 0$ (Both values correct)	A1 (2)
(d)	 Horizontal translation (touches x-axis still)	M1
	$k-2$ and k marked on positive x-axis	B1
	$k^2(2-k)$ (o.e) marked on negative y-axis	B1 (3)
		10 marks

	Notes
(a)	M1 for attempt to multiply out and then some attempt to differentiate $x^n \to x^{n-1}$ Do not award for $2x(x + 2)$ or $2x(1 + 2)$ etc
Prod Rule	Award M1 for a correct attempt: 2 products with a + and at least one product correct A1 for both terms correct. (If $+c$ or extra term is included score A0)
(b)	1st B1 for correct shape (anywhere). Must have 2 clear turning points. 2nd B1 for graph touching at origin (not crossing or ending) 3rd B1 for graph passing through (not stopping or touching at) -2 on x axis and -2 marked on axis
SC	B0B0B1 for $y = x^3$ <u>or</u> cubic with straight line between $(-2, 0)$ and $(0, 0)$
(c)	M1 for attempt at $y'(0)$ or $y'(-2)$. Follow through their 0 or – 2 and their $y'(x)$ <u>or</u> for a <u>correct</u> statement of zero gradient for an identified point on their curve that touches x-axis A1 for both correct answers
(d)	For the M1 in part (d) ignore any coordinates marked – just mark the shape. M1 for a horizontal translation of their (b). Should still touch x – axis if it did in (b) <u>Or</u> for a graph of correct shape with min. and intersection in correct order on +ve x-axis 1st B1 for k and k – 2 on the positive x-axis. Curve must pass through k – 2 and touch at k 2nd B1 for a correct intercept on negative y-axis in terms of k. Allow $(0, 2k^2 - k^3)$ (o.e.) seen in script if curve passes through –ve y-axis

Question Number	Scheme			Marks
90.	Either	Or		
	$y^2 = 4 - 4x + x^2$	$x^2 = 4 - 4y + y^2$		M1
	$4(4 - 4x + x^2) - x^2 = 11$ or $4(2-x)^2 - x^2 = 11$	$4y^2 - (4 - 4y + y^2) = 11$ or $4y^2 - (2-y)^2 = 11$		M1
	$3x^2 - 16x + 5 = 0$	$3y^2 + 4y - 15 = 0$	Correct 3 terms	A1
	$(3x-1)(x-5) = 0, \quad x =$	$(3y-5)(y+3) = 0, \quad y = ...$		M1
	$x = \frac{1}{3} \quad x = 5$	$y = \frac{5}{3} \quad y = -3$		A1
	$y = \frac{5}{3} \quad y = -3$	$x = \frac{1}{3} \quad x = 5$		M1 A1
				(7) **7**

Notes

1st M: Squaring to give 3 or 4 terms (need a middle term)

2nd M: Substitute to give quadratic in one variable (may have just two terms)

3rd M: Attempt to solve a **3 term** quadratic.

4th M: Attempt to find at least one y value (or x value). (The second variable)

This will be by substitution or by starting again.

If y solutions are given as x values, or vice-versa, penalise accuracy, so that it is possible to score M1 M1A1 M1 A0 M1 A0.

<u>"Non-algebraic" solutions:</u>

No working, and only one correct solution pair found (e.g. $x = 5$, $y = -3$):
M0 M0 A0 M1 A0 M1 A0

No working, and both correct solution pairs found, but not demonstrated:
M0 M0 A0 M1 A1 M1 A1

Both correct solution pairs found, and demonstrated: Full marks are possible (send to review)

Question Number	Scheme	Marks
91. **(a)**	Discriminant: $b^2 - 4ac = (k+3)^2 - 4k$ or equivalent	M1 A1 (2)
(b)	$(k+3)^2 - 4k = k^2 + 2k + 9 = (k+1)^2 + 8$	M1 A1 (2)
(c)	For real roots, $b^2 - 4ac \geq 0$ or $b^2 - 4ac > 0$ or $(k+1)^2 + 8 > 0$ $(k+1)^2 \geq 0$ for all k, so $b^2 - 4ac > 0$**,** so roots are real for all k (or equiv.)	M1 A1 cso (2) **6**

Notes

(a) M1: attempt to find discriminant – substitution is required

If formula $b^2 - 4ac$ is seen at least 2 of a, b and c must be correct

If formula $b^2 - 4ac$ is **not** seen all 3 of a, b and c must be correct

Use of $b^2 + 4ac$ is M0

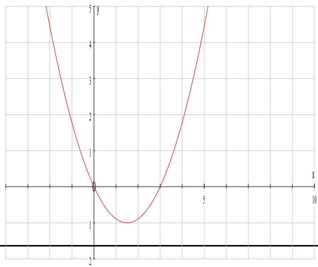
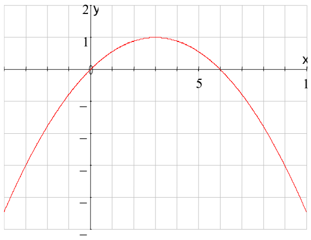
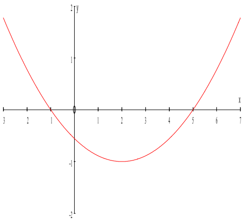
A1: correct unsimplified

(b) M1: Attempt at completion of square (see earlier notes)

A1: both correct (no ft for this mark)

(c) M1: States condition as on scheme **or** attempts to explain that their $(k+1)^2 + 8$ is greater than 0

A1: The final mark (A1cso) requires $(k+1)^2 \geq 0$ and conclusion. We will allow $(k+1)^2 > 0$ (or word positive) also allow $b^2 - 4ac \geq 0$ and conclusion.

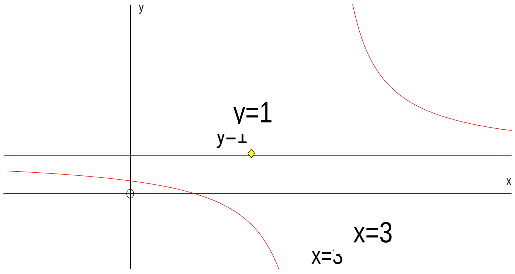
Question Number	Scheme		Marks
92. **(a)**		Shape  through (0, 0) (3, 0) (1.5, –1)	B1 B1 B1 (3)
(b)		Shape  (0, 0) and (6, 0) (3, 1)	B1 B1 B1 (3)
(c)		Shape  , <u>not</u> through (0, 0) Minimum in 4th quadrant $(-p, 0)$ and $(6-p, 0)$ $(3-p, -1)$	M1 A1 B1 B1 (4) **10**

<u>Notes</u>

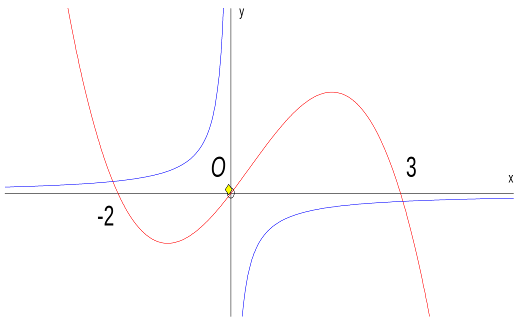
(a) B1: U shaped parabola through origin
B1: (3,0) stated or 3 labelled on x axis
B1: (1.5, -1) or equivalent e.g. (3/2, -1)
(b) B1: Cap shaped parabola in any position

B1: through origin (may not be labelled) and (6,0) stated or 6 labelled on x - axis
B1: (3,1) shown
(c) M1: U shaped parabola not through origin
A1: Minimum in 4th quadrant (depends on M mark having been given)
B1: Coordinates stated or shown on x axis
B1: Coordinates stated
Note: If values are taken for p, then it is possible to give M1A1B0B0 even if there are several attempts. (In this case all minima should be in fourth quadrant)

Question Number	Scheme	Marks
93.	$\frac{5-2\sqrt{3}}{\sqrt{3}-1}\times\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$	M1
	$=\frac{\dots}{2}$ denominator of 2	A1
	Numerator = $5\sqrt{3}+5-2\sqrt{3}\sqrt{3}-2\sqrt{3}$	M1
	So $\frac{5-2\sqrt{3}}{\sqrt{3}-1}=-\frac{1}{2}+\frac{3}{2}\sqrt{3}$	A1
		4
	Alternative: $(p+q\sqrt{3})(\sqrt{3}-1)=5-2\sqrt{3}$, and form simultaneous equations in *p* and *q*	M1
	$-p + 3q = 5$ and $p - q = -2$	A1
	Solve simultaneous equations to give $p = -\frac{1}{2}$ and $q=\frac{3}{2}$.	M1 A1
	Notes	
	1st M1 for multiplying numerator and denominator by same correct expression 1st A1 for a correct denominator as a single number (NB depends on M mark) 2nd M1 for an attempt to multiply the numerator by $(\sqrt{3}\pm1)$ and get 4 terms with at least 2 correct. 2nd A1 for the answer as written or $p = -\frac{1}{2}$ and $q=\frac{3}{2}$. Allow -0.5 and 1.5. (Apply isw if correct answer seen, then slip writing *p* =, *q* =)	
	Answer only (very unlikely) is full marks if correct – no part marks	

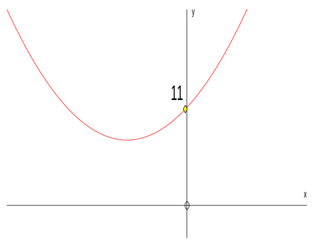
Question Number	Scheme	Marks
94. **(a)**	 Correct shape with a single crossing of each axis y = 1 labelled or stated x = 3 labelled or stated	B1 B1 B1 (3)
(b)	Horizontal translation so crosses the x-axis at (1, 0) New equation is $(y=)\dfrac{x\pm1}{(x\pm1)-2}$ When x = 0 y = $=\dfrac{1}{3}$	B1 M1 M1 A1 (4) **7**
	Notes	
(b)	B1 for point (1,0) identified - this may be marked on the sketch as 1 on x axis. Accept x = 1. 1st M1 for attempt at new equation and either numerator or denominator correct 2nd M1 for setting x = 0 in their new equation and solving as far as y =… A1 for $\frac{1}{3}$ or exact equivalent. Must see $y=\frac{1}{3}$ or (0, $\frac{1}{3}$) or point marked on y-axis. **Alternative** $f(-1)=\dfrac{-1}{-1-2}=\dfrac{1}{3}$ scores M1M1A0 unless x =0 is seen or they write the point as (0, $\frac{1}{3}$) or give y = 1/3 Answers only: x = 1, y = 1/3 is full marks as is (1,0) (0, 1/3) Just 1 and 1/3 is B0 M1 M1 A0 Special case : Translates 1 unit to left (a) B0, B1, B0 (b) Mark (b) as before May score B0 M1 M1 A0 so 3/7 or may ignore sketch and start again scoring full marks for this part.	

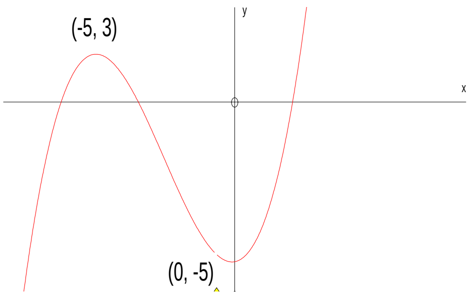
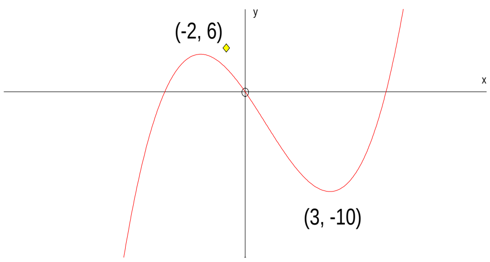
Question Number	Scheme	Marks
95. **(a)**	$b^2 - 4ac = (k-3)^2 - 4(3-2k)$	M1
	$k^2 - 6k + 9 - 4(3-2k) > 0$ or $(k-3)^2 - 12 + 8k > 0$ or better	M1
	$\underline{k^2 + 2k - 3 > 0}$ *	A1cso
		(3)
(b)	$(k+3)(k-1)[=0]$	M1
	Critical values are $k = 1$ or -3	A1
	(choosing "outside" region)	M1
	$\underline{k > 1 \text{ or } k < -3}$	A1 cao
		(4)
		7
	Notes	
(a)	1st M1 for attempt to find $b^2 - 4ac$ with one of b or c correct 2nd M1 for a correct inequality symbol and an attempt to expand. A1cso no incorrect working seen	
(b)	1st M1 for an attempt to factorize **or** solve leading to k = (2 values) 2nd M1 for a method that leads them to choose the "outside" region. Can follow through their critical values. 2nd A1 Allow "," instead of "or" $\geq$ loses the final A1 $1 < k < -3$ scores M1A0 unless a correct version is seen before or after this one.	

Question Number	Scheme	Marks
96. **(a)**	 (i) correct shape (-ve cubic) Crossing at (-2, 0) Through the origin Crossing at (3,0) (ii) 2 branches in correct quadrants not crossing axes One intersection with cubic on each branch	B1 B1 B1 B1 B1 B1 (6)
(b)	"2" solutions Since only "2" intersections	B1ft dB1ft (2) **8**
	Notes	
(b)	B1ft for a value that is compatible with their sketch dB1ft This mark is dependent on the value being compatible with their sketch. For a comment relating the number of solutions to the number of intersections. [Only allow 0, 2 or 4]	

Question Number	Scheme	Marks
97.	$\left(\sqrt{75}-\sqrt{27}\right)=5\sqrt{3}-3\sqrt{3}$ $= 2\sqrt{3}$	M1 A1 **2**
	Notes	
	M1 for $5\sqrt{3}$ from $\sqrt{75}$ or $3\sqrt{3}$ from $\sqrt{27}$ seen anywhere A1 for $2\sqrt{3}$; allow $\sqrt{12}$ or or $k=2, x=3$ allow $k=1, x=12$ Some Common errors $\sqrt{75}-\sqrt{27}=\sqrt{48}$ leading to $4\sqrt{3}$ is M0A0 $25\sqrt{3}-9\sqrt{3}=16\sqrt{3}$ is M0A0	

Question Number	Scheme	Marks
98. **(a)**	$3x-6<8-2x \rightarrow \ 5x<14$ (Accept $5x-14<0$ (o.e.))	M1
	$x<2.8$ or $\frac{14}{5}$ or $2\frac{4}{5}$ (condone $\le$)	A1 (2)
(b)	Critical values are $x=\frac{7}{2}$ and -1	B1
	Choosing "inside" $-1<x<\frac{7}{2}$	M1 A1 (3)
(c)	$-1<x<2.8$	B1ft (1)
	Accept any exact equivalents to -1, 2.8, 3.5	**6**
	<u>Notes</u>	
(a)	M1 for attempt to rearrange to $kx < m$ (o.e.) Either $k = 5$ or $m = 14$ should be correct Allow $5x = 14$ or even $5x > 14$	
(b)	B1 for both correct critical values. (May be implied by a correct inequality) M1 ft their values and choose the "inside" region A1 for fully correct inequality (Must be in part (b): do not give marks if only seen in (c)) Condone seeing $x < -1$ in working provided $-1 < x$ is in the final answer. e.g. $x>-1,\ x<\frac{7}{2}$ <u>or</u> $x>-1$ "or" $x<\frac{7}{2}$ <u>or</u> $x>-1$ "blank space" $x<\frac{7}{2}$ score M1A0 BUT allow $x>-1$ and $x<\frac{7}{2}$ to score M1A1 (the "and" must be seen) Also $\left(-1,\frac{7}{2}\right)$ will score M1A1 NB $x<-1, x<\frac{7}{2}$ is of course M0A0 and a number line even with "open" ends is M0A0 Allow 3.5 instead of $\frac{7}{2}$	
(c)	B1ft for $-1<x<2.8$ (ignoring their previous answers) <u>or</u> ft their answers to part (a) and part (b) provided both answers were regions and not single values. Allow use of "and" between inequalities as in part (b) If their set is empty allow a suitable description in words or the symbol $\varnothing$. <u>Common error:</u> If (a) is correct and in (b) they simply leave their answer as $x<-1$, $x < 3.5$ then in (c) $x < -1$ would get B1ft as this is a correct follow through of these 3 inequalities. Penalise use of $\le$ only on the A1 in part (b). [i.e. condone in part (a)]	

Question Number	Scheme	Marks
99. **(a)**	$(x+3)^2+2$ or $p=3$ or $\frac{6}{2}$ $q=2$	B1 B1 (2)
(b)	 U shape with min in 2nd quad (Must be above *x*-axis and not on *y*=axis) U shape crossing *y*-axis at (0, 11) only (Condone (11,0) marked on *y*-axis)	B1 B1 (2)
(c)	$b^2-4ac=6^2-4\times11$ $= \underline{-8}$	M1 A1 (2) **6**
	Notes	
(a)	Ignore an "= 0" so $(x+3)^2+2=0$ can score both marks	
(b)	The U shape can be interpreted fairly generously. Penalise an obvious V on 1st B1 only. The U needn't have equal "arms" as long as there is a clear min that "holds water" 1st B1 for U shape with minimum in 2nd quad. Curve need not cross the *y*-axis but minimum should NOT touch *x*-axis and should be left of (not on) *y*-axis 2nd B1 for U shaped curve crossing at (0, 11). Just 11 marked on *y*-axis is fine. The point must be marked on the sketch (do not allow from a table of values) Condone stopping at (0, 11)	
(c)	M1 for some correct substitution into b^2-4ac. This may be as part of the quadratic formula but must be in part (c) and must be only numbers (no *x* terms present). Substitution into $b^2<4ac$ or $b^2=4ac$ or $b^2>4ac$ is M0 A1 for -8 only. If they write $-8<0$ treat the <0 as ISW and award A1 If they write $-8\geq 0$ then score A0 A substitution in the quadratic formula leading to -8 inside the square root is A0. So substituting into $b^2-4ac<0$ leading to $-8<0$ can score M1A1. Only award marks for use of the discriminant in part (c)	

Question Number	Scheme		Marks	
100.				
(a)		Horizontal translation of ± 3	M1	
		$(-5, 3)$ marked on sketch **or in text**	B1	
		$(0, -5)$ and min intentionally on *y*-axis Condone $(-5, 0)$ if correctly placed on negative *y*-axis	A1	(3)
(b)		Correct shape and intentionally through (0,0) between the max and min	B1	
		$(-2, 6)$ marked on graph **or in text**	B1	
		$(3, -10)$ marked on graph **or in text**	B1	(3)
(c)	$(a =)$ <u>5</u>		B1	(1)

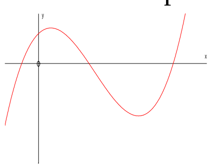
<u>Notes</u>

Turning points (not on axes) should have both co-ordinates given in form(x,y) .
Do not accept points marked on axes e.g. -5on *x*-axis and 3 on *y*-axis is not sufficient.
For repeated offenders apply this penalty **once only** at first offence and condone elsewhere.

In (a) and (b) no graphs means no marks.

In (a) and (b) the ends of the graphs do not need to cross the axes provided max and min are clear

(a)

M1 for a horizontal translation of ± 3 so accept  i.e max in 1st quad <u>and</u> coordinates of $(1, 3)$ <u>or</u> $(6, -5)$ seen.
[Horizontal translation to the left should have a min <u>on</u> the *y*-axis]
If curve passes through (0,0) then M0 (and A0) but they could score the B1 mark.

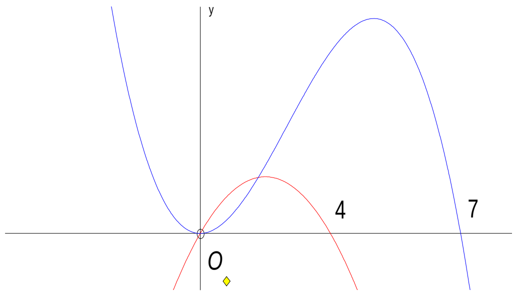
A1 for minimum clearly on negative *y*-axis and at least -5 marked on *y*-axis.
Allow this mark if the minimum is very close and the point $(0, -5)$ clearly indicated

(b)

1st B1 Ignore coordinates for this mark
Coordinates or points on sketch override coordinates given in the text.
Condone (y, x) confusion for points on axes only. So $(-5,0)$ for $(0, -5)$ is OK if the point is marked correctly but (3,10) is B0 even if in 4th quadrant.

(c)

This may be at the bottom of a page or in the question...make sure you scroll up and down!

Question Number	Scheme	Marks	
101. (a)	 (i) ∩ shape (anywhere on diagram)	B1	
	Passing through or stopping at (0, 0) and (4,0) only(Needn't be ∩ shape)	B1	
	(ii) correct shape (-ve cubic) with a max and min drawn anywhere	B1	
	Minimum or maximum at (0,0)	B1	
	Passes through or stops at (7,0) but <u>NOT</u> touching. (7, 0) should be to right of (4,0) or B0	B1	(5)
	Condone (0,4) or (0, 7) marked correctly on *x*-axis. Don't penalise poor overlap near origin. **Points must be marked on the sketch...not in the text**		
(b)	$x\left(4-x\right)=x^2\left(7-x\right)$ $(0=)x[7x-x^2-(4-x)]$	M1	
	$(0=)x[7x-x^2-(4-x)]$ (o.e.)	B1ft	
	$0=x\left(x^2-8x+4\right)$ *	A1 cso	(3)
(c)	$\left(0=x^2-8x+4\Rightarrow\right)x=\dfrac{8\pm\sqrt{64-16}}{2}$ or $\left(x\pm4\right)^2-4^2+4(=0)$	M1	
	$\left(x-4\right)^2=12$	A1	
	$=\dfrac{8\pm4\sqrt{3}}{2}$ or $\left(x-4\right)=\pm2\sqrt{3}$	B1	
	$x=4\pm2\sqrt{3}$	A1	
	From sketch *A* is $x=4-2\sqrt{3}$	M1	
	So $y=\left(4-2\sqrt{3}\right)\left(4-[4-2\sqrt{3}]\right)$ (dependent on 1st M1)	M1	
	$\underline{=-12+8\sqrt{3}}$	A1	(7)
			15
	Notes		
(b)	M1 for forming a suitable equation B1 for a common factor of *x* taken out legitimately. Can treat this as an M mark. Can ft their cubic = 0 found from an attempt at solving their equations e.g. $x^3-8x^2-4x=x(\ldots$ A1cso no incorrect working seen. The "= 0" is required but condone missing from some lines of working. Cancelling the *x* scores B0A0.		
(c)	1st M1 for some use of the correct formula or attempt to complete the square 1st A1 for a fully correct expression: condone + instead of <u>+</u> **or** for $\left(x-4\right)^2=12$ B1 for simplifying $\sqrt{48}=4\sqrt{3}$ or $\sqrt{12}=2\sqrt{3}$ .Can be scored independently of this expression 2nd A1 for correct solution of the form $p+q\sqrt{3}$: can be <u>+</u> or + or − 2nd M1 for selecting their answer in the interval (0,4). If they have no value in (0,4) score M0 3rd M1 for attempting *y* =…using their *x* in correct equation. An expression needed for M1A0 3rd A1 for correct answer. If 2 answers are given A0.		

Question number	Scheme	Marks
102.	(a) $(7+\sqrt{5})(3-\sqrt{5}) = 21 - 5 + 3\sqrt{5} - 7\sqrt{5}$ Expand to get 3 or 4 terms	M1
	$= 16, \ -4\sqrt{5}$ (1st A for 16, 2nd A for $-4\sqrt{5}$)	A1, A1
	(i.s.w. if necessary, e.g. $16 - 4\sqrt{5} \rightarrow 4 - \sqrt{5}$)	(3)
	(b) $\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ (This is sufficient for the M mark)	M1
	Correct denominator without surds, i.e. $9-5$ or 4	A1
	$4-\sqrt{5}$ or $4-1\sqrt{5}$	A1 (3) **[6]**
	(a) M1: Allowed for an attempt giving 3 or 4 terms, with at least 2 correct (even if unsimplified). e.g. $21-\sqrt{5}^2+\sqrt{15}$ scores M1. Answer only: $16-4\sqrt{5}$ scores full marks One term correct scores the M mark by implication, e.g. $26-4\sqrt{5}$ scores M1 A0 A1 (b) Answer only: $4-\sqrt{5}$ scores full marks One term correct scores the M mark by implication, e.g. $4+\sqrt{5}$ scores M1 A0 A0 $16-\sqrt{5}$ scores M1 A0 A0 Ignore subsequent working, e.g. $4-\sqrt{5}$ so $a=4, \ b=1$ Note that, as always, A marks are dependent upon the preceding M mark, so that, for example, $\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3+\sqrt{5}}{3-\sqrt{5}} = \frac{\ldots\ldots\ldots}{4}$ is M0 A0. <u>Alternative</u> $(a+b\sqrt{5})(3+\sqrt{5}) = 7+\sqrt{5}$, then form simultaneous equations in a and b. M1 Correct equations: $3a+5b=7$ and $3b+a=1$ A1 $a=4$ and $b=-1$ A1	

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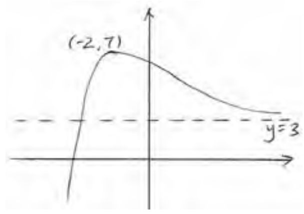
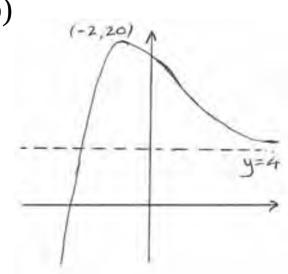
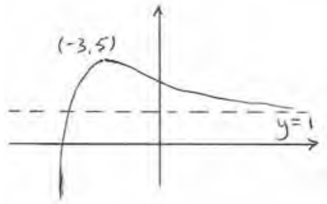
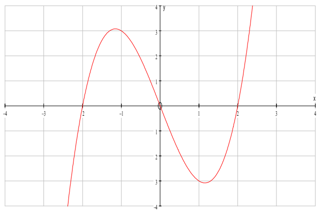
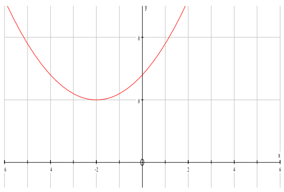
Question number	Scheme	Marks
103.	$y = 3x - 2 \qquad (3x-2)^2 - x - 6x^2 (= 0)$	M1
	$9x^2 - 12x + 4 - x - 6x^2 = 0$ $3x^2 - 13x + 4 = 0$ (or equiv., e.g. $3x^2 = 13x - 4$)	M1 A1cso
	$(3x-1)(x-4) = 0 \qquad x = \ldots \qquad x = \frac{1}{3}$ (or <u>exact</u> equivalent) $x = 4$	M1 A1
	$y = -1 \quad y = 10$ (Solutions need not be "paired")	M1 A1
		[7]
	1st M: Obtaining an equation in x only (or y only). Condone missing "= 0" Condone sign slips, e.g. $(3x+2)^2 - x - 6x^2 = 0$, but <u>not</u> other algebraic mistakes (such as squaring individual terms... see bottom of page). 2nd M: Multiplying out their $(3x-2)^2$, which must lead to a 3 term quadratic, i.e. $ax^2 + bx + c$, where $a \neq 0, b \neq 0, c \neq 0$, <u>and</u> collecting terms. 3rd M: Solving a 3-term quadratic (see general principles at end of scheme). 2nd A: Both values. 4th M: Using an x value, found algebraically, to attempt at least one y value (or using a y value, found algebraically, to attempt at least one x value)… allow b.o.d. for this mark in cases where the value is wrong but working is not shown. 3rd A: Both values. If y solutions are given as x values, or vice-versa, penalise at the end, so that it is possible to score M1 M1A1 M1 A1 M0 A0. <u>"Non-algebraic" solutions:</u> No working, and only one correct solution pair found (e.g. $x = 4$, $y = 10$): M0 M0 A0 M0 A0 M1 A0 No working, and both correct solution pairs found, but not demonstrated: M0 M0 A0 M1 A1 M1 A1 Both correct solution pairs found, and demonstrated: Full marks <u>Alternative</u>: $x = \frac{y+2}{3} \qquad y^2 - \frac{y+2}{3} - 6\left(\frac{y+2}{3}\right)^2 = 0$ M1 $y^2 - \frac{y+2}{3} - 6\left(\frac{y^2+4y+4}{9}\right) = 0 \qquad y^2 - 9y - 10 = 0$ M1 A1 $(y+1)(y-10) = 0 \qquad y = \ldots \qquad y = -1 \quad y = 10$ M1 A1 $x = \frac{1}{3} \quad x = 4$ M1 A1 <u>Squaring each term in the first equation</u>, e.g. $y^2 - 9x^2 + 4 = 0$, and using this to obtain an equation in x only could score at most 2 marks: M0 M0 A0 M1 A0 M1 A0.	

Question number	Scheme	Marks
104.	(a)  (b)  (c) 	
	(a) (–2, 7), $y = 3$ (Marks are dependent upon a sketch being attempted) See conditions below.	B1, B1 (2)
	(b) (–2, 20), $y = 4$ (Marks are dependent upon a sketch being attempted) See conditions below.	B1, B1 (2)
	(c) Sketch: Horizontal translation (either way)… (There must be evidence that $y = 5$ at the max and that the asymptote is still $y = 1$) (–3, 5), $y = 1$	B1 B1, B1 (3) **[7]**
	Parts (a) and (b): (i) If only one of the B marks is scored, there is no penalty for a wrong sketch. (ii) If both the maximum and the equation of the asymptote are correct, the sketch must be "correct" to score B1 B1. If the sketch is "wrong", award B1 B0. The (generous) conditions for a "correct" sketch are that the maximum must be in the 2nd quadrant and that the curve must not cross the positive x-axis... ignore other "errors" such as "curve appearing to cross its asymptote" and "curve appearing to have a minimum in the 1st quadrant". Special case: (b) Stretch $\frac{1}{4}$ instead of 4: Correct shape, with $\left(-2, \frac{5}{4}\right)$, $y = \frac{1}{4}$: B1 B0. Coordinates of maximum: If the coordinates are the wrong way round (e.g. (7, –2) in part (a)), or the coordinates are just shown as values on the x and y axes, penalise only once in the whole question, at first occurrence. Asymptote marks: If the equation of the asymptote is not given, e.g. in part (a), 3 is marked on the y-axis but $y = 3$ is not seen, penalise only once in the whole question, at first occurrence. Ignore extra asymptotes stated (such as $x = 0$).	

Question number	Scheme	Marks
105.	(a) $x(x^2-4)$ Factor x seen in a <u>correct</u> factorised form of the expression. $= x(x-2)(x+2)$ M: Attempt to factorise quadratic (general principles). Accept $(x-0)$ or $(x+0)$ instead of x at any stage. Factorisation must be seen in part (a) to score marks.	B1 M1 A1 (3)
	(b)  Shape (2 turning points required) Through (or touching) origin Crossing x-axis or "stopping at x-axis" (<u>not</u> a turning point) at (–2, 0) and (2, 0). Allow –2 and 2 on x-axis. Also allow (0, –2) and (0, 2) if marked on x-axis. Ignore extra intersections with x-axis.	B1 B1 B1 (3)
	(c) <u>Either</u> $y = 3$ (at $x = -1$) <u>or</u> $y = 15$ (at $x = 3$) Allow if seen elsewhere. Gradient $= \dfrac{"15-3"}{3-(-1)}$ $(=3)$ Attempt correct grad. formula with their y values. For gradient M mark, if correct formula not seen, allow one slip, e.g. $\dfrac{"15-3"}{3-1}$ $y-"15"=m(x-3)$ or $y-"3"=m(x-(-1))$, with any value for m. $y-15=3(x-3)$ or the <u>correct</u> equation in <u>any</u> form, e.g. $y-3=\dfrac{15-3}{3-(-1)}(x-(-1))$, $\dfrac{y-3}{x+1}=\dfrac{15-3}{3+1}$ $y = 3x+6$	B1 M1 M1 A1 A1 (5)
	(d) $AB = \sqrt{("15-3")^2+(3-(-1))^2}$ (With their <u>non-zero</u> y values)... Square root is required. $= \sqrt{160}$ $\left(=\sqrt{16}\sqrt{10}\right) = 4\sqrt{10}$ (Ignore ± if seen) ($\sqrt{16}\sqrt{10}$ need not be seen).	M1 A1 (2) **[13]**
	(a) $x^3-4x \rightarrow x(x^2-4) \rightarrow (x-2)(x+2)$ scores B1 M1 A0. $x^3-4x \rightarrow x^2-4 \rightarrow (x-2)(x+2)$ scores B0 M1 A0 (dividing by x). $x^3-4x \rightarrow x(x^2-4x) \rightarrow x^2(x-4)$ scores B0 M1 A0. $x^3-4x \rightarrow x(x^2-4) \rightarrow x(x-2)^2$ scores B1 M1 A0 Special cases: $x^3-4x \rightarrow (x-2)(x^2+2x)$ scores B0 M1 A0. $x^3-4x \rightarrow x(x-2)^2$ (with no intermediate step seen) scores B0 M1 A0 (b) The 2nd and 3rd B marks are not dependent upon the 1st B mark, but <u>are</u> dependent upon a sketch having been attempted. (c) 1st M: May be implicit in the equation of the line, e.g. $\dfrac{y-"15"}{3-"15"}=\dfrac{x-"3"}{-1-"3"}$ 2nd M: An equation of a line through (3, "15") or (–1, "3") <u>in any form</u>, with any gradient (except 0 or ∞). 2nd M: Alternative is to use one of the points in $y = mx + c$ to <u>find a value</u> for c, in which case $y = 3x + c$ leading to $c = 6$ is sufficient for both A marks. 1st A1: <u>Correct</u> equation in <u>any</u> form.	

