EXPERT TUITION

Maths Questions By Topic:

Algebra & Functions

A-Level Edexcel

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Table Of Contents

1. $$f(x) = ax^3 + 10x^2 - 3ax - 4$$

Given that $(x - 1)$ is a factor of $f(x)$, find the value of the constant a.

You must make your method clear.

(3)

Question 1 continued

(Total for Question 1 is 3 marks)

2. Given that

$$f(x) = x^2 - 4x + 5 \qquad x \in \mathbb{R}$$

(a) express $f(x)$ in the form $(x + a)^2 + b$ where a and b are integers to be found.

(2)

The curve with equation $y = f(x)$

- meets the y-axis at the point P
- has a minimum turning point at the point Q

(b) Write down

(i) the coordinates of P

(ii) the coordinates of Q

(2)

Question 2 continued

(Total for Question 2 is 4 marks)

3. The curve with equation $y = f(x)$ where

$$f(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$

Show that α is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0$$

(4)

Question 3 continued

(Total for Question 3 is 4 marks)

4. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Using algebra, solve the inequality

$$x^2 - x > 20$$

writing your answer in set notation.

(3)

Question 4 continued

(Total for Question 4 is 3 marks)

In this question you should show all stages of your working. Solutions relying on calculator technology are not acceptable.

5. (a) Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0$$

(3)

(b) Hence find all real solutions of

$$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$$

(3)

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Question 5 continued

(Total for Question 5 is 6 marks)

6. A curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = -3x^2 + 12x + 8$$

(a) Write $\mathrm{f}(x)$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The curve C has a maximum turning point at M.

(b) Find the coordinates of M.

(2)

Question 6 continued

(Total for Question 6 is 5 marks)

7. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = ax^3 + 15x^2 - 39x + b$$

and a and b are constants.

Given

- the point (2, 10) lies on C
- the gradient of the curve at (2, 10) is –3

(a) (i) show that the value of a is –2

(ii) find the value of b.

(4)

(b) Hence show that C has no stationary points.

(3)

(c) Write $\mathrm{f}(x)$ in the form $(x - 4)\mathrm{Q}(x)$ where $\mathrm{Q}(x)$ is a quadratic expression to be found.

(2)

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = \mathrm{f}(0.2x)$$

and the coordinate axes.

(2)

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8. The function f is defined by

$$f(x) = \frac{3x - 7}{x - 2} \qquad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$

(2)

(b) Show that $ff(x) = \dfrac{ax + b}{x - 3}$ where a and b are integers to be found.

(3)

Question 8 continued

(Total for Question 8 is 5 marks)

9.

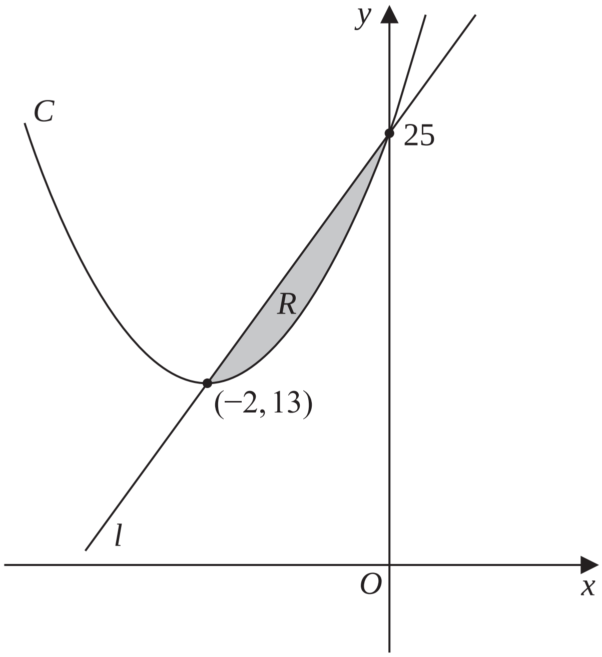


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ and a straight line l.

The curve C meets l at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region R is bounded by C and l as shown in Figure 1.

Given that

- $f(x)$ is a quadratic function in x
- $(-2, 13)$ is the minimum turning point of $y = f(x)$

use inequalities to define R.

(5)

Question 9 continued

(Total for Question 9 is 5 marks)

10.

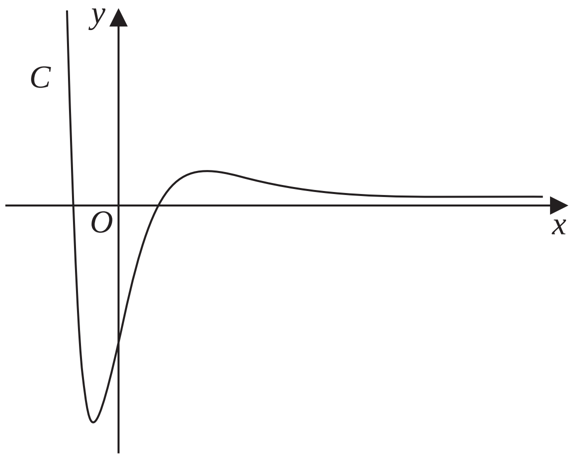


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 4(x^2 - 2)\mathrm{e}^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that $\mathrm{f}'(x) = 8(2 + x - x^2)\mathrm{e}^{-2x}$

(3)

(b) Hence find, in simplest form, the exact coordinates of the stationary points of C.

(3)

The function g and the function h are defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) \qquad x \in \mathbb{R}$$

$$\mathrm{h}(x) = 2\mathrm{f}(x) - 3 \qquad x \geqslant 0$$

(c) Find (i) the range of g

(ii) the range of h

(3)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 9 marks)

11. A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(5)

Question 11 continued

(Total for Question 11 is 5 marks)

12. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

Question 12 continued

(Total for Question 12 is 6 marks)

13. $$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that $g(x)$ is divisible by $(x - 5)$.

(2)

(b) Hence, showing all your working, write $g(x)$ as a product of three linear factors.

(4)

The finite region R is bounded by the curve with equation $y = g(x)$ and the x-axis, and lies below the x-axis.

(c) Find, using algebraic integration, the exact value of the area of R.

(4)

Question 13 continued

Question 13 continued

Question 13 continued

(Total for Question 13 is 10 marks)

14. A curve has equation $y = g(x)$.

Given that

- $g(x)$ is a cubic expression in which the coefficient of x^3 is equal to the coefficient of x
- the curve with equation $y = g(x)$ passes through the origin
- the curve with equation $y = g(x)$ has a stationary point at (2, 9)

(a) find $g(x)$,

(7)

(b) prove that the stationary point at (2, 9) is a maximum.

(2)

Question 14 continued

Question 14 continued

(Total for Question 14 is 9 marks)

Answer ALL questions. Write your answers in the spaces provided.

15. $$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

Question 15 continued

(Total for Question 15 is 3 marks)

16. $$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write f(x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

Question 16 continued

Question 16 continued

Question 16 continued

(Total for Question 16 is 10 marks)

17. Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

Question 17 continued

(Total for Question 17 is 8 marks)

18. The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C.

(3)

Question 18 continued

(Total for Question 18 is 8 marks)

19. $$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

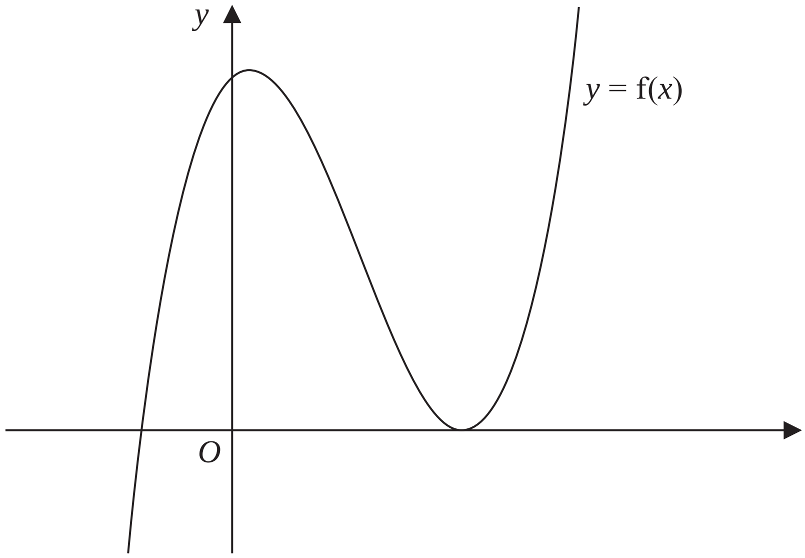


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

Question 19 continued

Question 19 continued

Question 19 continued

(Total for Question 19 is 10 marks)

20.

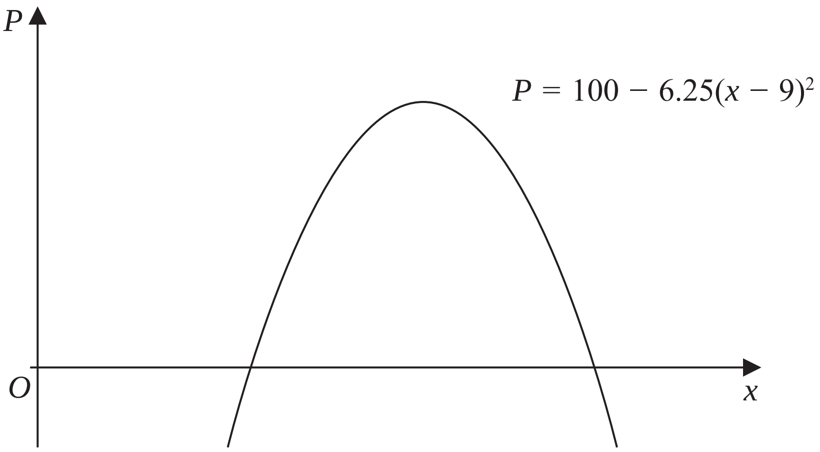


Figure 1

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2$$

where P is the profit measured in thousands of pounds and x is the selling price of the toy in pounds.

A sketch of P against x is shown in Figure 1.

Using the model,

(a) explain why £15 is not a sensible selling price for the toy.

(2)

Given that the company made an annual profit of more than £80 000

(b) find, according to the model, the least possible selling price for the toy.

(3)

The company wishes to maximise its annual profit.

State, according to the model,

(c) (i) the maximum possible annual profit,

(ii) the selling price of the toy that maximises the annual profit.

(2)

Question 20 continued

Question 20 continued

Question 20 continued

(Total for Question 20 is 7 marks)

21.

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

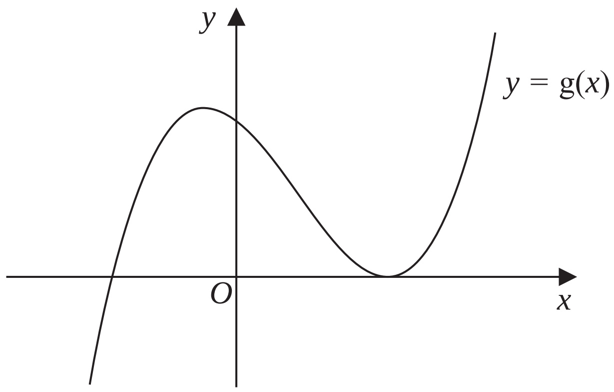


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

Question 21 continued

Question 21 continued

Question 21 continued

(Total for Question 21 is 9 marks)

22.

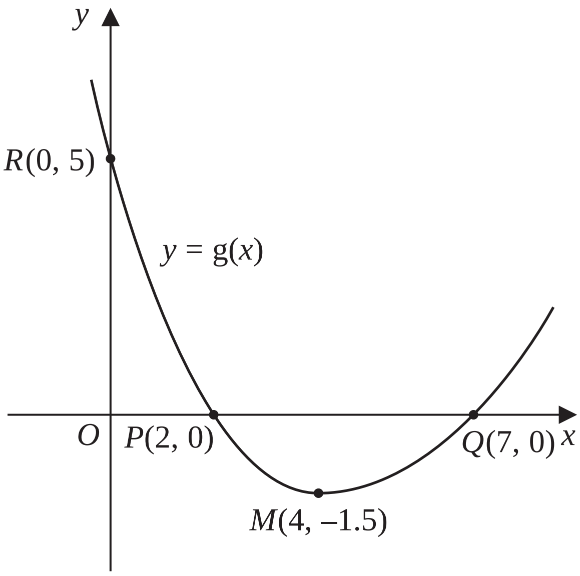


Figure 1

Figure 1 shows a sketch of the curve with equation $y = g(x)$.

The curve has a single turning point, a minimum, at the point $M(4, -1.5)$.

The curve crosses the x-axis at two points, $P(2, 0)$ and $Q(7, 0)$.

The curve crosses the y-axis at a single point $R(0, 5)$.

(a) State the coordinates of the turning point on the curve with equation $y = 2g(x)$.

(1)

(b) State the largest root of the equation

$$g(x + 1) = 0$$

(1)

(c) State the range of values of x for which $g'(x) \leqslant 0$

(1)

Given that the equation $g(x) + k = 0$, where k is a constant, has no real roots,

(d) state the range of possible values for k.

(1)

Question 22 continued

(Total for Question 22 is 4 marks)

23. $$f(x) = x^3 + 3x^2 - 4x - 12$$

(a) Using the factor theorem, explain why $f(x)$ is divisible by $(x + 3)$.

(2)

(b) Hence fully factorise $f(x)$.

(3)

(c) Show that $\dfrac{x^3 + 3x^2 - 4x - 12}{x^3 + 5x^2 + 6x}$ can be written in the form $A + \dfrac{B}{x}$ where A and B are integers to be found.

(3)

Question 23 continued

(Total for Question 23 is 8 marks)

24.

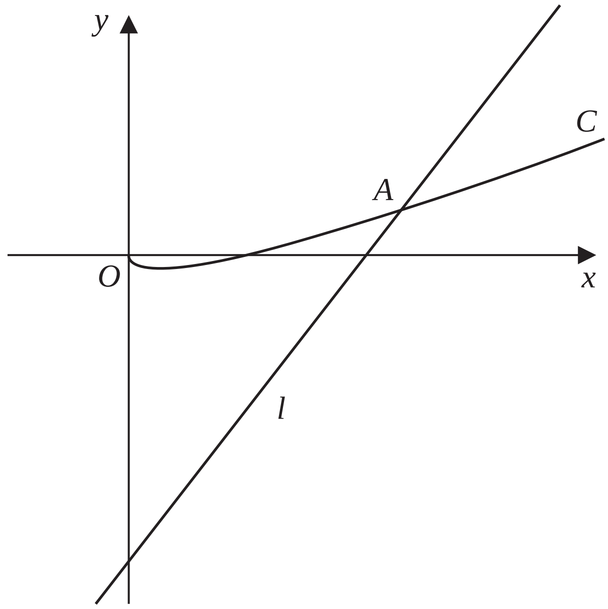


Figure 3

Figure 3 shows a sketch of the curve C with equation $y = 3x - 2\sqrt{x}$, $x \geqslant 0$ and the line l with equation $y = 8x - 16$

The line cuts the curve at point A as shown in Figure 3.

(a) Using algebra, find the x coordinate of point A.

(5)

(b)

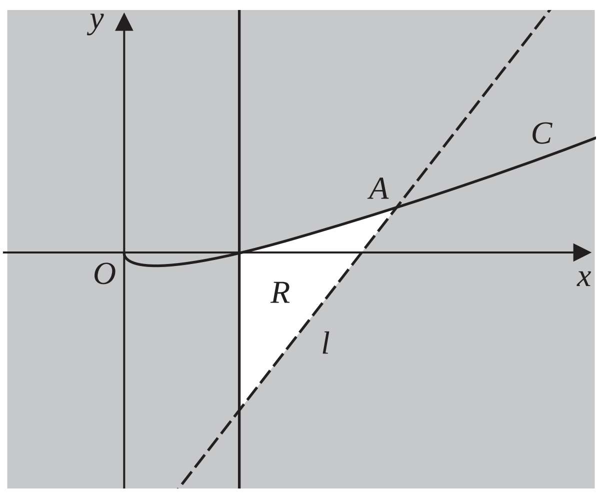


Figure 4

The region R is shown unshaded in Figure 4. Identify the inequalities that define R.

(3)

Question 24 continued

Question 24 continued

Question 24 continued

(Total for Question 24 is 8 marks)

25. $$f(x) = x^3 + ax^2 - ax + 48, \quad \text{where } a \text{ is a constant}$$

Given that $f(-6) = 0$

(a) (i) show that $a = 4$

(ii) express $f(x)$ as a product of two algebraic factors.

(4)

Given that $2\log_2(x + 2) + \log_2 x - \log_2(x - 6) = 3$

(b) show that $x^3 + 4x^2 - 4x + 48 = 0$

(4)

(c) hence explain why

$$2\log_2(x + 2) + \log_2 x - \log_2(x - 6) = 3$$

has no real roots.

(2)

Question 25 continued

(Total for Question 25 is 10 marks)

26.