Question number	Scheme	Marks
106.	(a) $(x+2k)^2$ or $\left(x+\frac{4k}{2}\right)^2$ $(x\pm F)^2 \pm G \pm 3 \pm 11k$ (where F and G are <u>any</u> functions of k, not involving x) $(x+2k)^2 - 4k^2 + (3+11k)$ Accept unsimplified equivalents such as $\left(x+\frac{4k}{2}\right)^2 - \left(\frac{4k}{2}\right)^2 + 3 + 11k$, <u>and i.s.w. if necessary</u>.	M1 M1 A1 (3)
	(b) Accept part (b) solutions seen in part (a). $"4k^2 - 11k - 3" = 0$ $(4k+1)(k-3)=0$ $k = \ldots,$ [Or, 'starting again', $b^2 - 4ac = (4k)^2 - 4(3+11k)$ and proceed to $k = \ldots$] $-\frac{1}{4}$ and 3 (Ignore any inequalities for the first 2 marks in (b)). Using $b^2 - 4ac < 0$ for no real roots, i.e. $"4k^2 - 11k - 3" < 0$, to establish inequalities involving their <u>two</u> critical values m and n (<u>even if the inequalities are wrong</u>, e.g. $k < m$, $k < n$). $-\frac{1}{4} < k < 3$ (See conditions below) Follow through their critical values. The final A1ft is still scored if the answer $m < k < n$ follows $k < m$, $k < n$. <u>Using x instead of k in the final answer</u> loses only the 2nd A mark, (condone use of x in earlier working).	M1 A1 M1 A1ft (4)
	(c)  Shape $\cup$ (seen in (c)) Minimum in correct quadrant, <u>not</u> touching the x-axis, <u>not</u> on the y-axis, and there must be no other minimum or maximum. (0, 14) or 14 on y-axis. Allow (14, 0) marked on y-axis. n.b. Minimum is at $(-2,10)$, (but there is no mark for this).	B1 B1 B1 (3) **[10]**
	(b) 1st M: Forming and solving a 3-term quadratic in k (usual rules.. see general principles at end of scheme). The quadratic must come from $"b^2 - 4ac"$, or from the $"q"$ in part (a). Using <u>wrong discriminant</u>, e.g. $"b^2 + 4ac"$ will score <u>no marks</u> in part (b). 2nd M: As defined in main scheme above. 2nd A1ft: $m < k < n$, where $m < n$, for their critical values m and n. Other possible forms of the answer (in each case $m < n$): (i) $n > k > m$ (ii) $k > m$ <u>and</u> $k < n$ In this case the word "and" must be seen (implying intersection). (iii) $k \in (m,n)$ (iv) $\{k : k > m\} \cap \{k : k < n\}$ <u>Not</u> just a number line. <u>Not</u> just $k > m$, $k < n$ (without the word "and"). (c) Final B1 is dependent upon a sketch having been attempted in part (c).	

Question Number	Scheme	Marks
107. (a)	$(3\sqrt{7})^2 = 63$	B1 (1)
(b)	$(8+\sqrt{5})(2-\sqrt{5}) = 16-5+2\sqrt{5}-8\sqrt{5}$	M1
	$= 11,\ -6\sqrt{5}$	A1, A1
		(3)
		[4]
(a) (b)	B1 for 63 only M1 for an attempt to expand <u>their</u> brackets with $\geq$ 3 terms correct. They may collect the $\sqrt{5}$ terms to get $16-5-6\sqrt{5}$ Allow $-\sqrt{5}\times\sqrt{5}$ or $-(\sqrt{5})^2$ or $-\sqrt{25}$ instead of the -5 These 4 values may appear in a list or table but they should have minus signs included **The next two marks should be awarded for the final answer but check that correct values follow from correct working. Do not use ISW rule** 1st A1 for 11 from 16 – 5 <u>or</u> $-6\sqrt{5}$ from $-8\sqrt{5}+2\sqrt{5}$ 2nd A1 for <u>both</u> 11 and $-6\sqrt{5}$. <u>S.C - Double sign error in expansion</u> For $16-5-2\sqrt{5}+8\sqrt{5}$ leading to 11 + … allow <u>one</u> mark	

Question Number	Scheme	Marks
108. (a)	$5x > 10$, $x > 2$ [Condone $x > \frac{10}{2} = 2$ for M1A1]	M1, A1 (2)
(b)	$(2x+3)(x-4) = 0$, 'Critical values' are $-\dfrac{3}{2}$ and 4	M1, A1
	$-\dfrac{3}{2} < x < 4$	M1 A1ft (4)
(c)	$2 < x < 4$	B1ft (1) **[7]**
(a)	M1 for attempt to collect like terms on each side leading to $ax > b$, or $ax < b$, or $ax = b$ Must have a or b correct so eg $3x > 4$ scores M0	
(b)	1st M1 for an attempt to factorize or solve to find critical values. Method must potentially give 2 critical values 1st A1 for $-\dfrac{3}{2}$ and 4 seen. They may write $x < -\dfrac{3}{2}$, $x < 4$ and still get this A1 2nd M1 for choosing the "inside region" for their critical values 2nd A1ft follow through their 2 distinct critical values Allow $x > -\dfrac{3}{2}$ with "or" "," "$\cup$" " " $x < 4$ to score M1A0 but "and" or "$\cap$" score M1A1 $x \in (-\frac{3}{2}, 4)$ is M1A1 but $x \in [-\frac{3}{2}, 4]$ is M1A0. Score M0A0 for a number line or graph only	
(c)	B1ft Allow if **a correct answer is seen** or follow through their answer to (a) and their answer to (b) but their answers to (a) and (b) must be regions. Do not follow through single values. If their follow through answer is the empty set accept $\varnothing$ or {} or equivalent in words If (a) or (b) are not given then score this mark for cao NB You may see x<4 (with anything or nothing in-between) $x < -1.5$ in (b) and empty set in (c) for B1ft **Do not award marks for part (b) if only seen in part (c)** Use of ≤ instead of < (or ≥ instead of >) loses one accuracy mark only, at first occurrence.	

Question Number	Scheme	Marks
109.	$b^2 - 4ac$ attempted, in terms of *p*. $(3p)^2 - 4p = 0$ o.e. Attempt to solve for *p* e.g. $p(9p-4)=0$ Must potentially lead to $p = k,\ k \neq 0$ $p = \frac{4}{9}$ (Ignore $p = 0$, if seen)	M1 A1 M1 A1cso **[4]**
	1st M1 for an attempt to substitute into $b^2 - 4ac$ or $b^2 = 4ac$ with *b* or *c* correct Condone *x*'s in one term only. This can be inside a square root as part of the quadratic formula for example. **Use of inequalities can score the M marks only** 1st A1 for any correct equation: $(3p)^2 - 4\times1\times p = 0$ or better 2nd M1 for an attempt to factorize or solve their quadratic expression in *p*. Method must be sufficient to lead to their $p = \frac{4}{9}$. Accept factors or use of quadratic formula or $\left(p \pm \frac{2}{9}\right)^2 = k^2$ (o.e. eg) $\left(3p \pm \frac{2}{3}\right)^2 = k^2$ or equivalent work on <u>their</u> eqn. $9p^2 = 4p \Rightarrow \frac{9p^{\not{2}}}{\not{p}} = 4$ which would lead to $9p = 4$ is OK for this 2nd M1 ALT <u>Comparing coefficients</u> M1 for $(x+\alpha)^2 = x^2 + \alpha^2 + 2\alpha x$ and A1 for a correct equation eg $3p = 2\sqrt{p}$ M1 for forming solving leading to $\sqrt{p} = \frac{2}{3}$ or better <u>Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark</u> If the formula is quoted accept some correct substitution leading to a partially correct expression. If the formula is not quoted only award for a fully correct expression using their values.	

Question Number	Scheme	Marks

110. (a)

$x(x^2 - 6x + 9)$ — B1

$= x(x-3)(x-3)$ — M1 A1 (3)

(b)

Shape  — B1

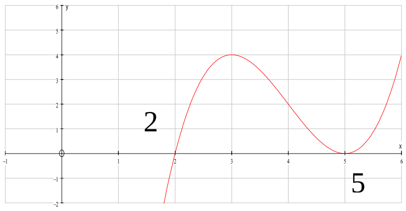
<u>Through</u> origin (<u>not</u> touching) — B1

Touching x-axis only once — B1

Touching at (3, 0), or 3 on x-axis — B1ft (4)

[Must be on graph not in a table]

(c)



Moved horizontally (either way) — M1

(2, 0) and (5, 0), or 2 and 5 on x-axis — A1 (2)

[9]

(a)

B1 for correctly taking out a factor of x

M1 for an attempt to factorize their 3TQ e.g. $(x+p)(x+q)$ where $|pq| = 9$.

So $(x-3)(x+3)$ will score M1 but A0

A1 for a fully correct factorized expression - accept $x(x-3)^2$

If they "solve" use ISW

S.C. If the only correct linear factor is (x - 3), perhaps from factor theorem, award B0M1A0

Do not award marks for factorising in part (b)

<u>**For the graphs**</u>

"Sharp points" will lose the 1st B1 in (b) but otherwise be generous on shape

Condone (0, 3) in (b) and (0, 2), (0,5) in (c) if the points are marked in the correct places.

(b) 2nd B1 for a curve that starts or terminates at (0, 0) score B0

4th B1ft for a curve that touches (not crossing or terminating) at (a, 0) where their $y = x(x-a)^2$

(c) M1 for their graph moved horizontally (only) <u>or</u> a fully correct graph

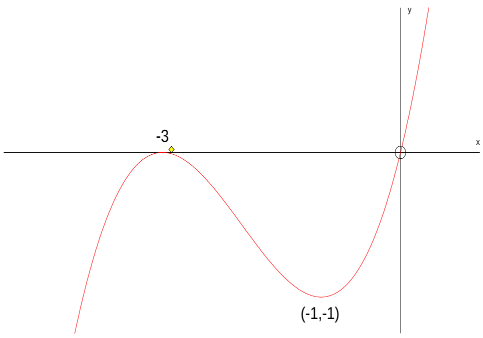
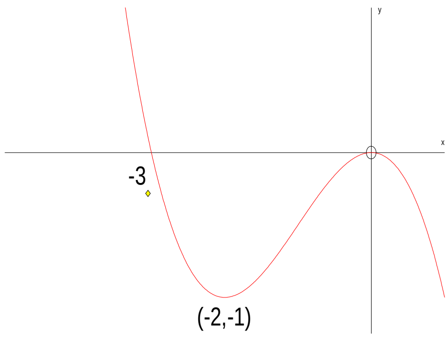
Condone a partial stretch if ignoring their values looks like a simple translation

A1 for their graph translated 2 to the right <u>and</u> crossing or touching the axis at 2 and 5 only

Allow a fully correct graph (as shown above) to score M1A1 whatever they have in (b)

Question Number	Scheme	Marks
111	$\sqrt{7}^2+2\sqrt{7}-2\sqrt{7}-2^2$, or $7-4$ or an exact equivalent such as $\sqrt{49}-2^2$ $=\mathbf{3}$	M1 A1 **[2]**
	M1 for an expanded expression. At worst, there can be one wrong term and one wrong sign, or two wrong signs. e.g. $7+2\sqrt{7}-2\sqrt{7}-2$ is M1 (one wrong term -2) $7+2\sqrt{7}+2\sqrt{7}+4$ is M1 (two wrong signs $+2\sqrt{7}$ and $+4$) $7+2\sqrt{7}+2\sqrt{7}+2$ is M1 (one wrong term $+2$, one wrong sign $+2\sqrt{7}$) $\sqrt{7}+2\sqrt{7}-2\sqrt{7}+4$ is M1 (one wrong term $\sqrt{7}$, one wrong sign $+4$) $\sqrt{7}+2\sqrt{7}-2\sqrt{7}-2$ is M0 (two wrong terms $\sqrt{7}$ and -2) $7+\sqrt{14}-\sqrt{14}-4$ is M0 (two wrong terms $\sqrt{14}$ and $-\sqrt{14}$) If only 2 terms are given, they must be correct, i.e. (7 – 4) or an equivalent unsimplified version to score M1. The terms can be seen separately for the M1. Correct answer with no working scores both marks.	

EXPERT TUITION

Question Number	Scheme	Marks
112. (a)		
	Shape  , touching the *x*-axis at its maximum.	M1
	Through (0,0) & -3 marked on *x*-axis, or $(-3,0)$ seen. Allow $(0,-3)$ if marked on the *x*-axis. Marked in the correct place, but 3, is A0.	A1
	Min at $(-1,-1)$	A1
		(3)
(b)		
	Correct shape  (top left - bottom right)	B1
	Through -3 and max at (0, 0). Marked in the correct place, but 3, is B0.	B1
	Min at $(-2,-1)$	B1
		(3)
		[6]
(a)	M1 as described above. Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 1st A1 for curve passing through -3 and the origin. Max at $(-3,0)$ 2nd A1 for minimum at $(-1,-1)$. Can simply be indicated on sketch.	
(b)	1st B1 for the correct shape. A negative cubic passing from top left to bottom right. Shape: Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 2nd B1 for curve passing through $(-3,0)$ having a max at (0, 0) and no other max. 3rd B1 for minimum at $(-2,-1)$ and no other minimum. If in correct quadrant but labelled, e.g. $(-2,1)$, this is B0. In each part the (0, 0) does not need to be written to score the second mark… having the curve pass through the origin is sufficient. The last mark (for the minimum) in each part is dependent on a sketch being attempted, and the sketch must show the minimum in approximately the correct place (not, for example, $(-2,-1)$ marked in the wrong quadrant). The mark for the minimum is not given for the coordinates just marked on the axes unless these are clearly linked to the minimum by vertical and horizontal lines.	

Question Number	Scheme	Marks
113. (a)	$b^2 - 4ac > 0 \Rightarrow \quad 16 - 4k(5-k) > 0$ or equiv., e.g. $16 > 4k(5-k)$	M1A1
	So $k^2 - 5k + 4 > 0$ (Allow any order of terms, e.g. $4 - 5k + k^2 > 0$) (*)	A1cso (3)
(b)	Critical Values $(k-4)(k-1) = 0 \quad k = \ldots.$	M1
	$k = 1$ or 4	A1
	Choosing "outside" region	M1
	$\underline{\boldsymbol{k < 1} \text{ or } \boldsymbol{k > 4}}$	A1 (4)
		[7]

For this question, ignore (a) and (b) labels and award marks wherever correct work is seen.

(a)

M1 for attempting to use the discriminant of the initial equation (> 0 not required, but substitution of a, b and c in the correct formula is required).
If the formula $b^2 - 4ac$ is seen, at least 2 of a, b and c must be correct.
If the formula $b^2 - 4ac$ is not seen, all 3 (a, b and c) must be correct.
This mark can still be scored if substitution in $b^2 - 4ac$ is within the quadratic formula.
This mark can also be scored by comparing b^2 and $4ac$ (with substitution).
However, use of $b^2 + 4ac$ is M0.

1st A1 for fully correct expression, possibly unsimplified, with > symbol. NB must appear before the last line, even if this is simply in a statement such as $b^2 - 4ac > 0$ or 'discriminant positive'.
Condone a bracketing slip, e.g. $16 - 4 \times k \times 5 - k$ if subsequent work is correct and convincing.

2nd A1 for a fully correct derivation with no incorrect working seen.
Condone a bracketing slip if otherwise correct and convincing.

Using $\sqrt{b^2 - 4ac} > 0$:
Only available mark is the first M1 (unless recovery is seen).

(b)

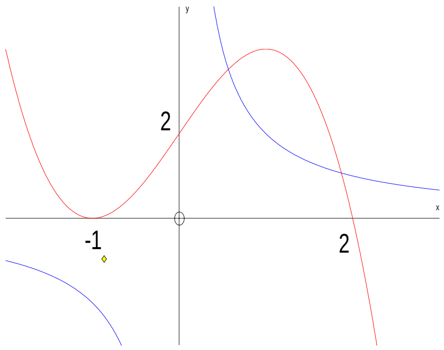
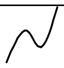
1st M1 for attempt to solve an appropriate 3TQ
1st A1 for both k = 1 and 4 (only the critical values are required, so accept, e.g. $k > 1$ and $k > 4$). **
2nd M1 for choosing the "outside" region. A diagram or table alone is not sufficient.
Follow through their values of k.
The set of values must be 'narrowed down' to score this M mark… listing everything $k < 1$, $1 < k < 4$, $k > 4$ is M0.
2nd A1 for correct answer only, condone "$k < 1$, $k > 4$" and even "$k < 1$ and $k > 4$", but "$1 > k > 4$" is A0.

** Often the statement $k > 1$ and $k > 4$ is followed by the correct final answer. Allow full marks.

Seeing 1 and 4 used as critical values gives the first M1 A1 by implication.

In part (b), condone working with x's except for the final mark, where the set of values must be a set of values of k (i.e. 3 marks out of 4).

Use of ≤ (or ≥) in the final answer loses the final mark.

Question Number	Scheme	Marks
114. (a)	$(a =)\ \ (1+1)^2(2-1) = \underline{\mathbf{4}}$ (1, 4) or $y = 4$ is also acceptable	B1 (1)
(b)	 (i) Shape  or  anywhere Min at $(-1, 0)$ … can be -1 on x-axis. Allow $(0, -1)$ if marked on the x-axis. Marked in the correct place, but 1, is B0. (2, 0) and (0, 2) can be 2 on axes (ii) Top branch in 1st quadrant with 2 intersections Bottom branch in 3rd quadrant (ignore any intersections)	B1 B1 B1 B1 B1 (5)
(c)	(2 intersections therefore) $\underline{\mathbf{2}}$ (roots)	B1ft (1) **[7]**
(b)	1st B1 for shape  or  Can be anywhere, but there must be one max. and one min. and no further max. and min. turning points. Shape: Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 2nd B1 for minimum at $(-1, 0)$ (even if there is an additional minimum point shown) 3rd B1 for the sketch meeting axes at (2, 0) and (0, 2). They can simply mark 2 on the axes. The marks for minimum and intersections are dependent upon having a sketch. Answers on the diagram for min. and intersections take precedence over answers seen elsewhere. 4th B1 for the branch fully within 1st quadrant having 2 intersections with (not just 'touching') the other curve. The curve can 'touch' the axes. A curve of (roughly) the correct shape is required, but be very generous, even when the arc appears to turn 'inwards' rather than approaching the axes, and when the curve looks like two straight lines with a small curve at the join. Allow, for example, shapes like these: 5th B1 for a branch fully in the 3rd quadrant (ignore any intersections with the other curve for this branch). The curve can 'touch' the axes. A curve of (roughly) the correct shape is required, but be very generous, even when the arc appears to turn 'inwards' rather than approaching the axes.	
(c)	B1ft for a statement about the number of roots - compatible with their sketch. No sketch is B0. The answer 2 <u>incompatible with the sketch</u> is B0 (ignore any algebra seen). If the sketch shows the 2 correct intersections <u>and</u>, for example, one other intersection, the answer here should be 3, not 2, to score the mark.	

Question number	Scheme	Marks
115.	$x(x^2-9)$ or $(x\pm 0)(x^2-9)$ or $(x-3)(x^2+3x)$ or $(x+3)(x^2-3x)$	B1
	$x(x-3)(x+3)$	M1A1 (3)
		3

B1 for first factor taken out correctly as indicated in line 1 above. So $x(x^2+9)$ is B0

M1 for attempting to factorise a relevant quadratic.

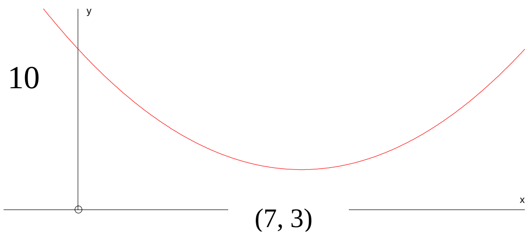
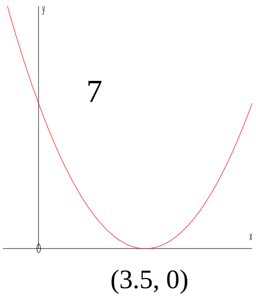
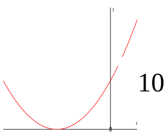
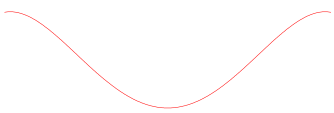
"Ends" correct so e.g. $(x^2-9)=(x\pm p)(x\pm q)$ where $pq = 9$ is OK.

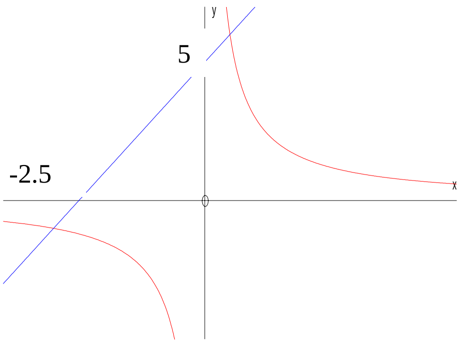
This mark can be scored for $\left(x^2-9\right)=\left(x+3\right)\left(x-3\right)$ seen anywhere.

A1 for a fully correct expression with all 3 factors.

Watch out for $-x(3-x)(x+3)$ which scores A1

Treat any working to solve the equation x^3-9x as ISW.

Question number	Scheme	Marks
116	(a) 	B1B1B1 (3)
	(b) 	B1B1 (2) **5**
	Allow "stopping at" (0, 10) or (0, 7) instead of "cutting"	
(a)	1st B1 for moving the given curve up. Must be U shaped curve, minimum in first quadrant, not touching x-axis but cutting positive y-axis. Ignore any values on axes. 2nd B1 for curve cutting y-axis at (0, 10) . Point 10(or even (10, 0) marked on positive y-axis is OK) 3rd B1 for minimum indicated at (7, 3). Must have both coordinates and in the right order.  If the curve flattens out to a turning point like this penalise <u>once</u> at first offence ie 1st B1 in (a) or in (b) but not in both. this would score B0B1B0 The U shape mark can be awarded if the sides are fairly straight as long as the vertex is rounded.	
(b)	1st B1 for U shaped curve, touching positive x-axis and crossing y-axis at (0, 7)[condone (7, 0) if marked on positive y axis] or 7 marked on y-axis 2nd B1 for minimum at (3.5, 0) or 3.5 or $\frac{7}{2}$ marked on x-axis. Do <u>not</u> condone (0, 3.5) here. Redrawing f(x) will score B1B0 in part (b). Points on sketch override points given in text/table. If coordinates are given elsewhere (text or table) marks can be awarded if they are compatible with the sketch.	

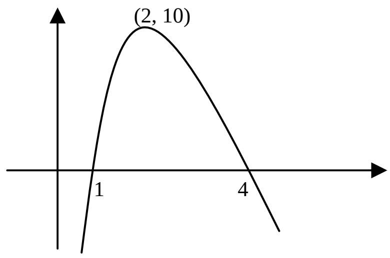
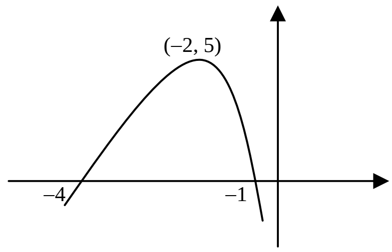
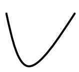
Question Number	Scheme	Marks
117. (a)		B1M1A1 (3)
(b)	$2x+5=\frac{3}{x}$	M1
	$2x^2+5x-3\,[=0]$ or $2x^2+5x=3$	A1
	$(2x-1)(x+3)\,[=0]$	M1
	$x=-3$ or $\frac{1}{2}$	A1
	$y=\frac{3}{-3}$ or $2\times(-3)+5$ or $y=\frac{3}{\frac{1}{2}}$ or $2\times\left(\frac{1}{2}\right)+5$	M1
	Points are $(-3,-1)$ and $\left(\frac{1}{2},6\right)$ (correct pairings)	A1ft
		9
(a)	B1 for curve of correct shape i.e 2 branches of curve, in correct quadrants, of roughly the correct shape and no touching or intersections with axes. Condone up to 2 inward bends but there must be some ends that are roughly asymptotic. M1 for a straight line <u>cutting</u> the positive *y*-axis and the negative *x*-axis. Ignore any values. A1 for (0,5) and (-2.5,0) or points correctly marked on axes. Do not give for values in tables. Condone mixing up (x, y) as (y, x) if one value is zero and other value correct.	
(b)	1st M1 for attempt to form a suitable equation and multiply by x (at least one of $2x$ or $+5$) should be multiplied. 1st A1 for correct 3TQ - condone missing = 0 2nd M1 for an attempt to solve a relevant 3TQ leading to 2 values for x =… 2nd A1 for both x = -3 and 0.5. T&I for x values <u>may</u> score 1st M1A1 otherwise no marks unless both values correct. Answer only of x = -3 and $x = \frac{1}{2}$ scores 4/4, then apply the scheme for the final M1A1ft 3rd M1 for an attempt to find at least one y value by substituting their x in either $\frac{3}{x}$ or $2x + 5$ 3rd A1ft follow through both their x values, in either equation but the same for each, correct pairings required but can be x = -3, y = -1 etc	

Question number	Scheme	Marks
118. (a)	[No real roots implies $b^2-4ac<0$.] $b^2-4ac=q^2-4\times 2q\times(-1)$	M1
	So $q^2-4\times 2q\times(-1)<0$ i.e. $q^2+8q<0$ **(*)**	A1cso (2)
(b)	$q(q+8)=0$ or $(q\pm 4)^2\pm 16=0$	M1
	$(q)=0$ or -8 (2 cvs)	A1
	$-8<q<0$ <u>or</u> $q\in$ (-8, 0) <u>or</u> q < 0 and q > -8	A1ft (3)
		5

(a)	M1 for attempting b^2-4ac with one of *b* or *a* correct. < 0 not needed for M1 This may be inside a square root. A1cso for simplifying to printed result with no incorrect working or statements seen. Need an intermediate step e.g. $q^2--8q<0$ or $q^2-4\times 2q\times -1<0$ or $q^2-4(2q)(-1)<0$ or $q^2-8q(-1)<0$ or $q^2-8q\times -1<0$ i.e. must have × or brackets on the 4*ac* term < 0 must be seen at least one line before the final answer.
(b)	M1 for factorizing or completing the square or attempting to solve $q^2\pm 8q=0$. A method that would lead to 2 values for *q*. The "= 0" may be implied by values appearing later. 1st A1 for $q=0$ and $q=-8$ 2nd A1 for $-8<q<0$. Can follow through their cvs but must choose "inside" region. $q<0$, $q>-8$ is A0, $q<0$ or $q>-8$ is A0, (-8, 0) on its own is A0 BUT "$q<0$ and $q>-8$" is A1 Do not accept a number line for final mark

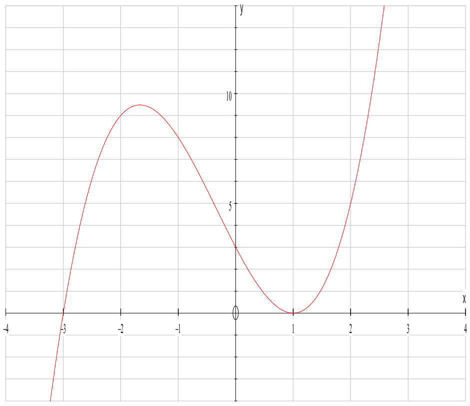
Question number	Scheme	Marks
119.	$\frac{(5-\sqrt{3})}{(2+\sqrt{3})}\times\frac{(2-\sqrt{3})}{(2-\sqrt{3})}$	M1
	$=\frac{10-2\sqrt{3}-5\sqrt{3}+(\sqrt{3})^2}{\ldots}$ $\left(=\frac{10-7\sqrt{3}+3}{\ldots}\right)$	M1
	$(=13-7\sqrt{3})$ $\left(\text{Allow } \frac{13-7\sqrt{3}}{1}\right)$ 13 (a = 13)	A1
	$-7\sqrt{3}$ (b = −7)	A1 (4)
		4
	1st M: Multiplying top and bottom by $(2-\sqrt{3})$. (As shown above is sufficient). 2nd M: Attempt to multiply out numerator $(5-\sqrt{3})(2-\sqrt{3})$. Must have at least 3 terms correct. Final answer: Although 'denominator = 1' may be <u>implied</u>, the $13-7\sqrt{3}$ must obviously be the final answer (not an intermediate step), to score full marks. (Also M0 M1 A1 A1 is <u>not</u> an option). The A marks cannot be scored unless the 1st M mark has been scored, but this 1st M mark <u>could</u> be implied by correct expansions of both numerator <u>and</u> denominator. It <u>is</u> possible to score M1 M0 A1 A0 or M1 M0 A0 A1 (after 2 correct terms in the numerator). <u>Special case</u>: If numerator is multiplied by $(2+\sqrt{3})$ instead of $(2-\sqrt{3})$, the 2nd M can still be scored for at least 3 of these terms correct: $10-2\sqrt{3}+5\sqrt{3}-(\sqrt{3})^2$. The maximum score in the special case is 1 mark: M0 M1 A0 A0. <u>Answer only</u>: Scores no marks. <u>Alternative method</u>: $5-\sqrt{3}=(a+b\sqrt{3})(2+\sqrt{3})$ $(a+b\sqrt{3})(2+\sqrt{3})=2a+a\sqrt{3}+2b\sqrt{3}+3$ M1: At least 3 terms correct. $5=2a+3b$ $-1=a+2b$ $a=\ldots$ or $b=\ldots$ M1: Form and attempt to solve simultaneous equations. $a=13$, $b=-7$ A1, A1	

EXPERT TUITION

Question number	Scheme		Marks	
120.	(a) 	Shape: Max in 1st quadrant and 2 intersections on positive x-axis	B1	
		1 and 4 labelled (in correct place) or clearly stated as coordinates	B1	
		(2, 10) labelled or clearly stated	B1	(3)
	(b) 	Shape: Max in 2nd quadrant and 2 intersections on negative x-axis	B1	
		–1 and –4 labelled (in correct place) or clearly stated as coordinates	B1	
		(–2, 5) labelled or clearly stated	B1	(3)
	(c) (a =) 2	May be implicit, i.e. $f(x+2)$	B1	(1)
	Beware: The answer to part (c) may be seen on the first page.			
				7
	(a) and (b): 1st B: 'Shape' is generous, providing the conditions are satisfied. 2nd and 3rd B marks are dependent upon a sketch having been drawn. 2nd B marks: Allow (0, 1), etc. (coordinates the wrong way round) <u>if</u> the sketch is correct. Points must be labelled correctly and be in appropriate place (e.g. (–2, 5) in the first quadrant is B0). (b) <u>Special case</u>: If the graph is reflected in the x-axis (instead of the y-axis), B1 B0 B0 can be scored. This requires shape and coordinates to be <u>fully correct</u>, i.e. Shape:  Minimum in 4th quadrant and 2 intersections on positive x-axis, 1 and 4 labelled (in correct place) or clearly stated as coordinates, (2, –5) labelled or clearly stated.			

EXPERT TUITION

Question number	Scheme	Marks	
121.	(a) $x^2 + kx + (8-k)$ $(=0)$ $\quad$ $8-k$ need not be bracketed	M1	
	$b^2 - 4ac = k^2 - 4(8-k)$	M1	
	$b^2 - 4ac < 0 \Rightarrow k^2 + 4k - 32 < 0$ $\quad$ (*)	A1cso	(3)
	(b) $(k+8)(k-4) = 0$ $\quad$ $k = ...$	M1	
	$k = -8$ $\quad$ $k = 4$	A1	
	Choosing 'inside' region (between the two *k* values)	M1	
	$-8 < k < 4$ or $4 > k > -8$	A1	(4)
			7
	(a) 1st M: Using the *k* from the right hand side to form 3-term quadratic in *x* ('= 0' can be implied), or… attempting to complete the square $\left(x + \frac{k}{2}\right)^2 - \frac{k^2}{4} + 8 - k$ $(=0)$ or equiv., using the *k* from the right hand side. For either approach, condone sign errors. 1st M may be implied when candidate moves straight to the discriminant. 2nd M: Dependent on the 1st M. Forming expressions in *k* (with no *x*'s) by using b^2 and $4ac$. (Usually seen as the discriminant $b^2 - 4ac$, but separate expressions are fine, and also allow the use of $b^2 + 4ac$. (For 'completing the square' approach, the expression must be clearly separated from the equation in *x*). If b^2 and $4ac$ are used in the quadratic formula, they must be clearly separated from the formula to score this mark. For any approach, condone sign errors. If the wrong statement $\sqrt{b^2 - 4ac} < 0$ is seen, maximum score is M1 M1 A0. (b) Condone the use of *x* (instead of *k*) in part (b). 1st M: Attempt to solve a 3-term quadratic equation in *k*. It might be different from the given quadratic in part (a). Ignore the use of < in solving the equation. The 1st M1 A1 can be scored if –8 and 4 are achieved, even if stated as $k < -8$, $k < 4$. Allow the first M1 A1 to be scored in part (a). N.B. '$k > -8$, $k < 4$' scores 2nd M1 A0 '$k > -8$ or $k < 4$' scores 2nd M1 A0 '$k > -8$ and $k < 4$' scores 2nd M1 A1 '$k = -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3$' scores 2nd M0 A0 Use of ≤ (in the answer) loses the final mark.		

Question number	Scheme	Marks
122.	(a) 	
	Shape  (drawn anywhere)	B1
	Minimum at (1, 0) (perhaps labelled 1 on x-axis)	B1
	$(-3, 0)$ (or -3 shown on –ve x-axis)	B1
	(0, 3) (or 3 shown on +ve y-axis)	B1 (4)
	N.B. The max. can be anywhere.	
	(b) $y = (x+3)(x^2 - 2x + 1)$	M1
	$= x^3 + x^2 - 5x + 3$ $(k = 3)$ [Marks can be awarded if this is seen in part (a)]	A1cso (2)
	(a) The individual marks are independent, <u>but</u> the 2nd, 3rd and 4th B's are dependent upon a sketch having been attempted. B marks for coordinates: Allow (0, 1), etc. (coordinates the wrong way round) <u>if</u> marked in the correct place on the sketch. (b) M: Attempt to multiply out $(x-1)^2$ and write as a product with $(x+3)$, or attempt to multiply out $(x+3)(x-1)$ and write as a product with $(x-1)$, or attempt to expand $(x+3)(x-1)(x-1)$ directly (at least 7 terms). The $(x-1)^2$ or $(x+3)(x-1)$ expansion must have 3 (or 4) terms, so should not, for example, be just $x^2 + 1$. A: It is not necessary to state explicitly 'k = 3'. Condone missing brackets if the intention seems clear and a fully correct expansion is seen.	

Question Number	Scheme		Marks
123. (a)	Way 1 Use f(1/2) or f (– 1/2) and put equal to 30 Stated $\frac{24}{8}+\frac{1}{4}A-\frac{3}{2}+B=30$ and $A + 4B = 114$ *	Way 2 Long division of f(x) by (2x -1) as far as remainder put = 30 Obtains $B+\frac{1}{4}A+\frac{3}{2}=30$ (o.e) and $A + 4B = 114$ *	M1 A1* **(2)**
(b)	Way 1 Used f(-1) or f(1) = 0 Stated -24+A+3+B = 0 so $A + B = 21$	Way 2 Long division of f(x) by (x +1) as far as remainder put = 0 Obtains $B – 21 + A = 0$	M1 A1 **(2)**
(c)	Solves to obtain one of A or B Obtains both A = -10 and B = 31		M1 A1 **(2)**
(d)	$f(x)=(x+1)(24x^2-34x+31)$ or factor is $(24x^2-34x+31)$		M1A1 **(2)** **(8 marks)**

Notes

(a) Way 1
M1: for attempting either $f(\frac{1}{2})$ or $f(-\frac{1}{2})$ – with **numbers substituted into expression and put = 30**
A1*: Obtaining correct equation correctly (Signs and powers of ½ need to be simplified correctly)
(a) Way 2
M1: for attempting long division of f(x) by (2x -1) obtaining $12x^2+...x+...$ as quotient and remainder term **put equal to 30**
A1*: Obtaining correct equation correctly
(b) Way 1
M1: for calculating $f(-1)$ or f(1) **and put equal to 0** (This may be implied by their equation in part (b))
A1:for obtaining a correct equivalent equation in part (b). (This mark may not be recovered in part (c))
Accept $A + B = 21$ or $-A - B = -21$ or $A + B – 21=0$ or $21 – A – B = 0$ or $B – 21 + A = 0$ and even $-24 + A + 3 + B = 0$ as a final answer to part (b).
(b) Way 2
M1: for attempting long division of f(x) by (x +1) obtaining $24x^2+...x+...$ as quotient and remainder term **put equal to 0** (This may be implied by their equation in part (b))
A1:for obtaining a correct equivalent equation in part (b). (This mark may not be recovered in part (c))
Accept $A + B = 21$ or $-A - B = -21$ or $A + B – 21=0$ or $21 – A – B = 0$ or $B – 21 + A = 0$ etc..
(c)
M1: Eliminate one variable and solve to obtain A or B
A1: Both correct
(d)
M1: Uses their values of A and B in the given cubic (even the wrong way round) and attempts to divide by $(x+1)$ leading to a 3TQ beginning with the correct term, usually $24x^2$ and including an x term and a constant term. This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. (If values of A and B were wrong there may be a remainder but this may be ignored) If they used division in part (b) they may substitute A and B into their quotient expression from (b).
A1: $24x^2-34x+31...$ Credit when seen and use isw if miscopied later or if attempt is made to solve

Question Number	Scheme	Marks
124. (a)	Attempt $\mathrm{f}(3)$ or $\mathrm{f}(-3)$ Use of long division is M0A0 as factor theorem was required. $\mathrm{f}(-3) = 162 - 63 - 120 + 21 = 0$ so $(x + 3)$ is a factor	M1 A1 **(2)**
(b)	Either (Way 1): $\mathrm{f}(x) = (x+3)(-6x^2 + 11x + 7)$ $= (x+3)(-3x+7)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$	M1A1 M1A1 **(4)**
	Or (Way 2) Uses trial or factor theorem to obtain $x = -1/2$ **or** $x = 7/3$ Uses trial or factor theorem to obtain both $x = -1/2$ **and** $x = 7/3$ Puts three factors together (see notes below) Correct factorisation : $(x+3)(7-3x)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$ oe	M1 A1 M1 A1 **(4)**
	Or (Way 3) No working three factors $(x+3)(-3x+7)(2x+1)$ otherwise need working	M1A1M1A1 **(4)**
(c)	$2^y = \frac{7}{3}$, $\rightarrow \log(2^y) = \log\left(\frac{7}{3}\right)$ or $y = \log_2\left(\frac{7}{3}\right)$ or $\dfrac{\log(7/3)}{\log 2}$ $\{y = 1.222392421...\} \Rightarrow y =$ awrt 1.22	B1, M1 A1 **(3)** **[9]**
	Notes	
(a)	M1 for attempting either $\mathrm{f}(3)$ or $\mathrm{f}(-3)$ – with **numbers substituted into expression** A1 for calculating $\mathrm{f}(-3)$ **correctly to 0**, and they must state $(x + 3)$ **is a factor** for A1 (or equivalent ie. QED, □ or a tick). A conclusion may be implied by a preamble, "if f(−3) = 0, $(x+3)$ is a factor". $-6(-3)^3-7(-3)^2 + 40(-3) + 21 = 0$ so $(x + 3)$ is a factor of $\mathrm{f}(x)$ is M1A1 providing bracketing is correct.	
(b)	1st M1: attempting to divide by $(x+3)$ leading to a 3TQ beginning with the correct term, usually $-6x^2$. This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. Allow for work in part (a) if the result is used in (b). 1st A1: usually for $(-6x^2 + 11x + 7)$ … Credit when seen and use isw if miscopied 2nd M1: for a ***valid**** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1) 2nd A1 is cao and needs all three factors together fully factorised. Accept e.g. $-3(x+3)(x-\frac{7}{3})(2x+1)$ but $(x+3)(x-\frac{7}{3})(-6x-3)$ and $(x+3)(3x-7)(-2x-1)$ are A0 as not fully factorised. Ignore subsequent work (such as a solution to a quadratic equation.) Way 2: The second M mark needs three roots together so $\pm 6(x-\alpha)(x-\beta)(x+3)$ or equivalent where they obtained α and β by trial, so if correct roots identified, then $(x+3)(3x-7)(2x+1)$ can gain M1A1M1A0. N.B. Replacing $(-6x^2 + 11x + 7)$ (already awarded M1A1) by $(6x^2 - 11x - 7)$ giving $(x+3)(3x-7)(2x+1)$ can have M1A0 for factorization so M1A1M1A0	
(c)	B1: $2^y = \frac{7}{3}$ M1: Attempt to take logs to solve $2^y = \alpha$ or $2^y = 1/\alpha$, where $\alpha > 0$ and α was a root of their factorization. A1: for an answer that rounds to 1.22. If other answers are included (and not "rejected") such as ln(−3) or −1 lose final A mark **Special case:** Those who deal **throughout** with $\mathrm{f}(x) = 6x^3 + 7x^2 - 40x - 21$ They may have full credit in part (a). In part (b) they can achieve a maximum of M1A0M1A0 unless they return the negative sign to give the correct answer. This is then full marks. Part (c) is fine. So they could lose 2 marks on the factorisation. (Like a misread)	

Question Number	Scheme		Marks
125.	$f(x) = 6x^3 + 13x^2 - 4$		
(a)	$f\left(-\frac{3}{2}\right) = 6\left(-\frac{3}{2}\right)^3 + 13\left(-\frac{3}{2}\right)^2 - 4 = 5$	Attempting $f\left(-\frac{3}{2}\right)$ or $f\left(\frac{3}{2}\right)$	M1
		5	A1 cao
			[2]
(b)	$f(-2) = 6(-2)^3 + 13(-2)^2 - 4$	Attempts $f(-2)$.	M1
	$= 0$, **and** so $(x+2)$ is a factor.	$f(-2) = 0$ with no sign or substitution errors **and for conclusion.**	A1
			[2]
(c)	$f(x) = \{(x+2)\}(6x^2 + x - 2)$		M1 A1
	$= (x+2)(2x-1)(3x+2)$		M1 A1
			[4]
			8

Question 125 Notes

	Note	**Long division scores no marks in part (a). The <u>remainder theorem</u> is required.**
(a)	**M1**	Attempting $f\left(-\frac{3}{2}\right)$ or $f\left(\frac{3}{2}\right)$. $6\left(-\frac{3}{2}\right)^3 + 13\left(-\frac{3}{2}\right)^2 - 4$ or $6\left(\frac{3}{2}\right)^3 + 13\left(\frac{3}{2}\right)^2 - 4$ is sufficient
	A1	5 cao
(b)	**M1**	Attempting $f(-2)$. (This is **not** given for f(2))
	A1	Must correctly show $f(-2) = 0$ **and** give a conclusion ***in part (b) only***. No simplification of terms is required here.
	Note	Stating "hence factor" or "it is a factor" or a "tick" or "QED" are possible conclusions. Also a conclusion can be implied from a <u>preamble</u>, eg: "If $f(-2) = 0$, $(x+2)$ is a factor…." **Long division scores no marks in part (b). The <u>factor theorem</u> is required.**
(c)	**1st M1**	Attempting to divide by $(x+2)$ leading to a quotient which is quadratic with at least two terms beginning with first term of $\pm 6x^2$ + linear or constant term. Or $f(x) = (x+2)(\pm 6x^2 + \text{linear and/or constant term})$ (This may be seen in part (b) where candidates did not use factor theorem and might be referred to here)
	1st A1	$(6x^2 + x - 2)$ seen as quotient or as factor. If there is an error in the division resulting in a remainder give A0, but allow recovery to gain next two marks if $(6x^2 + x - 2)$ is used
	2nd M1	For a ***valid*** attempt to factorise **their** three term quadratic.
	A1	$(x+2)(2x-1)(3x+2)$ and needs all three factors on the same line. Ignore subsequent work (such as a **solution** to a quadratic equation).
	Special cases	**Calculator methods:** Award M1A1M1A1 for correct answer $(x+2)(2x-1)(3x+2)$ with no working. Award M1A0M1A0 for either $(x+2)(2x+1)(3x+2)$ or $(x+2)(2x+1)(3x-2)$ or $(x+2)(2x-1)(3x-2)$ with no working. (At least one bracket incorrect) Award M1A1M1A1 for $x = -2, \frac{1}{2}, -\frac{2}{3}$ followed by $(x+2)(2x-1)(3x+2)$. Award M0A0M0A0 for a candidate who writes down $x = -2, \frac{1}{2}, -\frac{2}{3}$ giving no factors. Award M1A1M1A1 for $6(x+2)(x-\frac{1}{2})(x+\frac{2}{3})$ or $2(x+2)(x-\frac{1}{2})(3x+2)$ or equivalent Award SC: M1A0M1A0 for $x = -2, \frac{1}{2}, -\frac{2}{3}$ followed by $(x+2)(x-\frac{1}{2})(x+\frac{2}{3})$.

Question Number	Scheme	Marks
126.	$f(x) = 6x^3 + 3x^2 + Ax + B$	
Way 1 (a)	Attempting $f(1) = 45$ or $f(-1) = 45$	M1
	$f(-1) = -6 + 3 - A + B = 45$ or $-3 - A + B = 45 \Rightarrow B - A = 48$ * (allow 48 = $B - A$)	A1 * **cso** **(2)**
Way 1 (b)	Attempting $f(-\frac{1}{2}) = 0$	M1
	$6(-\frac{1}{2})^3 + 3(-\frac{1}{2})^2 + A(-\frac{1}{2}) + B = 0$ or $-\frac{1}{2}A + B = 0$ or $A = 2B$	A1 o.e.
	Solve to obtain $B = -48$ and $A = -96$	M1 A1 **(4)**
Way 2 (a)	Long Division $(6x^3 + 3x^2 + Ax + B) \div (x \pm 1) = 6x^2 + px + q$ and sets remainder = 45	M1
	Quotient is $6x^2 - 3x + (A + 3)$ and remainder is $B - A - 3$ =45 so $B - A$ = 48 *	A1*
Way 2 (b)	$(6x^3 + 3x^2 + Ax + B) \div (2x + 1) = 3x^2 + px + q$ and sets remainder = 0	M1
	Quotient is $3x^2 + \frac{A}{2}$ and remainder is $B - \frac{A}{2} = 0$	A1
	Then Solve to obtain $B = -48$ and $A = -96$ as in scheme above (Way 1)	M1 A1
(c)	Obtain $(3x^2 - 48)$, $(x^2 - 16)$, $(6x^2 - 96)$, $\left(3x^2 + \frac{A}{2}\right)$, $(3x^2 + B)$, $\left(x^2 + \frac{A}{6}\right)$ or $\left(x^2 + \frac{B}{3}\right)$ as factor or as quotient after division by (2x + 1) . Division by (x+4) or (x-4) see below	B1ft
	Factorises $(3x^2 - 48)$, $(x^2 - 16)$, $(48 - 3x^2)$, $(16 - x^2)$ or $(6x^2 - 96)$	M1
	= 3 (2x + 1)(x + 4)(x − 4) (if this answer follows from a wrong A or B then award A0) isw if they go on to solve to give x = 4, -4 and -1/2	A1cso **(3) [9]**

Notes

(a) Way 1: **M1:** 1 or −1 substituted into f(x) and expression put equal to ±45

A1*: Answer is given. Must have substituted − 1 and put expression equal to +45.
Correct equation with powers of − 1 evaluated and conclusion with no errors seen.

Way 2: **M1**: Long division as far as a remainder which is set equal to ±45

A1*: See correct quotient and correct remainder and printed answer obtained with no errors

(b) Way 1: **M1**: Must see $f(-\frac{1}{2})$ and "= 0" unless subsequent work implies this.

A1: Give credit for a correct equation **even unsimplified** when first seen, then isw.
A correct equation implies M1A1.

M1: Attempts to solve the **given equation from part (a)** and their simplified or unsimplified linear equation in A and B from part (b) as far as A =... or B = ...(must eliminate one of the constants but algebra need not be correct for this mark). May just write down the correct answers.

A1: **Both** A and B correct

Way 2: **M1**: Long division as far as a remainder which is set equal to 0

A1: See correct quotient and correct remainder put equal to 0

M1A1: As in Way 1

There may be a mixture of Way 1 for (a) and Way 2 for (b) or vice versa.

(c) **B1**: May be written straight down or from long division, inspection, comparing coefficients or pairing terms

M1: Valid attempt to factorise a **listed** quadratic (see general notes) so $(3x-16)(x+3)$ could get M1A0

A1cso: (Cannot be awarded if A or B is wrong) Needs the answer in the scheme or -3 (2x+1)(4+x)(4 − x) or equivalent but factor 3 must be shown and there must be all the terms together with brackets.

Way 2: A minority might divide by (x- 4) or (x + 4) obtaining $(6x^2 + 27x + 12)$ or $(6x^2 - 21x - 12)$ for B1

They then need to factorise $(6x^2 + 27x + 12)$ or $(6x^2 - 21x - 12)$ for M1

Then A1cso as before

Special cases:

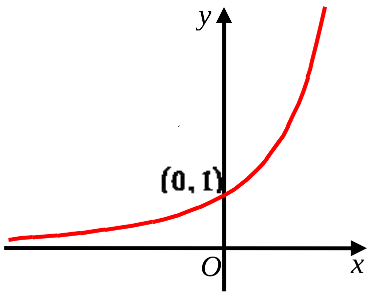
If they write down f(x) = 3 (2x+1)(x+4)(x - 4) with no working, this is B1 M1 A1

But if they give f(x) = (2x+1)(x+4)(x - 4) with no working (from calculator?) give B1M0A0

And f(x) = (2x + 1)(3x + 12)(x − 4) or f(x) = (6x + 3)(x + 4)(x − 4) or f(x) = (2x + 1)(x + 4)(3x − 12) is B1M1A0

Question Number	Scheme		Marks
	If there is no labelling, mark (a) and (b) in that order		
127(a)	$f(x) = 2x^3 - 7x^2 + 4x + 4$		
	$f(2) = 2(2)^3 - 7(2)^2 + 4(2) + 4$	Attempts $f(2)$ or $f(-2)$	M1
	$= 0$, and so $(x - 2)$ is a factor.	$f(\mathbf{2}) = 0$ with no sign or substitution errors ($2(2)^3 - 7(2)^2 + 4(2) + 4 = 0$ is sufficient) **and for conclusion.** Stating "hence factor" or "it is a factor" or a "tick" or "QED" or "no remainder" or "as required" are fine for the conclusion **but not = 0 just underlined and not hence (2 or f(2)) is a factor**. Note also that a conclusion can be implied from a preamble, eg: "If $f(2) = 0$, $(x - 2)$ is a factor…."	A1
	Note: Long division scores no marks in part (a). The factor theorem is required.		
			[2]
127(b)	$f(x) = \{(x - 2)\}(2x^2 - 3x - 2)$	M1: Attempts long division by $(x – 2)$ or other method using $(x – 2)$, to obtain $(2x^2 \pm ax \pm b)$, $a \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection."	M1 A1
		A1: $(2x^2 - 3x - 2)$	
	$= (x - 2)(x - 2)(2x + 1)$ or $(x - 2)^2(2x + 1)$ or equivalent e.g. $= 2(x - 2)(x - 2)(x + \frac{1}{2})$ or $2(x - 2)^2(x + \frac{1}{2})$	**d**M1: Factorises a 3 term quadratic. (see rule for factorising a quadratic in the General Principles for Core Maths Marking). **This is dependent** on the previous method mark being awarded but there must have been no remainder. Allow an attempt to solve the quadratic to determine the **factors**.	**d**M1 A1
		A1: cao – needs all three factors **on one line**. Ignore following work (such as a solution to a quadratic equation.)	
	Note $= (x - 2)(\frac{1}{2}x - 1)(4x + 2)$ would lose the last mark as it is not **fully** factorised		
	For correct answers only award full marks in (b)		
			[4]
			Total 6

EXPERT TUITION

Question Number	Scheme		Marks
128.	Graph of $y = 3^x$ and solving $3^{2x} - 9(3^x) + 18 = 0$		
(a)		At least two of the three criteria correct. **(See notes below.)**	B1
		All three criteria correct. **(See notes below.)**	B1
		Criteria number 1: Correct shape of curve for $x \geq 0$ and at least touches the positive y-axis. **Criteria number 2**: Correct shape of curve for $x < 0$. Must not touch the x-axis or have any turning points. **Criteria number 3**: $(0, 1)$ stated or in a table or 1 marked on the y-axis. Allow $(1, 0)$ rather than $(0, 1)$ if marked in the "correct" place on the y-axis.	
			[2]
(b)	$(3^x)^2 - 9(3^x) + 18 = 0$ or $y = 3^x \Rightarrow y^2 - 9y + 18 = 0$	Forms a quadratic of the correct form in 3^x or in "y" where $"y" = 3^x$ or even in x where $"x" = 3^x$	M1
	$\{ (y-6)(y-3) = 0$ or $(3^x - 6)(3^x - 3) = 0 \}$		
	$y = 6,\ y = 3$ or $3^x = 6,\ 3^x = 3$	**Both** $y = 6$ and $y = 3$.	A1
	$\{3^x = 6 \Rightarrow\}\ x\log 3 = \log 6$ or $x = \dfrac{\log 6}{\log 3}$ or $x = \log_3 6$	A valid method for solving $3^x = k$ where $k > 0, k \neq 1, k \neq 3$ **to give either** $x\log 3 = \log k$ or $x = \dfrac{\log k}{\log 3}$ or $x = \log_3 k$.	**d**M1
	$x = 1.63092...$	awrt 1.63	A1**cso**
	Provided the first M1A1 is scored, the second M1A1 can be implied by awrt 1.63		
	$x = 1$	$x = 1$ stated as a solution from ***any*** working.	B1
			[5]
			Total 7

EXPERT TUITION

Question Number	Scheme		Marks
	$f(x) = -4x^3 + ax^2 + 9x - 18$		
129. (a)	$f(2) = -32 + 4a + 18 - 18 = 0$	Attempts $f(2)$ or $f(-2)$	M1
	$\Rightarrow 4a = 32 \Rightarrow a = 8$	cso	A1
			[2]
(a) Way 2	$f(x) = (x-2)(px^2+qx+r)$		
	$= px^3 + (q-2p)x^2 + (r-2q)x - 2r$		
	$r = 9 \Rightarrow q = 0$ also $p = -4 \therefore a = -2p = 8$	Compares coefficients leading to $-2p = a$	M1
	$a = 8$	cso	A1
(a) Way 3	$(-4x^3 + ax^2 + 9x - 18) \div (x-2)$		
	$Q = -4x^2 + (a-8)x + 2a - 7$ $R = 4a - 32$	Attempt to divide ±f(x) by (x – 2) to give a quotient at least of the form $\pm 4x^2 + g(a)x$ and a remainder that is a function of a	M1
	$4a - 32 = 0 \Rightarrow a = 8$	cso	A1
(b)	$f(x) = (x-2)(-4x^2+9)$	Attempts long division or other method, to obtain $(-4x^2 \pm ax \pm b)$, $b \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection."	M1
	$= (x-2)(3-2x)(3+2x)$ or equivalent e.g. $= -(x-2)(2x-3)(2x+3)$ or $= (x-2)(2x-3)(-2x-3)$	dM1: A ***valid*** attempt to factorise their quadratic – see General Principles. **This is dependent** on the previous method mark being awarded, but there must have been no remainder. A1: cao – must have all 3 factors on the same line. Ignore subsequent work (such as a solution to a quadratic equation.)	**d**M1A1
			[3]
(c)	$f\left(\frac{1}{2}\right) = -4\left(\frac{1}{8}\right) + 8\left(\frac{1}{4}\right) + 9\left(\frac{1}{2}\right) - 18 = -12$	Attempts $f\left(\frac{1}{2}\right)$ or $f\left(-\frac{1}{2}\right)$ Allow A1ft for the correct numerical value of $\frac{\text{their } a}{4} - 14$	M1A1ft
			[2]
(c) Way 2	$\pm(-4x^3 + 8x^2 + 9x - 18) \div (2x-1)$		
	$Q = -2x^2 + 3x + 6$ $R = -12$	M1: Attempt long division to give a remainder that is independent of x A1: Allow A1ft for the correct numerical value of $\frac{\text{their } a}{4} - 14$.	M1A1ft
			Total 7

Question Number	Scheme	Marks
130. (a)	Either (Way 1) : Attempt $\text{f}(3)$ or $\text{f}(-3)$ $\text{f}(3) = 54 - 45 + 3a + 18 = 0 \Rightarrow 3a = -27 \Rightarrow a = -9$ * Or (Way 2): Assume *a* = -9 and attempt $\text{f}(3)$ or $\text{f}(-3)$ f(3) = 0 so (*x* -3) is factor	M1 A1 * cso **(2)**
	Or (Way 3): $\left(2x^3 - 5x^2 + ax + 18\right) \div \left(x - 3\right) = 2x^2 + px + q$ where *p* is a number and *q* is an expression in terms of *a* Sets the remainder 18+3*a*+9 = 0 and solves to give $a = -9$	M1 A1* cso **(2)**
(b)	Either (Way 1): $\text{f}(x) = (x - 3)(2x^2 + x - 6)$ $= (x - 3)(2x - 3)(x + 2)$	M1A1 M1A1 **(4)**
	Or (Way 2) Uses trial or factor theorem to obtain *x* = -2 **or** *x* = 3/2 Uses trial or factor theorem to obtain both *x* = -2 **and** *x* = 3/2 Puts three factors together (see notes below) Correct factorisation : $(x - 3)(2x - 3)(x + 2)$ or $(3 - x)(3 - 2x)(x + 2)$ or $2(x - 3)(x - \frac{3}{2})(x + 2)$ oe	M1 A1 M1 A1 **(4)**
	Or (Way 3) No working three factors $(x - 3)(2x - 3)(x + 2)$ otherwise need working	M1A1M1A1
(c)	$\{3^y = 3 \Rightarrow\}$ $\underline{y = 1}$ or g(1) = 0 $\{3^y = 1.5 \Rightarrow\} \log\left(3^y\right) = \log 1.5$ or $y = \log_3 1.5$ $\{y = 0.3690702...\} \Rightarrow y =$ awrt 0.37	B1 M1 A1 **(3)** **[9]**
	Notes for Question 130	
(a)	M1 for attempting either $\text{f}(3)$ or $\text{f}(-3)$ – with **numbers substituted into expression** A1 for applying $\text{f}(3)$ **correctly**, setting the result **equal to 0**, and manipulating this correctly to give the result given on the paper i.e. $a = -9$. (Do not accept ***x*** = -9) Note that the answer is given in part (a). If they **assume** *a* = -9 and **verify** by factor theorem or division they must state (*x* – 3) **is a factor** for A1 (or equivalent such as QED or a tick).	
(b)	1st M1: attempting to divide by $(x - 3)$ leading to a 3TQ beginning with the correct term, usually $2x^2$. (Could divide by $(3 - x)$, in which case the quadratic would begin $-2x^2$.) This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. 1st A1: usually for $2x^2 + x - 6$ … Credit when seen and use isw if miscopied 2nd M1: for a ***valid**** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1) 2nd A1 is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.) NB: $(x - 3)(x - \frac{3}{2})(x + 2)$ is M1A1M0A0, $(x - 3)(x - \frac{3}{2})(2x + 4)$ is M1A1M1A0, but $2(x - 3)(x - \frac{3}{2})(x + 2)$ is M1A1M1A1.	
(c)	B1: $\underline{y = 1}$ seen as a solution – may be spotted as answer – no working needed. Allow also for g(1) = 0. M1: Attempt to take logs to solve $3^y = \alpha$ or even $3^{ky} = \alpha$, but not $6^y = \alpha$ where $\alpha > 0$ and $\alpha \neq 3$ & was a root of f(*x*) = 0 (ft their factorization) A1: for an answer that rounds to 0.37. If a third answer is included (and not “rejected”) such as ln(-2) lose final A mark	

Question Number	Scheme		Marks
131.	$f(1) = a + b - 4 - 3 = 0$ **or** $a + b - 7 = 0$	Attempt f(±1)	M1
	$a + b = 7$ *	Must be f(1) and = **0** needs to be seen	A1
			(2)
	Long Division		
	$(ax^3 + bx^2 - 4x - 3) \div (x - 1) = ax^2 + px + q$ where p and q are in terms of a or b or both and sets their remainder = 0 NB Quotient = $ax^2 + (a+b)x + (a+b-4)$		M1
	a + b = 7 *		A1
			(2)

Question number	Scheme	Marks
132 (a)	$f(-2) = 2.(-2)^3 - 7.(-2)^2 - 10.(-2) + 24$	M1
	= 0 so (x+2) is a factor	A1
		(2)
(b)	$\mathrm{f}(x) = (x+2)(2x^2 - 11x + 12)$	M1 A1
	$\mathrm{f}(x) = (x+2)(2x-3)(x-4)$	dM1 A1
		(4)
		6 marks
Notes (a)	**M1** : Attempts f(± 2) (Long division is M0) **A1** : is for =0 **and conclusion** **Note**: Stating "hence factor" or "it is a factor" or a "√" (tick) or "QED" is fine for the conclusion. **Note** also that a conclusion can be implied from a <u>preamble</u>, eg: "If $\mathrm{f}(-2) = 0$, $(x + 2)$ is a factor…." (Not just f(-2)=0)	
(b)	**1st M1**: Attempts long division by correct factor or other method leading to obtaining $(2x^2 \pm ax \pm b)$, $a \neq 0, b \neq 0$, even with a remainder. Working need not be seen as could be done "by inspection." Or *Alternative Method* : **1st M1**: Use $(x+2)(ax^2 + bx + c) = 2x^3 - 7x^2 - 10x + 24$ with expansion and comparison of coefficients to obtain a = 2 and to obtain values for b and c **1st A1**: For seeing $(2x^2 - 11x + 12)$. [Can be seen here in (b) after work done in (a)] **2nd M1**: Factorises quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded and **needs factors** **2nd A1**: is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.) Note: Some candidates will go from $\{(x+2)\}(2x^2 - 11x + 12)$ to $\{x = -2\}$, $x = \frac{3}{2}$, 4, and not list all three factors. Award these responses M1A1M0A0. Finds x = 4 and x = 1.5 by factor theorem, formula or calculator and produces factors M1 $\mathrm{f}(x) = (x+2)(2x-3)(x-4)$ **or** $\mathrm{f}(x) = 2(x+2)(x-1.5)(x-4)$ **o.e. is full marks** $\mathrm{f}(x) = (x+2)(x-1.5)(x-4)$ **loses last A1**	

EXPERT TUITION

Question Number	Scheme		Marks
133. (a)	$f(x) = 2x^3 - 7x^2 - 5x + 4$ Remainder $= f(1) = 2 - 7 - 5 + 4 = -6$ $= -6$	Attempts $f(1)$ or $f(-1)$. -6	M1 A1 **[2]**
(b)	$f(-1) = 2(-1)^3 - 7(-1)^2 - 5(-1) + 4$ and so $(x+1)$ is a factor.	Attempts $f(-1)$. $f(-1) = 0$ with no sign or substitution errors **and for conclusion.**	M1 A1 **[2]**
(c)	$f(x) = \{(x+1)\}(2x^2 - 9x + 4)$ $= (x+1)(2x-1)(x-4)$ (**Note**: *Ignore the ePEN notation of (b) (should be (c)) for the final three marks in this part*).		M1 A1 dM1 A1 **[4]** **8**

(a) M1 for ***attempting*** either $f(1)$ or $f(-1)$. Can be implied. Only one slip permitted.

M1 can also be given for an attempt (at least two "subtracting" processes) at long division to give a remainder which is independent of *x*. A1 can be given also for -6 seen at the bottom of long division working. Award A0 for a candidate who finds -6 but then states that the remainder is 6.

Award M1A1 for -6 without any working.

(b) M1: attempting only $f(-1)$. A1: must correctly show $f(-1) = 0$ and give a conclusion ***in part (b) only***.

Note: Stating "hence factor" or "it is a factor" or a "tick" or "QED" is fine for the conclusion.

Note also that a conclusion can be implied from a preamble, eg: "If $f(-1) = 0$, $(x+1)$ is a factor…."

Note: Long division scores no marks in part (b). The factor theorem is required.

(c) 1st M1: Attempts long division or other method, to obtain $(2x^2 \pm ax \pm b)$, $a \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection." $(2x^2 \pm ax \pm b)$ must be seen ***in part (c) only***. Award 1st M0 if the quadratic factor is clearly found from dividing $f(x)$ by $(x-1)$. Eg. Some candidates use their $(2x^2 - 5x - 10)$ in part (c) found from applying a long division method in part (a).

1st A1: For seeing $(2x^2 - 9x + 4)$.

2nd dM1: Factorises a 3 term quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded. This mark can also be awarded if the candidate applies the quadratic formula correctly.

2nd A1: is cao and needs all three factors on one line. Ignore following work (such as a solution to a quadratic equation.)

Note: Some candidates will go from $\{(x+1)\}(2x^2 - 9x + 4)$ to $\{x = -1\}$, $x = \frac{1}{2}$, 4, and not list all three factors. Award these responses M1A1M1A0.

Alternative: 1st M1: For finding either $f(4) = 0$ or $f\left(\frac{1}{2}\right) = 0$.

1st A1: A second correct factor of usually $(x-4)$ or $(2x-1)$ found. Note that any one of the other correct factors found would imply the 1st M1 mark.

2nd dM1: For using two known factors to find the third factor, usually $(2x \pm 1)$.

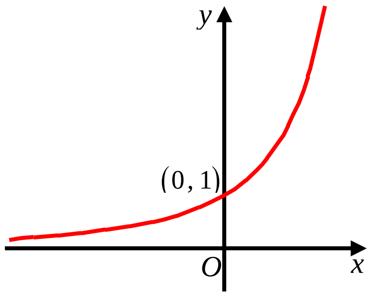
2nd A1 for correct answer of $(x+1)(2x-1)(x-4)$.

Alternative: (for the first two marks)

1st M1: Expands $(x+1)(2x^2 + ax + b)$ {giving $2x^3 + (a+2)x^2 + (b+a)x + b$ } then compare coefficients to find values for *a* and *b*. 1st A1: $a = -9$, $b = 4$

Not dealing with a factor of 2: $(x+1)(x - \frac{1}{2})(x-4)$ or $(x+1)(x - \frac{1}{2})(2x-8)$ scores M1A1M1A0.

Answer only, with one sign error: eg. $(x+1)(2x+1)(x-4)$ or $(x+1)(2x-1)(x+4)$ scores M1A1M1A0. **(c) Award M1A1M1A1 for Listing all three correct factors with no working**.

Question Number	Scheme	Marks
134.		
(a)	Graph of $y = 7^x$, $x \in \mathbb{R}$ and solving $7^{2x} - 4(7^x) + 3 = 0$	
		
	At least two of the three criteria correct. **(See notes below.)**	B1
	All three criteria correct. **(See notes below.)**	B1
		(2)
(b)	$y^2 - 4y + 3\{=0\}$ — Forming a quadratic {using $"y" = 7^x$ }.	M1
	$y^2 - 4y + 3\{=0\}$	A1
	$\{(y-3)(y-1) = 0$ or $(7^x - 3)(7^x - 1) = 0\}$	
	$y = 3$, $y = 1$ or $7^x = 3$, $7^x = 1$ — Both $y = 3$ and $y = 1$.	A1
	$\{7^x = 3 \Rightarrow\}$ $x\log 7 = \log 3$ or $x = \dfrac{\log 3}{\log 7}$ or $x = \log_7 3$ — A valid method for solving $7^x = k$ where $k > 0$, $k \neq 1$	dM1
	$x = 0.5645....$ — 0.565 or awrt 0.56	A1
	$x = 0$ — $x = 0$ stated as a solution.	B1
		(6)
		[8]

	Notes
(a)	B1B0: Any two of the following three criteria below correct. B1B1: All three criteria correct. Criteria number 1: Correct shape of curve for $x \geq 0$. Criteria number 2: Correct shape of curve for $x < 0$. Criteria number 3: $(0, 1)$ stated or 1 marked on the y-axis. Allow $(1, 0)$ rather than $(0, 1)$ if marked in the “correct” place on the y-axis.

Question Number	Scheme	Marks
(b)	1^{st} M1 is an attempt to form a quadratic equation {using $"y" = 7^x$.} 1^{st} A1 mark is for the correct quadratic equation of $y^2 - 4y + 3\{= 0\}$. Can use any variable here, eg: y, x or 7^x. Allow M1A1 for $x^2 - 4x + 3\{= 0\}$. Writing $(7^x)^2 - 4(7^x) + 3 = 0$ is also sufficient for M1A1. Award M0A0 for seeing $7^{x^2} - 4(7^x) + 3 = 0$ by itself without seeing $y^2 - 4y + 3\{= 0\}$ or $(7^x)^2 - 4(7^x) + 3 = 0$. 1^{st} A1 mark for both $y = 3$ and $y = 1$ or both $7^x = 3$ and $7^x = 1$. Do not give this accuracy mark for both $x = 3$ and $x = 1$, unless these are recovered in later working by candidate applying logarithms on these. Award M1A1A1 for $7^x = 3$ and $7^x = 1$ written down with no earlier working. 3^{rd} dM1 for solving $7^x = k$, $k > 0$, $k \neq 1$ to give either $x\ln 7 = \ln k$ or $x = \frac{\ln k}{\ln 7}$ or $x = \log_7 k$. dM1 is dependent upon the award of M1. 2^{nd} A1 for 0.565 or awrt 0.56. B1 is for the solution of $x = 0$, from ***any*** working.	