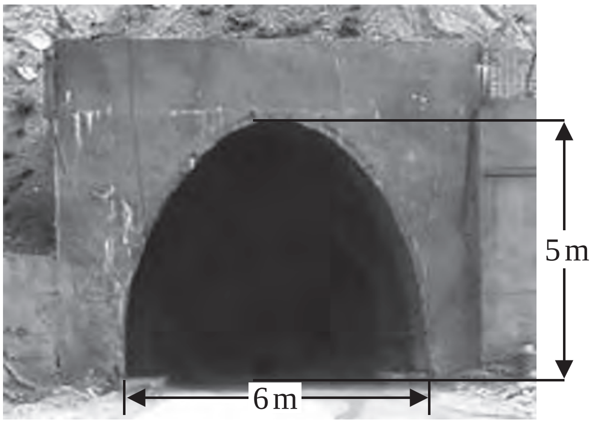


Figure 2

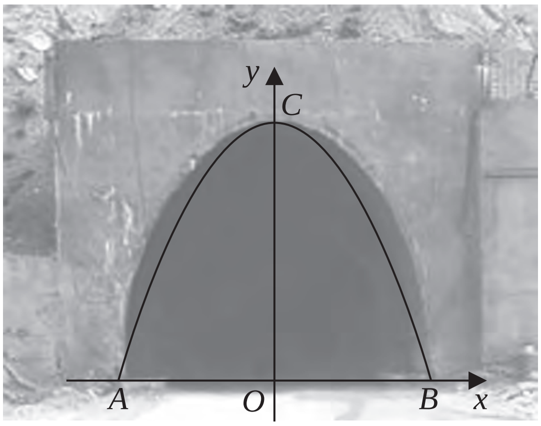


Figure 3

Figure 2 shows the entrance to a road tunnel. The maximum height of the tunnel is measured as 5 metres and the width of the base of the tunnel is measured as 6 metres.

Figure 3 shows a quadratic curve *BCA* used to model this entrance.

The points *A*, *O*, *B* and *C* are assumed to lie in the same vertical plane and the ground *AOB* is assumed to be horizontal.

(a) Find an equation for curve *BCA*.

(3)

A coach has height 4.1 m and width 2.4 m.

(b) Determine whether or not it is possible for the coach to enter the tunnel.

(2)

(c) Suggest a reason why this model may not be suitable to determine whether or not the coach can pass through the tunnel.

(1)

Question 26 continued

(Total for Question 26 is 6 marks)

27. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{3x-5}{x+1}, \quad x \in \mathbb{R}, \quad x \neq -1$$

(a) Find $\mathrm{f}^{-1}(x)$.

(3)

(b) Show that

$$\mathrm{ff}(x) = \frac{x+a}{x-1}, \quad x \in \mathbb{R}, \quad x \neq \pm 1$$

where a is an integer to be found.

(4)

The function g is defined by

$$\mathrm{g}: x \mapsto x^2 - 3x, \quad x \in \mathbb{R}, 0 \leqslant x \leqslant 5$$

(c) Find the value of $\mathrm{fg}(2)$.

(2)

(d) Find the range of g.

(3)

(e) Explain why the function g does not have an inverse.

(1)

Question 27 continued

(Total for Question 27 is 13 marks)

28.

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

(Total for Question 28 is 6 marks)

29. The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

(4)

(Total for Question 29 is 4 marks)

30. (a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^2 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

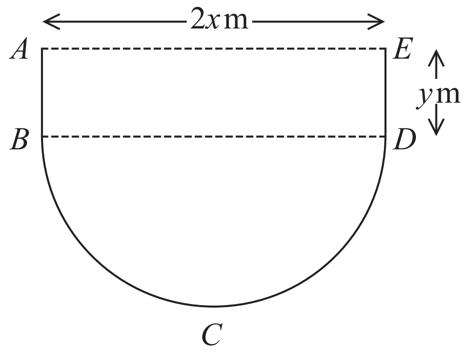
(c) Find the two possible values of a.

(3)

Question 30 continued

(Total for Question 30 is 7 marks)

31.



$%&)(+

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semicircular section *BCD* as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is $250\,\text{m}^2$,

(a) show that the perimeter, P metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(4)

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$

(2)

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

(4)

Question 31 continued

(Total for Question 31 is 10 marks)

32. An archer shoots an arrow.

The height, H metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.

(3)

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.

(1)

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form

$$A - B(d - C)^2$$

where A, B and C are constants to be found.

(3)

It is decided that the model should be adapted for a different archer.

The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

Hence or otherwise, find, for the adapted model

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.

(2)

Question 32 continued

(Total for Question 32 is 9 marks)

33. The functions f and g are defined by

$$f(x) = 7 - 2x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3x}{5x-1} \qquad x \in \mathbb{R} \qquad x \neq \frac{1}{5}$$

(a) State the range of f

(1)

(b) Find gf(1.8)

(2)

(c) Find $g^{-1}(x)$

(2)

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Question 33 continued

(Total for Question 33 is 5 marks)

34.

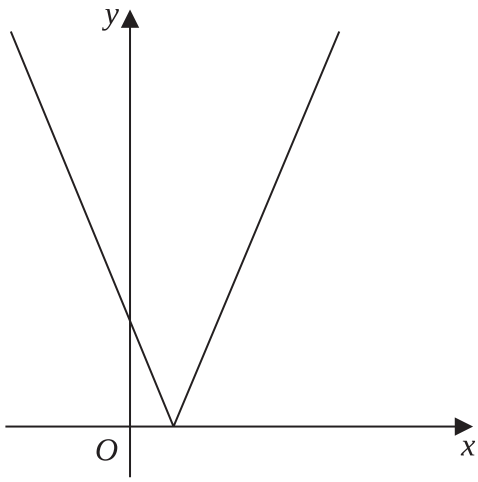


Figure 4

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where k is a positive constant.

(a) Sketch the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of k, the set of values of x for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

(4)

(c) Find, in terms of k, the coordinates of the minimum point of the graph with equation

$$y = 3 - 5\mathrm{f}\left(\frac{1}{2}x\right)$$

(2)

Question 34 continued

Question 34 continued

Question 34 continued

(Total for Question 34 is 10 marks)

35.

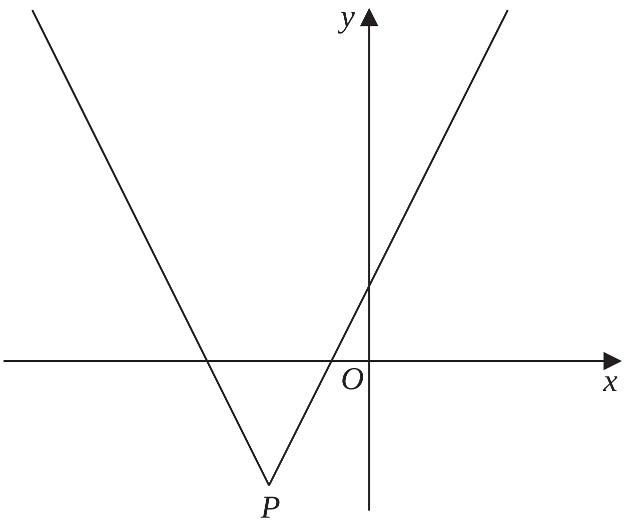


Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point P, shown in Figure 2.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

(c) find the range of possible values of a, writing your answer in set notation.

(3)

Question 35 continued

Question 35 continued

Question 35 continued

(Total for Question 35 is 7 marks)

36.

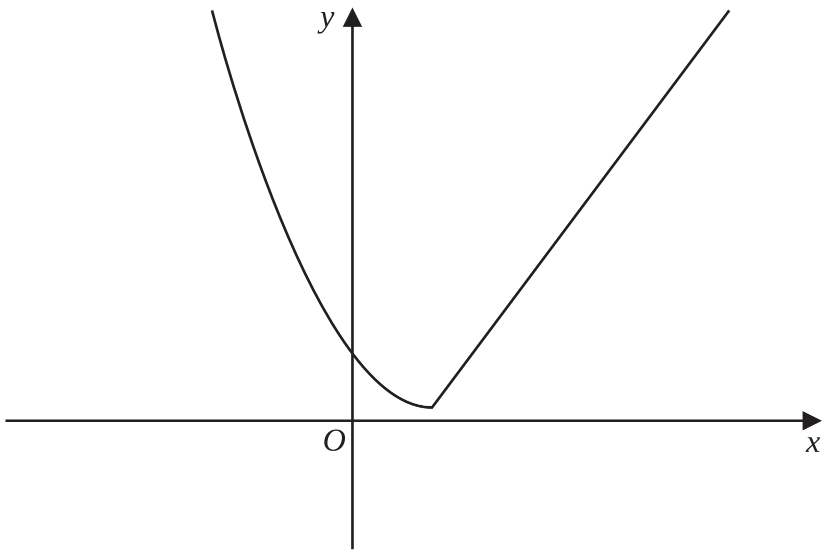


Figure 4

Figure 4 shows a sketch of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of $\mathrm{gg}(0)$.

(2)

(b) Find all values of x for which

$$\mathrm{g}(x) > 28$$

(4)

The function h is defined by

$$\mathrm{h}(x) = (x-2)^2 + 1 \qquad x \leqslant 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$\mathrm{h}^{-1}(x) = -\frac{1}{2}$$

(3)

Question 36 continued

Question 36 continued

Question 36 continued

(Total for Question 36 is 10 marks)

37.

$$g(x) = \frac{2x+5}{x-3} \qquad x \geqslant 5$$

(a) Find gg(5).

(2)

(b) State the range of g.

(1)

(c) Find $g^{-1}(x)$, stating its domain.

(3)

Question 37 continued

(Total for Question 37 is 6 marks)

38. (i) Sketch the graph of $y = |x| + 3$

(ii) Explain why $|x| + 3 \geqslant |x + 3|$ for all real values of x.

(3)

Question 38 continued

(Total for Question 38 is 3 marks)

39.

$$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$$

(a) (i) Calculate f(2)

(ii) Write f(x) as a product of two algebraic factors.

(3)

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

(c) deduce the number of real solutions, for $7\pi \leqslant \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0$$

(1)

Question 39 continued

Question 39 continued

Question 39 continued

(Total for Question 39 is 6 marks)

40.

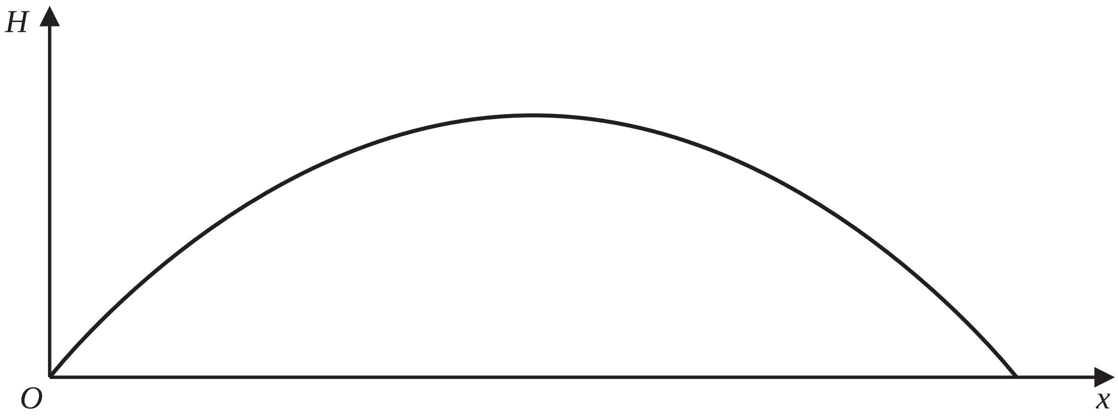


Figure 1

Figure 1 is a graph showing the trajectory of a rugby ball.

The height of the ball above the ground, H metres, has been plotted against the horizontal distance, x metres, measured from the point where the ball was kicked.

The ball travels in a vertical plane.

The ball reaches a maximum height of 12 metres and hits the ground at a point 40 metres from where it was kicked.

(a) Find a quadratic equation linking H with x that models this situation.

(3)

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from O.

(3)

(c) Give one limitation of the model.

(1)

Question 40 continued

Question 40 continued

Question 40 continued

(Total for Question 40 is 7 marks)

41. (a) Sketch the graph with equation

$$y = |2x - 5|$$

stating the coordinates of any points where the graph cuts or meets the coordinate axes.

(2)

(b) Find the values of x which satisfy

$$|2x - 5| > 7$$

(2)

(c) Find the values of x which satisfy

$$|2x - 5| > x - \frac{5}{2}$$

Write your answer in set notation.

(2)

Question 41 continued

(Total for Question 41 is 6 marks)

EXPERT TUITION

42. The line l has equation

$$3x - 2y = k$$

where k is a real constant.

Given that the line l intersects the curve with equation

$$y = 2x^2 - 5$$

at two distinct points, find the range of possible values for k.

(5)

Question 42 continued

(Total for Question 42 is 5 marks)

In this question you should show all stages of your working.

43. **Solutions relying on calculator technology are not acceptable.**

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

(3)

(Total for Question 43 is 3 marks)

44. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0,\ x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$

(3)

(Total for Question 44 is 5 marks)

45. Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0, \quad (a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. **(2)**				
(ii) If $ax > b$ then $x > \frac{b}{a}$ **(2)**				
(iii) The difference between consecutive square numbers is odd. **(2)**				

(Total for Question 45 is 6 marks)

46.

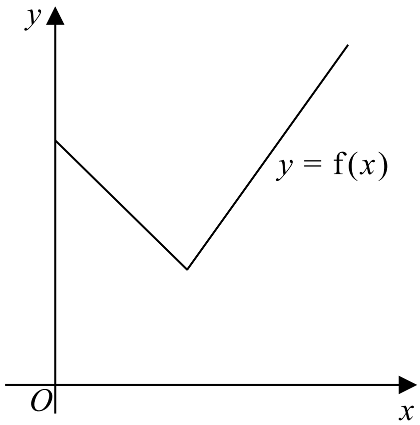


Figure 2

Figure 2 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$\mathrm{f}(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

Question 46 continued

(Total for Question 46 is 6 marks)

Leave blank

47. (i) Simplify

$$\sqrt{48} - \frac{6}{\sqrt{3}}$$

Write your answer in the form $a\sqrt{3}$, where a is an integer to be found.

(2)

(ii) Solve the equation

$$3^{6x-3} = 81$$

Write your answer as a rational number.

(3)

Leave blank

Question 47 continued

(Total 5 marks)

Leave blank

48. $$f(x) = x^2 - 10x + 23$$

(a) Express $f(x)$ in the form $(x + a)^2 + b$, where a and b are constants to be found.

(2)

(b) Hence, or otherwise, find the exact solutions to the equation

$$x^2 - 10x + 23 = 0$$

(2)

(c) Use your answer to part (b) to find the larger solution to the equation

$$y - 10y^{0.5} + 23 = 0$$

Write your solution in the form $p + q\sqrt{r}$, where p, q and r are integers.

(2)

Leave blank

Question 48 continued

(Total 6 marks)

Leave blank

49.

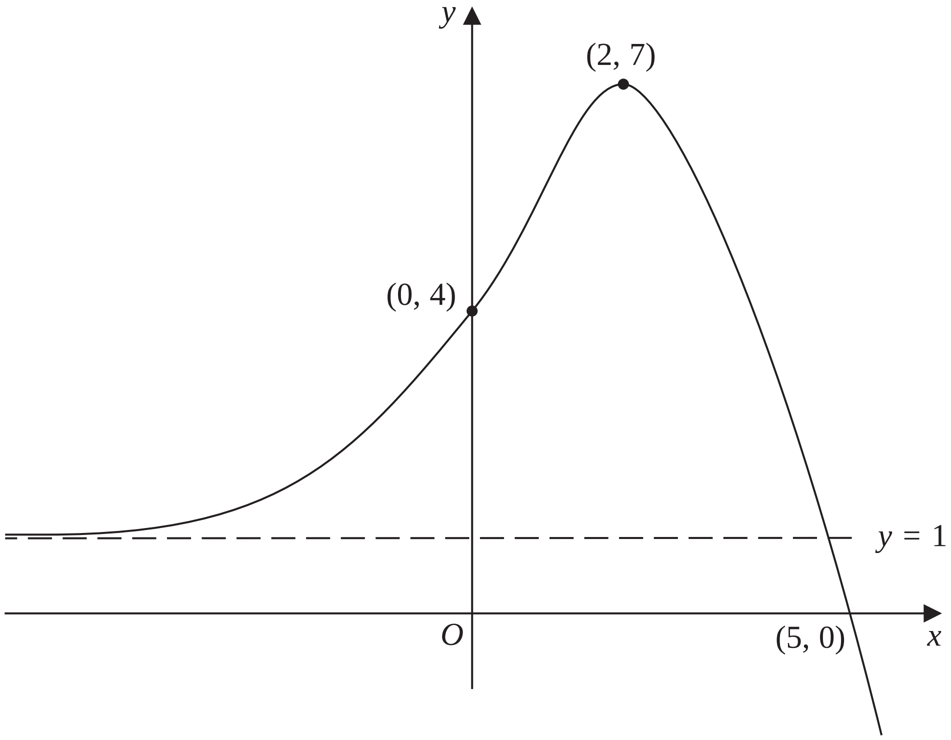


Figure 1

Figure 1 shows the sketch of a curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve crosses the y-axis at (0, 4) and crosses the x-axis at (5, 0).

The curve has a single turning point, a maximum, at (2, 7).

The line with equation $y = 1$ is the only asymptote to the curve.

(a) State the coordinates of the turning point on the curve with equation $y = f(x - 2)$.
(1)

(b) State the solution of the equation $f(2x) = 0$
(1)

(c) State the equation of the asymptote to the curve with equation $y = f(-x)$.
(1)

Given that the line with equation $y = k$, where k is a constant, meets the curve $y = f(x)$ at only one point,

(d) state the set of possible values for k.
(2)

Leave blank

Question 49 continued

EXPERT TUITION

Leave blank

Question 49 continued

Leave blank

Question 49 continued

(Total 5 marks)

Leave blank

50. The equation $20x^2 = 4kx - 13kx^2 + 2$, where k is a constant, has no real roots.

(a) Show that k satisfies the inequality

$$2k^2 + 13k + 20 < 0$$

(4)

(b) Find the set of possible values for k.