Question Number	Scheme	Marks
135	(a) Attempting to find $f(3)$ or $f(-3)$	M1
	$f(3) = 3(3)^3 - 5(3)^2 - (58 \times 3) + 40 = 81 - 45 - 174 + 40 = -98$	A1 (2)
	(b) $\{3x^3 - 5x^2 - 58x + 40 = (x-5)\}\ (3x^2 + 10x - 8)$	M1 A1
	Attempt to <u>factorise</u> 3-term quadratic, or to use the quadratic formula (see general principles at beginning of scheme). This mark may be implied by the correct solutions to the quadratic.	M1
	$(3x-2)(x+4) = 0$ $x = ...$ <u>or</u> $x = \frac{-10 \pm \sqrt{100+96}}{6}$	A1 ft
	$\frac{2}{3}$ (or exact equiv.), -4, 5 (Allow 'implicit' solns, e.g. f(5) = 0, etc.)	A1 (5)
	Completely correct solutions without working: full marks.	**7**

(a) <u>Alternative (long division):</u>

Divide by $(x - 3)$ to get $(3x^2 + ax + b)$, $a \neq 0, b \neq 0$. [M1]

$(3x^2 + 4x - 46)$, and -98 seen. [A1]

(If continues to say 'remainder = 98', isw)

<u>'Grid' method</u>

3	3	−5	−58	40
	0	9	12	−138
	3	4	−46	−98

(b) 1st M requires use of $(x-5)$ to obtain $(3x^2 + ax + b)$, $a \neq 0, b \neq 0$.

(Working need not be seen… this could be done 'by inspection'.)

<u>'Grid' method</u>

3	3	−5	−58	40
	0	15	50	−40
	3	10	−8	0

$(3x^2 + 10x - 8)$ ←

2nd M for the attempt to <u>factorise</u> their 3-term quadratic, or to solve it using the quadratic formula.

Factorisation: $(3x^2 + ax + b) = (3x + c)(x + d)$, where $|cd| = |b|$.

A1ft: Correct factors for their 3-term quadratic <u>followed by a solution</u> (at least one value, which might be incorrect), <u>or</u> numerically correct expression from the quadratic formula for their 3-term quadratic.

<u>Note</u> therefore that if the quadratic is correctly factorised but no solutions are given, the last 2 marks will be lost.

<u>Alternative (first 2 marks):</u>

$(x-5)(3x^2 + ax + b) = 3x^3 + (a-15)x^2 + (b-5a)x - 5b = 0$,

then compare coefficients to find <u>values</u> of a and b. [M1]

$a = 10$, $b = -8$ [A1]

<u>Alternative 1</u>: (factor theorem)

M1: Finding that $f(-4) = 0$

A1: Stating that $(x + 4)$ is a factor.

M1: Finding third factor $(x-5)(x+4)(3x \pm 2)$.

A1: Fully correct factors (no ft available here) <u>followed by a solution</u>, (which might be incorrect).

A1: All solutions correct.

<u>Alternative 2</u>: (direct factorisation)

M1: Factors $(x-5)(3x+p)(x+q)$ A1: $pq = -8$

M1: $(x-5)(3x \pm 2)(x \pm 4)$

Final A marks as in Alternative 1.

Throughout this scheme, allow $\left(x \pm \frac{2}{3}\right)$ as an alternative to $(3x \pm 2)$.

Question number	Scheme	Marks
136.	(a) Attempt to find f(–4) or f(4). $\left(f(-4) = 2(-4)^3 - 3(-4)^2 - 39(-4) + 20\right)$ $(= -128 - 48 + 156 + 20)$ = 0, so (*x* + 4) is a factor.	M1 A1 (2)
	(b) $2x^3 - 3x^2 - 39x + 20 = (x+4)(2x^2 - 11x + 5)$	M1 A1
	$.....(2x-1)(x-5)$ (The 3 brackets need not be written together) or $.....\left(x - \frac{1}{2}\right)(2x-10)$ or equivalent	M1 A1cso (4)
		6
	(a) Long division scores no marks in part (a). The <u>factor theorem</u> is required. However, the first two marks in (b) can be earned from division seen in (a)… … but if a different long division result is seen in (b), the work seen in (b) takes precedence for marks in (b). A1 requires zero and a simple <u>conclusion</u> (even just a tick, or Q.E.D.), or may be scored by a <u>preamble</u>, e.g. 'If f(–4) = 0, (*x* + 4) is a factor…..' (b) First M requires use of (*x* + 4) to obtain $(2x^2 + ax + b)$, $a \neq 0$, $b \neq 0$, even with a remainder. Working need not be seen… this could be done 'by inspection'. Second M for the attempt to factorise their three-term quadratic. Usual rule: $(kx^2 + ax + b) = (px + c)(qx + d)$, where $\|cd\| = \|b\|$ and $\|pq\| = \|k\|$. If 'solutions' appear before or after factorisation, ignore… … but factors must be seen to score the second M mark. <u>Alternative (first 2 marks):</u> $(x+4)(2x^2 + ax + b) = 2x^3 + (8+a)x^2 + (4a+b)x + 4b = 0$, then compare coefficients to find <u>values</u> of *a* and *b*. [M1] $a = -11$, $b = 5$ [A1] <u>Alternative:</u> Factor theorem: Finding that $f\left(\frac{1}{2}\right) = 0$ $\therefore$ factor is, $(2x - 1)$ [M1, A1] Finding that $f(5) = 0$ $\therefore$ factor is, $(x-5)$ [M1, A1] "Combining" all 3 factors is <u>not</u> required. If just one of these is found, score the <u>first 2 marks</u> M1 A1 M0 A0. <u>Losing a factor of 2:</u> $(x+4)\left(x - \frac{1}{2}\right)(x-5)$ scores M1 A1 M1 A0. <u>Answer only, one sign wrong:</u> e.g. $(x+4)(2x-1)(x+5)$ scores M1 A1 M1 A0	

Question Number	Scheme	Marks
137(a)	$4x^2-25 \to (2x+5)(2x-5)$	B1
	$\dfrac{6}{2x+5}+\dfrac{2}{2x-5}+\dfrac{60}{4x^2-25}=\dfrac{6(2x-5)+2(2x+5)+60}{(2x+5)(2x-5)}$	M1
	$=\dfrac{16x+40}{(2x+5)(2x-5)}$	A1
	$=\dfrac{8(2x+5)}{(2x+5)(2x-5)}=\dfrac{8}{(2x-5)}$	A1
		(4)
(b)	$\mathrm{f}(x)=\dfrac{8}{2x-5}\Rightarrow y=\dfrac{8}{2x-5}\Rightarrow 2xy-5y=8\Rightarrow x=\dfrac{8+5y}{2y}$	M1
	$\Rightarrow \mathrm{f}^{-1}(x)=\dfrac{8+5x}{2x}$ oe	A1
	$0<x<\dfrac{8}{3}$	B1ft
		(3)
		(7 marks)

Alternative solutions to part (a)

137(a) ALT I	$4x^2-25=(2x+5)(2x-5)$	B1
	$\dfrac{6}{2x+5}+\dfrac{2}{2x-5}=\dfrac{16x-20}{4x^2-25}$	
	$\dfrac{6}{2x+5}+\dfrac{2}{2x-5}+\dfrac{60}{4x^2-25}=\dfrac{16x-20+60}{4x^2-25}$	M1
	$=\dfrac{16x+40}{4x^2-25}$	A1
	$=\dfrac{8(2x+5)}{(2x+5)(2x-5)}=\dfrac{8}{2x-5}$	A1

137(a) ALT II	$4x^2-25=(2x+5)(2x-5)$	B1
	$\dfrac{60}{4x^2-25}=\dfrac{-6}{2x+5}+\dfrac{6}{2x-5}$	M1
	$\dfrac{6}{2x+5}+\dfrac{2}{2x-5}+\dfrac{60}{4x^2-25}=\dfrac{6}{2x+5}+\dfrac{2}{2x-5}+\dfrac{-6}{2x+5}+\dfrac{6}{2x-5}$	A1
	$=\dfrac{8}{(2x-5)}$	A1

(a)

B1: For **factorising** $4x^2-25 \to (2x+5)(2x-5)$ This can occur anywhere in the solution.

Note that it is possible to score this mark for expanding $(2x+5)(2x-5) \to 4x^2-25$ and then cancelling by $4x^2-25$. Both processes are required by this route. It can be implied if you see the correct intermediate form. (See A1)

M1: For combining the three fractions with a common denominator. The denominator must be correct and at least one numerator must have been adapted correctly . Accept as separate fractions. Condone missing brackets.

Accept $\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{6(2x-5)\left(4x^2-25\right)+2(2x+5)\left(4x^2-25\right)+60(2x+5)(2x-5)}{(2x+5)(2x-5)\left(4x^2-25\right)}$

Condone $\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{6(2x-5)+2+60}{(2x+5)(2x-5)}$ correct denominator, one numerator adapted correctly

Alternatively uses partial fractions $\frac{60}{4x^2-25}=\frac{A}{2x+5}+\frac{B}{2x-5}$ leading to values for A and B

A1: A correct intermediate form of $\frac{\text{simplified linear}}{\text{quadratic}}$ most likely to be $\frac{16x+40}{(2x+5)(2x-5)}$

Sometimes the candidate may write out the simplified numerator separately. In cases like this, you can award this A mark without explicitly seeing the fraction as long as a correct denominator is seen.

Using the partial fraction method, it is for $\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{-6}{2x+5}+\frac{6}{2x-5}$

A1: Further factorises and cancels (all of which may be implied) to reach the answer $\frac{8}{2x-5}$

This is not a given answer so condone slips in bracketing etc.

(b)

M1: Attempts to change the subject of the formula for a function of the form $y=\frac{A}{Bx+C}$

Condone attempts on an equivalent made up equation for candidates who don't progress in part (a). As a minimum expect to see multiplication by $(Bx+C)$ leading to x (or a replaced y) =

Alternatively award for 'inverting' Eg. $y=\frac{A}{Bx+C}$ to $\frac{Bx+C}{A}=\frac{1}{y}$ leading to x (or a replaced y) =

A1: $\mathrm{f}^{-1}(x)=\frac{8+5x}{2x}$ or $y=\frac{8+5x}{2x}$ or equivalent. Accept $y=\frac{4}{x}+\frac{5}{2}$ Condone $\mathrm{F}^{-1}(x)=\frac{8+5x}{2x}$

Condone $y=\frac{1}{2}\left(\frac{8}{x}+5\right)$ and $y=\frac{8}{2x}+\frac{5}{2}$ **BUT NOT** $y=\frac{\frac{8}{x}+5}{2}$ (fractions within fractions)

You may isw after a correct answer.

B1ft: $0<x<\frac{8}{3}$ or alternative forms such as $0<Domain<\frac{8}{3}$ Domain $=\left(0,\frac{8}{3}\right)$ or $\frac{8}{3}>x>0$

Do not accept $0<y<\frac{8}{3}$ or $0<\mathrm{f}^{-1}(x)<\frac{8}{3}$

Follow through on their $y=\frac{A}{Bx+C}$ so accept $0<x<\frac{A}{4B+C}$

Question Number	Scheme	Marks
138(a)	Either $k>13$ or $k=3$ Both $k>13$ $k=3$	B1 B1 **(2)**
(b)	Smaller solution: $2(5-x)+3=\frac{1}{2}x+10 \Rightarrow x=\frac{6}{5}$ Larger solution: $-2(5-x)+3=\frac{1}{2}x+10 \Rightarrow x=\frac{34}{3}$	M1 A1 M1 A1 **(4)**
(c)	$(6,12)$	B1B1 **(2)** **(8 marks)**

(a)

B1: Either $k>13$ or $k=3$ Condone $k \geqslant 13$ instead of $k>13$ for this mark only. Also condone $y \leftrightarrow k$

Do not accept $k \geq 3$ for B1

B1: Both $k>13$, $k=3$ with no other restrictions. Accept and / or / , between the two solutions

(b)

M1: An acceptable method of finding **the smaller intersection**. The initial equation must be of the correct form and it must lead to a value of *x*. For example $2(5-x)+3=\frac{1}{2}x+10 \Rightarrow x=...$ or $5-x=\left(\frac{1}{4}x+\frac{7}{2}\right)$

A1: For $x=\frac{6}{5}$ or equivalent such as 1.2 Ignore any reference to the y coordinate

M1: An acceptable method of finding **the larger intersection**. The initial equation must be of the correct form and it must lead to a value of *x*. For example $-2(5-x)+3=\frac{1}{2}x+10 \Rightarrow x=...$ or $5-x=-\left(\frac{1}{4}x+\frac{7}{2}\right)$

A1: For $x=\frac{34}{3}$ or equivalent such as $11.\dot{3}$ Ignore any reference to the y coordinate

If there are any extra solutions in addition to the correct two, then withhold the final A1 mark.
ISW if the candidate then refers back to the range in (a) and deletes a solution

....

Alt method by squaring

M1: $2|5-x|+3=\frac{1}{2}x+10 \Rightarrow 4(5-x)^2=\left(\frac{1}{2}x+7\right)^2$ oe. In the main scheme the equation must be correct of the correct form but in this case you may condone '2' not being squared

A1: Correct 3TQ. The $=0$ may be implied by subsequent work. $\frac{15}{4}x^2-47x+51=0$ oe

M1: Solves using an appropriate method $15x^2-188x+204=0 \Rightarrow (5x-6)(3x-34)=0 \Rightarrow x=..$

A1: Both $x=\frac{6}{5}$ $x=\frac{34}{3}$ and no others.

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(c)

B1: Accept p = 6 **or** q = 12. Allow in coordinates as x = 6 **or** y = 12.

B1: For both p = 6 **and** q = 12. Allow in coordinates as x = 6 **and** y = 12

Allow embedded within a single coordinate $(6,12)$. So for example $(2,12)$ is scored B1 B0

Question Number	Scheme	Marks
139.	$x^2-9=(x+3)(x-3)$	B1
	$\frac{4x}{x^2-9}-\frac{2}{(x+3)}=\frac{4x-2(x-3)}{(x+3)(x-3)}$	M1
	$=\frac{2x+6}{(x+3)(x-3)}$	A1
	$=\frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$	
	$=\frac{2}{(x-3)}$	A1
		(4)

B1 $x^2-9=(x+3)(x-3)$ This can occur anywhere.

M1 For combining the two fractions with a common denominator. The denominator must be correct and at least one numerator must have been adapted. Accept as separate fractions. Condone missing brackets.

For example accept $\frac{4x}{x^2-9}-\frac{2}{x+3}=\frac{4x(x+3)-2(x^2-9)}{(x+3)(x^2-9)}$

accept separately $\frac{4x}{(x+3)(x-3)}-\frac{2}{(x+3)}=\frac{4x}{(x+3)(x-3)}-\frac{2x-3}{(x+3)(x-3)}$ condoning missing bracket

condone $\frac{4x}{x^2-9}-\frac{2}{x+3}=\frac{4x(x+3)-2}{(x+3)(x^2-9)}$ ....as only one numerator has been adapted

A1 A correct intermediate form of $\frac{\text{simplified linear}}{\text{simplified quadratic}}$

Accept $\frac{2x+6}{(x+3)(x-3)}$, $\frac{2x+6}{x^2-9}$, and even $\frac{(2x+6)\cancel{(x+3)}}{(x^2-9)\cancel{(x+3)}}$,

A1 Further factorises and cancels (which may be implied) to reach the answer $\frac{2}{x-3}$

Do not penalise correct solutions that include incomplete lines Eg $\frac{4x-2(x-3)}{(x+3)(x-3)}=\frac{4x-2x+6}{\ldots}=\frac{2x+6}{(x+3)(x-3)}=\frac{2}{x-3}$

This is not a "show that" question.

Note: Watch out for an answer of $\frac{2}{x+3}$ probably scored from $\frac{4x-2(x-3)}{(x+3)(x-3)}=\frac{2x-6}{(x+3)(x-3)}=\frac{2(x-3)}{(x+3)(x-3)}$

This would score B1 M1 A0 A0

Question Number	Scheme	Marks
140(a)	$y \geq 3$	B1
		(1)
(b)	$y = 3 + \sqrt{x+2} \Rightarrow y - 3 = \sqrt{x+2} \Rightarrow x = (y-3)^2 - 2$	M1 A1
	$\Rightarrow g^{-1}(x) = (x-3)^2 - 2, \quad \text{with } x..3$	A1
		(3)
(c)	$g(x) = x \Rightarrow 3 + \sqrt{x+2} = x$	
	$\Rightarrow x + 2 = (x-3)^2 \Rightarrow x^2 - 7x + 7 = 0$	M1, A1
	$\Rightarrow x = \dfrac{7 \pm \sqrt{21}}{2} \Rightarrow x = \dfrac{7 + \sqrt{21}}{2}$ only	M1, A1
		(4)
(d)	$a = \dfrac{7 + \sqrt{21}}{2}$	B1 ft
		(1)
		9 marks
(c) Alt	Solves $g^{-1}(x) = x \Rightarrow (x-3)^2 - 2 = x$	
	$\Rightarrow x^2 - 7x + 7 = 0$	M1, A1
	$\Rightarrow x = \dfrac{7 \pm \sqrt{21}}{2} \Rightarrow x = \dfrac{7 + \sqrt{21}}{2}$ only	dM1, A1
		(4)

(a)

B1 States the correct range for g Accept $g(x)..3, g..3, \text{Range}..3, [3, \infty)$ Range is greater than or equal to 3

Condone $f..3$ Do not accept $g(x) > 3, x..3, (3, \infty)$

(b)

M1 Attempts to make *x* or a swapped *y* the subject of the formula. The minimum expectation is that the 3 is moved over followed by an attempt to square both sides. Condone for this mark $\sqrt{x+2} = y \pm 3 \Rightarrow x + 2 = y^2 \pm 9$

A1 Achieves $x = (y-3)^2 - 2$ or if swapped $y = (x-3)^2 - 2$ or equivalent such as $x = y^2 - 6y + 7$

A1 Requires a correct function in *x* + correct domain **or** a correct function in *x* with a correct follow through on the range in (a) but do not follow through on $x \in \mathbb{R}$

Accept for example $g^{-1}(x)=(x-3)^2-2, \quad x..3$ Condone $f^{-1}(x)=(x-3)^2-2, \quad x..3$

or variations such as $y=(x-3)^2-2, \quad x>3$ if (a) was y >3

Accept expanded versions such as $g^{-1}(x)=x^2-6x+7, \quad x..3$ but remember to isw after a correct answer (Condone $f^{-1}(x)=x^2-6x+7, \quad x..3$)

(c)

M1 Sets $3+\sqrt{x+2}=x$, moves the 3 over and then attempts to square both sides.

Can be scored for $\sqrt{x+2}=x-3 \Rightarrow x+2=x^2 \pm 9$

A1 $x^2-7x+7=0$. The = 0 may be implied by subsequent working

M1 Correct method of solving their 3TQ by the formula/ completing the square. The equation must have real roots.

It is dependent upon them having attempted to set $3+\sqrt{x+2}=x$ and proceeding to a quadratic.

You may just see both roots written down which is fine.

Allow for this mark decimal answers Eg 5.79 and 1.21 for $x^2-7x+7=0$ You may need to check with a calc.

A1 $(x)=\dfrac{7+\sqrt{21}}{2}$ or exact equivalent **only**.

This answer following the correct quadratic would imply the previous M

Allow $x=\dfrac{7}{2}+\sqrt{\dfrac{21}{4}}$ but **DO NOT** allow $x=\dfrac{7\pm\sqrt{21}}{2}$

....

(c) can of course be attempted by solving

$$3+\sqrt{x+2}="(x-3)^2-2" \Rightarrow x^4-12x^3+44x^2-49x+14=0$$
$$\vdots \qquad \Rightarrow (x^2-7x+7)(x^2-5x+2)=0$$

The scheme can be applied to this

....

(d)

B1ft $(a)=\dfrac{7+\sqrt{21}}{2}$ oe . You may condone $\boldsymbol{x}=\dfrac{7+\sqrt{21}}{2}$. You may allow this following a re - start.

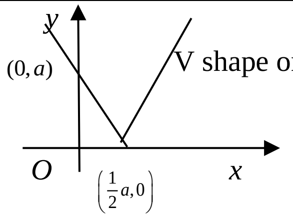
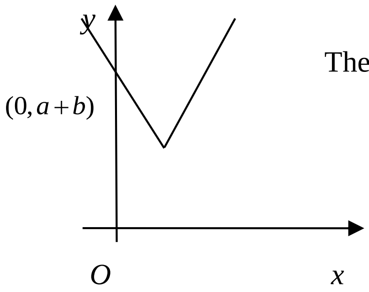
You may allow the correct decimal answer, awrt 5.79, following exact/decimal work in part (c) or a restart.

Follow through on their root, including decimals, coming from the **positive** root with the **positive** sign in (c).

Eg In (c) . $x^2-7x+11=0 \Rightarrow x=\dfrac{7\pm\sqrt{5}}{2}$ So the correct follow through would be $x=\dfrac{7+\sqrt{5}}{2}$

If they only had one root in (c) then follow through on this as long as it is positive.

SC. If they give the correct roots in parts (c) and (d) without considering the correct answer then award B1 in (d) following the A0 in (c). So $(x)=\dfrac{7\pm\sqrt{21}}{2}$ as their answer in part (c), allow $(x/a)=\dfrac{7\pm\sqrt{21}}{2}$ for B1 in (d).

Question Number	Scheme	Marks
141(a)(i)	 V shape on x - axis **or** coordinates $\left(\frac{1}{2}a,0\right)$ **and** $(0,a)$	B1
	Correct shape, position and coordinates	B1
(ii)	 Their "V" shape translated up or $(0,a+b)$	B1ft
	Correct shape, position and $(0,a+b)$	B1
		(4)
(b)	States or uses $a+b=8$	B1
	Attempts to solve $\lvert 2x-a\rvert+b=\frac{3}{2}x+8$ in either x or with $x=c$	
	$2c-a+b=\frac{3}{2}c+8\Rightarrow kc=\text{f}(a,b)$	M1
	Combines $kc=\text{f}(a,b)$ with $a+b=8$ $\Rightarrow c=4a$	dM1 A1
		(4)
		(8 marks)

(a)(i)

B1 V shape sitting anywhere on the x- axis **or** for $\left(\frac{1}{2}a,0\right)$ and $(0,a)$ lying on the curve.

Condone non -symmetrical graphs and ones lying on just one side of the y -axis

B1 V shape sitting on the positive x-axis at $\left(\frac{1}{2}a,0\right)$, cutting the y-axis at $(0,a)$ and lying in both quadrants 1 and 2

Accept $\frac{1}{2}a$ and a marked on the correct axis. Condone say $(a,0)$ for $(0,a)$ as long as it is on the correct axis.

Condone a dotted line appearing on the diagram as many reflect $y=2x-a$ to sketch $y=\lvert 2x-a\rvert$

If it is a solid line then it would not score the shape mark.

(a)(ii)

B1ft Follow through on (a)(i). Their graph translated up. Allow on U shapes and non symmetrical graphs.

Alternatively score for the $(0,a+b)$ lying on the curve

B1 V shape lying in quadrants 1 and 2 with the vertex in quadrant 1 cutting the y- axis at $(0,a+b)$

Ignore any coordinates given for the vertex.

(b)

B1 States or uses $a+b=8$ or exact equivalent. Condone use of capital letters throughout

It is not scored for just $|0-a|+b=8$

M1 This M is for an understanding of the modulus.

It is scored for an attempt at solving $(2x-a)+b=\frac{3}{2}x+8$ or $-(2x-a)+b=\frac{3}{2}x+8$ in either *x* or with *x* replaced by *c*. The signs of the 2*x* and the *a* must be different. $|2x-a|\neq 2x+a$

You may see $(2x-a)+b=\frac{3}{2}x+8 \Rightarrow kx=\mathrm{f}(a,b)$

You may see $-2x+a+b=\frac{3}{2}x+8 \Rightarrow kx=\mathrm{f}(a,b)$

You may see $(2x-a)+b=\frac{3}{2}x+8 \Rightarrow kx=\mathrm{f}(a,b)$ being solved with *b* replaced with **their** $a+b=8$

You may see $-2c+a+b=\frac{3}{2}c+8 \Rightarrow kc=\mathrm{f}(a,b)$ being solved with *b* replaced with **their** $a+b=8$

dM1 This dM mark is scored for combining $b=8-a$ with $(2x-a)+b=\frac{3}{2}x+8$ (or their $kx=\mathrm{f}(a,b)$ resulting from that equation) resulting in a link between *x* and *a* **Both equations must have been correct initially.**

Alternatively for combining $b=8-a$ with their $2c-a+b=\frac{3}{2}c+8$ (or their $kc=\mathrm{f}(a,b)$ resulting from that equation) resulting in a link between *c* and *a*

You may condone sign slips in finding the link between *x* (or *c*) and *a*

If you see an approach that involves making $|2x-a|$ the subject followed by squaring, and you feel that it deserves credit, please send to review. The solution proceeds as follows

Look for $|2x-a|=\frac{3}{2}x+8-b \Rightarrow |2x-a|=\frac{3}{2}x+a \Rightarrow (2x-a)^2=\left(\frac{3}{2}x+a\right)^2 \Rightarrow 7x\left(\frac{1}{4}x-a\right)=0$

A1 $c=4a$ ONLY

....

Special Case where they have the roots linked with the incorrect branch of the curve.

They have $x=0$ as the solution to $2x-a+b=\frac{3}{2}x+8 \Rightarrow -a+b=8$..................(1)

They have $x=c$ as the solution to $-2x+a+b=\frac{3}{2}x+8 \Rightarrow \frac{7}{2}x=a+b-8$............(2)

Solve (1) and (2) $\Rightarrow x=\frac{4}{7}a$

Hence $\Rightarrow c=\frac{4}{7}a$

This would score B0 M1 dM0 A0 anyway but should be awarded SC B0, M1 dM1, A0 for above work leading to either $x=\frac{4}{7}a$ or $c=\frac{4}{7}a$

Question	Scheme		Marks
142(a)	$\text{fg}(x)=\dfrac{28}{x-2}-1$ $\quad\left(=\dfrac{30-x}{x-2}\right)$		M1
	Sets $\text{fg}(x)=x\Rightarrow\dfrac{28}{x-2}-1=x$ $\Rightarrow 28=(x+1)(x-2)$ $\Rightarrow x^2-x-30=0$		M1
	$\Rightarrow (x-6)(x+5)=0$ $\Rightarrow x=6,\ x=-5$		dM1 A1 **(4)**
(b)	$a=6$		B1 ft **(1)** **5 marks**
Alt 1(a)	$\text{fg}(x)=x\Rightarrow \text{g}(x)=\text{f}^{-1}(x)$ $\dfrac{4}{x-2}=\dfrac{x+1}{7}$		M1
	$\Rightarrow x^2-x-30=0$		M1
	$\Rightarrow (x-6)(x+5)=0$ $\Rightarrow x=6,\ x=-5$		dM1 A1 **4 marks**
S. Case	Uses $\text{gf}(x)$ instead $\text{fg}(x)$ $\dfrac{4}{7x-1-2}=x$	Makes an error on $\text{fg}(x)$ Sets $\text{fg}(x)=x\Rightarrow\dfrac{7\times4}{7\times(x-2)}-1=x$	M0
	$\Rightarrow 7x^2-3x-4=0$ $\Rightarrow (7x+4)(x-1)=0$	$\Rightarrow x^2-x-6=0$ $\Rightarrow (x+2)(x-3)=0$	M1
	$\Rightarrow x=-\dfrac{4}{7},\quad x=1$	$\Rightarrow x=-2,\quad x=3$	dM1 A0 **2 out of 4 marks**

(a)

M1 Sets or implies that $\text{fg}(x)=\dfrac{28}{x-2}-1$ Eg accept $\text{fg}(x)=7\left(\dfrac{4}{x-2}\right)-1$ followed by $\text{fg}(x)=\dfrac{7\times4}{x-2}-1$

Alternatively sets $\text{g}(x)=\text{f}^{-1}(x)$ where $\text{f}^{-1}(x)=\dfrac{x\pm1}{7}$

Note that $\text{fg}(x)=7\left(\dfrac{4}{x-2}\right)-1=\dfrac{28}{7(x-2)}-1$ is M0

M1 Sets up a 3TQ (= 0) from an attempt at $\text{fg}(x)=x$ or $\text{g}(x)=\text{f}^{-1}(x)$

dM1 Method of solving 3TQ (= 0) to find at least one value for x. See "General Priciples for Core Mathematics" on page 3 for the award of the mark for solving quadratic equations
This is dependent upon the previous M. You may just see the answers following the 3TQ.

A1 Both $x=6$ and $x=-5$

(b)

B1ft For $a=6$ but you may follow through on the largest solution from part (a) provided more than one answer was found in (a). Accept 6, $a=6$ and even $x=6$
Do not award marks for part (a) for work in part (b).

Question	Scheme	Marks
143	(i) 21	B1
	(ii) $4e^{2x}-25=0 \Rightarrow e^{2x}=\frac{25}{4} \Rightarrow 2x=\ln\left(\frac{25}{4}\right) \Rightarrow x=\frac{1}{2}\ln\left(\frac{25}{4}\right), \Rightarrow x=\ln\left(\frac{5}{2}\right)$	M1A1, A1
	(iii) 25	B1
		(5)
		(5 marks)

(i)

B1 Sight of 21. Accept $(0, 21)$

Do not accept just $|4-25|$ or $(21, 0)$

(ii)

M1 Sets $4e^{2x}-25=0$ and proceeds via $e^{2x}=\frac{25}{4}$ or $e^{x}=\frac{5}{2}$ to $x=..$

Alternatively sets $4e^{2x}-25=0$ and proceeds via $(2e^{x}-5)(2e^{x}+5)=0$ to $e^{x}=..$

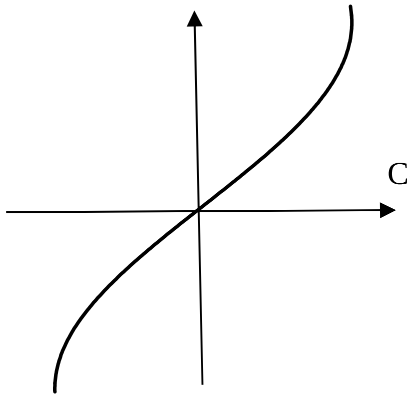
A1 $\frac{1}{2}\ln\left(\frac{25}{4}\right)$ or awrt 0.92

A1 cao $\ln\left(\frac{5}{2}\right)$ or $\ln 5-\ln 2$. Accept $\left(\ln\left(\frac{5}{2}\right), 0\right)$

(iii)

B1 $k=25$ Accept also 25 or $y=25$

Do not accept just $|-25|$ or $x=25$ or $y=\pm 25$

Question	Scheme	Marks
144(a)	 Correct position **or** curvature Correct position **and** curvature	M1 A1 **(2)**
(b)	$3\arcsin(x+1)+\pi=0 \Rightarrow \arcsin(x+1)=-\frac{\pi}{3}$ $\Rightarrow (x+1)=\sin\left(-\frac{\pi}{3}\right)$ $\Rightarrow x=-1-\frac{\sqrt{3}}{2}$	M1 dM1A1 **(3)** **(5 marks)**

(a) Ignore any scales that appear on the axes

M1 Accept for the method mark
Either one of the two sections with correct curvature passing through (0,0),
Or both sections condoning dubious curvature passing through (0,0) (but do not accept any negative gradients)
Or a curve with a different range or an "extended range"
See the next page for a useful guide for clarification of this mark.

A1 A curve only in quadrants one and three passing through the point (0,0) with a gradient that is always positive. The gradient should appear to be approx ∞ at each end. If you are unsure use review
If range and domain are given then ignore.

(b)

M1 Substitutes $g(x+1)=\arcsin(x+1)$ in $3g(x+1)+\pi=0$ and attempts to make $\arcsin(x+1)$ the subject

Accept $\arcsin(x+1)=\pm\frac{\pi}{3}$ or even $g(x+1)=\pm\frac{\pi}{3}$. Condone $\frac{\pi}{3}$ in decimal form awrt1.047

dM1 Proceeds by evaluating $\sin\left(\pm\frac{\pi}{3}\right)$ and making x the subject.

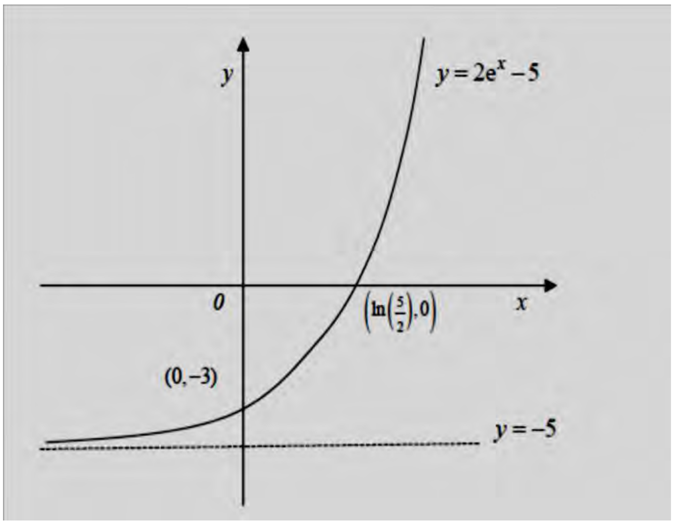
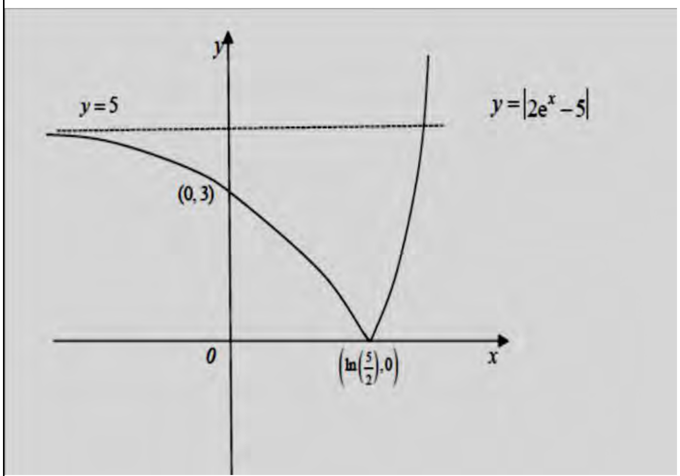
Accept for this mark $\Rightarrow x=\pm\frac{\sqrt{3}}{2}\pm 1$. Accept decimal such as -1.866

Do not allow this mark if the candidate works in mixed modes (radians and degrees)
You may condone invisible brackets for both M's as long as the candidate is working correctly with the function

A1 $-1-\frac{\sqrt{3}}{2}$ oe with no other solutions. Remember to isw after a correct answer

Be careful with single fractions. $-\frac{2-\sqrt{3}}{2}$ and $\frac{-2+\sqrt{3}}{2}$ are incorrect but $-\frac{2+\sqrt{3}}{2}$ is correct

Note: It is possible for a candidate to change $\frac{\pi}{3}$ to $60°$ and work in degrees for all marks

Question Number	Scheme		Marks
145.(ai)		Shape	B1
		$\left(\ln\left(\frac{5}{2}\right),0\right)$ and $(0,-3)$	B1
		$y=-5$	B1
			(3)
(aii)		Shape inc cusp	B1ft
		$\left(\ln\left(\frac{5}{2}\right),0\right)$ *and* $(0,3)$	B1ft
		$y=5$	B1ft
			(3)
(b)	$x \geq \ln\left(\frac{5}{2}\right)$		B1 ft
			(1)
(c)	$2e^x - 5 = -2 \Rightarrow (x) = \ln\left(\frac{3}{2}\right)$		M1A1
	$(x) = \ln\left(\frac{7}{2}\right)$		B1
			(3)
			(10 marks)

(a)(i)

B1 — For an exponential (growth) shaped curve in any position. For this mark be tolerant on slips of the pen at either end. See Practice and Qualification for examples.

B1 — Intersections with the axes at $\left(\ln\left(\frac{5}{2}\right),0\right)$ and $(0,-3)$.

Allow $\ln\left(\frac{5}{2}\right)$ and -3 being marked on the correct axes.

Condone $\left(0,\ln\left(\frac{5}{2}\right)\right)$ and $(-3,0)$ being marked on the x and y axes respectively.

Do not allow $\left(\ln\left(\frac{5}{2}\right),0\right)$ appearing as awrt (0.92, 0) for this mark unless seen elsewhere. Allow if seen in body of script. If they are given in the body of the script and differently on the curve (save for the decimal equivalent) then **the ones on the curve take precedence.**

B1 — Equation of the asymptote given as $y=-5$. Note that the curve must appear to have an asymptote at $y=-5$, not necessarily drawn. It is not enough to have -5 marked on the axis or indeed $x=-5$. An extra asymptote **with an equation** gets B0

(a)(ii)

B1ft — For **either** the correct shape **or** a reflection of their curve from (a)(i) in the x- axis. For this to be scored it must have appeared both above and below the x - axis. The shape must be correct including the cusp. The curve to the lhs of the cusp must appear to have the correct curvature

B1ft — Score for both intersections or follow through on both the intersections given in part (a)(i), including decimals, as long as the curve appeared both above and below the x - axis. See part (a) for acceptable forms

B1ft — Score for an asymptote of $y=5$ or follow through on an asymptote of $y=-C$ from part (a)(i). Note that the curve must appear to have an asymptote at $y=C$ but do not penalise if the first mark in (a)(ii) has been withheld for incorrect curvature on the lhs.

(b)

B1ft — Score for $x\ldots\ln\left(\frac{5}{2}\right)$, $x\ldots$awrt 0.92 or follow through on the x intersection in part (a)

(c)

M1 — Accept $2e^x-5=-2$ or $-2e^x+5=2 \Rightarrow x=..\ln(..)$

Allow squaring so $\left(2e^x-5\right)^2=4 \Rightarrow e^x=..\text{and}..\Rightarrow x=\ln(..),\ln(..)$

A1 — $x=\ln\left(\frac{3}{2}\right)$ or exact equivalents such as $x=\ln 1.5$. You do not need to see the x.

Remember to isw a subsequent decimal answer 0.405

B1 — $x=\ln\left(\frac{7}{2}\right)$ or exact equivalents such as $x=\ln 3.5$. You do not need to see the x.

Remember to isw a subsequent decimal answer 1.25

If both answers are given in decimals and there is no working $x=$awrt 1.25, 0.405 award SC 100

Question Number	Scheme	Marks
146.(a)	Applies $vu'+uv'$ to $\left(x^2-x^3\right)e^{-2x}$	
	$g'(x)=\left(x^2-x^3\right)\times\text{-}2e^{-2x}+\left(2x-3x^2\right)\times e^{-2x}$	M1 A1
	$g'(x)=\left(2x^3-5x^2+2x\right)e^{-2x}$	A1
		(3)
(b)	Sets $\left(2x^3-5x^2+2x\right)e^{-2x}=0\Rightarrow 2x^3-5x^2+2x=0$	M1
	$x\left(2x^2-5x+2\right)=0\Rightarrow x=(0),\frac{1}{2},2$	M1,A1
	Sub $x=\frac{1}{2},2$ into $g(x)=\left(x^2-x^3\right)e^{-2x}\Rightarrow g\left(\frac{1}{2}\right)=\frac{1}{8e}$, $g(2)=-\frac{4}{e^4}$	dM1,A1
	Range $-\frac{4}{e^4}\leqslant g(x)\leqslant\frac{1}{8e}$	A1 **(6)**
(c)	Accept $g(x)$ is NOT a ONE to ONE function	
	Accept $g(x)$ is a MANY to ONE function	B1
	Accept $g^{-1}(x)$ would be ONE to MANY	**(1)**
		(10 marks)

Note that parts (a) and (b) can be scored together. Eg accept work in part (b) for part (a)

(a)

M1 **Uses the product rule** $vu'+uv'$ with $u=x^2-x^3$ and $v=e^{-2x}$ or vice versa. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $vu'+uv'$. If the rule is not quoted nor implied only accept expressions of the form $\left(x^2-x^3\right)\times\pm Ae^{-2x}+\left(Bx\pm Cx^2\right)\times e^{-2x}$ condoning bracketing issues

Method 2: multiplies out and **uses the product rule** on each term of $x^2e^{-2x}-x^3e^{-2x}$
Condone issues in the signs of the last two terms for the method mark
Uses the product rule for $uvw=u'vw+uv'w+uvw'$ applied as in method 1

Method 3:Uses **the quotient rule** with $u=x^2-x^3$ and $v=e^{2x}$. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $\frac{vu'-uv'}{v^2}$ If the rule is not quoted nor implied accept expressions of the form $\frac{e^{2x}\left(Ax-Bx^2\right)-\left(x^2-x^3\right)\times Ce^{2x}}{\left(e^{2x}\right)^2}$

condoning missing brackets on the numerator and e^{2x^2} on the denominator.

Method 4: Apply implicit differentiation to $ye^{2x}=x^2-x^3\Rightarrow e^{2x}\times\frac{dy}{dx}+y\times 2e^{2x}=2x-3x^2$

Condone errors on coefficients and signs

A1 A correct (unsimplified form) of the answer

$g'(x) = \left(x^2 - x^3\right) \times -2e^{-2x} + \left(2x - 3x^2\right) \times e^{-2x}$ by one use of the product rule

or $g'(x) = x^2 \times -2e^{-2x} + 2xe^{-2x} - x^3 \times -2e^{-2x} - 3x^2 \times e^{-2x}$ using the first alternative

or $g'(x) = 2x(1-x)e^{-2x} + x^2 \times -1 \times e^{-2x} + x^2(1-x) \times -2e^{-2x}$ using the product rule on 3 terms

or $g'(x) = \dfrac{e^{2x}\left(2x - 3x^2\right) - \left(x^2 - x^3\right) \times 2e^{2x}}{\left(e^{2x}\right)^2}$ using the quotient rule.

A1 Writes $g'(x) = \left(2x^3 - 5x^2 + 2x\right)e^{-2x}$. You do not need to see $f(x)$ stated and award even if a correct $g'(x)$ is followed by an incorrect $f(x)$. If the f(x) is not simplified at this stage you need to see it simplified later for this to be awarded.

(b) Note: The last mark in e-pen has been changed from a 'B' to an A mark

M1 For setting their f(x) =0. The = 0 may be implied by subsequent working.
Allow even if the candidate has failed to reach a 3TC for f(x).
Allow for $f(x) \geq 0$ or $f(x) \leq 0$ as they can use this to pick out the relevant sections of the curve

M1 For solving their 3TC = 0 by ANY correct method.
Allow for division of x or factorising out the x followed by factorisation of 3TQ. Check first and last terms of the 3TQ. Allow for solutions from either $f(x) \geq 0$ or $f(x) \leq 0$
Allow solutions **from the cubic equation** just appearing from a Graphical Calculator

A1 $x = \frac{1}{2}, 2$. Correct answers from a correct $g'(x)$ would imply all 3 marks so far in (b)

dM1 Dependent upon both previous M's being scored. For substituting their **two** (non zero) values of x into g(x) to find both y values. Minimal evidence is required $x = .. \Rightarrow y = ..$ is OK.

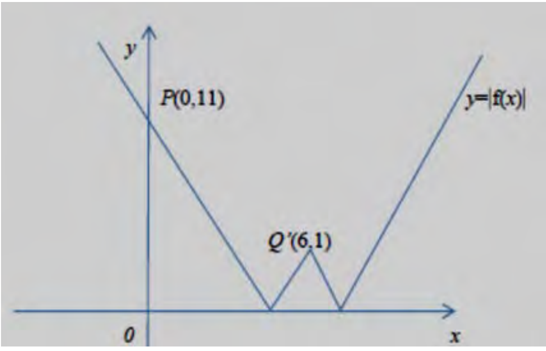
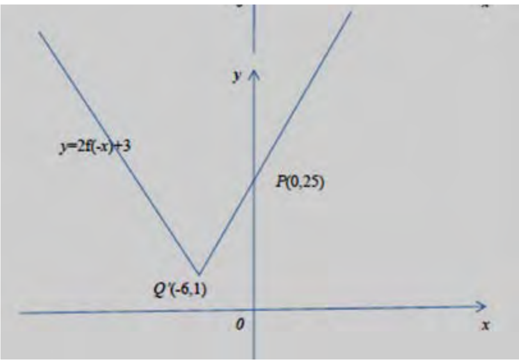
A1 Accept decimal answers for this mark. $g\left(\frac{1}{2}\right) = \frac{1}{8e}$ =awrt 0.046 **AND** $g(2) = -\frac{4}{e^4}$ =awrt -0.073

A1 CSO Allow $-\frac{4}{e^4} \leq$ Range $\leq \frac{1}{8e}$, $-\frac{4}{e^4} \leq y \leq \frac{1}{8e}$, $\left[-\frac{4}{e^4}, \frac{1}{8e}\right]$. Condone $y \geq -\frac{4}{e^4}$ $y \leq \frac{1}{8e}$

Note that the question states hence and part (a) must have been used for all marks. Some students will just write down the answers for the range from a graphical calculator.
Seeing just $-\frac{4}{e^4} \leq g(x) \leq \frac{1}{8e}$ or $-0.073 \leq g(x) \leq 0.046$ Special case 100000.
They know what a range is!

(c)

B1 If the candidate states 'NOT ONE TO ONE' then accept unless the explicitly link it to $g^{-1}(x)$.
So accept 'It is not a one to one function'. 'The function is not one to one' '$g(x)$ is not one to one'
If the candidate states 'IT IS MANY TO ONE' then accept unless the candidate explicitly links it to $g^{-1}(x)$. So accept 'It is a many to one function.' 'The function is many to one' '$g(x)$ is many to one'
If the candidate states 'IT IS ONE TO MANY' then accept unless the candidate explicitly links it to $g(x)$
Accept an explanation like " one value of x would map/ go to more than one value of y"
Incorrect statements scoring B0 would be $g^{-1}(x)$ is not one to one, $g^{-1}(x)$ is many to one and $g(x)$ is one to many.

Question Number	Scheme		Marks
147(a)		'W' Shape (0, 11) and (6, 1)	B1 B1 (2)
(b)		'V' shape (-6,1) (0,25)	B1 B1 B1 (3)
(c)	One of $a = 2$ or $b = 6$ $a = 2$ and $b = 6$		B1 B1 (2)
			(7 marks)

(a)

B1 A W shape in any position. The arms of the W do not need to be symmetrical but the two bottom points must appear to be at the same height. Do not accept rounded W's.
A correct sketch of $y = \text{f}(|x|)$ would score this mark.

B1 A W shape in quadrants 1 and 2 sitting on the x axis with $P' = (0,11)$ **and** $Q' = (6,1)$. It is not necessary to see them labelled. Accept 11 being marked on the y axis for P'. Condone $P' = (11,0)$ marked on the correct axis, but $Q' = (1,6)$ is B0

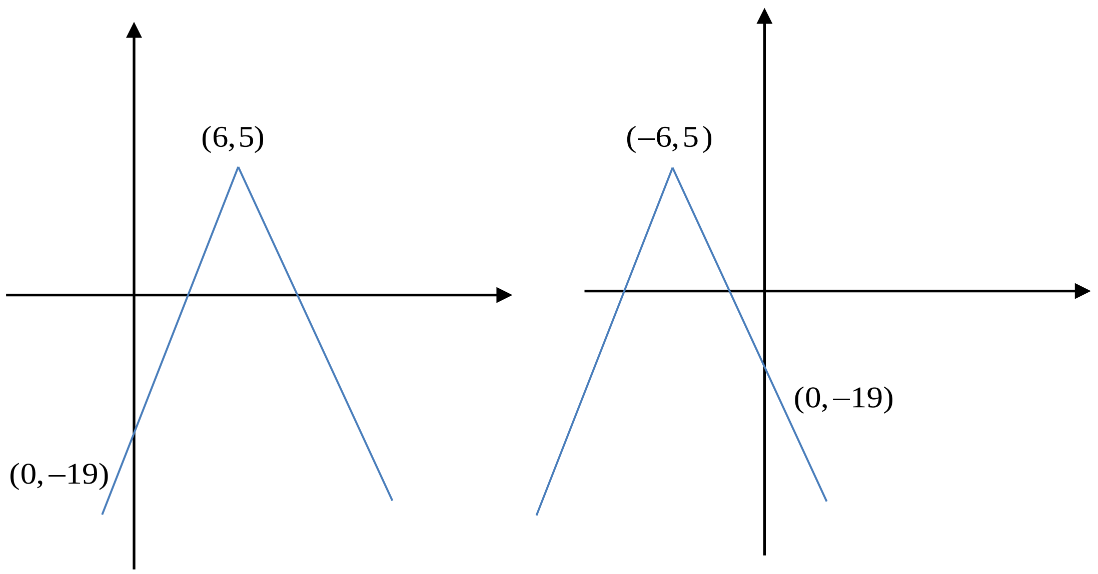
(b)

B1 Score for a V shape in any position on the grid. The arms of the V do not need to be symmetrical. Do not accept rounded or upside down V's for this mark.

B1 $Q' = (-6,1)$. It does not need to be labelled but it must correspond to the minimum point on the curve and be in the correct quadrant.

B1 $P' = (0,25)$. It does not need to be labelled but it must correspond to the y intercept and the line must cross the axis. Accept 25 marked on the correct axis. Condone $P' = (25,0)$ marked on the positive y axis.

Special case: A candidate who mistakenly sketches $y = -2\text{f}(x) + 3$ or $y = -2\text{f}(-x) + 3$ will arrive at one of the following. They can be awarded SC B1B0B0



(c)

B1 Either states a = 2 **or** b = 6.
This can be implied (if there are no stated answers given) by the candidate writing that $y = ..|x-6|-1$ or $y = 2|x-..|-1$. If they are both stated and written, the stated answer takes precedence.

B1 States both a = 2 **and** b = 6
This can be implied by the candidate stating that $y = 2|x-6|-1$
If they are both stated and written, the stated answer takes precedence.

Question Number	Scheme	Marks
148.(a)	$x^2+x-6=(x+3)(x-2)$	B1
	$\frac{x}{x+3}+\frac{3(2x+1)}{(x+3)(x-2)}=\frac{x(x-2)+3(2x+1)}{(x+3)(x-2)}$	M1
	$=\frac{x^2+4x+3}{(x+3)(x-2)}$	A1
	$=\frac{(x+3)(x+1)}{(x+3)(x-2)}$	
	$=\frac{(x+1)}{(x-2)}$ cso	A1*
		(4)
(b)	One end either $(y)>1,(y)\geqslant 1$ or $(y)<4,(y)\leqslant 4$	B1
	$1<y<4$	B1
		(2)
(c)	Attempt to set Either $g(x)=x$ or $g(x)=g^{-1}(x)$ or $g^{-1}(x)=x$ or $g^2(x)=x$	
	$\frac{(x+1)}{(x-2)}=x$ $\quad\frac{x+1}{x-2}=\frac{2x+1}{x-1}$ $\quad\frac{2x+1}{x-1}=x$ $\quad\frac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2}=x$	M1
	$x^2-3x-1=0\Rightarrow x=...$	A1, dM1
	$a=\frac{3+\sqrt{13}}{2}$ oe $\left(1.5+\sqrt{3.25}\right)$ cso	A1
		(4) (10 marks)

EXPERT TUITION

(a)

B1 $x^2+x-6=(x+3)(x-2)$ This can occur anywhere in the solution.

M1 For combining the two fractions with a common denominator. The denominator must be correct for their fractions and at least one numerator must have been adapted. Accept as separate fractions. Condone missing brackets.

Accept $\frac{x}{x+3}+\frac{3(2x+1)}{x^2+x-6}=\frac{x(x^2+x-6)+3(2x+1)(x+3)}{(x+3)(x^2+x-6)}$

Condone $\frac{x}{x+3}+\frac{3(2x+1)}{(x+3)(x-2)}=\frac{x\times x-2}{(x+3)(x-2)}+\frac{3(2x+1)}{(x+3)(x-2)}$

A1 A correct intermediate form of $\frac{\text{simplified quadratic}}{\text{simplified quadratic}}$

Accept $\frac{x^2+4x+3}{(x+3)(x-2)}, \frac{x^2+4x+3}{x^2+x-6}$, OR $\frac{x^3+7x^2+15x+9}{(x+3)(x^2+x-6)} \rightarrow \frac{(x+1)(x+3)\cancel{(x+3)}}{\cancel{(x+3)}(x^2+x-6)}$

As in question one they can score this mark having 'invisible' brackets on line 1.

A1* Further factorises and cancels (which may be implied) to complete the proof to reach the given answer $=\frac{(x+1)}{(x-2)}$. All aspects including bracketing must be correct. If a cubic is formed then it needs to be correct.

(b)

B1 States either end of the range. Accept either $y<4$, $y\leqslant 4$ or $y>1$, $y\geqslant 1$ with or without the y's.

B1 Correct range. Accept $1<y<4$, $1<\text{g}<4$, $y>1$ and $y<4$, $(1,4)$, $1<\text{Range}<4$, even $1<\text{f}<4$,

Do not accept $1<x<4$, $1<y\le 4$, $[1,4)$ etc.

Special case, allow B1B0 for $1<x<4$

(c)

M1 Attempting to set $g(x)=x$, $g^{-1}(x)=x$ or $g(x)=g^{-1}(x)$ or $g^2(x)=x$.

If $g^{-1}(x)$ has been used then a full attempt must have been made to make x the subject of the formula.

A full attempt would involve cross multiplying, collecting terms, factorising and ending with division.

As a result, it must be in the form $g^{-1}(x)=\frac{\pm 2x\pm 1}{\pm x\pm 1}$

Accept as evidence $\frac{(x+1)}{(x-2)}=x$ OR $\frac{x+1}{x-2}=\frac{\pm 2x\pm 1}{\pm x\pm 1}$ OR $\frac{\pm 2x\pm 1}{\pm x\pm 1}=x$ OR $\frac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2}=x$

A1 $x^2-3x-1=0$ or exact equivalent. The =0 may be implied by subsequent work.

dM1 For solving a 3TQ=0. It is dependent upon the first M being scored. Do not accept a method using factors unless it clearly factorises. Allow the answer written down awrt 3.30 (from a graphical calculator).

A1 a or $x=\frac{3+\sqrt{13}}{2}$. Ignore any reference to $\frac{3-\sqrt{13}}{2}$

Withhold this mark if additional values are given for x, $x>3$

Question Number	Scheme	Marks
149.	Factorise $4x^2-9=(2x-3)(2x+3)$ Use of common denominator $\frac{3}{2x+3}-\frac{1}{2x-3}+\frac{6}{4x^2-9}=\frac{3(2x-3)-1(2x+3)+6}{(2x+3)(2x-3)}$ $=\frac{4x-6}{(2x+3)(2x-3)}$ $=\frac{2\cancel{(2x-3)}}{(2x+3)\cancel{(2x-3)}}=\frac{2}{2x+3}$	B1 M1 A1 A1 **(4)** **4 marks**
	Alternative where $4x^2-9$ is not factorised $\frac{3}{2x+3}-\frac{1}{2x-3}+\frac{6}{4x^2-9}=\frac{3(2x-3)(4x^2-9)-1(2x+3)(4x^2-9)+6(2x+3)(2x-3)}{(2x+3)(2x-3)(4x^2-9)}$ $=\frac{2(2x-3)(4x^2-9)}{(2x+3)(2x-3)(4x^2-9)}$ or $\frac{(4x-6)(4x^2-9)}{(2x+3)(2x-3)(4x^2-9)}$ or $\frac{(2x-3)(8x^2-18)}{(2x+3)(2x-3)(4x^2-9)}$ $=\frac{(4x-6)\cancel{(4x^2-9)}}{(2x+3)(2x-3)\cancel{(4x^2-9)}}$ or $\frac{2\cancel{(4x-6)}(4x^2-9)}{(2x+3)\cancel{(2x-3)}(4x^2-9)}$ $=\frac{2}{2x+3}$	 M1 B1 A1 A1

B1 For **factorising** $4x^2-9$ to $(2x-3)(2x+3)$ at any point. Note that this is not scored for combining the terms $(2x-3)(2x+3)$ and writing the product as $4x^2-9$

M1 Use of common denominator – combines three fractions to form one. The denominator must be correct for their fractions and at least one numerator must have been adapted. Condone missing brackets.

$\frac{16x^3-24x^2-36x+54}{(4x^2-9)^2}$ is a correct intermediate stage but needs to be factorised and cancelled before A1

Examples of incorrect fractions scoring this mark are: $\frac{3(2x-3)-2x+3+6}{(2x+3)(2x-3)}$ missing bracket

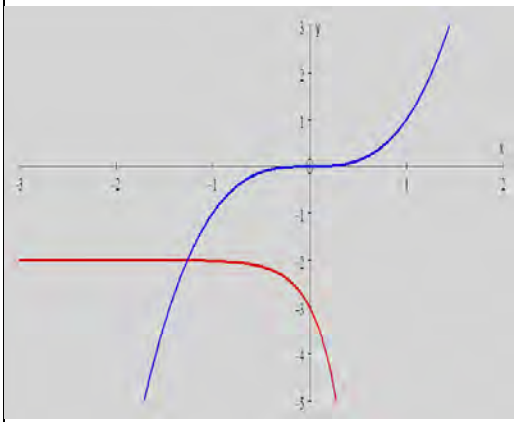
$\frac{3(4x^2-9)-4x^2-9+6(2x+3)(2x-3)}{(2x+3)(2x-3)(4x^2-9)}$ denominator correct and at least one numerator has been adapted.

A1 Correct simplified intermediate answer. It must be a CORRECT $\frac{\text{Linear}}{\text{Quadratic}}$ or $\frac{\text{Quadratic}}{\text{Cubic}}$

Accept versions of $\frac{4x-6}{(2x+3)(2x-3)}$ or $\frac{8x^2-18}{(2x+3)(4x^2-9)}$

A1 cao $=\frac{2}{2x+3}$

Allow recovery from invisible brackets for all 4 marks as the answer is not given.

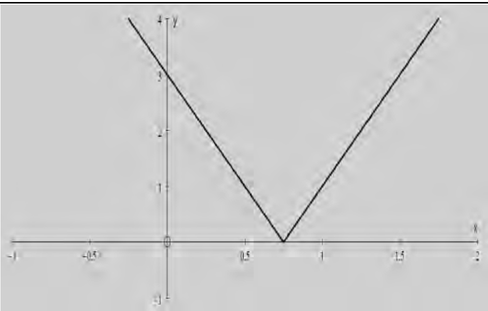
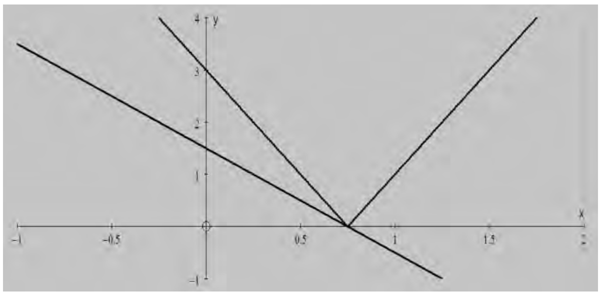
Question Number	Scheme	Marks
150		
	$y = x^3$	B1
	Shape of $y = -2 - e^{4x}$	B1
	$y = -2 - e^{4x}$ cuts y axis at (0,-3)	B1
	$y = -2 - e^{4x}$ has asymptote at $y = -2$	B1
		(4)
		4 marks

B1 Correct shape and position for $y = x^3$. It must appear to go through the origin.
It must only appear in Quadrants 1 and 3 and have a gradient that is always $\geqslant 0$. The gradient should appear large at either end. Tolerate slips of the pen.See practice and qualification for acceptable curves.

B1 Correct shape for $y = -2 - e^{4x}$, its position is not important for this mark. The gradient must be approximately zero at the left hand end and increase negatively as you move from left to right along the curve. See practice and qualification for acceptable curves.

B1 Score for $y = -2 - e^{4x}$ cutting or meeting the y axis at (0,-3). Its shape is not important.
Accept for the intention of (0,-3), -3 being marked on the y – axis as well as (-3,0)
Do not accept 3 being marked on the negative y axis.

B1 Score for $y = -2 - e^{4x}$ having an asymptote stated as $y = -2$. This is dependent upon the curve appearing to have an asymptote there. Do not accept the asymptote marked as '-2' or indeed $x = -2$. See practice and qualification for acceptable solutions.

Question Number	Scheme	Marks
151 (a)	 V shaped graph	B1
	Touches x axis at ¾ and cuts y axis at 3	B1
		(2)
(b)	Solves $4x-3=2-2x$ or $3-4x=2-2x$ to give either value of x	M1
	Both $x=\frac{5}{6}$ and $x=\frac{1}{2}$ or $x>\frac{5}{6}$ or $x<\frac{1}{2}$	A1
	$x<\frac{1}{2}$ or $x>\frac{5}{6}$	dM1A1
		(4)
(c)	 Draws graph Or solves $\lvert 4x-3\rvert=1\tfrac{1}{2}-2x$ to give one soln x = ¾	M1
	Accept for all values of x except $x=\frac{3}{4}$ Or $(x\in\mathbb{R},)$ $x\neq\frac{3}{4}$, or $x<\frac{3}{4}$, $x>\frac{3}{4}$	A1
		(2)
		(8 marks)

(a)

B1 A 'V' shaped graph. The position is not important. Do not accept curves. See practice and qualification items for clarity. Accept a V shape with a 'dotted' extension of $y = 4x - 3$ appearing under the x axis.

B1 The graph **meets** the x axis at $x = \frac{3}{4}$ and **crosses** the y axis at $y = 3$. Do not allow multiple meets or crosses

If they have lost the previous B1 mark for an extra section of graph underneath the x axis allow for **crossing** the x axis at $x = \frac{3}{4}$ and **crosses** the y axis at $y = 3$.

Accept marked elsewhere on the page with A and B marked on the graph and $A = \left(\frac{3}{4}, 0\right)$ and $B = (0, 3)$

Condone $\left(0, \frac{3}{4}\right)$ and (3, 0) marked on the correct axis

(b)

M1 Attempts to solve $|4x - 3| \ldots 2 - 2x$ finding at least one solution. You may see … replaced by either = or >

Accept as evidence $\pm 4x \pm 3 = 2 - 2x \Rightarrow x = ..$

Accept as evidence $\pm 4x \pm 3 > 2 - 2x \Rightarrow x > ..,$ or $x < ..$

A1 Both critical values $x = \frac{5}{6}$ and $x = \frac{1}{2}$, or one inequality, accept $x > \frac{5}{6}$ or $x < \frac{1}{2}$

Accept $x = 0.83$ and $x = 0.5$ for the critical values

Accept both of these answers with no incorrect working for both marks

dM1 Dependent upon the previous M, this is scored for selecting the outside region of their two points.

Eg if M1 has been scored for $4x - 3 = 2 - 2x \Rightarrow x = 0.83$ and $-4x - 3 = 2 - 2x \Rightarrow x = -2.5$

A correct application of M1 would be $x < -2.5, x > 0.83$

A1 Correct answer only $x < \frac{1}{2}$ or $x > \frac{5}{6}$.

Accept $x < 0.5, x > 0.8\dot{3}$

(c)

M1 **Either** sketch both lines showing a single intersection at the point $x = \frac{3}{4}$

Or solves $|4x - 3| = 1\frac{1}{2} - 2x$ using both $4x - 3 = 1\frac{1}{2} - 2x$ and $-4x + 3 = 1\frac{1}{2} - 2x$ **giving one solution** $x = \frac{3}{4}$

Accept $|4x - 3| > 1\frac{1}{2} - 2x$ using both $4x - 3 > 1\frac{1}{2} - 2x$ and $-4x + 3 > 1\frac{1}{2} - 2x$ **giving one solution** $x \ldots \frac{3}{4}$

If two values are obtained using either method it is M0A0

A1 States that the solution set is all values apart from $x = \frac{3}{4}$. Do not isw in this question. Score their final statement. Accept versions of all values of x except $x = \frac{3}{4}$ or $x \in \mathbb{R}, \ x \neq \frac{3}{4}$, or $x < \frac{3}{4}, \ x > \frac{3}{4}$

Question Number	Scheme	Marks
152(a)	$\mathrm{f}(x) > k^2$	B1
		(1)
(b)	$y = \mathrm{e}^{2x} + k^2 \Rightarrow \mathrm{e}^{2x} = y - k^2$	M1
	$\Rightarrow x = \frac{1}{2}\ln(y - k^2)$	dM1
	$\Rightarrow \mathrm{f}^{-1}(x) = \frac{1}{2}\ln(x - k^2), \quad x > k^2$	A1
		(3)
(c)	$\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$	M1
	$\Rightarrow \ln 8x^6 = 6$	M1
	$\Rightarrow 8x^6 = \mathrm{e}^6 \Rightarrow x = ..$	M1
	$\Rightarrow x = \left(\frac{\mathrm{e}}{\sqrt[6]{8}}\right) = \frac{\mathrm{e}}{\sqrt{2}}$ (Ignore any reference to $-\frac{\mathrm{e}}{\sqrt{2}}$)	A1
		(4)
(d)	$\mathrm{fg}(x) = e^{2\times\ln(2x)} + k^2$	M1
	$\Rightarrow \mathrm{fg}(x) = (2x)^2 + k^2 = 4x^2 + k^2$	A1
		(2)
(e)	$\mathrm{fg}(x) = 2k^2 \Rightarrow 4x^2 + k^2 = 2k^2$	
	$\Rightarrow 4x^2 = k^2 \Rightarrow x = ..$	M1
	$\Rightarrow x = \frac{k}{2}$ **only**	A1
		(2) **12 marks**
(alt c)	$\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$	M1
	$\Rightarrow \ln 2 + \ln x + \ln 2 + 2\ln x + \ln 2 + 3\ln x = 6$	
	$\Rightarrow 3\ln 2 + 6\ln x = 6$	
	$\Rightarrow \ln x = 1 - \frac{1}{2}\ln 2$	M1
	$\Rightarrow x = e^{1-\frac{1}{2}\ln 2}, = \frac{\mathrm{e}}{\sqrt{2}}$ (Ignore any reference to $-\frac{\mathrm{e}}{\sqrt{2}}$)	M1, A1
(alt e)		(4)
	$\Rightarrow 2\ln(2x) = \ln(2k^2 - k^2)$	
	$\Rightarrow \ln(2x)^2 = \ln(k^2), \Rightarrow 4x^2 = k^2 \Rightarrow x = \frac{k}{2}$	M1, A1

(a)

B1 States the correct range for f. Accept $\mathrm{f}(x) > k^2, \mathrm{f} > k^2, Range > k^2, (k^2, \infty), y > k^2$ Range is greater than k^2

Do not accept $\mathrm{f}(x) \geq k^2, x > k^2, [k^2, \infty)$

(b)

M1 Attempts to make x or a swapped y the subject of the formula. Score for $y = \mathrm{e}^{2x} + k^2 \Rightarrow \mathrm{e}^{2x} = y \pm k^2$ and proceeding to $x = \ln ...$. The minimum expectation is that e^{2x} is made the subject before taking ln's

dM1 Dependent upon the previous M having been scored. It is for proceeding by firstly taking ln's of the whole rhs, not the individual elements, and then dividing by 2. Score M1, dM1 for writing down

$x = \frac{1}{2}\ln(y \pm k^2)$ or alternatively $y = \frac{1}{2}\ln(x \pm k^2)$. Condone missing brackets for this mark.

A1 The correct answer in terms of x including the bracket **and** the domain $\mathrm{f}^{-1}(x) = \frac{1}{2}\ln(x - k^2), \quad x > k^2$.

Accept equivalent answers like $y = 0.5\ln\left|x - k^2\right|,$ Domain greater than k^2, (k^2, ∞)

(c)

M1 Attempts to solve equation by writing down $\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$

M1 Uses addition laws of logs to write in the form $\ln Ax^n = 6$

M1 Takes exp's (correctly) and proceeds to a solution for $x = ..$

A1 Correct solution and correct answer. $x = \frac{\mathrm{e}}{\sqrt{2}}$. You may ignore any reference to $x = -\frac{\mathrm{e}}{\sqrt{2}}$

Special caseS. Candidate who solve (and treat it as though it was bracketed)

S. Case 1 $\ln 2x + \ln 2x^2 + \ln 2x^3 = 6 \Rightarrow \ln 2x + 2\ln 2x + 3\ln 2x = 6 \Rightarrow 6\ln 2x = 6 \Rightarrow x = \frac{\mathrm{e}}{2}$

S. Case 2 $\ln 2x + \ln(2x)^2 + \ln(2x)^3 = 6 \Rightarrow 6\ln 2x = 6 \Rightarrow \ln 2x = 1 \Rightarrow x = \frac{\mathrm{e}}{2}$

S. Case 3 $\ln 2x + \ln(2x)^2 + \ln(2x)^3 = 6 \Rightarrow \ln 2x + \ln 4x^2 + \ln 8x^3 = 6 \Rightarrow \ln 64x^6 = 6 \Rightarrow 64x^6 = \mathrm{e}^6 \Rightarrow x = \frac{\mathrm{e}}{2}$

scores M0 (Incorrect statement/ may be implied by subsequent work), M1 (Correct ln laws), M1 (Correct method of arriving at x=) , A0

(d) For the purposes of marking you can score (d) and (e) together

M1 Correct order of applying g before f to give a correct unsimplified answer. Accept y =

Accept versions of $\mathrm{fg}(x) = e^{2\times\ln(2x)} + k^2, y = e^{\ln(2x)^2} + k^2$

A1 A correct simplified answer $\mathrm{fg}(x) = (2x)^2 + k^2$, or $\mathrm{fg}(x) = 4x^2 + k^2$. Accept y =

(e)

M1 Sets the answer to (d) in the form $Ax^2 + k^2 = 2k^2$, where $A = 2$ or 4 and proceeds in the correct order to reach an equation of the form $Ax^2 = k^2$.