(4)

Leave blank

Question 50 continued

(Total 8 marks)

EXPERT TUITION

Leave blank

51. $$f(x) = x^2 - 8x + 19$$

(a) Express f(x) in the form $(x + a)^2 + b$, where a and b are constants.

(2)

The curve C with equation $y = f(x)$ crosses the y-axis at the point P and has a minimum point at the point Q.

(b) Sketch the graph of C showing the coordinates of point P and the coordinates of point Q.

(3)

(c) Find the distance PQ, writing your answer as a simplified surd.

(3)

Leave blank

Question 51 continued

(Total 8 marks)

Leave blank

52. (a) On separate axes sketch the graphs of

(i) $y = -3x + c$, where c is a positive constant,

(ii) $y = \frac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the y-axis and the equation of any horizontal asymptote.

(4)

Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \frac{1}{x} + 5$ at two distinct points,

(b) show that $(5 - c)^2 > 12$

(3)

(c) Hence find the range of possible values for c.

(4)

Leave blank

Question 52 continued

Leave blank

Question 52 continued

Leave blank

Question 52 continued

(Total 11 marks)

EXPERT TUITION

Leave blank

53. (a) Simplify

$$\sqrt{50} - \sqrt{18}$$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

(2)

(b) Hence, or otherwise, simplify

$$\frac{12\sqrt{3}}{\sqrt{50} - \sqrt{18}}$$

giving your answer in the form $b\sqrt{c}$, where b and c are integers and $b \neq 1$

(3)

(Total 5 marks)

Leave blank

54.

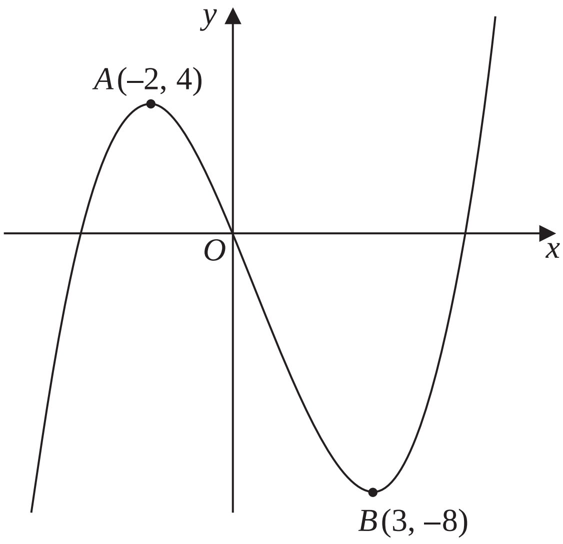


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$. The curve has a maximum point A at (–2, 4) and a minimum point B at (3, –8) and passes through the origin O.

On separate diagrams, sketch the curve with equation

(a) $y = 3f(x)$,

(2)

(b) $y = f(x) - 4$

(3)

On each diagram, show clearly the coordinates of the maximum and the minimum points and the coordinates of the point where the curve crosses the y-axis.

(Total 5 marks)

Leave blank

55. Solve the simultaneous equations

$$y + 4x + 1 = 0$$

$$y^2 + 5x^2 + 2x = 0$$

(6)

(Total 6 marks)

Leave blank

56. The straight line with equation $y = 3x - 7$ does not cross or touch the curve with equation $y = 2px^2 - 6px + 4p$, where p is a constant.

(a) Show that $4p^2 - 20p + 9 < 0$

(4)

(b) Hence find the set of possible values of p.

(4)

(Total 8 marks)

Leave blank

57. Simplify

(a) $(2\sqrt{5})^2$

(1)

(b) $\dfrac{\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}}$ giving your answer in the form $a + \sqrt{b}$, where a and b are integers.

(4)

(Total 5 marks)

EXPERT TUITION

Leave blank

58. Solve the simultaneous equations

$$y - 2x - 4 = 0$$

$$4x^2 + y^2 + 20x = 0$$

(7)

(Total 7 marks)

EXPERT TUITION

Leave blank

59. The equation

$$(p - 1)x^2 + 4x + (p - 5) = 0, \text{ where } p \text{ is a constant}$$

has no real roots.

(a) Show that p satisfies $p^2 - 6p + 1 > 0$

(3)

(b) Hence find the set of possible values of p.

(4)

Leave blank

Question 59 continued

(Total 7 marks)

Leave blank

60. (a) Factorise completely $9x - 4x^3$

(3)

(b) Sketch the curve C with equation

$$y = 9x - 4x^3$$

Show on your sketch the coordinates at which the curve meets the x-axis.

(3)

The points A and B lie on C and have x coordinates of -2 and 1 respectively.

(c) Show that the length of AB is $k\sqrt{10}$ where k is a constant to be found.

(4)

Leave blank

Question 60 continued

(Total 10 marks)

Leave blank

61. Find the set of values of x for which

(a) $3x - 7 > 3 - x$

(2)

(b) $x^2 - 9x \leqslant 36$

(4)

(c) **both** $3x - 7 > 3 - x$ **and** $x^2 - 9x \leqslant 36$

(1)

(Total 7 marks)

EXPERT TUITION

Leave blank

62.

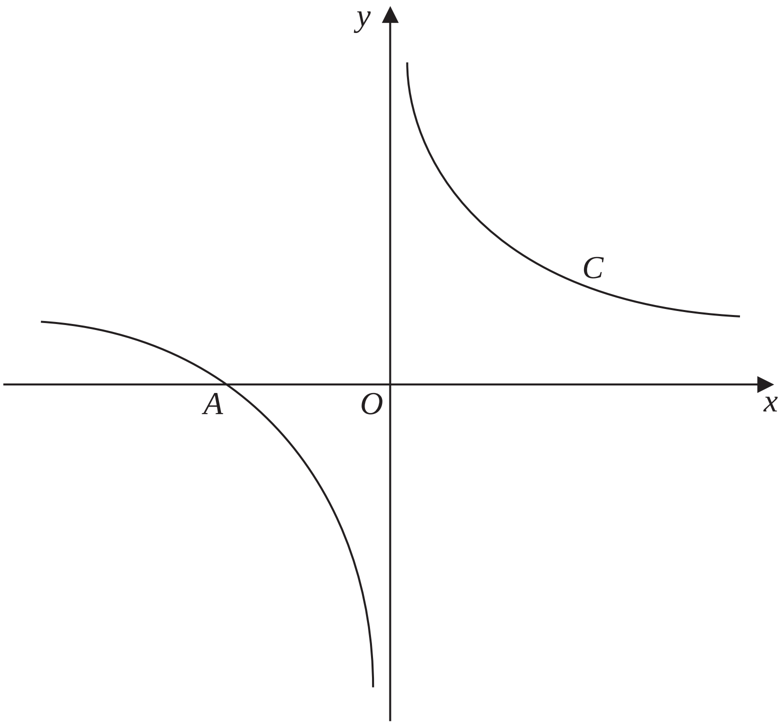


Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{1}{x} + 1, \qquad x \neq 0$$

The curve C crosses the x-axis at the point A.

(a) State the x coordinate of the point A.

(1)

The curve D has equation $y = x^2(x - 2)$, for all real values of x.

(b) A copy of Figure 1 is shown on the next page.
On this copy, sketch a graph of curve D.
Show on the sketch the coordinates of each point where the curve D crosses the coordinate axes.

(3)

(c) Using your sketch, state, giving a reason, the number of real solutions to the equation

$$x^2(x - 2) = \frac{1}{x} + 1$$

(1)

Leave blank

Question 62 continued

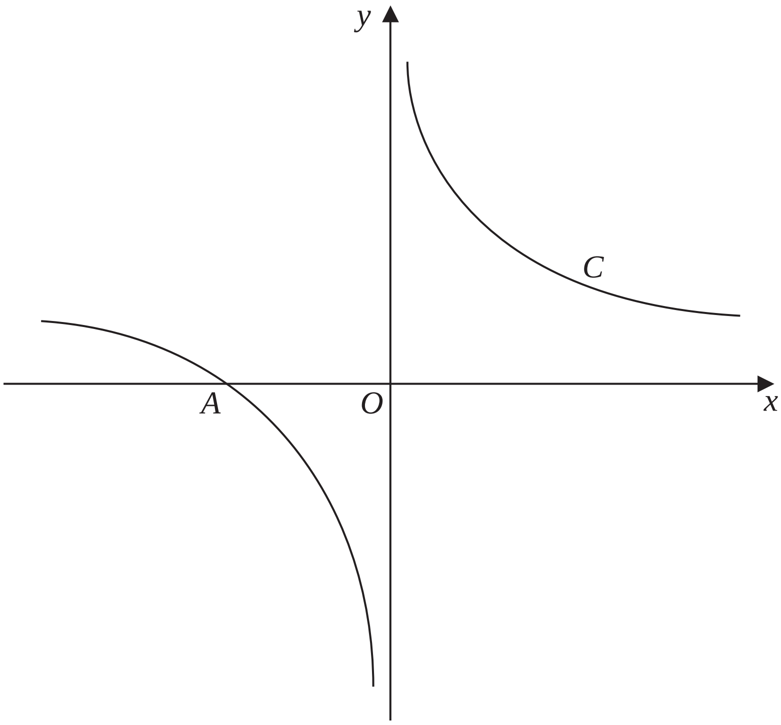


Figure 1

(Total 5 marks)

EXPERT TUITION

Leave blank

63. (a) Write $\sqrt{80}$ in the form $c\sqrt{5}$, where c is a positive constant.

(1)

A rectangle R has a length of $(1 + \sqrt{5})$ cm and an area of $\sqrt{80}$ cm^2.

(b) Calculate the width of R in cm. Express your answer in the form $p + q\sqrt{5}$, where p and q are integers to be found.

(4)

(Total 5 marks)

Leave blank

64. Given that

$$f(x) = 2x^2 + 8x + 3$$

(a) find the value of the discriminant of $f(x)$.

(2)

(b) Express $f(x)$ in the form $p(x + q)^2 + r$ where p, q and r are integers to be found.

(3)

The line $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = f(x)$.

(c) Calculate the value of c.

(5)

Leave blank

Question 64 continued

(Total 10 marks)

Leave blank

65. Factorise fully $25x - 9x^3$

(3)

(Total 3 marks)

EXPERT TUITION

Leave blank

66. Solve the equation

$$10 + x\sqrt{8} = \frac{6x}{\sqrt{2}}$$

Give your answer in the form $a\sqrt{b}$ where a and b are integers.

(4)

(Total 4 marks)

Leave blank

67.

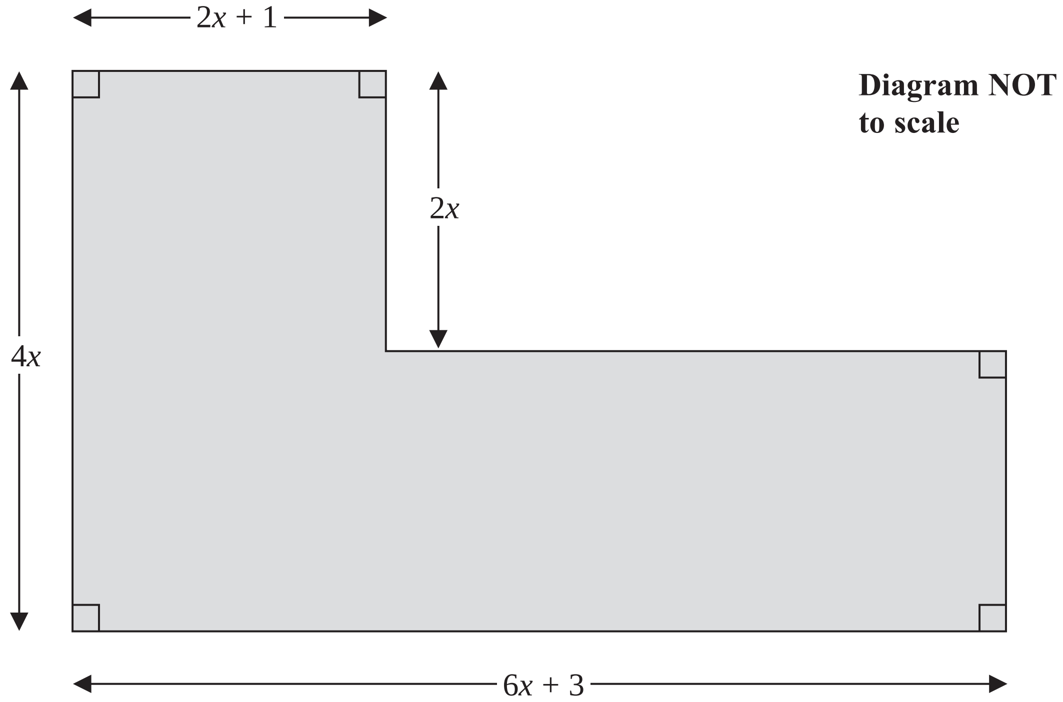


Figure 1

Figure 1 shows the plan of a garden. The marked angles are right angles.

The six edges are straight lines.

The lengths shown in the diagram are given in metres.

Given that the perimeter of the garden is greater than 40 m,

(a) show that $x > 1.7$

(3)

Given that the area of the garden is less than 120 m^2,

(b) form and solve a quadratic inequality in x.

(5)

(c) Hence state the range of the possible values of x.

(1)

Leave blank

Question 67 continued

(Total 9 marks)

Leave blank

68. The curve C has equation $y = \frac{1}{3}x^2 + 8$

The line L has equation $y = 3x + k$, where k is a positive constant.

(a) Sketch C and L on separate diagrams, showing the coordinates of the points at which C and L cut the axes.

(4)

Given that line L is a tangent to C,

(b) find the value of k.

(5)

Leave blank

Question 68 continued

(Total 9 marks)

Leave blank

69. Simplify

$$\frac{7 + \sqrt{5}}{\sqrt{5} - 1}$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are integers.

(4)

(Total 4 marks)

Leave blank

70. Find the set of values of x for which

(a) $2(3x + 4) > 1 - x$

(2)

(b) $3x^2 + 8x - 3 < 0$

(4)

Leave blank

Question 70 continued

(Total 6 marks)

Leave blank

71.

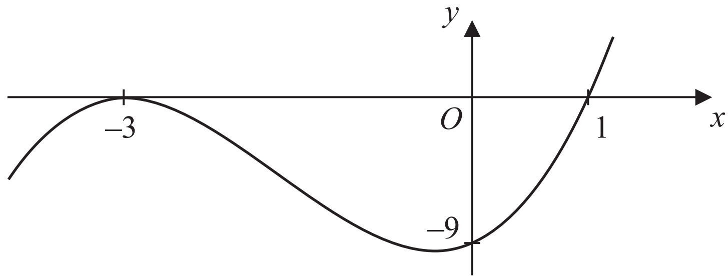


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \text{f}(x)$ where

$$\text{f}(x) = (x + 3)^2 (x - 1), \qquad x \in \mathbb{R}.$$

The curve crosses the x-axis at (1, 0), touches it at (–3, 0) and crosses the y-axis at (0, –9)

(a) In the space below, sketch the curve C with equation $y = \text{f}(x + 2)$ and state the coordinates of the points where the curve C meets the x-axis.

(3)

(b) Write down an equation of the curve C.

(1)

(c) Use your answer to part (b) to find the coordinates of the point where the curve C meets the y-axis.

(2)

Leave blank

Question 71 continued

Leave blank

Question 71 continued

(Total 6 marks)

EXPERT TUITION

Leave blank

72.

$$f'(x) = \frac{(3 - x^2)^2}{x^2}, \quad x \neq 0$$

Show that $$f'(x) = 9x^{-2} + A + Bx^2,$$

where A and B are constants to be found.

(3)

Leave blank

Question 72 continued

(Total 3 marks)

EXPERT TUITION

Leave blank

73. Given the simultaneous equations

$$2x + y = 1$$
$$x^2 - 4ky + 5k = 0$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0$$

(2)

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k.

(3)

(c) For this value of k, find the solution of the simultaneous equations.

(3)

EXPERT TUITION

Leave blank

Question 73 continued

Leave blank

Question 73 continued

Leave blank

Question 73 continued

(Total 8 marks)

Leave blank

74.

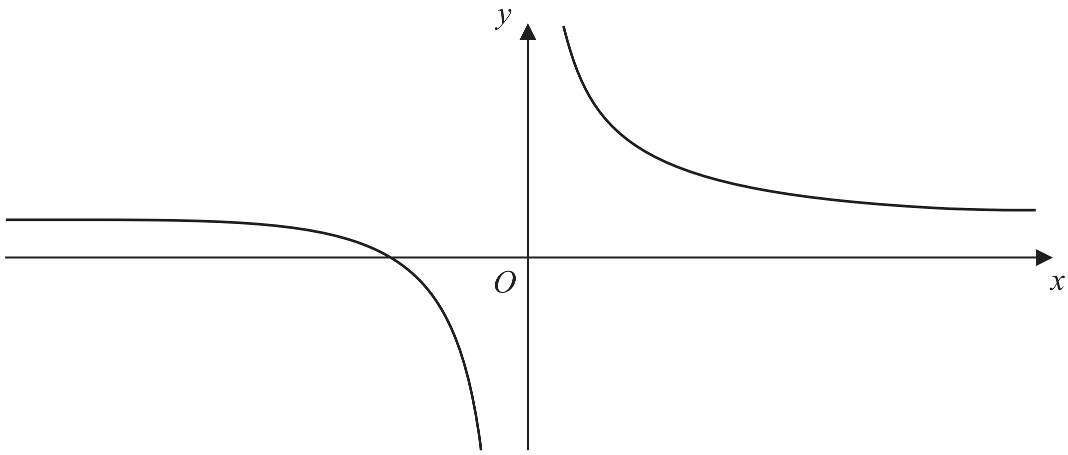


Figure 2

Figure 2 shows a sketch of the curve H with equation $y = \frac{3}{x} + 4$, $x \neq 0$.

(a) Give the coordinates of the point where H crosses the x-axis.

(1)

(b) Give the equations of the asymptotes to H.

(2)

Leave blank

Question 74 continued

(Total 3 marks)

Leave blank

75. Express $\frac{15}{\sqrt{3}} - \sqrt{27}$ in the form $k\sqrt{3}$, where k is an integer.

(4)

(Total 4 marks)

Leave blank

76. A rectangular room has a width of x m.

The length of the room is 4 m longer than its width.

Given that the perimeter of the room is greater than 19.2 m,

(a) show that $x > 2.8$

(3)

Given also that the area of the room is less than 21 m^2,

(b) (i) write down an inequality, in terms of x, for the area of the room.

(ii) Solve this inequality.

(4)

(c) Hence find the range of possible values for x.

(1)

Leave blank

Question 76 continued

Leave blank

Question 76 continued

Leave blank

Question 76 continued

(Total 8 marks)

Leave blank

77.

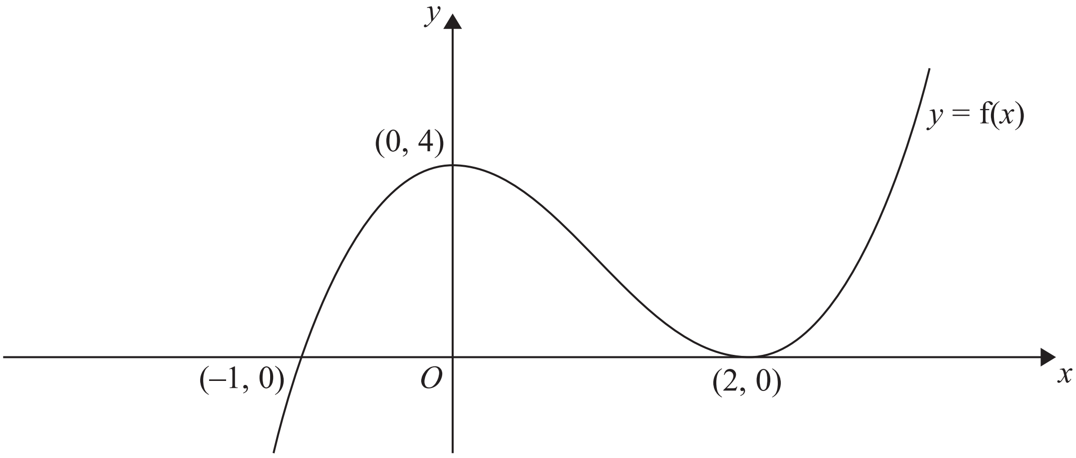


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.

The curve C passes through the point $(-1, 0)$ and touches the x-axis at the point $(2, 0)$.

The curve C has a maximum at the point $(0, 4)$.

(a) The equation of the curve C can be written in the form

$$y = x^3 + ax^2 + bx + c$$

where a, b and c are integers.

Calculate the values of a, b and c.

(5)

(b) Sketch the curve with equation $y = \mathrm{f}(\frac{1}{2}x)$ in the space provided on page 24

Show clearly the coordinates of all the points where the curve crosses or meets the coordinate axes.

(3)

Leave blank

Question 77 continued

Leave blank

Question 77 continued

(Total 8 marks)

EXPERT TUITION

Leave blank

78. Factorise completely $x - 4x^3$

(3)

(Total 3 marks)

EXPERT TUITION

Leave blank

79. (i) Express

$$\left(5-\sqrt{8}\right)\left(1+\sqrt{2}\right)$$

in the form $a+b\sqrt{2}$, where a and b are integers.

(3)

(ii) Express

$$\sqrt{80}+\frac{30}{\sqrt{5}}$$

in the form $c\sqrt{5}$, where c is an integer.

(3)

(Total 6 marks)

Leave blank

80.

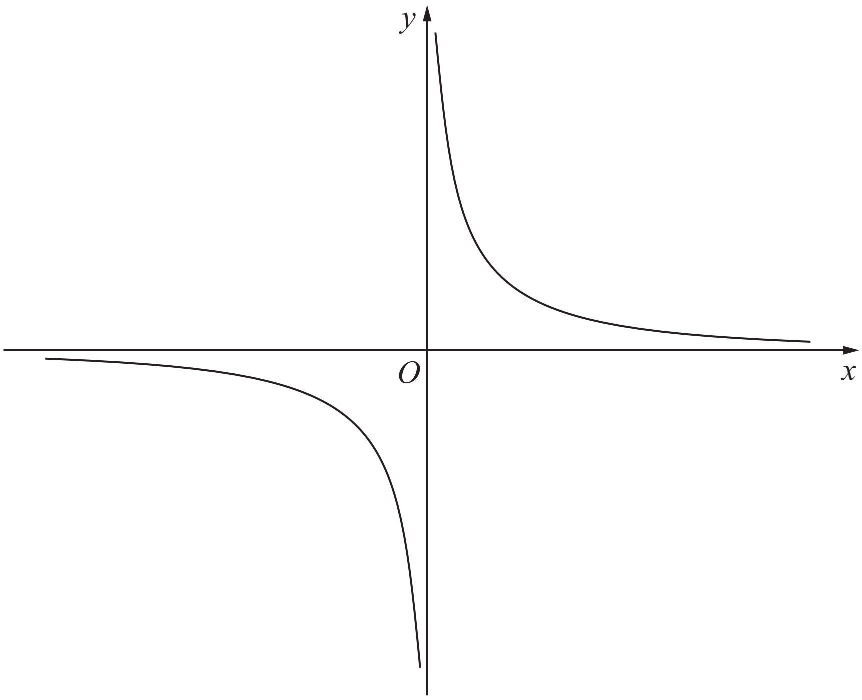


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{2}{x}$, $x \neq 0$

The curve C has equation $y = \frac{2}{x} - 5$, $x \neq 0$, and the line l has equation $y = 4x + 2$

(a) Sketch and clearly label the graphs of C and l on a single diagram.

On your diagram, show clearly the coordinates of the points where C and l cross the coordinate axes.

(5)

(b) Write down the equations of the asymptotes of the curve C.

(2)

(c) Find the coordinates of the points of intersection of $y = \frac{2}{x} - 5$ and $y = 4x + 2$

(5)

Leave blank

Question 80 continued

(Total 12 marks)

Leave blank

81. The equation

$$(k + 3)x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x.

(a) Show that k satisfies

$$k^2 - 2k - 24 < 0$$

(4)

(b) Hence find the set of possible values of k.

(3)

(Total 7 marks)

Leave blank

82. $$4x^2 + 8x + 3 \equiv a(x + b)^2 + c$$

(a) Find the values of the constants a, b and c.

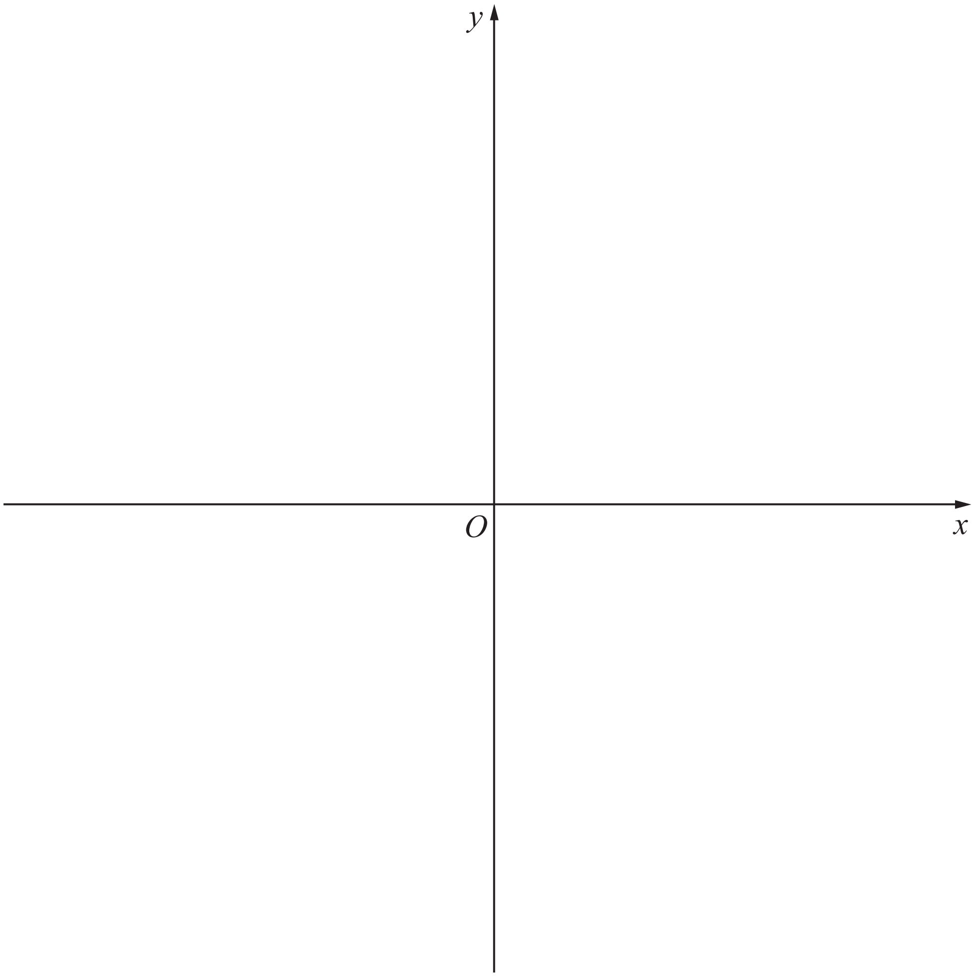
(3)

(b) On the axes on page 27, sketch the curve with equation $y = 4x^2 + 8x + 3$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(4)

Leave blank

Question 82 continued



(Total 7 marks)

EXPERT TUITION

Leave blank

83. Show that $\dfrac{2}{\sqrt{(12)}-\sqrt{(8)}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers.
(5)

Leave blank

Question 83 continued

(Total 5 marks)

Leave blank

84. $$4x - 5 - x^2 = q - (x + p)^2$$

where p and q are integers.

(a) Find the value of p and the value of q.

(3)

(b) Calculate the discriminant of $4x - 5 - x^2$

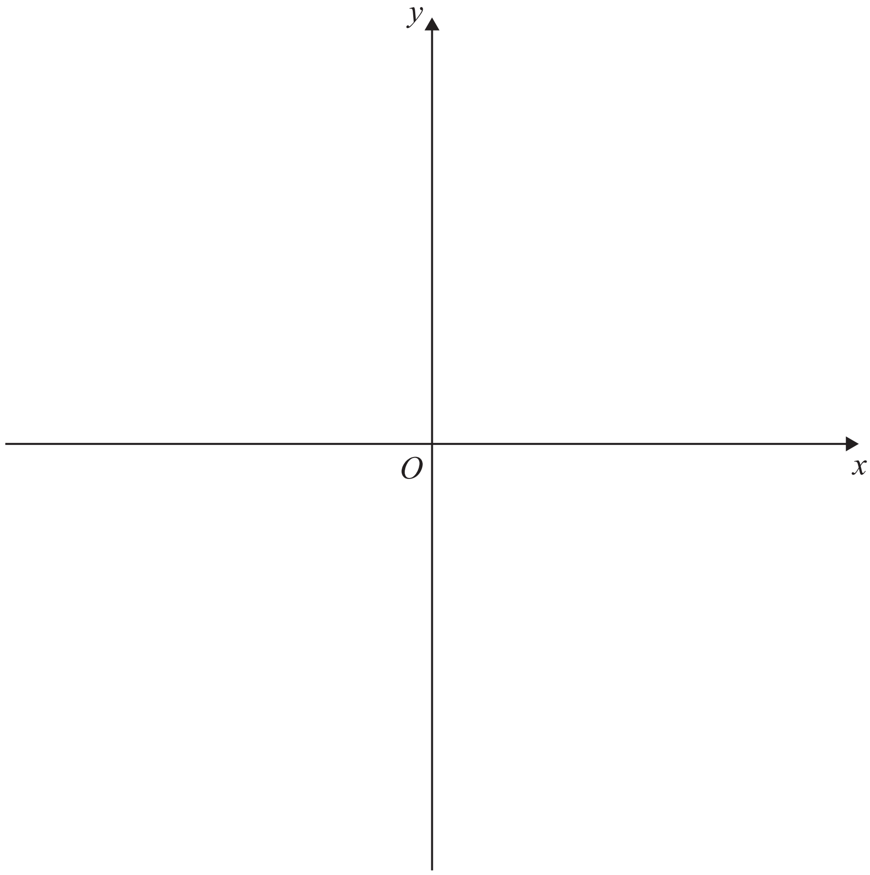
(2)

(c) On the axes on page 17, sketch the curve with equation $y = 4x - 5 - x^2$ showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3)

Leave blank

Question 84 continued



(Total 8 marks)

EXPERT TUITION

Leave blank

85.

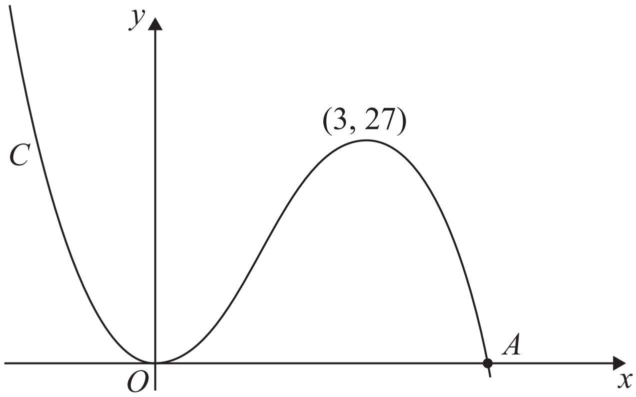


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = x^2(9 - 2x)$$

There is a minimum at the origin, a maximum at the point (3, 27) and C cuts the x-axis at the point A.

(a) Write down the coordinates of the point A.

(1)

(b) On separate diagrams sketch the curve with equation

(i) $y = \mathrm{f}(x + 3)$

(ii) $y = \mathrm{f}(3x)$

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes.

(6)

The curve with equation $y = \mathrm{f}(x) + k$, where k is a constant, has a maximum point at (3, 10).

(c) Write down the value of k.

(1)

Leave blank

Question 85 continued

Leave blank

Question 85 continued

(Total 8 marks)

Leave blank

86. (a) Simplify

$$\sqrt{32}+\sqrt{18}$$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

(2)

(b) Simplify

$$\frac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}$$

giving your answer in the form $b\sqrt{2}+c$, where b and c are integers.

(4)

(Total 6 marks)

Leave blank

87. Find the set of values of x for which

(a) $4x - 5 > 15 - x$

(2)

(b) $x(x - 4) > 12$

(4)

(Total 6 marks)

Leave blank

88. The curve C has equation $y = x(5-x)$ and the line L has equation $2y = 5x + 4$

(a) Use algebra to show that C and L do not intersect.

(4)

(b) In the space on page 11, sketch C and L on the same diagram, showing the coordinates of the points at which C and L meet the axes.

(4)

(Total 8 marks)

Leave blank

89. The curve C_1 has equation

$$y = x^2(x+2)$$

(a) Find $\frac{dy}{dx}$

(2)

(b) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis.

(3)

(c) Find the gradient of C_1 at each point where C_1 meets the x-axis.

(2)

The curve C_2 has equation

$$y = (x-k)^2(x-k+2)$$

where k is a constant and $k > 2$

(d) Sketch C_2, showing the coordinates of the points where C_2 meets the x and y axes.

(3)

Leave blank

Question 89 continued

(Total 10 marks)

EXPERT TUITION

Leave blank

90. Solve the simultaneous equations

$$x + y = 2$$

$$4y^2 - x^2 = 11$$

(7)

(Total 7 marks)

Leave blank

91. $$f(x) = x^2 + (k + 3)x + k$$

where k is a real constant.

(a) Find the discriminant of $f(x)$ in terms of k.

(2)

(b) Show that the discriminant of $f(x)$ can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation $f(x) = 0$ has real roots.

(2)

(Total 6 marks)

92.

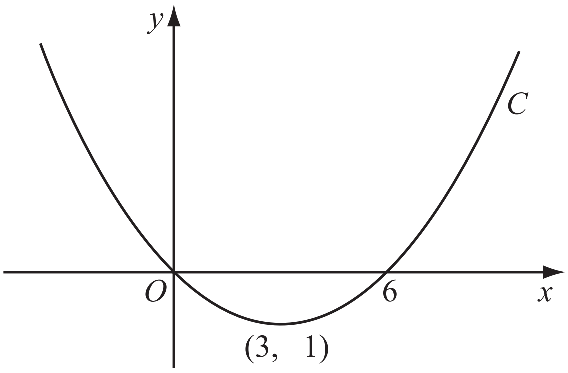


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.
The curve C passes through the origin and through $(6, 0)$.
The curve C has a minimum at the point (3,   1).

On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(2x)$,

(3)

(b) $y = -\mathrm{f}(x)$,

(3)

(c) $y = \mathrm{f}(x + p)$, where p is a constant and $0 < p < 3$.