In the alternative method it would be for reaching $\ln(Ax^2) = \ln(k^2)$, $A = 2$ or 4 or any equivalent form ln…= ln…

A1 $x = \frac{k}{2}$ **only.** The answer $x = \pm\frac{k}{2}$ is A0.

Question Number	Scheme	Marks
153	$$\begin{array}{r} 3x^2-2x+7 \\ x^2(+0x)-4 \overline{\big)\,3x^4-2x^3-5x^2+(0x)-4} \\ \underline{3x^4+0x^3-12x^2} \\ -2x^3+7x^2+0x \\ \underline{-2x^3+0x^2+8x} \\ 7x^2-8x-4 \\ \underline{7x^2+0x-28} \\ -8x+24 \end{array}$$	
By Division		
	$a=3$	B1
	Long division as far as $$\begin{array}{r} 3x^2-2x....... \\ x^2(+0x)-4 \overline{\big)\,3x^4-2x^3-5x^2+(0x)-4} \\ \underline{3x^4+0x^3-12x^2} \\ -2x^3+............... \\ \underline{-2x^3+................} \end{array}$$	M1
	Two of $b=-2$ $c=7$ $d=-8$ $e=24$	A1
	All four of $b=-2$ $c=7$ $d=-8$ $e=24$	A1
		(4 marks)

Notes for Question 153

B1 Stating $a=3$. This can also be scored by the coefficient of x^2 in $3x^2-2x+7$

M1 Using long division by x^2-4 and getting as far as the 'x' term. The coefficients need not be correct.
Award if you see the whole number part as $...x^2+...x$ following some working. You may also see this in a table/ grid.
Long division by $(x+2)$ will not score anything until $(x-2)$ has been divided into the new quotient. It is very unlikely to score full marks and the mark scheme can be applied.

A1 Achieving two of $b=-2$ $c=7$ $d=-8$ $e=24$.
The answers may be embedded within the division sum and can be implied.

A1 Achieving all of $b=-2$ $c=7$ $d=-8$ and $e=24$

Accept a correct long division for 3 out of the 4 marks scoring B1M1A1A0

Need to see a=…, b=…, or the values embedded in the rhs for all 4 marks

Question Number	Scheme	Marks
Alt 1 **By Multiplication**	$* \ 3x^4 - 2x^3 - 5x^2 - 4 \equiv (ax^2 + bx + c)(x^2 - 4) + dx + e$	
	Compares the x^4 terms $a = 3$	B1
	Compares coefficients to obtain a numerical value of one **furthe**r constant $-2 = b, \quad -5 = -4a + c \Rightarrow c = ..,$	M1
	Two of $b = -2 \quad c = 7 \quad d = -8 \quad e = 24$	A1
	All four of $b = -2 \quad c = 7 \quad d = -8 \quad e = 24$	A1
		(4 marks)

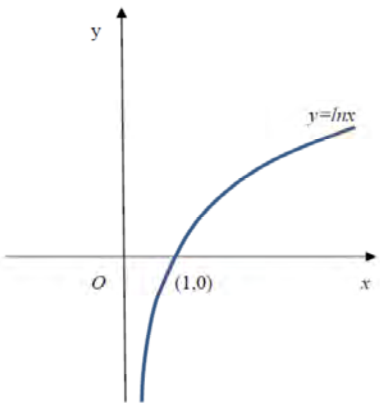
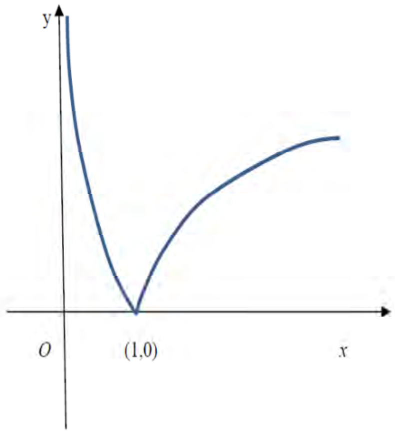
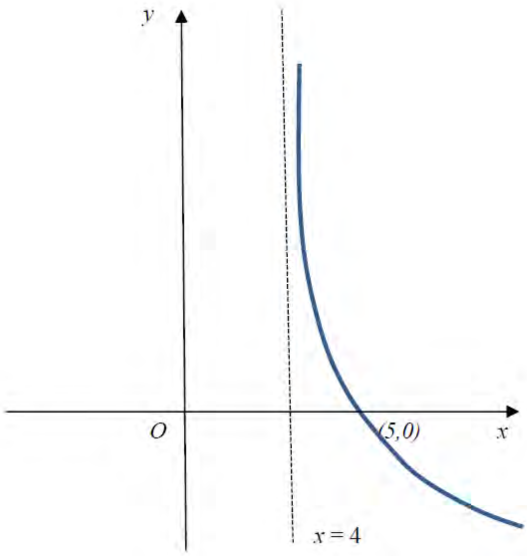
Notes for Question 153

B1 Stating $a = 3$. This can also be scored for writing $3x^4 = ax^4$

M1 Multiply out expression given to get *. Condone slips only on signs of either expression.
Then compare the coefficient of any term (other than x^4) to obtain a numerical value of one further constant. In reality this means a valid attempt at either *b* or *c*
The method may be implied by a correct additional constant to *a*.

A1 Achieving two of $b = -2 \ c = 7 \ d = -8 \ e = 24$

A1 Achieving all of $b = -2 \ c = 7 \ d = -8$ and $e = 24$

Question Number	Scheme		Marks
154(i)		*ln* graph crossing *x* axis at (1,0) and asymptote at *x*=0	B1
154(ii)		Shape including cusp	B1ft
		Touches or crosses the *x* axis at (1,0)	B1ft
		Asymptote given as *x*=0	B1
154(iii)		Shape	B1
		Crosses at (5, 0)	B1ft
		Asymptote given as *x*=4	B1
			(7 marks)

Notes for Question 154

(i) B1 Correct shape, correct position and passing through (1, 0).
Graph must 'start' to the rhs of the y - axis in quadrant 4 with a gradient that is large. The gradient then decreases as it moves through (1, 0) into quadrant 1. There must not be an obvious maximum point but condone 'slips'. Condone the point marked (0,1) on the correct axis. See practice and qualification for clarification. **Do not with hold this mark if (x=0) the asymptote is incorrect or not given**.

(ii) B1ft Correct shape **including the cusp** wholly contained in quadrant 1.
The shape to the rhs of the cusp should have a decreasing gradient and must not have an obvious maximum.. The shape to the lhs of the cusp should not bend backwards past (1,0)
Tolerate a 'linear' looking section here but not one with incorrect curvature (See examples sheet (ii) number 3. For further clarification see practice and qualification items.
Follow through on an incorrect sketch in part (i) as long as it was above and below the x axis.

B1ft The curve touches or crosses the x axis at (1, 0). Allow for the curve passing through a point marked '1' on the x axis. Condone the point marked on the correct axis as (0, 1)
Follow through on an incorrect intersection in part (i).

B1 Award for the asymptote to the curve given/ marked as $x = 0$. Do not allow for it given/ marked as 'the y axis'. There must be a graph for this mark to be awarded, and there must be an asymptote on the graph at $x = 0$. Accept if x=0 is drawn separately to the y axis.

(iii)

B1 Correct shape.
The gradient should always be negative and becoming less steep. It must be approximately infinite at the *lh* end and not have an obvious minimum. The lh end must not bend 'forwards' to make a C shape. The position is not important for this mark. See practice and qualification for clarification.

B1ft The graph crosses (or touches) the x axis at (5, 0). Allow for the curve passing through a point marked '5' on the x axis. Condone the point marked on the correct axis as (0, 5)
Follow through on an incorrect intersection in part (i). Allow for $\left((i)+4,0\right)$

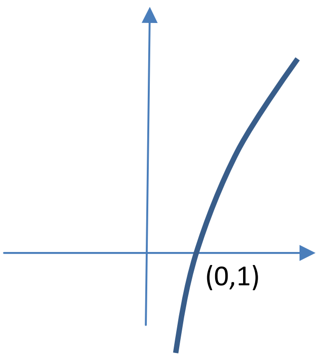
B1 The asymptote is given/ marked as $x = 4$. There must be a graph for this to be awarded and there must be an asymptote on the graph (in the correct place to the rhs of the y axis).

If the graphs are not labelled as (i), (ii) and (iii) mark them in the order that they are given.

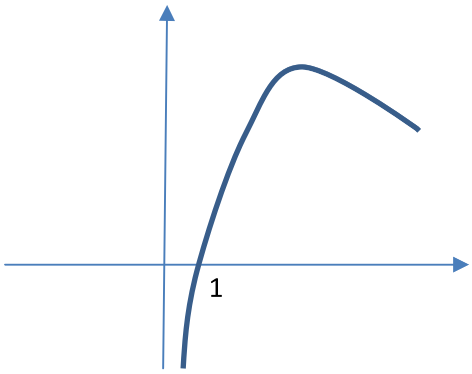
Examples of graphs in number 154

Part (i)

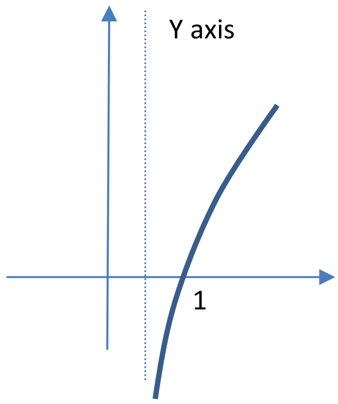
Condoned



Not condoned

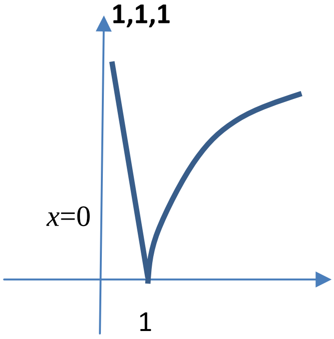


Condone

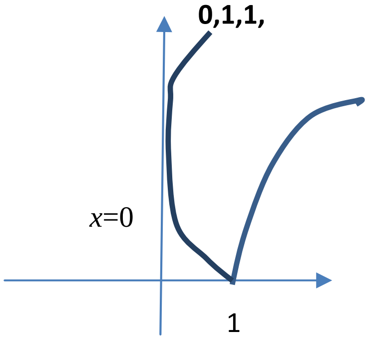


Part (ii)

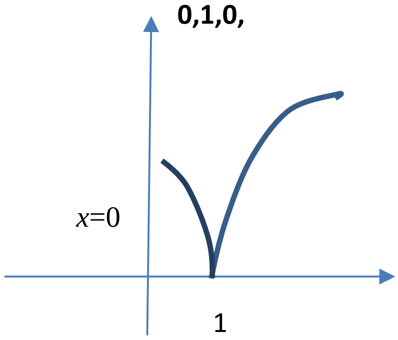
1,1,1



0,1,1,

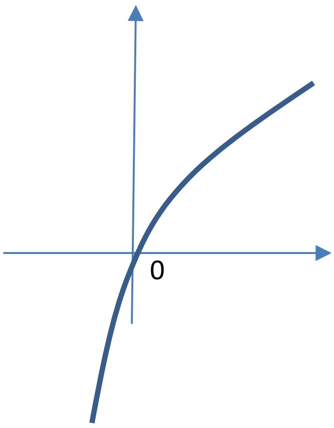


0,1,0,

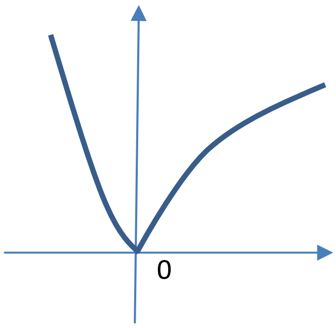


Example of follow through in part (ii) and (iii)

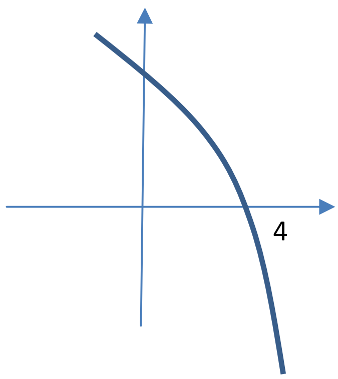
(i) B0



(ii) B1ftB1ftB0



(iii) B0B1ftB0



Question Number	Scheme	Marks
155(a)	$0 \leqslant \mathrm{f}(x) \leqslant 10$	B1
		(1)
(b)	ff(0) = f(5), = 3	B1,B1
		(2)
(c)	$y = \dfrac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$	
	$\Rightarrow 5y-4 = xy+3x$	M1
	$\Rightarrow 5y-4 = x(y+3) \Rightarrow x = \dfrac{5y-4}{y+3}$	dM1
	$\mathrm{g}^{-1}(x) = \dfrac{5x-4}{3+x}$	A1
		(3)
(d)	$\mathrm{gf}(x) = 16 \Rightarrow \mathrm{f}(x) = \mathrm{g}^{-1}(16) = 4$ oe	M1A1
	$\mathrm{f}(x) = 4 \Rightarrow x = 6$	B1
	$\mathrm{f}(x) = 4 \Rightarrow 5-2.5x = 4 \Rightarrow x = 0.4$ oe	M1A1
		(5)
		(11 marks)
Alt 1 to 155(d)	$\mathrm{gf}(x) = 16 \Rightarrow \dfrac{4+3(ax+b)}{5-(ax+b)} = 16$	M1
	$ax+b = x-2$ *or* $5-2.5x$	A1
	$\Rightarrow x = 6$	B1
	$\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = \ldots$	M1
	$\Rightarrow x = 0.4$ oe	A1 **(5)**

Notes for Question 155

(a)

B1 Correct range. Allow $0 \leqslant f(x) \leqslant 10$, $0 \leqslant f \leqslant 10$, $0 \leqslant y \leqslant 10$, $0 \leqslant \text{range} \leqslant 10$, $[0,10]$

Allow $f(x) \geqslant 0$ and $f(x) \leqslant 10$ but not $f(x) \geqslant 0$ or $f(x) \leqslant 10$

Do Not Allow $0 \leqslant x \leqslant 10$. The inequality must include BOTH ends

(b)

B1 For correct one application of the function at x=0

Possible ways to score this mark are f(0)=5, f(5) $0 \to 5 \to ...$

B1: 3 ('3' can score both marks as long as no incorrect working is seen.)

(c)

M1 For an attempt to make x or a replaced y the subject of the formula. This can be scored for putting y = g(x), multiplying across, expanding and collecting x terms on one side of the equation. Condone slips on the signs

dM1 Take out a common factor of x (or a replaced y) and divide, to make x subject of formula. Only allow **one sign error** for this mark

A1 Correct answer. No need to state the domain. Allow $g^{-1}(x) = \dfrac{5x-4}{3+x}$ $y = \dfrac{5x-4}{3+x}$

Accept alternatives such as $y = \dfrac{4-5x}{-3-x}$ and $y = \dfrac{5-\frac{4}{x}}{1+\frac{3}{x}}$

(d)

M1 Stating or implying that $f(x) = g^{-1}(16)$. For example accept $\dfrac{4+3f(x)}{5-f(x)} = 16 \Rightarrow f(x) = ..$

A1 Stating $f(x) = 4$ or implying that solutions are where $f(x) = 4$

B1 x = 6 and may be given if there is no working

M1 Full method to obtain other value from line $y = 5 - 2.5x$

$5 - 2.5x = 4 \Rightarrow x = ...$.

Alternatively this could be done by similar triangles. Look for $\dfrac{2}{5} = \dfrac{2-x}{4}(oe) \Rightarrow x = ..$

A1 0.4 or 2/5

Alt 1 to (d)

M1 Writes $gf(x) = 16$ with a linear $f(x)$. The order of gf(x) must be correct

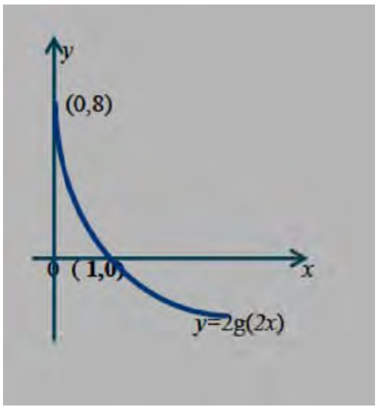
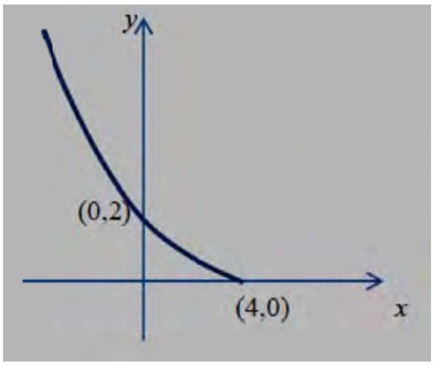
Condone invisible brackets. Even accept if there is a modulus sign.

A1 Uses $f(x) = x - 2$ or $f(x) = 5 - 2.5x$ in the equation $gf(x) = 16$

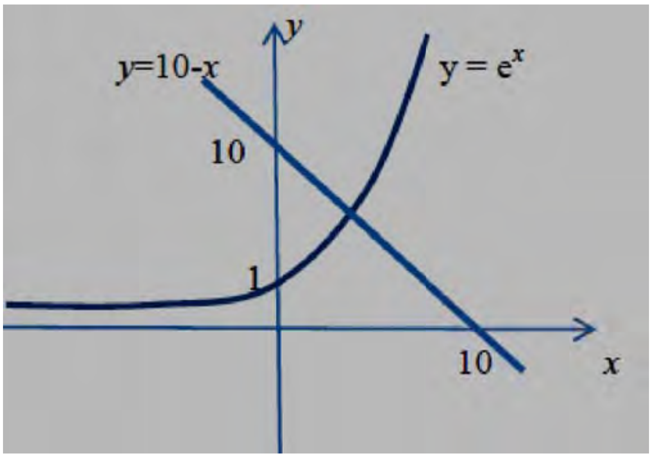
B1 x = 6 and may be given if there is no working

M1 Attempt at solving $\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = ...$. The bracketing must be correct and there must be no more than one error in their calculation

A1 $x = 0.4, \dfrac{2}{5}$ or equivalent

Question Number	Scheme	Marks
156.	(a) $x^2+3x+2=(x+2)(x+1)$	B1
	Attempt as a single fraction $\frac{6x+12-2(x^2+3x+2)}{x^2+3x+2}$ or $\frac{6-2(x+1)}{x+1}$	M1
	$=\frac{4-2x}{x+1}$	A1
		(3)
	(b)(i)  Same shape	B1
	intercept at (0,8)	B1
	x intercept at (1,0)	B1
		(3)
	(b)(ii)  Correct shape in quadrants 1& 2	B1
	Both (0,2) and (4,0)	B1
		(2) **(8 marks)**

Question Number	Scheme	Marks
157.	(a) (2.5,0) (0,-5)	B1B1
		(2)
	(b) $2x-5=3-x \Rightarrow x=\frac{8}{3}$ oe.	B1
	$-2x-5=3-x \Rightarrow x=-8$	M1,A1
		(3)
		(5 marks)

Question Number	Scheme		Marks
158	(a)		
		Shape for $y = 10 - x$	B1
		Shape for $y = e^x$	B1
		co- ordinates correct (0,10),(10,0) and (0,1)	B1
			(3)
	(b) One solution as there is one point of intersection		B1√
			(1)
	(c) Sub x=2 and x=3 into $f(x) = e^x - 10 + x$		
	f(2)=-0.61, f(3)=(+)13.1		M1
	Both correct to 1sf, reason (change of sign) and conclusion (hence root)		A1
			(2)

Question Number	Scheme	Marks
159.	(a) $f(x) \geq -3$	B1
		(1)
	(b) $f(0) = 5$ or attempts to put their f(0) into $e^{2x-8} - 4$	M1
	Correct answer ff(0)= $e^2 - 4$	A1
		(2)
	(c) Either $5 - 2x = 21 \Rightarrow x = -8$	M1A1
	Or $e^{2x-8} - 4 = 21$	M1
	Correct order and ln work $\Rightarrow x = \dfrac{\ln 25 + 8}{2}$ oe. $\ln 5 + 4$	M1A1
		(5)
	(d) f does not have an inverse as it is a 'many to one' function	
	Accept f does not have an inverse as it is not a 'one to one' function	B1
		(1)
		(9 marks)

Question Number	Scheme	Marks
160.	(a) $x^2+x-12=(x+4)(x-3)$	B1
	Attempt as a single fraction $\dfrac{(3x+5)(x-3)-2(x^2+x-12)}{(x^2+x-12)(x-3)}$ or $\dfrac{3x+5-2(x+4)}{(x+4)(x-3)}$	M1
	$=\dfrac{x-3}{(x+4)(x-3)}$, $=\dfrac{1}{(x+4)}$ cao	A1, A1
		(4 marks)

Notes for Question 160

B1 For correctly factorising $x^2+x-12=(x+4)(x-3)$. It could appear anywhere in their solution

M1 For an attempt to combine two fractions. The denominator must be correct for 'their' fractions.
The terms could be separate but one term must have been modified.
Condone invisible brackets.
Examples of work scoring this mark are;

$\dfrac{(3x+5)(x-3)}{(x^2+x-12)(x-3)}-\dfrac{2(x^2+x-12)}{(x^2+x-12)(x-3)}$ Two separate terms

$\dfrac{3x+5-2x+4}{(x+4)(x-3)}$ Single term, invisible bracket

$\dfrac{(3x+5)}{(x^2+x-12)(x-3)}-\dfrac{2(x^2+x-12)}{(x^2+x-12)(x-3)}$ Separate terms, only one numerator modified

A1 Correct un simplified answer $\dfrac{x-3}{(x+4)(x-3)}$

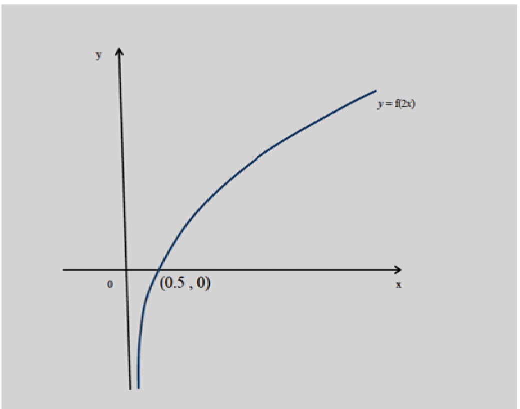
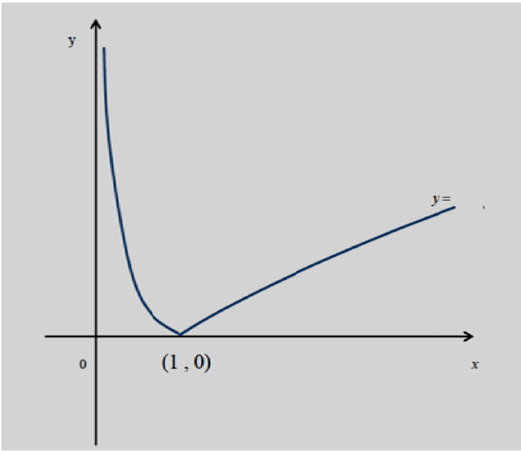
If $\dfrac{x^2-6x-9}{(x^2+x-12)(x-3)}$ scored M1 the fraction must be subsequently be reduced to a correct $\dfrac{x-3}{x^2+x-12}$ or $\dfrac{(x-3)(x-3)}{(x+4)(x-3)(x-3)}$ to score this mark.

A1 cao $\dfrac{1}{(x+4)}$

Do Not isw in this question.

The method of partial fractions is perfectly acceptable and can score full marks

$$\underbrace{\frac{3x+5}{(x+4)(x-3)}}_{B1}-\frac{2}{x-3}=\underbrace{\frac{1}{x+4}+\frac{2}{x-3}}_{M1A1}-\frac{2}{x-3}=\underbrace{\frac{1}{x+4}}_{A1}$$

Question Number	Scheme	Marks
161.(a)		
	Shape	B1
	(0.5, 0)	B1
		(2)
161(b)		
	Shape	B1
	(1,0)	B1
	Cusp at (1,0)	B1
		(3)
		(5 marks)

Notes for Question 161
(a) B1 Award for the correct shape. Look for an increasing function with decreasing gradient. Condone linear looking functions in the first quadrant. It needs to look asymptotic at the *y* axis and have no obvious maximum point. It must be wholly contained in quadrants 1 and 4 See practice and qualification items for clarification. B1 Crosses *x* axis at $\left(\frac{1}{2},0\right)$. Accept $\frac{1}{2}$, 0.5 or even $\left(0,\frac{1}{2}\right)$ marked on the correct axis. There must be a graph for this mark to be scored. (b) B1 Correct shape wholly contained in quadrant 1. The shape to the rhs of the cusp must not have an obvious maximum. Accept linear, or positive with decreasing gradient. The gradient of the curve to the lhs of the cusp/minimum should always be negative. The curve in this section should not 'bend' back past (1, 0) forming a 'C' shape or have incorrect curvature. See practice and qualification for clarification. B1 The curve touches or crosses the *x* axis at (1, 0). Allow for the curve passing through a point marked '1' on the *x* axis. Condone the point marked on the correct axis as (0, 1) B1 Award for a cusp, not a minimum at (1,0) Note that $f(\lvert x\rvert)$ scores B0 B1 B0 under the scheme. If the graphs are not labelled (a) and (b), then they are to be marked in the order they are presented

Question Number	Scheme	Marks
162(a)	$f(x) \geqslant 3$	M1A1
		(2)
(b)	An attempt to find $2\|3-4x\|+3$ when $x=1$	M1
	Correct answer $fg(1)=5$	A1
		(2)
(c)	$y=3-4x \Rightarrow 4x=3-y \Rightarrow x=\dfrac{3-y}{4}$	M1
	$g^{-1}(x)=\dfrac{3-x}{4}$	A1
		(2)
(d)	$\left[g(x)\right]^2=(3-4x)^2$	B1
	$gg(x)=3-4(3-4x)$	M1
	$gg(x)+\left[g(x)\right]^2=0 \Rightarrow -9+16x+9-24x+16x^2=0$	
	$16x^2-8x=0$	A1
	$8x(2x-1)=0 \Rightarrow x=0,\ 0.5$ oe	M1A1
		(5)
		(11 marks)

Notes for Question 162

(a)

M1 Attempt at calculating f at x=0. Sight of 3 is sufficient. Accept $\mathrm{f}(x) > 3$ and $x > 3$ for M1,

A1 $\mathrm{f}(x) \geqslant 3$. Accept $y \geqslant 3$, range$\geqslant 3$, $[3, \infty)$

Do not accept $\mathrm{f}(x) > 3$, $x \geqslant 3$

The correct answer is sufficient for both marks.

(b)

M1 A full method of finding fg(1). The order of substituting into the expressions must be correct and $2|x|+3$ must be used as opposed to $2x+3$

Accept an attempt to calculate $2|x|+3$ when x= -1.

Accept an attempt to put x=1 into $3-4x$ and then substituting their answer to $3-4x|_{x=1}$ into $2|x|+3$

Do not accept the substitution of x=1 into $2|x|+3$, followed by their result into '3-4x'

This is evidence of incorrect order.

A1 fg(1)=5.

Watch for $1 \xrightarrow{3-4x} 1 \xrightarrow{2|x|+3} 5$ **which is M1A0**

(c)

M1 Award for an attempt to make x or a swapped y the subject of the formula. It must be a full method and cannot finish $4x = ..$

You can condone at most one 'arithmetic' error for this method mark.

$y = 3-4x \Rightarrow 4x = 3+y \Rightarrow x = \frac{3+y}{4}$ is fine for the M1 as there is only one error

$y = 3-4x \Rightarrow 4x = 3-y \Rightarrow x = \frac{3}{4} - y$ is fine for the M1 as there is only one error

$y = 3-4x \Rightarrow 4x = 3+y \Rightarrow x = \frac{3}{4} + y$ is M0 as there are two arithmetic errors

A1 Obtaining a correct expression $\mathrm{g}^{-1}(x) = \frac{3-x}{4}$ oe such as $\mathrm{g}^{-1}(x) = \frac{x-3}{-4}$, $\mathrm{g}^{-1}(x) = \frac{3}{4} - \frac{x}{4}$

It must be in terms of x, but could be expressed 'y=' or $g^{-1}(x) \rightarrow$

(d)

B1 Sight of $[g(x)]^2 = (3-4x)^2$. If only the expanded version appears it must be correct

M1 A full attempt to find $\mathrm{gg}(x) = 3-4(3-4x)$

Condone invisible brackets. Note that it may appear in an equation

A1 $16x^2 - 8x = 0$ Accept other alternatives such as $2x^2 = x$

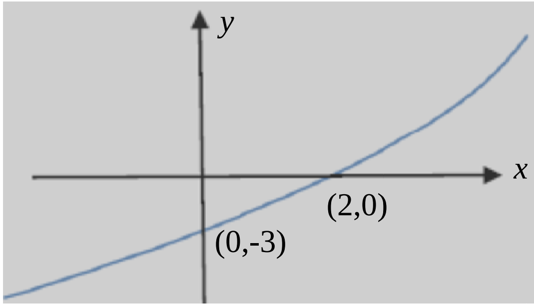
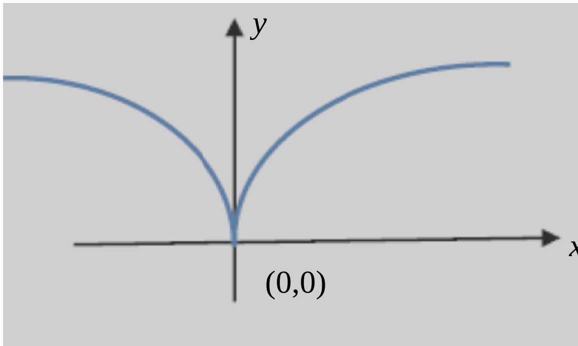
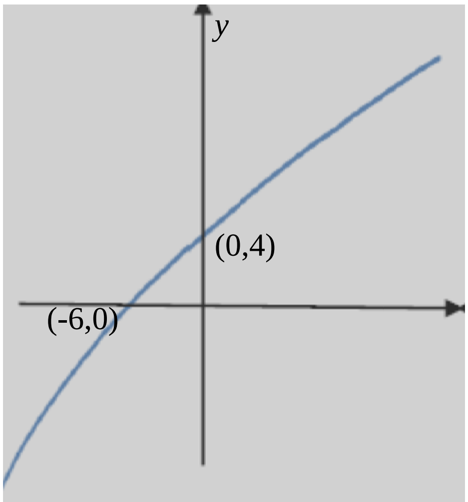
M1 For factorising their quadratic or cancelling their $Ax^2 = Bx$ by x to get ≥ 1 value of x

If they have a 3TQ then usual methods are applicable.

A1 Both values correct $x = 0, 0.5$ oe

Question Number	Scheme	Marks
163	$f(x)=0 \Rightarrow x^2+3x+1=0$ $\Rightarrow x=\frac{-3\pm\sqrt{5}}{2}=$ awrt -0.382, -2.618	M1A1 **(2)**

Notes for Question 163	
M1	Solves $x^2+3x+1=0$ by completing the square or the formula, producing two 'non integer answers. **Do not accept factorisation here** . Accept awrt -0.4 and -2.6 for this mark
A1	Answers correct. Accept awrt -0.382, -2.618.
	Accept just the answers for both marks. Don't withhold the marks for incorrect labelling.

Question Number	Scheme		Marks
164.	(a) ff(-3)= f(0),=2		M1,A1
			(2)
	(b)  $y = f^{-1}(x)$		
		Shape	B1
		(0,-3) and (2,0)	B1
			(2)
	(c)  y=f(|x|)-2		
		Shape	B1
		(0,0)	B1
			(2)
	(d) 		
		Shape	B1
		(-6,0) or (0,4)	B1
		(-6,0) and (0,4)	B1
			(3)
			(9 marks)

(a) M1 A full method of finding ff(-3). f(0) is acceptable but f(-3)=0 is not.

Accept a solution obtained from two substitutions into the equation $y = \frac{2}{3}x + 2$ as the line passes through both points. Do not allow for $y = \ln(x+4)$, which only passes through one of the points.

A1 Cao ff(-3)=2. Writing down 2 on its own is enough for both marks provided no incorrect working is seen.

(b)

B1 For the correct shape. Award this mark for an increasing function in quadrants 3, 4 and 1 only. Do not award if the curve bends back on itself or has a clear minimum

B1 This is independent to the first mark and for the graph passing through (0,-3) and (2, 0)

Accept -3 and 2 marked on the correct axes.
Accept (-3,0) and (0,2) instead of (0,-3) and (2,0) as long as they are on the correct axes
Accept P'=(0,-3), Q'=(2,0) stated elsewhere as long as P'and Q' are marked in the correct place on the graph
There must be a graph for this to be awarded

(c)

B1 Award for a correct shape 'roughly' symmetrical about the *y*- axis. It must have a cusp and a gradient that 'decreases' either side of the cusp. Do not award if the graph has a clear maximum

B1 (0,0) lies on their graph. Accept the graph passing through the origin without seeing (0, 0) marked

(d) B1 Shape. The position is not important. The gradient should be always positive but decreasing There should not be a clear maximum point.