(4)

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

Leave blank

Question 92 continued

(Total 10 marks)

Leave blank

93. Simplify

$$\frac{5-2\sqrt{3}}{\sqrt{3}-1}$$

giving your answer in the form $p+q\sqrt{3}$, where p and q are rational numbers.

(4)

(Total 4 marks)

Leave blank

94.

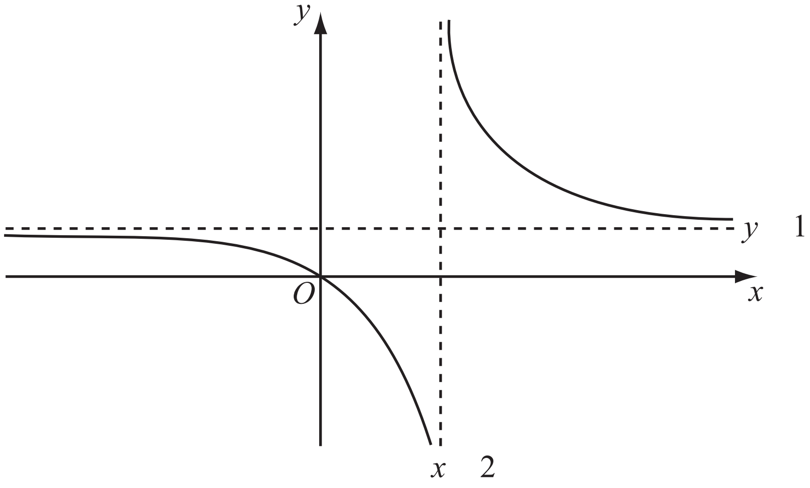


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{x-2}, \qquad x \neq 2$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(a) In the space below, sketch the curve with equation $y = \mathrm{f}(x-1)$ and state the equations of the asymptotes of this curve.

(3)

(b) Find the coordinates of the points where the curve with equation $y = \mathrm{f}(x-1)$ crosses the coordinate axes.

(4)

Leave blank

Question 94 continued

(Total 7 marks)

Leave blank

95. The equation $x^2 + (k-3)x + (3-2k) = 0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2 + 2k - 3 > 0$$

(3)

(b) Find the set of possible values of k.

(4)

(Total 7 marks)

Leave blank

96. (a) On the axes below, sketch the graphs of

(i) $y = x(x+2)(3-x)$

(ii) $y = -\frac{2}{x}$

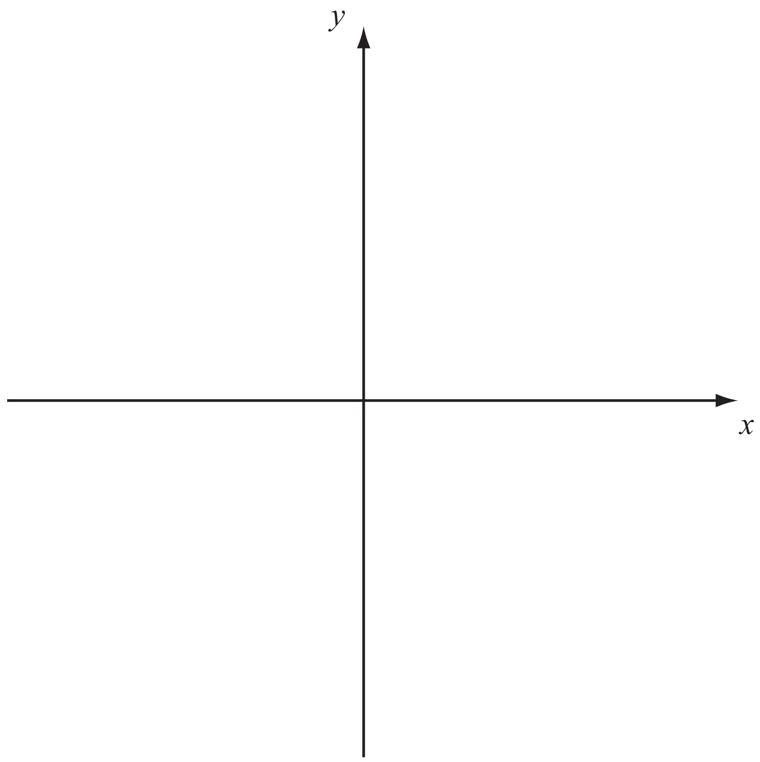
showing clearly the coordinates of all the points where the curves cross the coordinate axes.

(6)

(b) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x+2)(3-x) + \frac{2}{x} = 0$$

(2)



(Total 8 marks)

Leave blank

97. Write

$$\sqrt{(75)} - \sqrt{(27)}$$

in the form $k\sqrt{x}$, where k and x are integers.

(2)

(Total 2 marks)

Leave blank

98. Find the set of values of x for which

(a) $3(x-2) < 8-2x$

(2)

(b) $(2x-7)(1+x) < 0$

(3)

(c) both $3(x-2) < 8-2x$ **and** $(2x-7)(1+x) < 0$

(1)

(Total 6 marks)

Leave blank

99. (a) Show that $x^2 + 6x + 11$ can be written as

$$(x + p)^2 + q$$

where p and q are integers to be found.

(2)

(b) In the space at the top of page 7, sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes.

(2)

(c) Find the value of the discriminant of $x^2 + 6x + 11$

(2)

Leave blank

Question 99 continued

(Total 6 marks)

Leave blank

100.

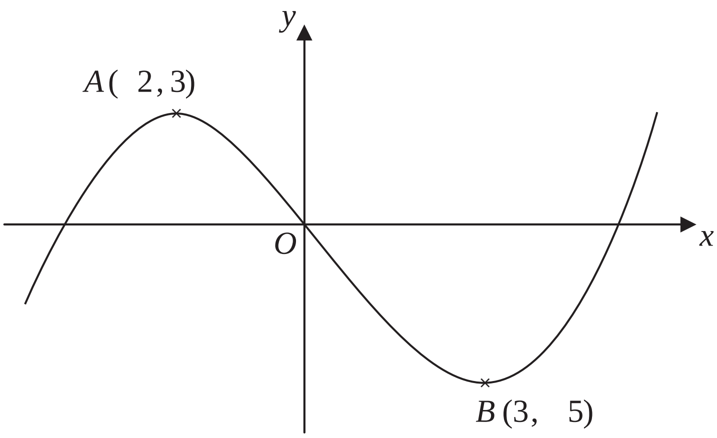


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve has a maximum point A at (–2, 3) and a minimum point B at (3, –5).

On separate diagrams sketch the curve with equation

(a) $y = \mathrm{f}(x+3)$

(3)

(b) $y = 2\mathrm{f}(x)$

(3)

On each diagram show clearly the coordinates of the maximum and minimum points.

The graph of $y = \mathrm{f}(x) + a$ has a minimum at (3, 0), where a is a constant.

(c) Write down the value of a.

(1)

Leave blank

Question 100 continued

(Total 7 marks)

Leave blank

101. (a) On the axes below sketch the graphs of

(i) $y = x(4 - x)$

(ii) $y = x^2(7 - x)$

showing clearly the coordinates of the points where the curves cross the coordinate axes.

(5)

(b) Show that the x-coordinates of the points of intersection of

$$y = x(4 - x) \quad \text{and} \quad y = x^2(7 - x)$$

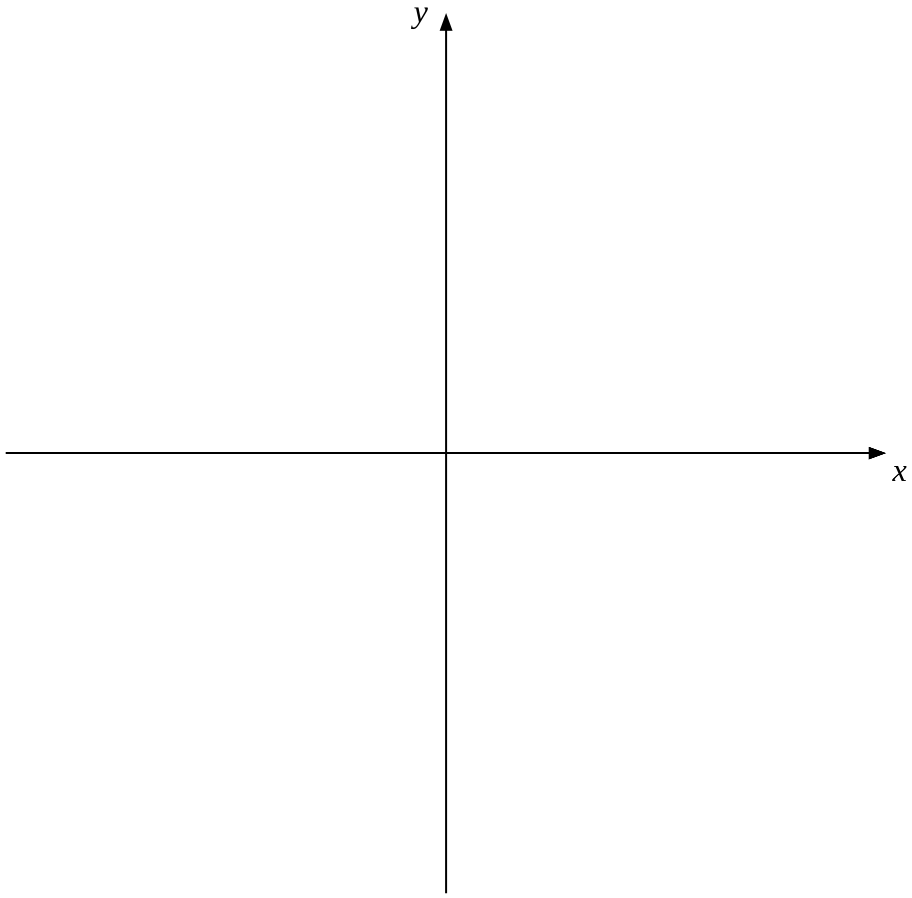
are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$

(3)

The point A lies on both of the curves and the x and y coordinates of A are both positive.

(c) Find the exact coordinates of A, leaving your answer in the form $(p + q\sqrt{3},\ r + s\sqrt{3})$, where p, q, r and s are integers.

(7)



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Question 101 continued

(Total 15 marks)

EXPERT TUITION

Leave blank

102. (a) Expand and simplify $(7 + \sqrt{5})(3 - \sqrt{5})$.

(3)

(b) Express $\dfrac{7+\sqrt{5}}{3+\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers.

(3)

(Total 6 marks)

Leave blank

103. Solve the simultaneous equations

$$y - 3x + 2 = 0$$

$$y^2 - x - 6x^2 = 0$$

(7)

Leave blank

Question 103 continued

(Total 7 marks)

Leave blank

104.

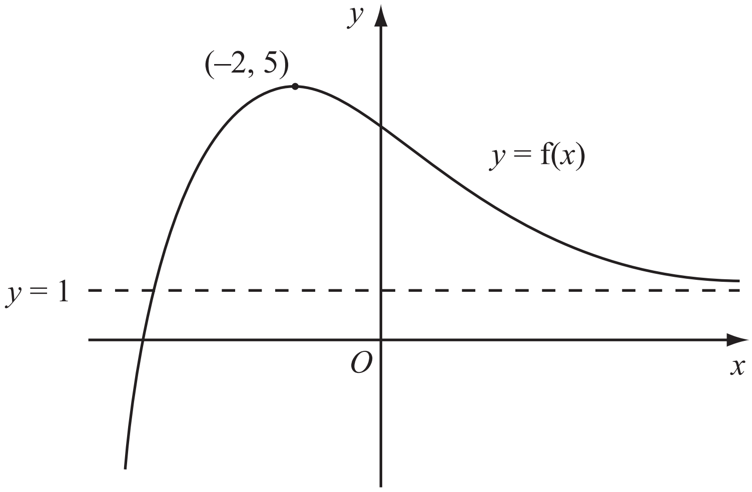


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$.

The curve has a maximum point $(-2, 5)$ and an asymptote $y = 1$, as shown in Figure 1.

On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(x) + 2$

(2)

(b) $y = 4\mathrm{f}(x)$

(2)

(c) $y = \mathrm{f}(x + 1)$

(3)

On each diagram, show clearly the coordinates of the maximum point and the equation of the asymptote.

Leave blank

Question 104 continued

Leave blank

Question 104 continued

Leave blank

Question 104 continued

(Total 7 marks)

Leave blank

105. (a) Factorise completely $x^3 - 4x$

(3)

(b) Sketch the curve C with equation

$$y = x^3 - 4x,$$

showing the coordinates of the points at which the curve meets the x-axis.

(3)

The point A with x-coordinate -1 and the point B with x-coordinate 3 lie on the curve C.

(c) Find an equation of the line which passes through A and B, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

(d) Show that the length of AB is $k\sqrt{10}$, where k is a constant to be found.

(2)

Leave blank

Question 105 continued

Leave blank

Question 105 continued

Leave blank

Question 105 continued

(Total 13 marks)

Leave blank

106. $$f(x) = x^2 + 4kx + (3 + 11k), \quad \text{where } k \text{ is a constant.}$$

(a) Express f(x) in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k.

(3)

Given that the equation f(x) = 0 has no real roots,

(b) find the set of possible values of k.

(4)

Given that $k = 1$,

(c) sketch the graph of y = f(x), showing the coordinates of any point at which the graph crosses a coordinate axis.

(3)

Leave blank

Question 106 continued

Leave blank

Question 106 continued

Leave blank

Question 106 continued

(Total 10 marks)

Leave blank

107. Simplify

(a) $(3\sqrt{7})^2$

(1)

(b) $(8+\sqrt{5})(2-\sqrt{5})$

(3)

(Total 4 marks)

Leave blank

108. Find the set of values of x for which

(a) $4x - 3 > 7 - x$

(2)

(b) $2x^2 - 5x - 12 < 0$

(4)

(c) **both** $4x - 3 > 7 - x$ **and** $2x^2 - 5x - 12 < 0$

(1)

(Total 7 marks)

Leave blank

109. The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

(4)

(Total 4 marks)

Leave blank

110. (a) Factorise completely $x^3 - 6x^2 + 9x$

(3)

(b) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis.

(4)

Using your answer to part (b), or otherwise,

(c) sketch, on a separate diagram, the curve with equation

$$y = (x - 2)^3 - 6(x - 2)^2 + 9(x - 2)$$

showing the coordinates of the points at which the curve meets the x-axis.

(2)

Leave blank

Question 110 continued

(Total 9 marks)

Leave blank

111. Expand and simplify $(\sqrt{7} + 2)(\sqrt{7} - 2)$.

(2)

(Total 2 marks)

112.

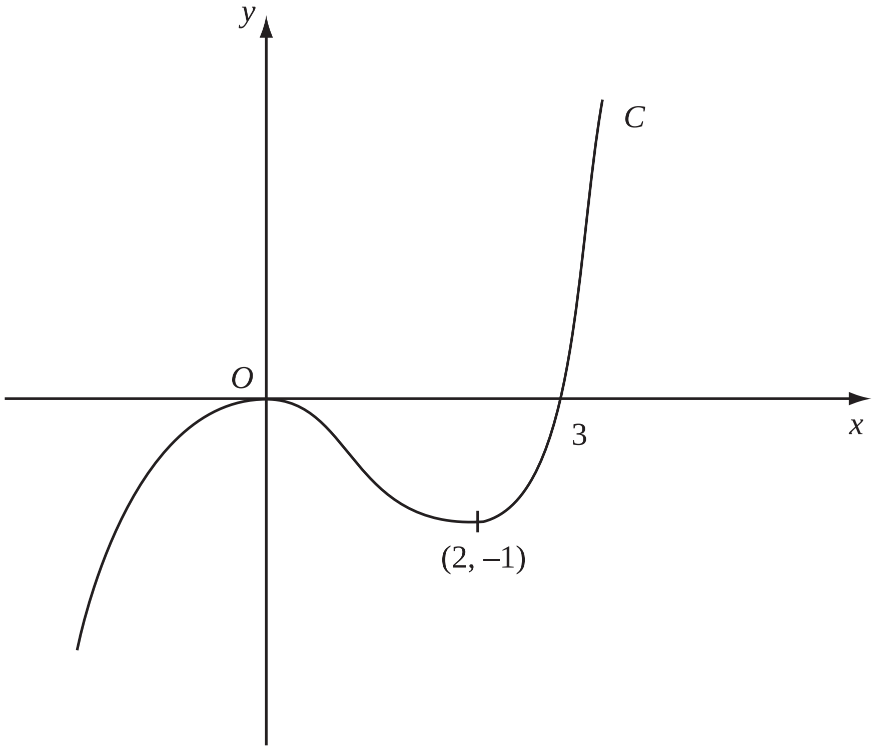


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$. There is a maximum at (0, 0), a minimum at (2, –1) and C passes through (3, 0).

On separate diagrams sketch the curve with equation

(a) $y = \mathrm{f}(x + 3)$,

(3)

(b) $y = \mathrm{f}(-x)$.

(3)

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the x-axis.

Leave blank

Question 112 continued

(Total 6 marks)

Leave blank

113. The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(3)

(b) Hence find the set of possible values of k.

(4)

(Total 7 marks)

Leave blank

114. The point P (1, a) lies on the curve with equation $y = (x + 1)^2(2 - x)$.

(a) Find the value of a.

(1)

(b) On the axes below sketch the curves with the following equations:

(i) $y = (x + 1)^2(2 - x)$,

(ii) $y = \dfrac{2}{x}$.

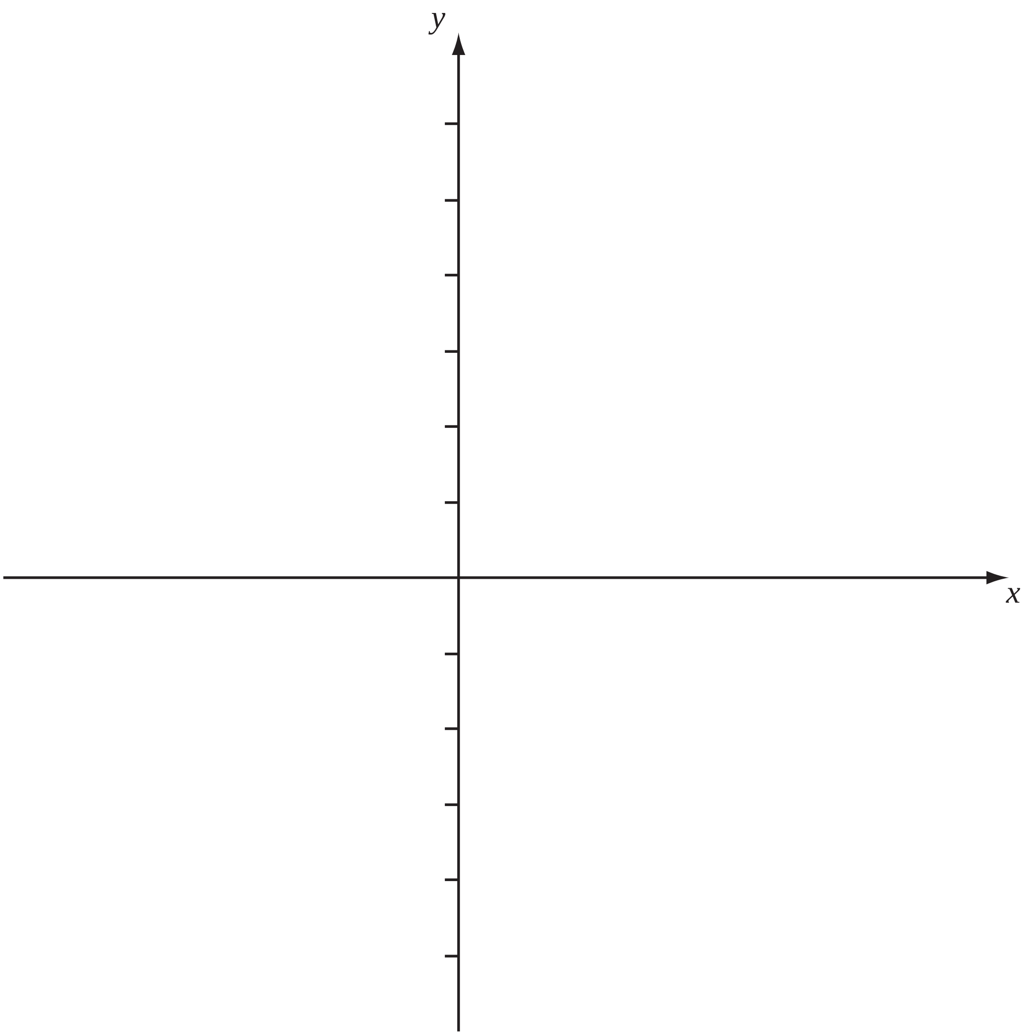
On your diagram show clearly the coordinates of any points at which the curves meet the axes.

(5)

(c) With reference to your diagram in part (b) state the number of real solutions to the equation

$$(x+1)^2(2-x) = \frac{2}{x}.$$

(1)



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Question 114 continued

(Total 7 marks)

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115. Factorise completely

$$x^3 - 9x.$$

(3)

(Total 3 marks)

116.

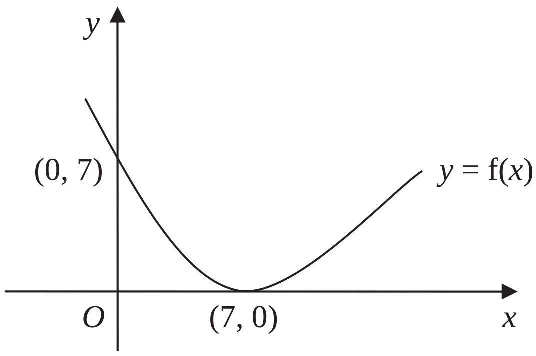


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve passes through the point (0, 7) and has a minimum point at (7, 0).

On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(x) + 3$,

(3)

(b) $y = \mathrm{f}(2x)$.

(2)

On each diagram, show clearly the coordinates of the minimum point and the coordinates of the point at which the curve crosses the y-axis.

(Total 5 marks)

Leave blank

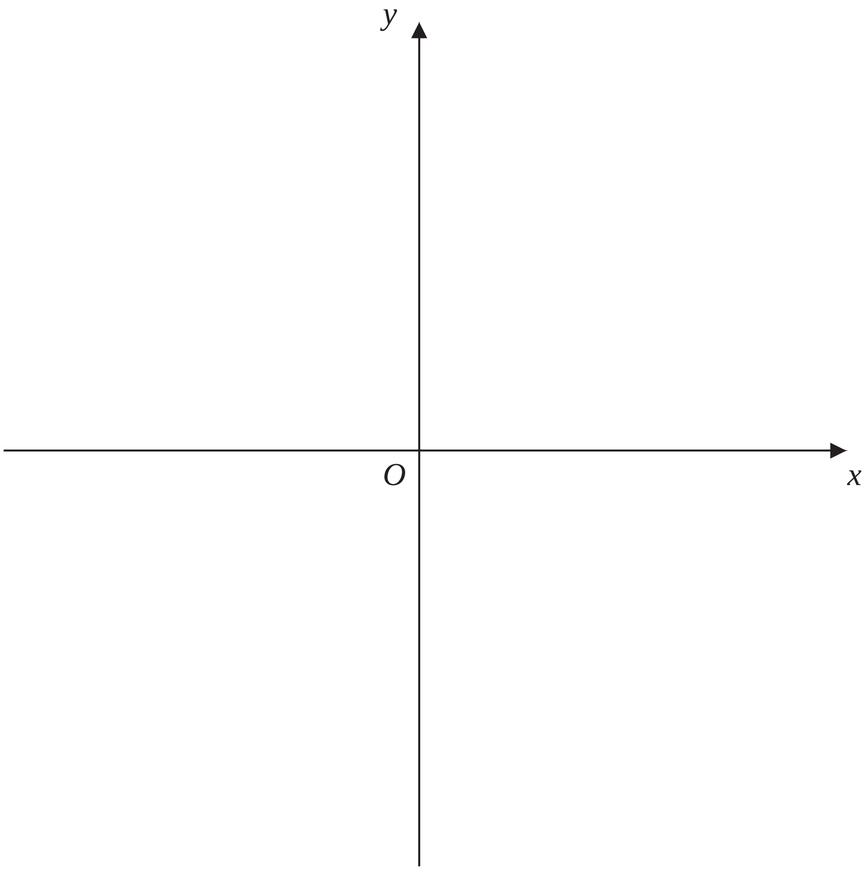
117. The curve C has equation $y = \frac{3}{x}$ and the line l has equation $y = 2x + 5$.

(a) On the axes below, sketch the graphs of C and l, indicating clearly the coordinates of any intersections with the axes.

(3)

(b) Find the coordinates of the points of intersection of C and l.

(6)



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Question 117 continued

(Total 9 marks)

Leave blank

118. Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q.

(3)

(Total 5 marks)

Leave blank

119. Simplify

$$\frac{5-\sqrt{3}}{2+\sqrt{3}},$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are integers.

(4)

(Total 4 marks)

Leave blank

120.

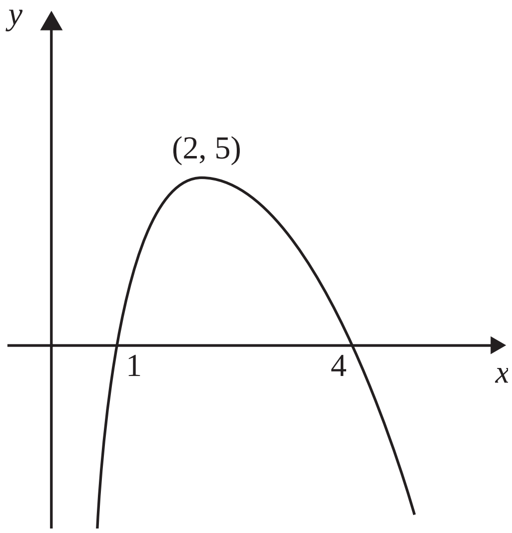


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve crosses the x-axis at the points (1, 0) and (4, 0). The maximum point on the curve is (2, 5).
In separate diagrams sketch the curves with the following equations.
On each diagram show clearly the coordinates of the maximum point and of each point at which the curve crosses the x-axis.

(a) $y = 2\mathrm{f}(x)$,

(3)

(b) $y = \mathrm{f}(-x)$.

(3)

The maximum point on the curve with equation $y = \mathrm{f}(x + a)$ is on the y-axis.

(c) Write down the value of the constant a.

(1)

Leave blank

Question 120 continued

(Total 7 marks)

Leave blank

121. The equation

$$x^2 + kx + 8 = k$$

has no real solutions for x.

(a) Show that k satisfies $k^2 + 4k - 32 < 0$.

(3)

(b) Hence find the set of possible values of k.

(4)

(Total 7 marks)

Leave blank

122. The curve C has equation

$$y = (x + 3)(x - 1)^2 .$$

(a) Sketch C showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

(b) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k.

(2)

Leave blank

Question 122 continued

(Total 6 marks)

Leave blank

123.

$$f(x) = 24x^3 + Ax^2 - 3x + B$$

where A and B are constants.

When $f(x)$ is divided by $(2x - 1)$ the remainder is 30

(a) Show that $A + 4B = 114$

(2)

Given also that $(x + 1)$ is a factor of $f(x)$,

(b) find another equation in A and B.

(2)

(c) Find the value of A and the value of B.

(2)

(d) Hence find a quadratic factor of $f(x)$.

(2)

Leave blank

Question 123 continued

Leave blank

Question 123 continued

Leave blank

Question 123 continued

(Total 8 marks)

Leave blank

124. $$f(x) = -6x^3 - 7x^2 + 40x + 21$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$

(2)

(b) Factorise $f(x)$ completely.

(4)

(c) Hence solve the equation

$$6(2^{3y}) + 7(2^{2y}) = 40(2^y) + 21$$

giving your answer to 2 decimal places.

(3)

Leave blank

Question 124 continued

Leave blank

Question 124 continued

Leave blank

Question 124 continued

(Total 9 marks)

Leave blank

125. $$f(x) = 6x^3 + 13x^2 - 4$$

(a) Use the remainder theorem to find the remainder when $f(x)$ is divided by $(2x + 3)$.
(2)

(b) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.
(2)

(c) Factorise $f(x)$ completely.
(4)

(Total 8 marks)

Leave blank

126. $f(x) = 6x^3 + 3x^2 + Ax + B$, where A and B are constants.

Given that when $f(x)$ is divided by $(x + 1)$ the remainder is 45,

(a) show that $B - A = 48$

(2)

Given also that $(2x + 1)$ is a factor of $f(x)$,

(b) find the value of A and the value of B.

(4)

(c) Factorise $f(x)$ fully.

(3)

Leave blank

Question 126 continued

(Total 9 marks)

Leave blank

127. $$f(x) = 2x^3 - 7x^2 + 4x + 4$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

(Total 6 marks)

EXPERT TUITION

Leave blank

128. (a) Sketch the graph of

$$y = 3^x, \quad x \in \mathbb{R}$$

showing the coordinates of any points at which the graph crosses the axes.

(2)

(b) Use algebra to solve the equation

$$3^{2x} - 9(3^x) + 18 = 0$$

giving your answers to 2 decimal places where appropriate.

(5)

Leave blank

Question 128 continued

(Total 7 marks)

Leave blank

129. $f(x) = -4x^3 + ax^2 + 9x - 18$, where a is a constant.

Given that $(x - 2)$ is a factor of $f(x)$,

(a) find the value of a,

(2)

(b) factorise $f(x)$ completely,

(3)

(c) find the remainder when $f(x)$ is divided by $(2x - 1)$.

(2)

(Total 7 marks)

Leave blank

130. $$f(x) = 2x^3 - 5x^2 + ax + 18$$

where a is a constant.

Given that $(x - 3)$ is a factor of $f(x)$,

(a) show that $a = -9$

(2)

(b) factorise $f(x)$ completely.

(4)

Given that

$$g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^{y}) + 18$$

(c) find the values of y that satisfy $g(y) = 0$, giving your answers to 2 decimal places where appropriate.

(3)

Leave blank

Question 130 continued

(Total 9 marks)

EXPERT TUITION

Leave blank

131. $f(x) = ax^3 + bx^2 - 4x - 3$, where a and b are constants.

Given that $(x - 1)$ is a factor of $f(x)$,

Show that

$$a + b = 7$$

(2)

(Total 2 marks)

EXPERT TUITION

Leave blank

132. $$f(x) = 2x^3 - 7x^2 - 10x + 24$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

(Total 6 marks)

Leave blank

133. $$f(x) = 2x^3 - 7x^2 - 5x + 4$$

(a) Find the remainder when $f(x)$ is divided by $(x-1)$.

(2)

(b) Use the factor theorem to show that $(x+1)$ is a factor of $f(x)$.

(2)

(c) Factorise $f(x)$ completely.

(4)

(Total 8 marks)

Leave blank

134. (a) Sketch the graph of $y = 7^x$, $x \in \mathbb{R}$, showing the coordinates of any points at which the graph crosses the axes.

(2)

(b) Solve the equation

$$7^{2x} - 4(7^x) + 3 = 0$$

giving your answers to 2 decimal places where appropriate.

(6)

(Total 8 marks)

EXPERT TUITION

Leave blank

135. $$f(x) = 3x^3 - 5x^2 - 58x + 40$$

(a) Find the remainder when $f(x)$ is divided by $(x - 3)$.

(2)

Given that $(x - 5)$ is a factor of $f(x)$,

(b) find all the solutions of $f(x) = 0$.

(5)

(Total 7 marks)

EXPERT TUITION

Leave blank

136. $$f(x) = 2x^3 - 3x^2 - 39x + 20$$

(a) Use the factor theorem to show that $(x + 4)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

(Total 6 marks)

Leave blank

137. The function f is defined by

$$f(x) = \frac{6}{2x+5} + \frac{2}{2x-5} + \frac{60}{4x^2-25}, \qquad x > 4$$

(a) Show that $f(x) = \dfrac{A}{Bx + C}$ where A, B and C are constants to be found.

(4)

(b) Find $f^{-1}(x)$ and state its domain.

(3)

Leave blank

Question 137 continued

Leave blank

Question 137 continued

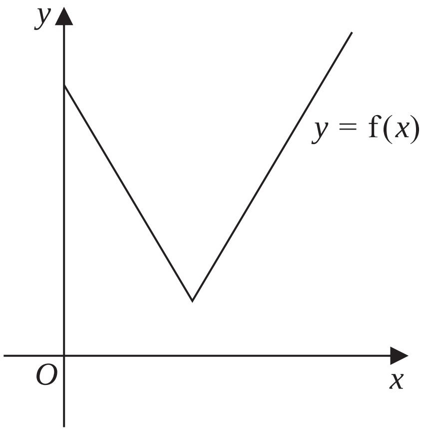
Leave blank

Question 137 continued

(Total 7 marks)

Leave blank

138.



Not to scale

Figure 2

Figure 2 shows part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|5 - x| + 3, \qquad x \geqslant 0$$

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly one root,

(a) state the set of possible values of k.

(2)

(b) Solve the equation $\mathrm{f}(x) = \frac{1}{2}x + 10$

(4)

The graph with equation $y = \mathrm{f}(x)$ is transformed onto the graph with equation $y = 4\mathrm{f}(x - 1)$.
The vertex on the graph with equation $y = 4\mathrm{f}(x - 1)$ has coordinates (p, q).

(c) State the value of p and the value of q.

(2)

EXPERT TUITION

Leave blank

Question 138 continued

Leave blank

Question 138 continued

Leave blank

Question 138 continued

(Total 8 marks)

EXPERT TUITION

Leave blank

139. Express $\dfrac{4x}{x^2 - 9} - \dfrac{2}{x + 3}$ as a single fraction in its simplest form.

(4)

Leave blank

Question 139 continued

(Total 4 marks)

Leave blank

140.

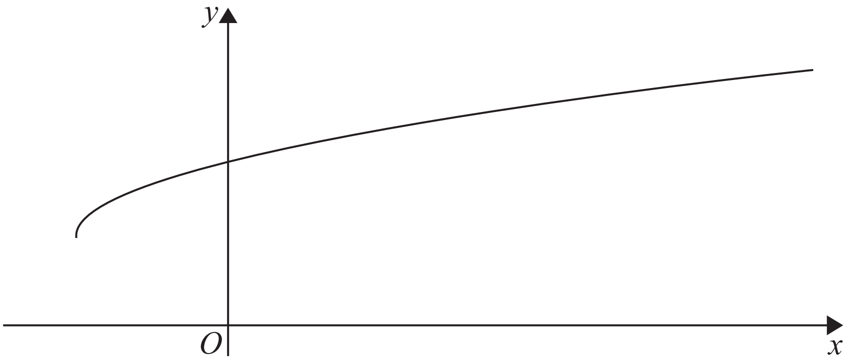


Figure 1

Figure 1 shows a sketch of part of the graph of $y = \text{g}(x)$, where

$$\text{g}(x) = 3 + \sqrt{x + 2}\,, \qquad x \geqslant -2$$

(a) State the range of g.