B1 The graph passes through (0,4) **or** (-6,0). See part (b) for allowed variations

B1 The graph passes through (0,4) **and** (-6,0). See part (b) for allowed variations

Question Number	Scheme	Marks
165.	(a) $\frac{2}{x+2}+\frac{4}{x^2+5}-\frac{18}{(x+2)(x^2+5)}=\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$	M1A1
	$=\frac{2x(x+2)}{(x+2)(x^2+5)}$	M1
	$=\frac{2x}{(x^2+5)}$	A1*
		(4)
	(b) $\mathrm{h}'(x)=\frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$	M1A1
	$\mathrm{h}'(x)=\frac{10-2x^2}{(x^2+5)^2}$ cso	A1
		(3)
	(c) Maximum occurs when $\mathrm{h}'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$	M1
	$\Rightarrow x=\sqrt{5}$	A1
	When $x=\sqrt{5}\Rightarrow \mathrm{h}(x)=\frac{\sqrt{5}}{5}$	M1,A1
	Range of h(x) is $0\le h(x)\le\frac{\sqrt{5}}{5}$	A1ft
		(5) **(12 marks)**

(a) M1 Combines the three fractions to form a single fraction with a common denominator.
Allow errors on the numerator but at least one must have been adapted.
Condone 'invisible' brackets for this mark.
Accept three separate fractions with the same denominator.
Amongst possible options allowed for this method are

$\frac{2x^2+5+4x+2-18}{(x+2)(x^2+5)}$ Eg 1 An example of 'invisible' brackets

$\frac{2(x^2+5)}{(x+2)(x^2+5)}+\frac{4}{(x+2)(x^2+5)}-\frac{18}{(x+2)(x^2+5)}$ Eg 2An example of an error (on middle term), 1st term has been adapted

$\frac{2(x^2+5)^2(x+2)+4(x+2)^2(x^2+5)-18(x^2+5)(x+2)}{(x+2)^2(x^2+5)^2}$ Eg 3 An example of a correct fraction with a different denominator

A1 Award for a correct un simplified fraction with the correct (lowest) common denominator.

$\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$

Accept if there are three separate fractions with the correct (lowest) common denominator.

Eg $\frac{2(x^2+5)}{(x+2)(x^2+5)}+\frac{4(x+2)}{(x+2)(x^2+5)}-\frac{18}{(x+2)(x^2+5)}$

Note, Example 3 would score M1A0 as it does not have the correct lowest common denominator

M1 There must be a single denominator. Terms must be collected on the numerator. A factor of (x+2) must be taken out of the numerator and then cancelled with one in the denominator. The cancelling may be assumed if the term 'disappears'

A1* Cso $\frac{2x}{(x^2+5)}$ This is a given solution and this mark should be withheld if there are any errors

(b) M1 Applies the quotient rule to $\frac{2x}{(x^2+5)}$, a form of which appears in the formula book.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{(x^2+5)\times A-2x\times Bx}{(x^2+5)^2} \quad \text{where } A,B>0$$

A1 Correct unsimplified answer $\mathrm{h}'(x)=\frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$

A1 $\mathrm{h}'(x)=\frac{10-2x^2}{(x^2+5)^2}$ The correct simplified answer. Accept $\frac{2(5-x^2)}{(x^2+5)^2}$ $\frac{-2(x^2-5)}{(x^2+5)^2}$, $\frac{10-2x^2}{(x^4+10x^2+25)}$

DO NOT ISW FOR PART (b). INCORRECT SIMPLIFICATION IS A0

(c) M1 Sets their h'(x)=0 and proceeds with a correct method to find x. There must have been an attempt to differentiate. Allow numerical errors but do not allow solutions from 'unsolvable' equations.

A1 Finds the correct x value of the maximum point $x=\sqrt{5}$.
Ignore the solution $x=-\sqrt{5}$ but withhold this mark if other positive values found.

M1 Substitutes their answer into their h'(x)=0 in h(x) to determine the maximum value

A1 Cso-the maximum value of h(x) = $\frac{\sqrt{5}}{5}$. Accept equivalents such as $\frac{2\sqrt{5}}{10}$ **but not** 0.447

A1ft Range of h(x) is $0\le \mathrm{h}(x)\le\frac{\sqrt{5}}{5}$. Follow through on their maximum value if the M's have been scored. Allow $0\le y\le\frac{\sqrt{5}}{5}$, $0\le Range\le\frac{\sqrt{5}}{5}$, $\left[0,\frac{\sqrt{5}}{5}\right]$ but not $0\le x\le\frac{\sqrt{5}}{5}$, $\left(0,\frac{\sqrt{5}}{5}\right)$

If a candidate attempts to work out $h^{-1}(x)$ in (b) and does all that is required for (b) in (c), then allow. Do not allow $h^{-1}(x)$ to be used for h'(x) in part (c). For this question (b) and (c) can be scored together.

Alternative to (b) using the product rule

M1 Sets $\mathrm{h}(x)=2x(x^2+5)^{-1}$ and applies the product rule vu'+uv' with terms being 2x and $(x^2+5)^{-1}$
If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$(x^2+5)^{-1}\times A+2x\times\pm Bx(x^2+5)^{-2}$$

A1 Correct un simplified answer $(x^2+5)^{-1}\times 2+2x\times -2x(x^2+5)^{-2}$

A1 The question asks for h'(x) to be put in its simplest form. Hence in this method the terms need to be combined to form a single correct expression.

For a correct simplified answer accept

$$\mathrm{h}'(x)=\frac{10-2x^2}{(x^2+5)^2}=\frac{2(5-x^2)}{(x^2+5)^2}=\frac{-2(x^2-5)}{(x^2+5)^2}=(10-2x^2)(x^2+5)^{-2}$$

Question Number	Scheme	Marks
166.	$9x^2-4=(3x-2)(3x+2)$ At any stage	B1
	Eliminating the common factor of (3x+2) at any stage $\frac{2(3x+2)}{(3x-2)(3x+2)}=\frac{2}{3x-2}$	B1
	Use of a common denominator $\frac{2(3x+2)(3x+1)}{(9x^2-4)(3x+1)}-\frac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ or $\frac{2(3x+1)}{(3x-2)(3x+1)}-\frac{2(3x-2)}{(3x+1)(3x-2)}$	M1
	$\frac{6}{(3x-2)(3x+1)}$ or $\frac{6}{9x^2-3x-2}$	A1
		(4 marks)

Notes

B1 For factorising $9x^2-4=(3x-2)(3x+2)$ using difference of two squares. It can be awarded at any stage of the answer but it must be scored on E pen as the first mark

B1 For eliminating/cancelling out a factor of (3x+2) at any stage of the answer.

M1 For combining two fractions to form a single fraction with a common denominator. Allow slips on the numerator but at least one must have been adapted. Condone invisible brackets. Accept two separate fractions with the same denominator as shown in the mark scheme. Amongst possible (incorrect) options scoring method marks are

$\frac{2(3x+2)}{(9x^2-4)(3x+1)}-\frac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ Only one numerator adapted, separate fractions

$\frac{2\times3x+1-2\times3x-2}{(3x-2)(3x+1)}$ Invisible brackets, single fraction

A1 $\frac{6}{(3x-2)(3x+1)}$

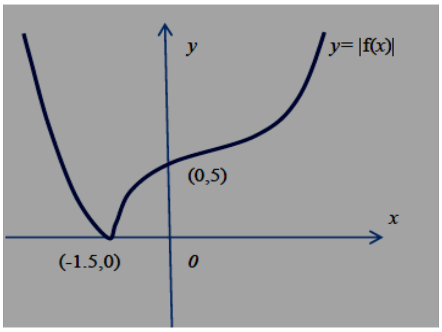
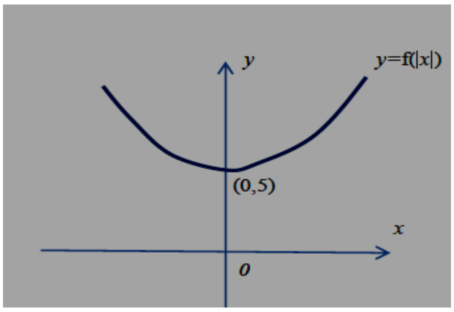
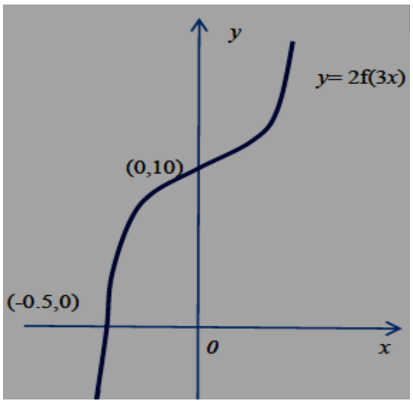
This is not a given answer so you can allow recovery from 'invisible' brackets.

Alternative method

$\frac{2(3x+2)}{(9x^2-4)}-\frac{2}{(3x+1)}=\frac{2(3x+2)(3x+1)-2(9x^2-4)}{(9x^2-4)(3x+1)}=\frac{18x+12}{(9x^2-4)(3x+1)}$ has scored 0,0,1,0 so far

$=\frac{6(3x+2)}{(3x+2)(3x-2)(3x+1)}$ is now 1,1,1,0

$=\frac{6}{(3x-2)(3x+1)}$ and now 1,1,1,1

Question Number	Scheme		Marks
167.(a)		Shape including cusp	B1
		(-1.5, 0) **and** (0, 5)	B1
			(2)
(b)		Shape	B1
		(0,5)	B1
			(2)
(c)		Shape	B1
		(0,10)	B1
		(-0.5, 0)	B1
			(3)
			(7 marks)

(a) **Note that this appears as M1A1 on EPEN**

B1 Shape (inc cusp) with graph in just quadrants 1 and 2. Do not be overly concerned about relative gradients, but the left hand section of the curve should not bend back beyond the cusp

B1 This is independent, and for the curve touching the x-axis at (-1.5, 0) **and** crossing the y-axis at (0,5)

(b) **Note that this appears as M1A1 on EPEN**

B1 For a U shaped curve symmetrical about the y- axis

B1 (0,5) lies on the curve

(c) **Note that this appears as M1B1B1 on EPEN**

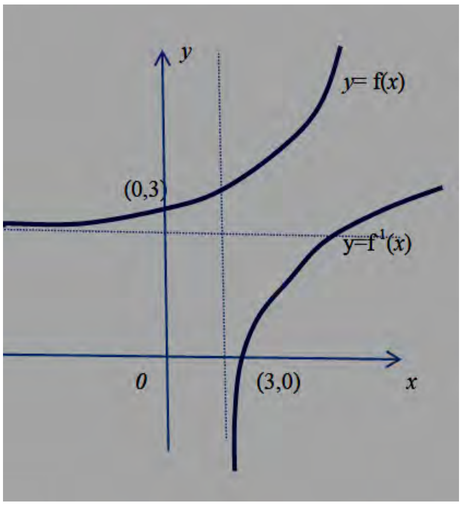
B1 Correct shape- do not be overly concerned about relative gradients. Look for a similar shape to f(x)

B1 Curve **crosses** the y axis at (0, 10). The curve must appear in both quadrants 1 and 2

B1 Curve **crosses** the x axis at (-0.5, 0). The curve must appear in quadrants 3 and 2.

In all parts accept the following for any co-ordinate. Using (0,3) as an example, accept both (3,0) or 3 written on the y axis (as long as the curve passes through the point)

Special case with (a) and (b) completely correct but the wrong way around mark - SC(a) 0,1 SC(b) 0,1 Otherwise follow scheme

Question Number	Scheme	Marks
168.	(a) $\quad f(x)>2$	B1 (1)
	(b) $\quad fg(x)=e^{\ln x}+2, = x+2$	M1,A1 (2)
	(c) $\quad e^{2x+3}+2=6 \Rightarrow e^{2x+3}=4$	M1A1
	$\Rightarrow 2x+3=\ln 4$	
	$\Rightarrow x=\dfrac{\ln 4-3}{2}$ or $\ln 2-\dfrac{3}{2}$	M1A1 (4)
	(d) Let $y=e^x+2 \Rightarrow y-2=e^x \Rightarrow \ln(y-2)=x$	M1
	$f^{-1}(x)=\ln(x-2), \quad x>2.$	A1 , **B1ft** (3)
	(e) 	
	Shape for $f(x)$	B1
	(0, 3)	B1
	Shape for $f^{-1}(x)$	B1
	(3, 0)	B1 (4)
		(14 marks)

(a) B1 Range of $f(x)>2$. Accept $y>2$, $(2,\infty)$, $f>2$, as well as 'range is the set of numbers bigger than 2' but **don't accept** $x>2$

(b) M1 For applying the correct order of operations. Look for $e^{\ln x}+2$. Note that $\ln e^x+2$ is M0

A1 Simplifies $e^{\ln x}+2$ to $x+2$. Just the answer is acceptable for both marks

(c) M1 Starts with $e^{2x+3}+2=6$ and proceeds to $e^{2x+3}=...$

A1 $e^{2x+3}=4$

M1 Takes ln's both sides, $2x+3=\ln..$ and proceeds to $x=....$

A1 $x=\dfrac{\ln 4-3}{2}$ oe. eg $\ln 2-\dfrac{3}{2}$ Remember to isw any incorrect working after a correct answer

(d) **Note that this is marked M1A1A1 on EPEN**

M1 Starts with $y = e^x + 2$ or $x = e^y + 2$ and attempts to change the subject.
All ln work must be correct. The 2 must be dealt with first.
Eg. $y = e^x + 2 \Rightarrow \ln y = x + \ln 2 \Rightarrow x = \ln y - \ln 2$ is M0

A1 $f^{-1}(x) = \ln(x-2)$ or $y = \ln(x-2)$ or $y = \ln|x-2|$ There must be some form of bracket

B1ft Either $x>2$, or follow through on their answer to part (a), provided that it wasn't $y \in \Re$
Do not accept y>2 or $f^{-1}(x)>2$.

(e) B1 Shape for $y=e^x$. The graph should only lie in quadrants 1 and 2. It should start out with a gradient that is approx. 0 above the *x* axis in quadrant 2 and increase in gradient as it moves into quadrant 1. You should not see a minimum point on the graph.

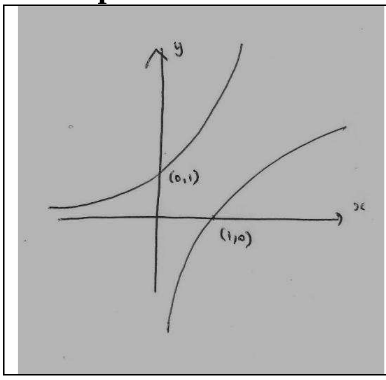
B1 (0, 3) lies on the curve. Accept 3 written on the y axis as long as the point lies on the curve

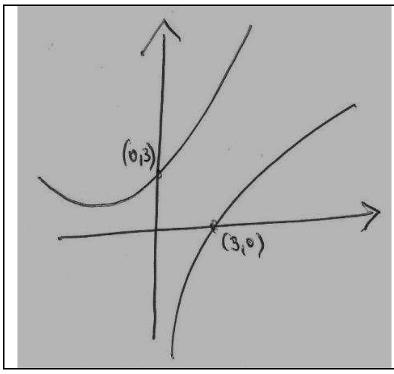
B1 Shape for $y=\ln x$. The graph should only lie in quadrants 4 and 1. It should start out with gradient that is approx. infinite to the right of the *y* axis in quadrant 4 and decrease in gradient as it moves into quadrant 1. You should not see a maximum point. Also with hold this mark if it intersects $y=e^x$

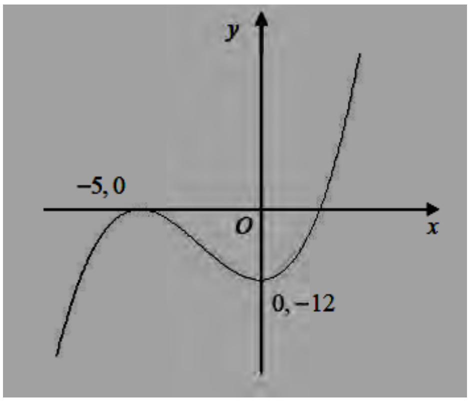
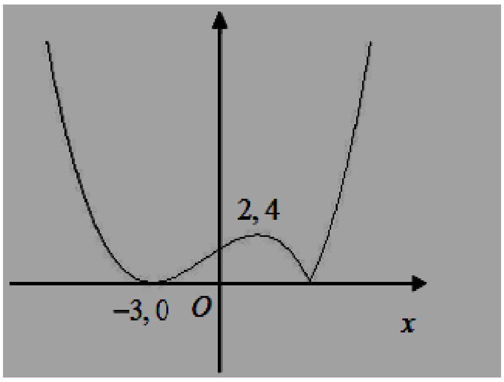
B1 (3, 0) lies on the curve. Accept 3 written on the *x* axis as long as the point lies on the curve

Condone lack of labels in this part

Examples

	Scores **1,0,1,0.** Both shapes are fine, do not be concerned about asymptotes appearing at x=2, y=2. (See notes) Both co-ordinates are incorrect

	Scores **0,1,1,1** Shape for $y = e^x$ is incorrect, there is a minimum point on the graph. All other marks an be awarded

Question No	Scheme		Marks
169	(a)		
		Shape	B1
		x coordinates correct	B1
		y coordinates correct	B1
			(3)
	(b)		
		Shape	B1
		Max at (2,4)	B1
		Min at (-3,0)	B1
			(3)
			6 marks

(a)

B1 Shape unchanged. The positioning of the curve is not significant for this mark. The right hand section of the curve does not have to cross *x* axis.

B1 The x- coordinates of P' and Q' are -5 and 0 respectively. This is for translating the curve 2 units left. The minimum point Q' must be on the y axis. Accept if -5 is marked on the x axis for P' with Q' on the *y* axis (marked -12).

B1 The y- coordinates of P' and Q' are 0 and -12 respectively. This is for the stretch ×3 parallel to the *y* axis. The maximum P' must be on the *x* axis. Accept if -12 is marked on the y axis for Q' with P' on the x axis (marked -5)

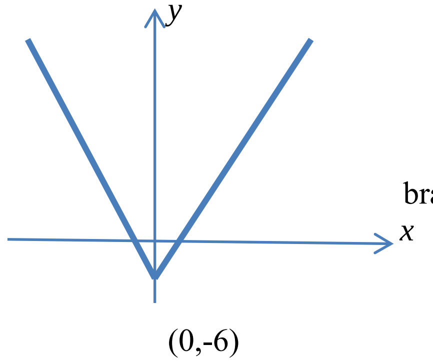
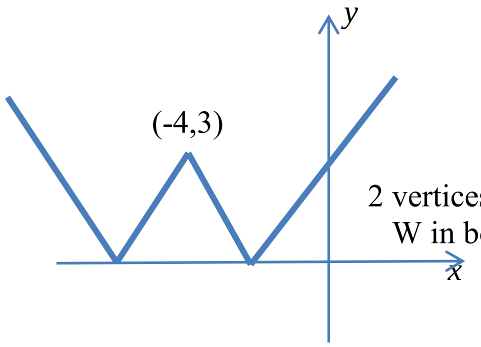
(b)

B1 The curve below the x axis reflected in the x axis and the curve above the x axis is unchanged. Do not accept if the curve is clearly rounded off with a zero gradient at the x axis but allow small curvature issues. Use the same principles on the lhs- do not accept if this is a cusp.

B1 Both the x- and y- coordinates of Q', (2,4) given correctly and associated with the maximum point in the first quadrant. To gain this mark there must be a graph and it must only have one maximum.
Accept as 2 and 4 marked on the correct axes or in the script as long as there is no ambiguity.

B1 Both the x- and y- coordinates of P', (-3,0) given correctly and associated with the minimum point in the second quadrant. To gain this mark there must be a graph. Tolerate two cusps if this mark has been lost earlier in the question. Accept (0, -3) marked on the correct axis.

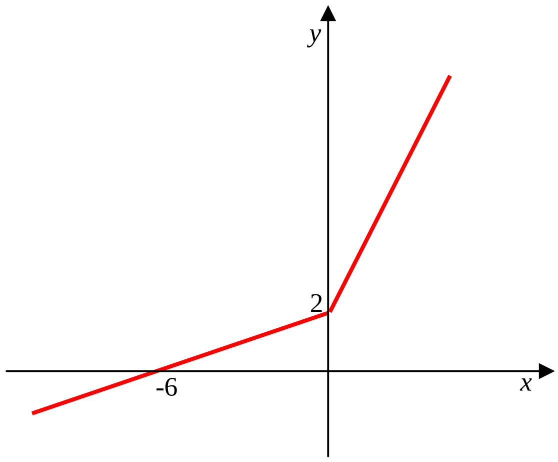
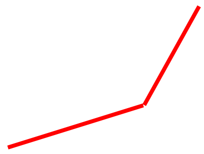
Question No	Scheme	Marks
170	(a) $2x^2+7x-4=(2x-1)(x+4)$	B1
	$\dfrac{3(x+1)}{(2x-1)(x+4)}-\dfrac{1}{(x+4)}=\dfrac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$	M1
	$=\frac{x+4}{(2x-1)(x+4)}$	M1
	$=\frac{1}{2x-1}$	A1* **(4)**
	(b) $y=\frac{1}{2x-1}\Rightarrow y(2x-1)=1\Rightarrow 2xy-y=1$	
	$2xy=1+y\Rightarrow x=\dfrac{1+y}{2y}$	M1M1
	$y\ OR\ f^{-1}(x)=\dfrac{1+x}{2x}$	A1 **(3)**
	(c) x>0	B1 **(1)**
	(d) $\frac{1}{2\ln(x+1)-1}=\frac{1}{7}$	M1
	$\ln(x+1)=4$	A1
	$x=e^4-1$	M1A1 **(4)**
		12 Marks

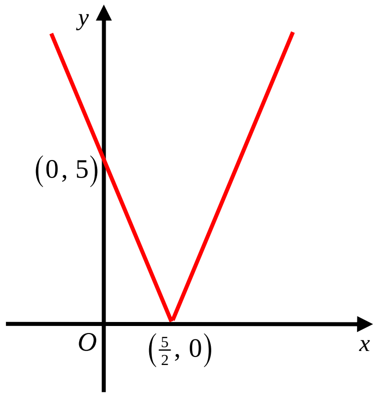
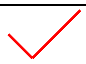
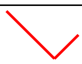
Question Number	Scheme	Marks
171 (a)		
	V shape	B1
	vertex on y axis &both branches of graph cross x axis	B1
	„y" co-ordinate of R is -6	B1
		(3)
(b)		
	W shape	B1
	2 vertices on the negative x axis. W in both quad 1 & quad 2.	B1dep
	R"=(-4,3)	B1
		(3)
		6 Marks
172 (a)	$y = 4 - \ln(x+2)$ $\ln(x+2) = 4 - y$ $x + 2 = e^{4-y}$ $x = e^{4-y} - 2$	M1
	$f^{-1}(x) = e^{4-x} - 2$ oe	M1A1
		(3)
(b)	$x \leq 4$	B1
		(1)
(c)	$fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$	M1
	$fg(x) = 4 - x^2$	dM1A1
		(3)
(d)	$fg(x) \leq 4$	B1ft
		(1)
		8 Marks

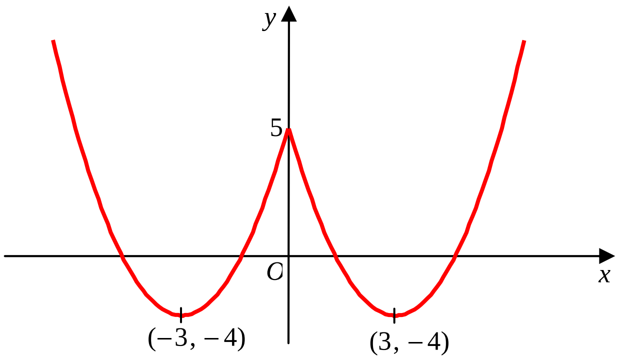
EXPERT TUITION

Question Number	Scheme		Marks
173.			
(a)	$\dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)}$		
	$= \dfrac{(4x-1)(2x-1)-3}{2(x-1)(2x-1)}$	An attempt to form a single fraction	M1
	$= \dfrac{8x^2-6x-2}{\{2(x-1)(2x-1)\}}$	Simplifies to give a correct quadratic numerator over a correct quadratic denominator	A1 aef
	$= \dfrac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$	An attempt to factorise a 3 term quadratic numerator	M1
	$= \dfrac{4x+1}{2x-1}$		A1 (4)
(b)	$f(x) = \dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)} - 2, \quad x > 1$		
	$f(x) = \dfrac{(4x+1)}{(2x-1)} - 2$		
	$= \dfrac{(4x+1)-2(2x-1)}{(2x-1)}$	An attempt to form a single fraction	M1
	$= \dfrac{4x+1-4x+2}{(2x-1)}$		
	$= \dfrac{3}{(2x-1)}$	Correct result	A1 * (2)

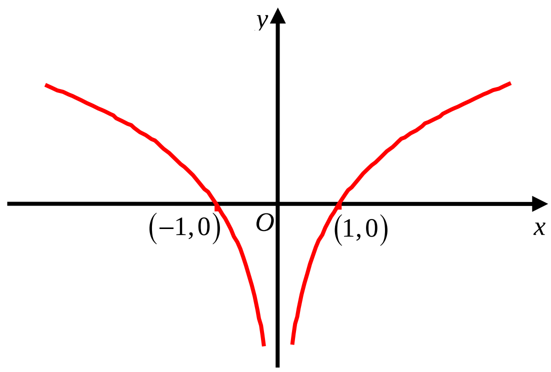
Question Number	Scheme		Marks
174.			
(a)	$y = \dfrac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$	Attempt to make x (or swapped y) the subject	M1
	$xy - 5y = 3 - 2x$		
	$\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3 + 5y$	Collect x terms together and factorise.	M1
	$\Rightarrow x = \dfrac{3+5y}{y+2} \quad \therefore f^{-1}(x) = \underline{\dfrac{3+5x}{x+2}}$	$\underline{\dfrac{3+5x}{x+2}}$	A1 oe
			(3)
(b)	Range of g is $-9 \le g(x) \le 4$ or $-9 \le y \le 4$	<u>Correct Range</u>	B1 (1)
(c)		Deduces that $g(2)$ is 0. Seen or implied.	M1
	g g(2)= g (0) $= -6$, from sketch.	-6	A1 (2)
(d)	$fg(8) = f(4)$	Correct order g followed by f	M1
	$= \dfrac{3-4(2)}{4-5} = \dfrac{-5}{-1} = \underline{5}$	5	A1
			(2)

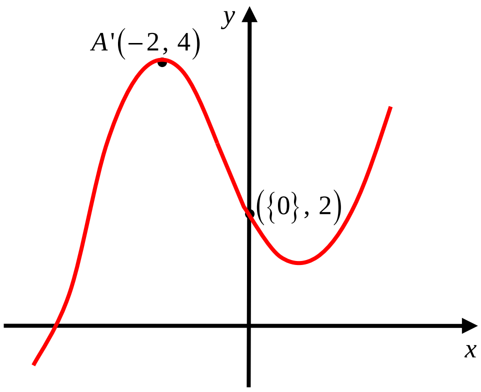
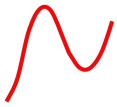
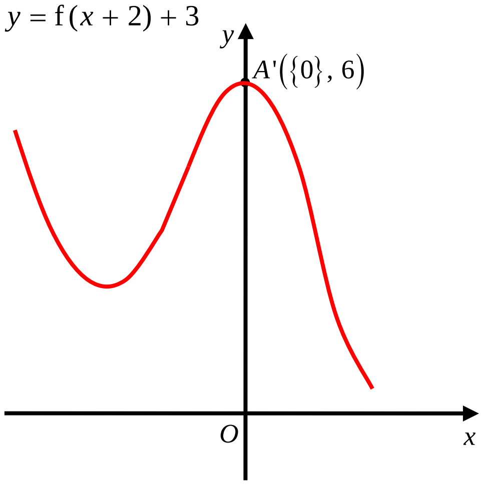
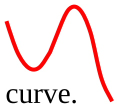
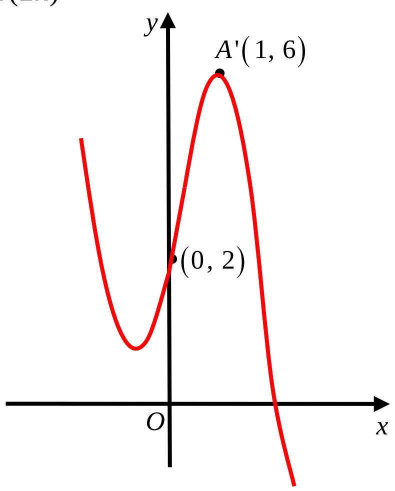
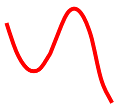
Question Number	Scheme		Marks
(e)(ii)		Correct shape  Graph goes through $(\{0\}, 2)$ and $(-6, \{0\})$ which are marked.	B1 B1 (4)
(f)	Domain of g^{-1} is $-9 \le x \le 4$	Either correct answer or a follow through from part (b) answer	B1 (1) **[13]**

Question Number	Scheme	Marks
175.(a)		M1A1 (2)
(b)	$\underline{x = 20}$ $2x-5 = -(15+x)$; $\Rightarrow$ $\underline{x = -\frac{10}{3}}$	B1 M1;A1 oe. (3)
(c)	$\text{fg}(2) = \text{f}(-3) = \lvert 2(-3)-5 \rvert; = \lvert -11 \rvert = 11$	M1;A1 (2)
(d)	$\text{g}(x) = x^2 - 4x + 1 = (x-2)^2 - 4 + 1 = (x-2)^2 - 3$. Hence $\text{g}_{\text{min}} = -3$ Either $\text{g}_{\text{min}} = -3$ or $\text{g}(x) \geqslant -3$ or $\text{g}(5) = 25 - 20 + 1 = 6$ $\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$	M1 B1 A1 (3) **[10]**
	(a) M1: V or  or  graph with vertex on the *x*-axis. A1: $\left(\frac{5}{2}, \{0\}\right)$ and $\left(\{0\}, 5\right)$ seen and the graph appears in both the first and second quadrants. (b) M1: Either $2x-5 = -(15+x)$ or $-(2x-5) = 15+x$ (c) M1: ***Full method*** of inserting $\text{g}(2)$ into $\text{f}(x) = \lvert 2x - 5 \rvert$ or for inserting $x = 2$ into $\lvert 2(x^2 - 4x + 1) - 5 \rvert$. There must be evidence of the modulus being applied. (d) M1: **Full method** to establish the minimum of g. Eg: $(x \pm \alpha)^2 + \beta$ leading to $\text{g}_{\text{min}} = \beta$. Or for candidate to differentiate the quadratic, set the result equal to zero, find *x* and insert this value of *x* back into $\text{f}(x)$ in order to find the minimum. B1: For either finding the correct minimum value of g (can be implied by $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$) or for stating that $\text{g}(5) = 6$. A1: $\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$ or $\underline{-3 \leqslant \text{g} \leqslant 6}$. **Note that:** $-3 \leqslant x \leqslant 6$ is A0. **Note that:** $-3 \leqslant \text{f}(x) \leqslant 6$ is A0. **Note that**: $-3 \geqslant \text{g}(x) \geqslant 6$ is A0. **Note that:** $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$ or $x \geqslant -3$ or $x > -3$ with no working gains M1B1A0. **Note that for the final Accuracy Mark**: If a candidate writes down $-3 < \text{g}(x) < 6$ or $-3 < y < 6$, then award M1B1A0. If, however, a candidate writes down $\text{g}(x) \geqslant -3$, $\text{g}(x) \leqslant 6$, then award A0. If a candidate writes down $\text{g}(x) \geqslant -3$ or $\text{g}(x) \leqslant 6$, then award A0.	

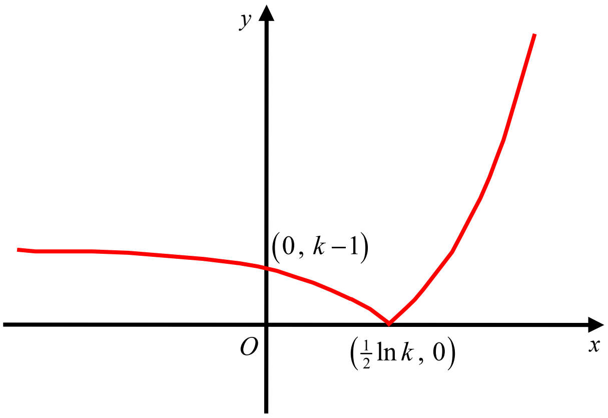
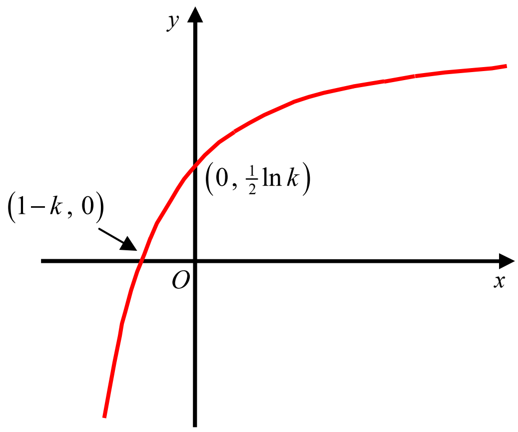
Question Number	Scheme	Marks
176.(a) (i)	$(3, 4)$	B1 B1
(ii)	$(6, -8)$	B1 B1
		(4)
(b)		B1 B1 B1
		(3)
(c)	$\mathrm{f}(x) = (x-3)^2 - 4$ or $\mathrm{f}(x) = x^2 - 6x + 5$	M1A1
		(2)
(d)	Either: The function f is a many-one {mapping}. Or: The function f is not a one-one {mapping}.	B1
		(1)
		[10]
	(b) B1: Correct shape for $x \geqslant 0$, with the curve meeting the positive y-axis and the turning point is found below the x-axis. (providing candidate does not copy the whole of the original curve and adds nothing else to their sketch.). B1: Curve is symmetrical about the y-axis or correct shape of curve for $x < 0$. **Note:** The first two B1B1 can only be awarded if the curve has the correct shape, with a cusp on the positive y-axis and with both turning points located in the correct quadrants. Otherwise award B1B0. B1: Correct turning points of $(-3, -4)$ and $(3, -4)$. Also, $(\{0\}, 5)$ is marked where the graph cuts through the y-axis. Allow $(5, 0)$ rather than $(0, 5)$ if marked in the "correct" place on the y-axis. (c) M1: Either states $\mathrm{f}(x)$ in the form $(x \pm \alpha)^2 \pm \beta$; $\alpha, \beta \neq 0$ Or uses a complete method on $\mathrm{f}(x) = x^2 + ax + b$, with $\mathrm{f}(0) = 5$ and $\mathrm{f}(3) = -4$ to find both *a* and *b*. A1: Either $(x-3)^2 - 4$ or $x^2 - 6x + 5$ (d) B1: Or: The inverse is a one-many {mapping and not a function}. Or: Because $\mathrm{f}(0) = 5$ and also $\mathrm{f}(6) = 5$. Or: One y-coordinate has 2 corresponding x-coordinates {and therefore cannot have an inverse}.	