(1)

(b) Find $\text{g}^{-1}(x)$ and state its domain.

(3)

(c) Find the exact value of x for which

$$\text{g}(x) = x$$

(4)

(d) Hence state the value of a for which

$$\text{g}(a) = \text{g}^{-1}(a)$$

(1)

Leave blank

Question 140 continued

Leave blank

Question 140 continued

Leave blank

Question 140 continued

(Total 9 marks)

Leave blank

141. Given that a and b are positive constants,

(a) on separate diagrams, sketch the graph with equation

(i) $y = |2x - a|$

(ii) $y = |2x - a| + b$

Show, on each sketch, the coordinates of each point at which the graph crosses or meets the axes.

(4)

Given that the equation

$$|2x - a| + b = \frac{3}{2}x + 8$$

has a solution at $x = 0$ and a solution at $x = c$,

(b) find c in terms of a.

(4)

Leave blank

Question 141 continued

Leave blank

Question 141 continued

Leave blank

Question 141 continued

(Total 8 marks)

Leave blank

142. The functions f and g are defined by

$$f : x \rightarrow 7x - 1, \qquad x \in \mathbb{R}$$

$$g : x \rightarrow \frac{4}{x - 2}, \qquad x \neq 2, x \in \mathbb{R}$$

(a) Solve the equation $fg(x) = x$

(4)

(b) Hence, or otherwise, find the largest value of a such that $g(a) = f^{-1}(a)$

(1)

(Total 5 marks)

Leave blank

143.

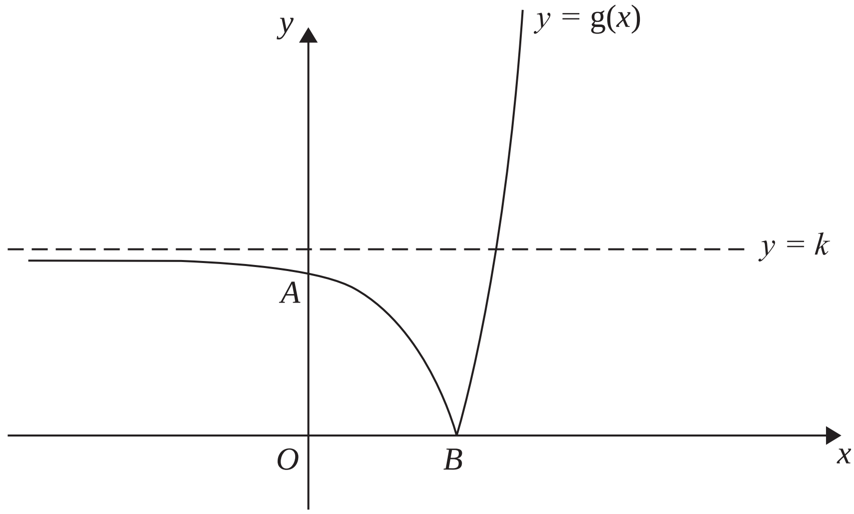


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = |4e^{2x} - 25|, \qquad x \in \mathbb{R}$$

The curve cuts the y-axis at the point A and meets the x-axis at the point B. The curve has an asymptote $y = k$, where k is a constant, as shown in Figure 1

Find, giving each answer in its simplest form,

(i) the y coordinate of the point A,

(ii) the exact x coordinate of the point B,

(iii) the value of the constant k.

(5)

Leave blank

Question 143 continued

(Total 5 marks)

EXPERT TUITION

Leave blank

144. (a) For $-\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$, sketch the graph of $y = \text{g}(x)$ where

$$\text{g}(x) = \arcsin x \qquad -1 \leqslant x \leqslant 1$$

(2)

(b) Find the exact value of x for which

$$3\text{g}(x + 1) + \pi = 0$$

(3)

(Total 5 marks)

Leave blank

145. Given that

$$f(x) = 2e^x - 5, \quad x \in \mathbb{R}$$

(a) sketch, on separate diagrams, the curve with equation

(i) $y = f(x)$

(ii) $y = |f(x)|$

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

(6)

(b) Deduce the set of values of x for which $f(x) = |f(x)|$

(1)

(c) Find the exact solutions of the equation $|f(x)| = 2$

(3)

Leave blank

Question 145 continued

(Total 10 marks)

EXPERT TUITION

Leave blank

146.

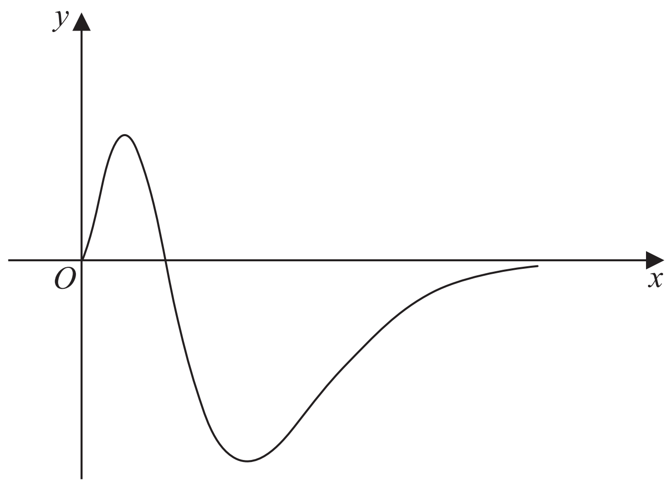


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$g(x) = x^2(1 - x)e^{-2x}, \quad x \geqslant 0$$

(a) Show that $g'(x) = f(x)e^{-2x}$, where $f(x)$ is a cubic function to be found.

(3)

(b) Hence find the range of g.

(6)

(c) State a reason why the function $g^{-1}(x)$ does not exist.

(1)

Leave blank

Question 146 continued

(Total 10 marks)

EXPERT TUITION

Leave blank

147.

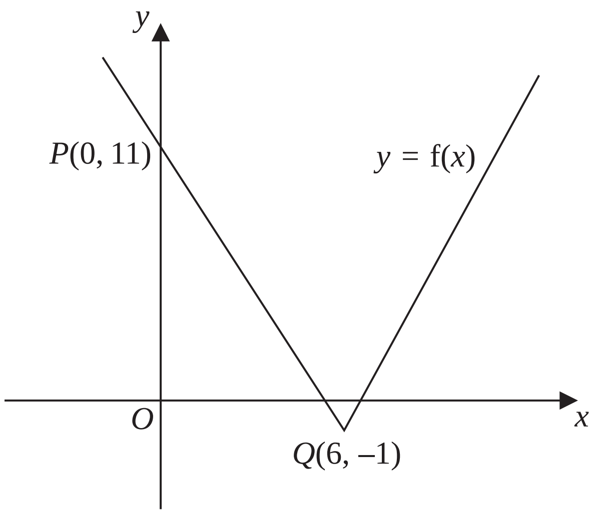


Figure 1

Figure 1 shows part of the graph with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $Q(6, -1)$.

The graph crosses the y-axis at the point $P(0, 11)$.

Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$

(2)

(b) $y = 2\mathrm{f}(-x) + 3$

(3)

On each diagram, show the coordinates of the points corresponding to P and Q.

Given that $\mathrm{f}(x) = a|x - b| - 1$, where a and b are constants,

(c) state the value of a and the value of b.

(2)

Leave blank

Question 147 continued

(Total 7 marks)

Leave blank

148.

$$g(x) = \frac{x}{x+3} + \frac{3(2x+1)}{x^2+x-6}, \quad x > 3$$

(a) Show that $g(x) = \dfrac{x+1}{x-2}, \quad x > 3$

(4)

(b) Find the range of g.

(2)

(c) Find the exact value of a for which $g(a) = g^{-1}(a)$.

(4)

Leave blank

Question 148 continued

(Total 10 marks)

Leave blank

149. Express

$$\frac{3}{2x+3} - \frac{1}{2x-3} + \frac{6}{4x^2-9}$$

as a single fraction in its simplest form.

(4)

(Total 4 marks)

EXPERT TUITION

Leave blank

150. A curve C has equation $y = e^{4x} + x^4 + 8x + 5$

On the axes given on the next page, sketch, on a single diagram, the curves with equations

(i) $y = x^3$,

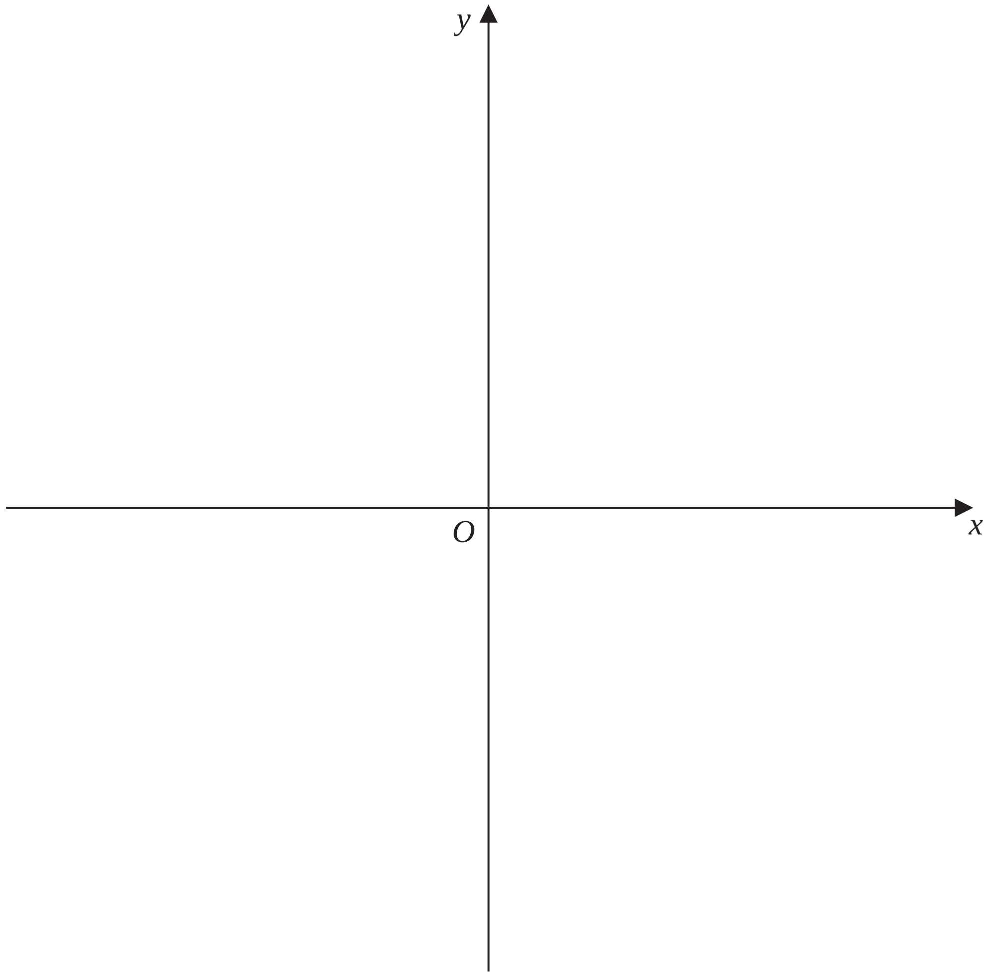
(ii) $y = -2 - e^{4x}$

On your diagram give the coordinates of the points where each curve crosses the y-axis and state the equation of any asymptotes.

(4)

Leave blank

Question 150 continued



(Total 4 marks)

Leave blank

151. (a) Sketch the graph with equation

$$y = |4x - 3|$$

stating the coordinates of any points where the graph cuts or meets the axes.

(2)

Find the complete set of values of x for which

(b)

$$|4x - 3| > 2 - 2x$$

(4)

(c)

$$|4x - 3| > \frac{3}{2} - 2x$$

(2)

Leave blank

Question 151 continued

(Total 8 marks)

Leave blank

152. The function f is defined by

$$f : x \to e^{2x} + k^2, \quad x \in \mathbb{R}, \quad k \text{ is a positive constant.}$$

(a) State the range of f.

(1)

(b) Find f^{-1} and state its domain.

(3)

The function g is defined by

$$g : x \to \ln(2x), \quad x > 0$$

(c) Solve the equation

$$g(x) + g(x^2) + g(x^3) = 6$$

giving your answer in its simplest form.

(4)

(d) Find $fg(x)$, giving your answer in its simplest form.

(2)

(e) Find, in terms of the constant k, the solution of the equation

$$fg(x) = 2k^2$$

(2)

Leave blank

Question 152 continued

(Total 12 marks)

EXPERT TUITION

Leave blank

153. Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

find the values of the constants a, b, c, d and e.

(4)

(Total 4 marks)

Leave blank

154. Given that

$$f(x) = \ln x, \quad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x-axis.
In each case, state the equation of the asymptote.

(7)

(Total 7 marks)

155. The function f has domain $-2 \leqslant x \leqslant 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = \mathrm{f}(x)$ is shown in Figure 1.

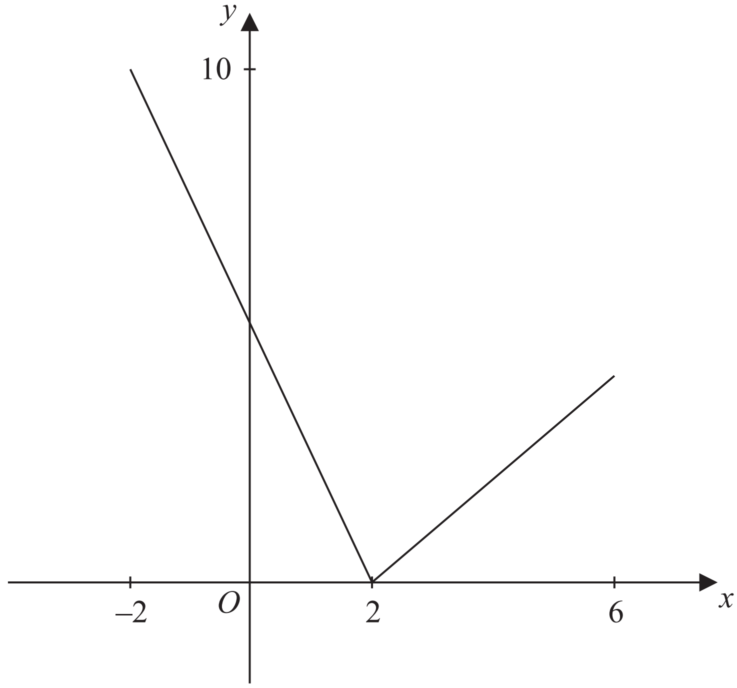


Figure 1

(a) Write down the range of f.

(1)

(b) Find ff(0).

(2)

The function g is defined by

$$\mathrm{g} : x \to \frac{4 + 3x}{5 - x}, \qquad x \in \mathbb{R}, \qquad x \neq 5$$

(c) Find $\mathrm{g}^{-1}(x)$

(3)

(d) Solve the equation $\mathrm{gf}(x) = 16$

(5)

Leave blank

Question 155 continued

(Total 11 marks)

Leave blank

156. $$g(x) = \frac{6x + 12}{x^2 + 3x + 2} - 2, \quad x \geqslant 0$$

(a) Show that $g(x) = \dfrac{4 - 2x}{x + 1}, \; x \geqslant 0$

(3)

(b)

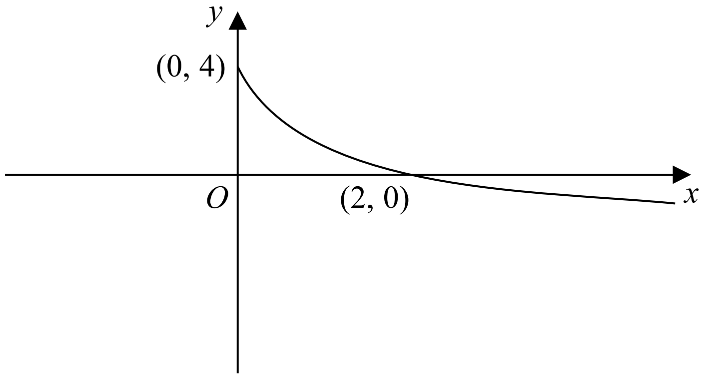


Figure 1

Figure 1 shows a sketch of the curve with equation $y = g(x), \; x \geqslant 0$

The curve meets the y-axis at (0, 4) and crosses the x-axis at (2, 0).

On separate diagrams sketch the graph with equation

(i) $y = 2g(2x)$,

(ii) $y = g^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or crosses the axes.

(5)

Leave blank

Question 156 continued

(Total 8 marks)

157.

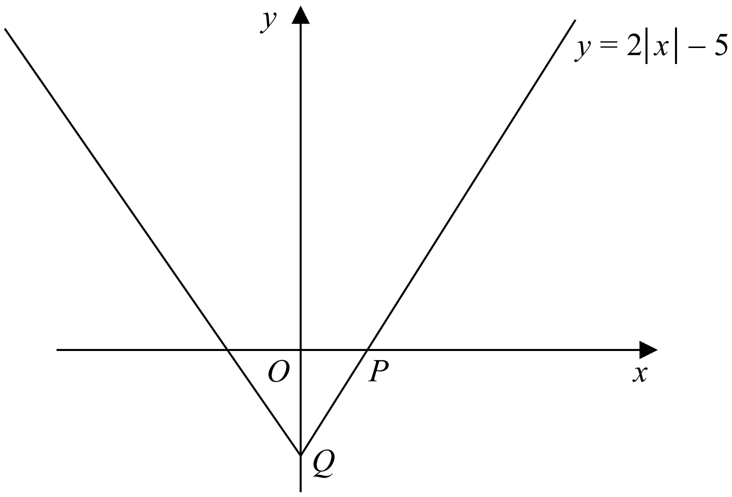


Figure 2

Figure 2 shows a sketch of the graph with equation $y = 2|x| - 5$.