Question Number	Scheme		Marks
177.	$\dfrac{x+1}{3x^2-3} - \dfrac{1}{3x+1}$		
	$= \dfrac{x+1}{3(x^2-1)} - \dfrac{1}{3x+1}$		
	$= \dfrac{x+1}{3(x+1)(x-1)} - \dfrac{1}{3x+1}$	$x^2-1 \to (x+1)(x-1)$ or $3x^2-3 \to (x+1)(3x-3)$ or $3x^2-3 \to (3x+3)(x-1)$ seen or implied anywhere in candidate's working.	**Award below**
	$= \dfrac{1}{3(x-1)} - \dfrac{1}{3x+1}$		
	$= \dfrac{3x+1-3(x-1)}{3(x-1)(3x+1)}$	Attempt to combine.	M1
	or $\dfrac{3x+1}{3(x-1)(3x+1)} - \dfrac{3(x-1)}{3(x-1)(3x+1)}$	Correct result.	A1
		Decide to award M1 here!!	**M1**
	$= \dfrac{4}{3(x-1)(3x+1)}$	Either $\dfrac{4}{3(x-1)(3x+1)}$ or $\dfrac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\dfrac{4}{(3x-3)(3x+1)}$ or $\dfrac{4}{9x^2-6x-3}$	A1 **aef**
			[4]

Question Number	Scheme		Marks
178.	$y = \ln\lvert x\rvert$		
		Right-hand branch in quadrants 4 and 1. Correct shape.	B1
		Left-hand branch in quadrants 2 and 3. Correct shape.	B1
		Completely correct sketch and both $(-1,\{0\})$ and $(1,\{0\})$	B1
			(3)
			[3]

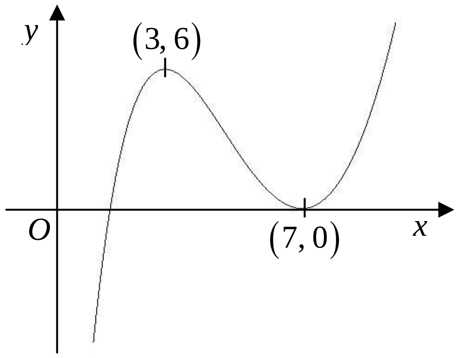
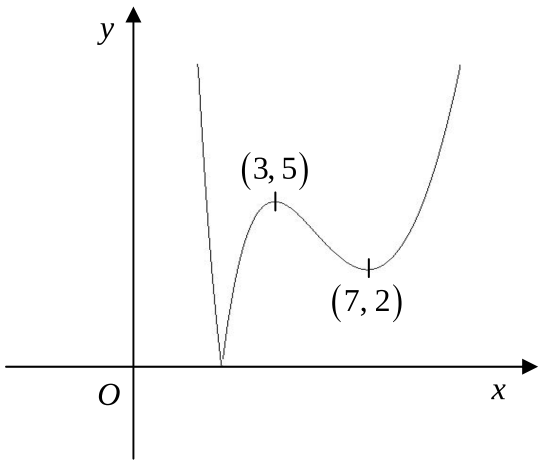
Question Number	Scheme	Marks
179. (i)	$y = \mathrm{f}(-x) + 1$	
		
	Shape of  and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive y-axis.	B1
	Either $(\{0\}, 2)$ or $A'(-2, 4)$	B1
	Both $(\{0\}, 2)$ and $A'(-2, 4)$	B1
		(3)
(ii)	$y = \mathrm{f}(x + 2) + 3$	
		
	Any translation  of the original curve.	B1
	The ***translated maximum*** has either x-coordinate of 0 (can be implied) or y-coordinate of 6.	B1
	The translated curve has maximum $(\{0\}, 6)$ and is in the correct position on the Cartesian axes.	B1
		(3)
(iii)	$y = 2\mathrm{f}(2x)$	
		
	Shape of  with a minimum in quadrant 2 and a maximum in quadrant 1.	B1
	Either $(\{0\}, 2)$ or $A'(1, 6)$	B1
	Both $(\{0\}, 2)$ and $A'(1, 6)$	B1
		(3)
		[9]

Question Number	Scheme		Marks
180 (a)	$\mathrm{f}(x) = \mathrm{e}^{2x} + 3,\ x \in \mathbb{R}$		
	$y = \mathrm{e}^{2x} + 3 \Rightarrow y - 3 = \mathrm{e}^{2x}$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \ln(y-3) = 2x$ $\Rightarrow \frac{1}{2}\ln(y-3) = x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $\mathrm{f}^{-1}(x) = \underline{\frac{1}{2}\ln(x-3)}$	$\underline{\frac{1}{2}\ln(x-3)}$ or $\underline{\ln\sqrt{(x-3)}}$ or $\underline{\mathrm{f}^{-1}(y) = \frac{1}{2}\ln(y-3)}$ (see appendix)	A1 **cao**
	$\mathrm{f}^{-1}(x)$: Domain: $\underline{x > 3}$ or $\underline{(3, \infty)}$	Either $\underline{x > 3}$ or $\underline{(3, \infty)}$ or $\underline{\text{Domain} > 3}$.	B1
			(4)
(b)	$\mathrm{g}(x) = \ln(x-1),\ x \in \mathbb{R},\ x > 1$		
	$\mathrm{fg}(x) = \mathrm{e}^{2\ln(x-1)} + 3 \quad \{= (x-1)^2 + 3\}$	An attempt to put function g into function f.	M1
		$\mathrm{e}^{2\ln(x-1)} + 3$ or $(x-1)^2 + 3$ or $x^2 - 2x + 4$.	A1 isw
	$\mathrm{fg}(x)$: Range: $\underline{y > 3}$ or $\underline{(3, \infty)}$	Either $\underline{y > 3}$ or $\underline{(3, \infty)}$ or $\underline{\text{Range} > 3}$ or $\underline{\mathrm{fg}(x) > 3}$.	B1
			(3)

Question Number	Scheme		Marks
181 (a)		Curve retains shape when $x > \frac{1}{2}\ln k$	B1
		Curve reflects through the x-axis when $x < \frac{1}{2}\ln k$	B1
		$(0,\ k-1)$ and $(\frac{1}{2}\ln k,\ 0)$ marked in the correct positions.	B1
			(3)
(b)		Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)	B1
		$(1-k,\ 0)$ and $(0,\ \frac{1}{2}\ln k)$	B1
			(2)
(c)	Range of f: $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$	Either $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$ or $\underline{f > -k}$ or $\underline{\text{Range} > -k}$.	B1
			(1)
(d)	$y = e^{2x} - k \Rightarrow y + k = e^{2x}$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \ln(y+k) = 2x$ $\Rightarrow \frac{1}{2}\ln(y+k) = x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x+k)}$	$\underline{\frac{1}{2}\ln(x+k)}$ or $\underline{\ln\sqrt{(x+k)}}$	A1 cao
			(3)
(e)	$f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$	Either $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$ or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer	B1 $\sqrt{}$
			(1)
			[10]

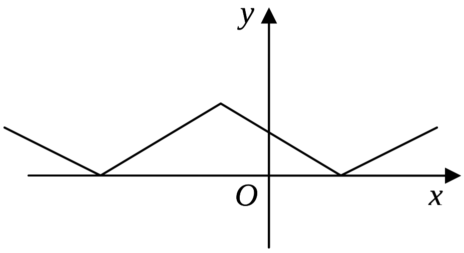
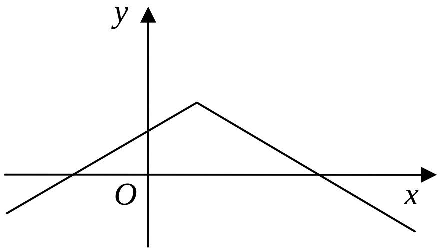
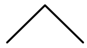
EXPERT TUITION

Question Number	Scheme	Marks
182.	$\frac{2x+2}{x^2-2x-3}-\frac{x+1}{x-3}=\frac{2x+2}{(x-3)(x+1)}-\frac{x+1}{x-3}$	
	$=\frac{2x+2-(x+1)(x+1)}{(x-3)(x+1)}$	M1 A1
	$=\frac{(x+1)(1-x)}{(x-3)(x+1)}$	M1
	$=\frac{1-x}{x-3}$ Accept $-\frac{x-1}{x-3}$, $\frac{x-1}{3-x}$	A1 **(4)**
	Alternative	
	$\frac{2x+2}{x^2-2x-3}=\frac{2(x+1)}{(x-3)(x+1)}=\frac{2}{x-3}$	M1 A1
	$\frac{2}{x-3}-\frac{x+1}{x-3}=\frac{2-(x+1)}{x-3}$	M1
	$=\frac{1-x}{x-3}$	A1 **(4)**

Question Number	Scheme		Marks	
183.	(a)			
		Shape $(3, 6)$ $(7, 0)$	B1 B1 B1	**(3)**
	(b)			
		Shape $(3, 5)$ $(7, 2)$	B1 B1 B1	**(3)** **[6]**

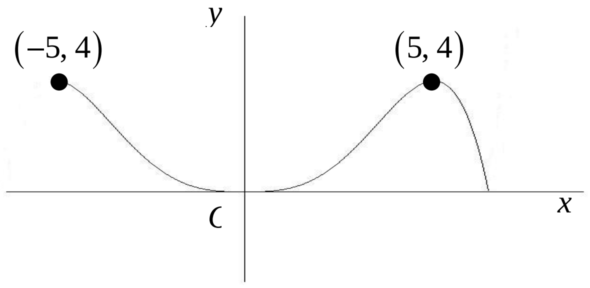
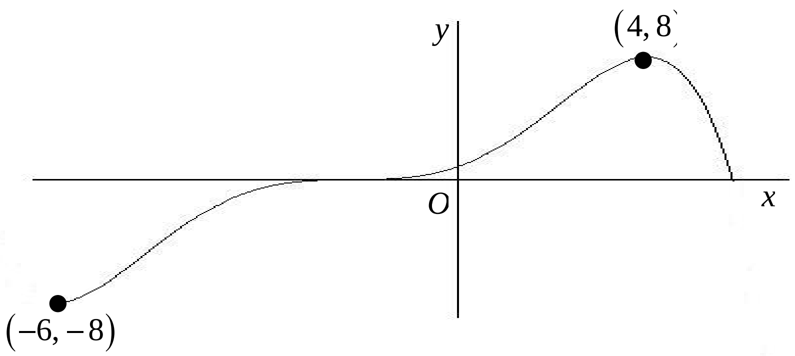
Question Number	Scheme		Marks	
184.	$x = \cos(2y+\pi)$			
	$\dfrac{dx}{dy} = -2\sin(2y+\pi)$		M1 A1	
	$\dfrac{dy}{dx} = -\dfrac{1}{2\sin(2y+\pi)}$	Follow through their $\dfrac{dx}{dy}$ before or after substitution	A1ft	
	At $y = \dfrac{\pi}{4}$, $\dfrac{dy}{dx} = -\dfrac{1}{2\sin\dfrac{3\pi}{2}} = \dfrac{1}{2}$		B1	
	$y - \dfrac{\pi}{4} = \dfrac{1}{2}x$		M1	
	$y = \dfrac{1}{2}x + \dfrac{\pi}{4}$		A1	**(6)**
				[6]

Question Number	Scheme	Marks
185.	(a) $\quad g(x) \geq 1$	B1 **(1)**
	(b) $\quad fg(x) = f\left(e^{x^2}\right) = 3e^{x^2} + \ln e^{x^2}$	M1
	$= x^2 + 3e^{x^2}$ ∗	A1 **(2)**
	$\left(fg : x \mapsto x^2 + 3e^{x^2}\right)$	
	(c) $\quad fg(x) \geq 3$	B1 **(1)**
	(d) $\quad \dfrac{d}{dx}\left(x^2 + 3e^{x^2}\right) = 2x + 6xe^{x^2}$	M1 A1
	$2x + 6xe^{x^2} = x^2e^{x^2} + 2x$	
	$e^{x^2}\left(6x - x^2\right) = 0$	M1
	$e^{x^2} \neq 0, \quad 6x - x^2 = 0$	A1
	$x = 0, 6$	A1 A1 **(6)**
		[10]

Question Number	Scheme		Marks	
186.	(a) 	 shape	B1	
		Vertices correctly placed	B1	(2)
	(b) 	 shape	B1	
		Vertex and intersections with axes correctly placed	B1	(2)
	(c) $P:(-1, 2)$		B1	
	$Q:(0,1)$		B1	
	$R:(1, 0)$		B1	(3)
	(d) $x > -1$; $2 - x - 1 = \frac{1}{2}x$		M1 A1	
	Leading to $x = \frac{2}{3}$		A1	
	$x < -1$; $2 + x + 1 = \frac{1}{2}x$		M1	
	Leading to $x = -6$		A1	(5)
				[12]

Question Number	Scheme		Marks	
187.	(a) $x^2-2x-3=(x-3)(x+1)$		B1	
	$f(x)=\dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left(or\ \dfrac{2(x-1)}{(x-3)(x+1)}-\dfrac{x+1}{(x-3)(x+1)} \right)$		M1 A1	
	$=\dfrac{x-3}{(x-3)(x+1)}=\dfrac{1}{x+1}$ ✱	cso	A1	(4)
	(b) $\left(0,\dfrac{1}{4}\right)$ Accept $0<y<\dfrac{1}{4},\ 0<f(x)<\dfrac{1}{4}$ etc.		B1 B1	(2)
	(c) Let $y=f(x)$ $y=\dfrac{1}{x+1}$			
	$x=\dfrac{1}{y+1}$			
	$yx+x=1$			
	$y=\dfrac{1-x}{x}$	or $\dfrac{1}{x}-1$	M1 A1	
	$f^{-1}(x)=\dfrac{1-x}{x}$			
	Domain of f^{-1} is $\left(0,\dfrac{1}{4}\right)$	ft their part (b)	B1 ft	(3)
	(d) $fg(x)=\dfrac{1}{2x^2-3+1}$			
	$\dfrac{1}{2x^2-2}=\dfrac{1}{8}$		M1	
	$x^2=5$		A1	
	$x=\pm\sqrt{5}$	both	A1	(3)
				[12]

Question Number	Scheme	Marks
188.	Long division of $2x^4 - 3x^2 + x + 1$ by $x^2 - 1$: Quotient: $2x^2 \quad -1$ $2x^4 - 3x^2 + x + 1$ $2x^4 - 2x^2$ $-x^2 + x + 1$ $-x^2 \quad +1$ x	M1
	$a = 2$ stated or implied	A1
	$c = -1$ stated or implied	A1
	$2x^2 - 1 + \dfrac{x}{x^2 - 1}$ $a = 2, b = 0, c = -1, d = 1, e = 0$	
	$d = 1$ and $b = 0, e = 0$ stated or implied	A1
		[4]

Question Number	Scheme	Marks	
189.	(a) 		
	Shape	B1	
	$(5, 4)$	B1	
	$(-5, 4)$	B1	(3)
	(b) For the purpose of marking this paper, the graph is identical to (a)		
	Shape	B1	
	$(5, 4)$	B1	
	$(-5, 4)$	B1	(3)
	(c) 		
	General shape – unchanged	B1	
	Translation to left	B1	
	$(4, 8)$	B1	
	$(-6, -8)$	B1	(4)
	In all parts of this question ignore any drawing outside the domains shown in the diagrams above.		**[10]**

EXPERT TUITION

Question Number	Scheme	Marks
190.	(a) $x = 1 - 2y^3 \Rightarrow y = \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ *or* $\sqrt[3]{\frac{1-x}{2}}$	M1 A1 (2)
	$f^{-1} : x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ Ignore domain	
	(b) $gf(x) = \frac{3}{1-2x^3} - 4$	M1 A1
	$= \frac{3 - 4(1-2x^3)}{1-2x^3}$	M1
	$= \frac{8x^3 - 1}{1-2x^3}$ * cso	A1 (4)
	$gf : x \mapsto \frac{8x^3-1}{1-2x^3}$ Ignore domain	
	(c) $8x^3 - 1 = 0$ Attempting solution of numerator = 0	M1
	$x = \frac{1}{2}$ Correct answer and no additional answers	A1 (2)
		[8]

Question Number	Scheme	Marks
191.	$x = 2\sin t, \quad y = 1 - \cos 2t \quad \left\{= 2\sin^2 t\right\}, \qquad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$	
(a)	$y = 1 - \cos 2t = 1 - (1 - 2\sin^2 t)$ $= 2\sin^2 t$	M1
	So, $y = 2\left(\frac{x}{2}\right)^2$ or $y = \frac{x^2}{2}$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$ $\quad y = \frac{x^2}{2}$ or equivalent.	A1 **cso isw**
	Either $k = 2$ or $-2 \leqslant x \leqslant 2$	B1 **[3]**
(b)	Range: $0 \leqslant \text{f}(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant \text{f} \leqslant 2$ $\quad$ See notes	B1 B1 **[2]** **5**

Notes for Question 191	
191. (a)	**M1:** Uses the **correct** double angle formula $\cos 2t = 1 - 2\sin^2 t$ or $\cos 2t = 2\cos^2 t - 1$ or $\cos 2t = \cos^2 t - \sin^2 t$ in an attempt to get y in terms of $\sin^2 t$ or get y in terms of $\cos^2 t$ or get y in terms of $\sin^2 t$ and $\cos^2 t$. Writing down $y = 2\sin^2 t$ is fine for M1. **A1:** Achieves $y = \frac{x^2}{2}$ or un-simplified equivalents **in the form $y = f(x)$**. For example: $y = \frac{2x^2}{4}$ or $y = 2\left(\frac{x}{2}\right)^2$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$ or $y = 1 - \frac{4 - x^2}{4} + \frac{x^2}{4}$ and you can ignore subsequent working if a candidate states a correct version of the Cartesian equation. **IMPORTANT: Please check working as this result can be fluked from an incorrect method.** **Award A0** if there is a $+c$ added to their answer. **B1:** Either $k = 2$ or a candidate writes down $-2 \leqslant x \leqslant 2$. Note: $-2 \leqslant k \leqslant 2$ unless k stated as 2 is B0.
(b)	**Note: The values of 0 and/or 2 need to be evaluated in this part** **B1:** Achieves an inclusive upper **or** lower limit, using acceptable notation. Eg: $f(x) \geqslant 0$ or $f(x) \leqslant 2$ **B1:** $0 \leqslant f(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant f \leqslant 2$ **Special Case: SC: B1B0** for either $0 < f(x) < 2$ or $0 < f < 2$ or $0 < y < 2$ or $(0, 2)$ **Special Case: SC: B1B0** for $0 \leqslant x \leqslant 2$. **IMPORTANT: Note that:** Therefore candidates can use either y or f in place of $f(x)$ **Examples:** $0 \leqslant x \leqslant 2$ is SC: B1B0 — $0 < x < 2$ is B0B0 $x \geqslant 0$ is B0B0 — $x \leqslant 2$ is B0B0 $f(x) > 0$ is B0B0 — $f(x) < 2$ is B0B0 $x > 0$ is B0B0 — $x < 2$ is B0B0 $0 \geqslant f(x) \geqslant 2$ is B0B0 — $0 < f(x) \leqslant 2$ is B1B0 $0 \leqslant f(x) < 2$ is B1B0. — $f(x) \geqslant 0$ is B1B0 $f(x) \leqslant 2$ is B1B0 — $f(x) \geqslant 0$ and $f(x) \leqslant 2$ is B1B1. Must state AND {or} $\cap$ $2 \leqslant f(x) \leqslant 2$ is B0B0 — $f(x) \geqslant 0$ or $f(x) \leqslant 2$ is B1B0. $\lvert f(x) \rvert \leqslant 2$ is B1B0 — $\lvert f(x) \rvert \geqslant 2$ is B0B0 $1 \leqslant f(x) \leqslant 2$ is B1B0 — $1 < f(x) < 2$ is B0B0 $0 \leqslant f(x) \leqslant 4$ is B1B0 — $0 < f(x) < 4$ is B0B0 $0 \leqslant$ Range $\leqslant 2$ is B1B0 — Range is in between 0 and 2 is B1B0 $0 <$ Range < 2 is B0B0. — Range $\geqslant 0$ is B1B0 Range $\leqslant 2$ is B1B0 — Range $\geqslant 0$ and Range $\leqslant 2$ is B1B0. $[0, 2]$ is B1B1 — $(0, 2)$ is SC B1B0

Notes for Question 191 Continued		
Aliter 191. (a) Way 2	$y = 1 - \cos 2t = 1 - (2\cos^2 t - 1)$	M1
	$y = 2 - 2\cos^2 t \Rightarrow \cos^2 t = \frac{2-y}{2} \Rightarrow 1 - \sin^2 t = \frac{2-y}{2}$	
	$1 - \left(\frac{x}{2}\right)^2 = \frac{2-y}{2}$	(Must be in the form $y = \mathrm{f}(x)$).
	$y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	A1

Aliter 191. (a) Way 3	$x = 2\sin t \Rightarrow t = \sin^{-1}\left(\frac{x}{2}\right)$ So, $y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	Rearranges to make *t* the subject and substitutes the result into *y*. $y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	M1 A1 **oe**
Aliter 191. (a) Way 4	$y = 1 - \cos 2t \Rightarrow \cos 2t = 1 - y \Rightarrow t = \frac{1}{2}\cos^{-1}(1-y)$ So, $x = \pm 2\sin\left(\frac{1}{2}\cos^{-1}(1-y)\right)$ So, $y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	Rearranges to make *t* the subject and substitutes the result into *y*. $y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	M1 A1 **oe**
Aliter 191. (a) Way 5	$\frac{dy}{dx} = 2\sin t = x \Rightarrow y = \frac{1}{2}x^2 + c$ Eg: when eg: $t = 0$ (nb : $-\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$), $x = 0,\ y = 1 - 1 = 0 \Rightarrow c = 0 \Rightarrow y = \frac{1}{2}x^2$ **Note:** $\frac{dy}{dx} = 2\sin t = x \Rightarrow y = \frac{1}{2}x^2$, with no attempt to find *c* is M1A0.	$\frac{dy}{dx} = x \Rightarrow y = \frac{1}{2}x^2 + c$ Full method of finding $y = \frac{1}{2}x^2$ using a value of t: $-\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$	M1 A1

Question Number	Scheme		Marks
192.	$\dfrac{5x+3}{(2x+1)(x+1)^2} \equiv \dfrac{A}{(2x+1)} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+1)^2}$	At least one of "A" or "C" are correct.	B1
	$A = 2, \quad C = 2$	Breaks up their partial fraction correctly into three terms **and** both $"A" = 2$ and $"C" = 2$.	B1 **cso**
	$5x + 3 \equiv A(x+1)^2 + B(2x+1)(x+1) + C(2x+1)$ $x = -1 \Rightarrow -2 = -C \Rightarrow C = 2$ $x = -\dfrac{1}{2} \Rightarrow -\dfrac{5}{2} + 3 = \dfrac{1}{4}A \Rightarrow \dfrac{1}{2} = \dfrac{1}{4}A \Rightarrow A = 2$	Writes down ***a correct identity*** and attempts to find the value of either one "A" or "B" or "C".	M1
	Either $x^2: 0 = A + 2B$, constant: $3 = A + B + C$ $x: 5 = 2A + 3B + 2C$		
	leading to $B = -1$	Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.	A1 **cso**
			[4] **4**
	So, $\dfrac{5x+3}{(2x+1)(x+1)^2} \equiv \dfrac{2}{(2x+1)} - \dfrac{1}{(x+1)} + \dfrac{2}{(x+1)^2}$		

Notes for Question 192

BE CAREFUL! Candidates will assign ***their own*** "A, B and C" for this question.

B1: At least one of "A" or "C" are correct.

B1: Breaks up their partial fraction correctly into three terms **and** both $"A" = 2$ and $"C" = 2$.

M1: Writes down ***a correct identity*** (although this can be implied) and attempts to find the value of either one of "A" or "B" or "C".
This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously.

A1: Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.
Note: If a candidate does not give partial fraction decomposition then:

- the 2nd B1 mark can follow from a correct identity.
- the final A1 mark can be awarded for a correct "B" if a candidate goes writes out their partial fractions at the end.

Note: The correct partial fraction from no working scores B1B1M1A1.

Note: A number of candidates will start this problem by writing out the correct identity and then attempt to find "A" or "B" or "C". Therefore the B1 marks can be awarded from this method.

Question Number	Scheme		Marks
193.	**<u>Method 1: Using one identity</u>**		
	$\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv A + \dfrac{B}{(x+2)} + \dfrac{C}{(3x-1)}$		
	$A = 3$	their constant term $= 3$	B1
	$9x^2+20x-10 \equiv A(x+2)(3x-1) + B(3x-1) + C(x+2)$	Forming a correct identity.	B1
	Either $x^2: \ 9 = 3A, \quad x: \ 20 = 5A + 3B + C$ constant: $-10 = -2A - B + 2C$ or $x = -2 \Rightarrow 36 - 40 - 10 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$	Attempts to find the value of either one of their B or their C from their identity.	M1
	$x = \dfrac{1}{3} \Rightarrow 1 + \dfrac{20}{3} - 10 = \dfrac{7}{3}C \Rightarrow -\dfrac{7}{3} = \dfrac{7}{3}C \Rightarrow C = -1$	Correct values for their B and their C, which are found using a correct identity.	A1
			[4]
	<u>Method 2: Long Division</u>		
	$\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{5x-4}{(x+2)(3x-1)}$	their constant term $= 3$	B1
	So, $\dfrac{5x-4}{(x+2)(3x-1)} \equiv \dfrac{B}{(x+2)} + \dfrac{C}{(3x-1)}$		
	$5x - 4 \equiv B(3x-1) + C(x+2)$	Forming a correct identity.	B1
	Either $x: \ 5 = 3B + C$, constant: $-4 = -B + 2C$ or $x = -2 \Rightarrow -10 - 4 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$	Attempts to find the value of either one of their B or their C from their identity.	M1
	$x = \dfrac{1}{3} \Rightarrow \dfrac{5}{3} - 4 = \dfrac{7}{3}C \Rightarrow -\dfrac{7}{3} = \dfrac{7}{3}C \Rightarrow C = -1$	Correct values for their B and their C, which are found using $5x - 4 \equiv B(3x-1) + C(x+2)$	A1
	So, $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{2}{(x+2)} - \dfrac{1}{(3x-1)}$		**[4]**
			4

1st B1: Their constant term must be equal to 3 for this mark.
2nd B1 (M1 on epen): Forming a correct identity. This can be implied by later working.
M1 (A1 on epen): Attempts to find the value of either one of their B or their C from their identity. This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously.
A1: Correct values for their B and their C, which are found using a correct identity.
Note : $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv \dfrac{A}{(x+2)} + \dfrac{B}{(3x-1)}$, leading to $9x^2 + 20x - 10 \equiv A(3x-1) + B(x+2)$, leading to $A = 2$ and $B = -1$ will gain a maximum of B0B0M1A0

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193. ctd	**Note:** You can imply the 2nd B1 from either $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv \frac{A(x+2)(3x-1)+B(3x-1)+C(x+2)}{(x+2)(3x-1)}$ or $\frac{5x-4}{(x+2)(3x-1)} \equiv \frac{B(3x-1)+C(x+2)}{(x+2)(3x-1)}$ **Alternative Method 1: Initially dividing by (x + 2)** $\frac{9x^2+20x-10}{"(x+2)"(3x-1)} \equiv \frac{9x+2}{(3x-1)} - \frac{14}{(x+2)(3x-1)}$ $\equiv 3 + \frac{5}{(3x-1)} - \frac{14}{(x+2)(3x-1)}$ — **B1:** their constant term $= 3$ So, $\frac{-14}{(x+2)(3x-1)} \equiv \frac{B}{(x+2)} + \frac{C}{(3x-1)}$ $-14 \equiv B(3x-1)+C(x+2)$ — **B1:** Forming a correct identity. $\Rightarrow B=2, C=-6$ — **M1:** Attempts to find either one of their B or their C from their identity. So, $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{5}{(3x-1)} + \frac{2}{(x+2)} - \frac{6}{(3x-1)}$ and $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{2}{(x+2)} - \frac{1}{(3x-1)}$ — **A1:** Correct answer in partial fractions. **Alternative Method 2: Initially dividing by (3x - 1)** $\frac{9x^2+20x-10}{(x+2)"(3x-1)"} \equiv \frac{3x+\frac{23}{3}}{(x+2)} - \frac{\frac{7}{3}}{(x+2)(3x-1)}$ $\equiv 3 + \frac{\frac{5}{3}}{(x+2)} - \frac{\frac{7}{3}}{(x+2)(3x-1)}$ — **B1:** their constant term $= 3$ So, $\frac{-\frac{7}{3}}{(x+2)(3x-1)} \equiv \frac{B}{(x+2)} + \frac{C}{(3x-1)}$ $-\frac{7}{3} \equiv B(3x-1)+C(x+2)$ — **B1:** Forming a correct identity. $\Rightarrow B=\frac{1}{3}, C=-1$ — **M1:** Attempts to find either one of their B or their C from their identity. So, $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{\frac{5}{3}}{(x+2)} + \frac{\frac{1}{3}}{(x+2)} - \frac{1}{(3x-1)}$ and $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{2}{(x+2)} - \frac{1}{(3x-1)}$ — **A1:** Correct answer in partial fractions.

Question Number	Scheme	Marks
194.	$9x^2 = A(x-1)(2x+1)+B(2x+1)+C(x-1)^2$	B1
	$x \to 1$ $\quad 9 = 3B \;\Rightarrow\; B = 3$	M1
	$x \to -\frac{1}{2}$ $\quad \frac{9}{4} = \left(-\frac{3}{2}\right)^2 C \;\Rightarrow\; C = 1$ $\quad$ Any two of *A*, *B*, *C*	A1
	x^2 terms $\quad 9 = 2A + C \;\Rightarrow\; A = 4$ $\quad$ All three correct	A1 **(4)** **[4]**
	Alternatives for finding A.	
	x terms $\quad 0 = -A + 2B - 2C \;\Rightarrow A = 4$ Constant terms $\quad 0 = -A + B + C \;\Rightarrow A = 4$	

Question Number	Scheme	Marks
195 (a)	Use of $\cos 2t = 1 - 2\sin^2 t$	M1
	$\cos 2t = \frac{x}{2}$, $\sin t = \frac{y}{6}$	
	$\frac{x}{2} = 1 - 2\left(\frac{y}{6}\right)^2$	M1
	Leading to $y = \sqrt{(18 - 9x)}$ $\left(= 3\sqrt{(2 - x)}\right)$ cao	A1
	$-2 \le x \le 2$ $k = 2$	B1 (4)
(b)	$0 \le f(x) \le 6$ either $0 \le f(x)$ or $f(x) \le 6$	B1
	Fully correct. Accept $0 \le y \le 6$, $[0, 6]$	B1 (2)
		[6]

Question Number	Scheme		Marks
196.	$x^3 - 4y^2 = 12xy \quad (\text{ eqn } *)$		
	$x = -8 \Rightarrow \; -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$	Substitutes $x = -8$ (at least once) into * to obtain a three term quadratic in y. Condone the loss of $= 0$.	M1
	$4y^2 - 96y + 512 = 0$ $y^2 - 24y + 128 = 0$		
	$(y-16)(y-8) = 0$ $y = \dfrac{24 \pm \sqrt{576 - 4(128)}}{2}$	An attempt to solve the quadratic in y by either factorising or by the formula or by ***completing the square***.	dM1
	$y = 16 \quad \text{or} \quad y = 8.$	Both $\underline{y = 16}$ and $\underline{y = 8}$. or $\underline{(-8, 8)}$ and $\underline{(-8, 16)}$.	A1
			[3]
			3 marks

Question Number	Scheme		Marks
	$x=\ln(t+2)$, $y=\frac{1}{t+1}$		
197. (a)	$e^x=t+2 \Rightarrow t=e^x-2$	Attempt to make t =… the subject giving $t=e^x-2$	M1 A1
	$y=\frac{1}{e^x-2+1} \Rightarrow y=\frac{1}{e^x-1}$	Eliminates t by substituting in y giving $y=\frac{1}{e^x-1}$	dM1 A1
			[4]
Aliter **197.** (a) **Way 2**	$t+1=\frac{1}{y} \Rightarrow t=\frac{1}{y}-1$ or $t=\frac{1-y}{y}$ $y(t+1)=1 \Rightarrow yt+y=1 \Rightarrow yt=1-y \Rightarrow t=\frac{1-y}{y}$	Attempt to make t =… the subject Giving either $t=\frac{1}{y}-1$ or $t=\frac{1-y}{y}$	M1 A1
	$x=\ln\left(\frac{1}{y}-1+2\right)$ or $x=\ln\left(\frac{1-y}{y}+2\right)$	Eliminates t by substituting in x	dM1
	$x=\ln\left(\frac{1}{y}+1\right)$		
	$e^x=\frac{1}{y}+1$		
	$e^x-1=\frac{1}{y}$		
	$y=\frac{1}{e^x-1}$	giving $y=\frac{1}{e^x-1}$	A1
			[4]
(b)	Domain: $\underline{x>0}$	$\underline{x>0}$ or just > 0	B1
			[1]
			5 marks

Question Number	Scheme		Marks
Aliter **197.** (a) **Way 3**	$e^x = t+2 \Rightarrow t+1 = e^x - 1$	Attempt to make $t+1 = \ldots$ the subject	M1
		giving $t+1 = e^x - 1$	A1
	$y = \dfrac{1}{t+1} \Rightarrow y = \dfrac{1}{e^x - 1}$	Eliminates t by substituting in y	dM1
		giving $y = \dfrac{1}{e^x - 1}$	A1
			[4]
Aliter **197.** (a) **Way 4**	$t+1 = \dfrac{1}{y} \Rightarrow t+2 = \dfrac{1}{y} + 1$ or $t+2 = \dfrac{1+y}{y}$	Attempt to make $t+2 = \ldots$ the subject	M1
		Either $t+2 = \dfrac{1}{y} + 1$ or $t+2 = \dfrac{1+y}{y}$	A1
	$x = \ln\left(\dfrac{1}{y} + 1\right)$ or $x = \ln\left(\dfrac{1+y}{y}\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\dfrac{1}{y} + 1\right)$		
	$e^x = \dfrac{1}{y} + 1 \Rightarrow e^x - 1 = \dfrac{1}{y}$		
	$y = \dfrac{1}{e^x - 1}$	giving $y = \dfrac{1}{e^x - 1}$	A1
			[4]