The graph intersects the positive x-axis at the point P and the negative y-axis at the point Q.

(a) State the coordinates of P and the coordinates of Q.

(2)

(b) Solve the equation

$$2|x| - 5 = 3 - x$$

(3)

(Total 5 marks)

Leave blank

Leave blank

158. (a) On the same diagram, sketch and clearly label the graphs with equations

$$y = e^x \quad \text{and} \quad y = 10 - x$$

Show on your sketch the coordinates of each point at which the graphs cut the axes.

(3)

(b) Explain why the equation $e^x - 10 + x = 0$ has only one solution.

(1)

(c) Show that the solution of the equation

$$e^x - 10 + x = 0$$

lies between $x = 2$ and $x = 3$

(2)

Leave blank

Question 158 continued

(Total 6 marks)

EXPERT TUITION

Leave blank

159.

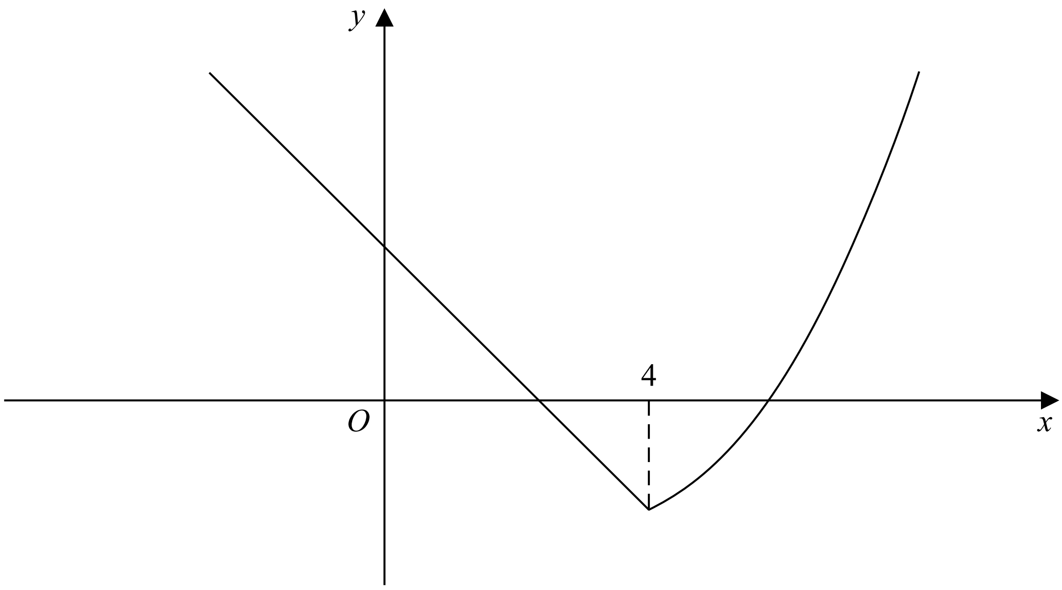


Figure 3

Figure 3 shows a sketch of the graph of $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \begin{cases} 5 - 2x, & x \leqslant 4 \\ e^{2x-8} - 4, & x > 4 \end{cases}$$

(a) State the range of $\mathrm{f}(x)$.

(1)

(b) Determine the exact value of $\mathrm{ff}(0)$.

(2)

(c) Solve $\mathrm{f}(x) = 21$

Give each answer as an exact answer.

(5)

(d) Explain why the function f does not have an inverse.

(1)

Leave blank

Question 159 continued

(Total 9 marks)

Leave blank

160. Express

$$\frac{3x+5}{x^2+x-12} - \frac{2}{x-3}$$

as a single fraction in its simplest form.

(4)

(Total 4 marks)

Leave blank

161.

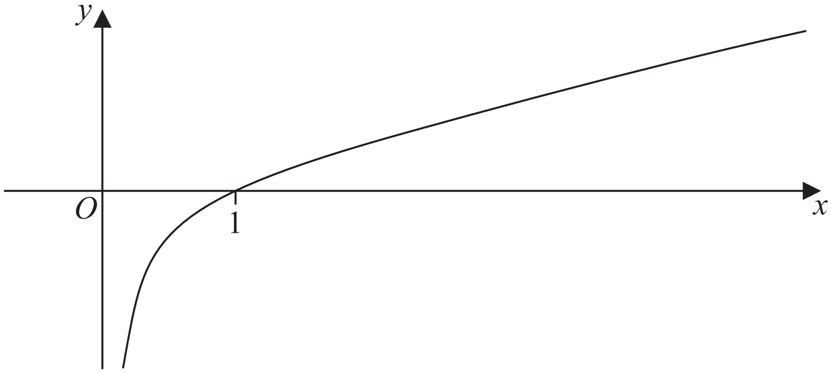


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, $x > 0$, where f is an increasing function of x. The curve crosses the x-axis at the point (1, 0) and the line $x = 0$ is an asymptote to the curve.

On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(2x)$, $x > 0$

(2)

(b) $y = |\mathrm{f}(x)|$, $x > 0$

(3)

Indicate clearly on each sketch the coordinates of the point at which the curve crosses or meets the x-axis.

(Total 5 marks)

Leave blank

162. The functions f and g are defined by

$$f : x \mapsto 2|x| + 3, \qquad x \in \mathbb{R},$$

$$g : x \mapsto 3 - 4x, \qquad x \in \mathbb{R}$$

(a) State the range of f.

(2)

(b) Find fg(1).

(2)

(c) Find g^{-1}, the inverse function of g.

(2)

(d) Solve the equation

$$gg(x) + [g(x)]^2 = 0$$

(5)

Leave blank

Question 162 continued

(Total 11 marks)

163.

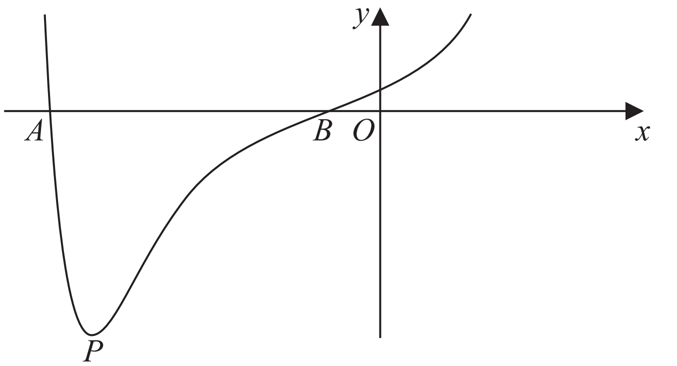


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (x^2 + 3x + 1)\,\mathrm{e}^{x^2}$$

The curve cuts the x-axis at points A and B as shown in Figure 2.

Calculate the x coordinate of A and the x coordinate of B, giving your answers to 3 decimal places.

(2)

(Total 2 marks)

Leave blank

164.

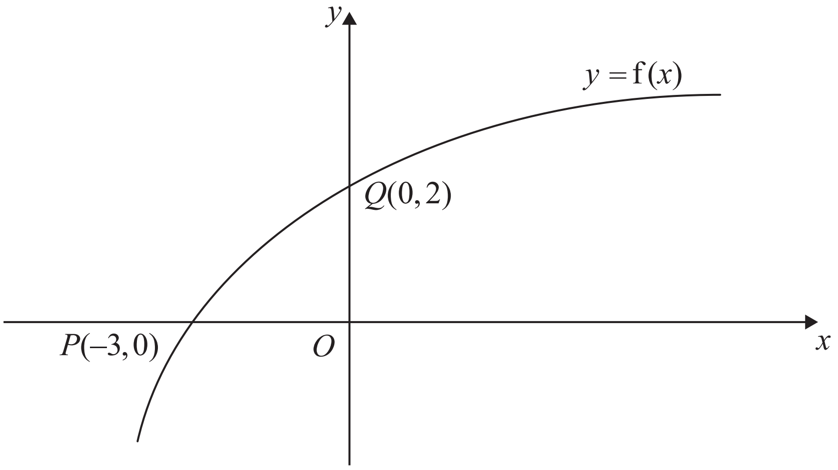


Figure 1

Figure 1 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

(a) Find the value of $\mathrm{ff}(-3)$.

(2)

On separate diagrams, sketch the curve with equation

(b) $y = \mathrm{f}^{-1}(x)$,

(2)

(c) $y = \mathrm{f}(|x|) - 2$,

(2)

(d) $y = 2\mathrm{f}\left(\frac{1}{2}x\right)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

Leave blank

Question 164 continued

(Total 9 marks)

Leave blank

165. $$\mathrm{h}(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geqslant 0$$

(a) Show that $\mathrm{h}(x) = \dfrac{2x}{x^2+5}$

(4)

(b) Hence, or otherwise, find $\mathrm{h}'(x)$ in its simplest form.

(3)

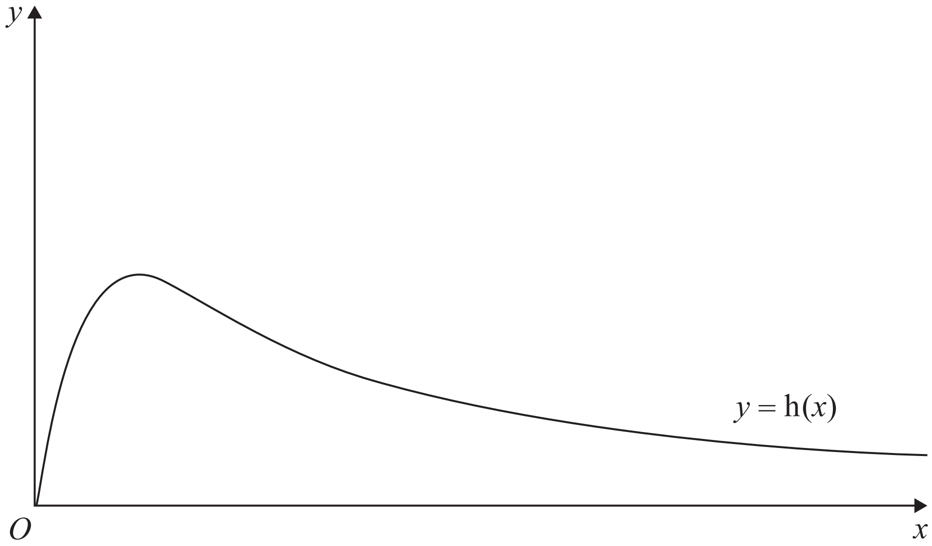


Figure 2

Figure 2 shows a graph of the curve with equation $y = \mathrm{h}(x)$.

(c) Calculate the range of $\mathrm{h}(x)$.

(5)

Leave blank

Question 165 continued

(Total 12 marks)

Leave blank

166. Express

$$\frac{2(3x+2)}{9x^2-4} - \frac{2}{3x+1}$$

as a single fraction in its simplest form.

(4)

(Total 4 marks)

Leave blank

167.

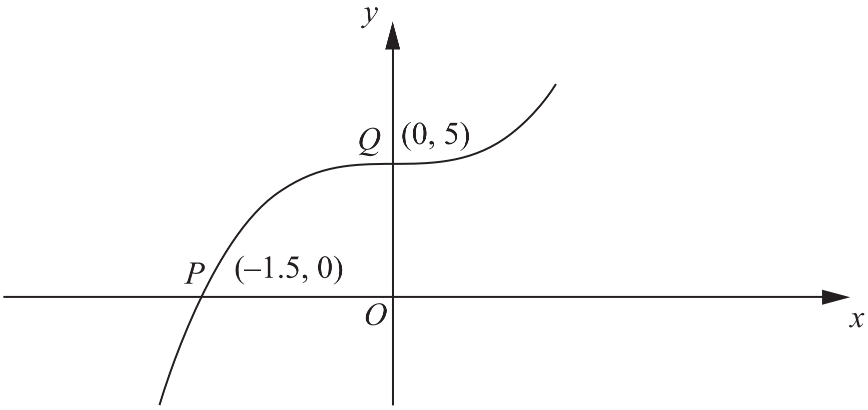


Figure 2

Figure 2 shows part of the curve with equation $y = f(x)$
The curve passes through the points $P(-1.5,\ 0)$ and $Q(0,\ 5)$ as shown.

On separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$

(2)

(b) $y = f(|x|)$

(2)

(c) $y = 2f(3x)$

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

Leave blank

Question 167 continued

(Total 4 marks)

Leave blank

168. The functions f and g are defined by

$$f : x \mapsto e^x + 2, \qquad x \in \mathbb{R}$$

$$g : x \mapsto \ln x, \qquad x > 0$$

(a) State the range of f.

(1)

(b) Find $fg(x)$, giving your answer in its simplest form.

(2)

(c) Find the exact value of x for which $f(2x+3) = 6$

(4)

(d) Find f^{-1}, the inverse function of f, stating its domain.

(3)

(e) On the same axes sketch the curves with equation $y = f(x)$ and $y = f^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes.

(4)

Leave blank

Question 168 continued

(Total 14 marks)

EXPERT TUITION

169.

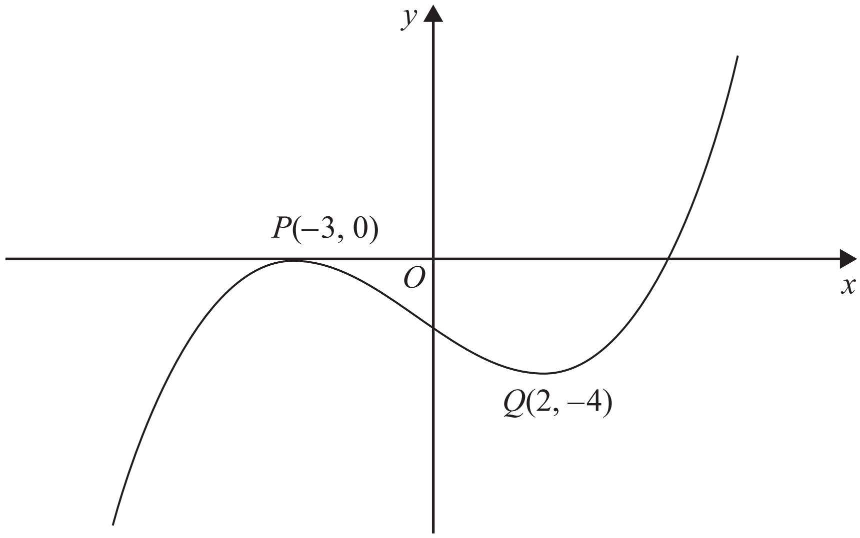


Figure 1

Figure 1 shows the graph of equation $y = \mathrm{f}(x)$.

The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 3\mathrm{f}(x + 2)$

(3)

(b) $y = |\mathrm{f}(x)|$

(3)

On each diagram, show the coordinates of any stationary points.

(Total 6 marks)

Leave blank

170. The function f is defined by

$$f : x \mapsto \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4}, \qquad x \in \mathbb{R}, x > \frac{1}{2}$$

(a) Show that $f(x) = \dfrac{1}{2x-1}$ **(4)**

(b) Find $f^{-1}(x)$ **(3)**

(c) Find the domain of f^{-1} **(1)**

$$g(x) = \ln(x+1)$$

(d) Find the solution of $fg(x) = \dfrac{1}{7}$, giving your answer in terms of e. **(4)**

Leave blank

Question 170 continued

(Total 12 marks)

Leave blank

171.

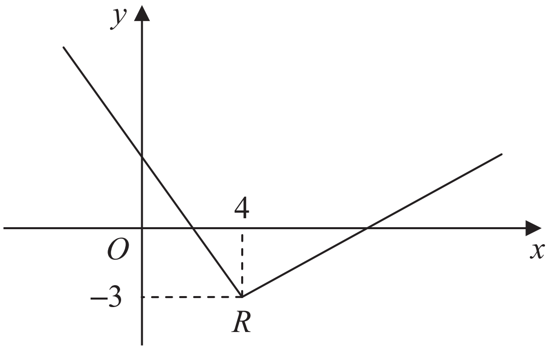


Figure 1

Figure 1 shows part of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $R(4, -3)$, as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(a) $y = 2\mathrm{f}(x+4)$,

(3)

(b) $y = |\mathrm{f}(-x)|$.

(3)

On each diagram, show the coordinates of the point corresponding to R.

(Total 6 marks)

Leave blank

172. The function f is defined by

$$f: x \mapsto 4 - \ln(x+2), \quad x \in \mathbb{R}, \ x \geqslant -1$$

(a) Find $f^{-1}(x)$.

(3)

(b) Find the domain of f^{-1}.

(1)

The function g is defined by

$$g: x \mapsto e^{x^2} - 2, \quad x \in \mathbb{R}$$

(c) Find $fg(x)$, giving your answer in its simplest form.

(3)

(d) Find the range of fg.

(1)

(Total 8 marks)

Leave blank

173. (a) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \qquad x > 1,$$

(b) show that

$$f(x) = \frac{3}{2x-1}$$

(2)

Leave blank

Question 173 continued

(Total 6 marks)

Leave blank

174. The function f is defined by

$$f: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R}, \ x \neq 5$$

(a) Find $f^{-1}(x)$.

(3)

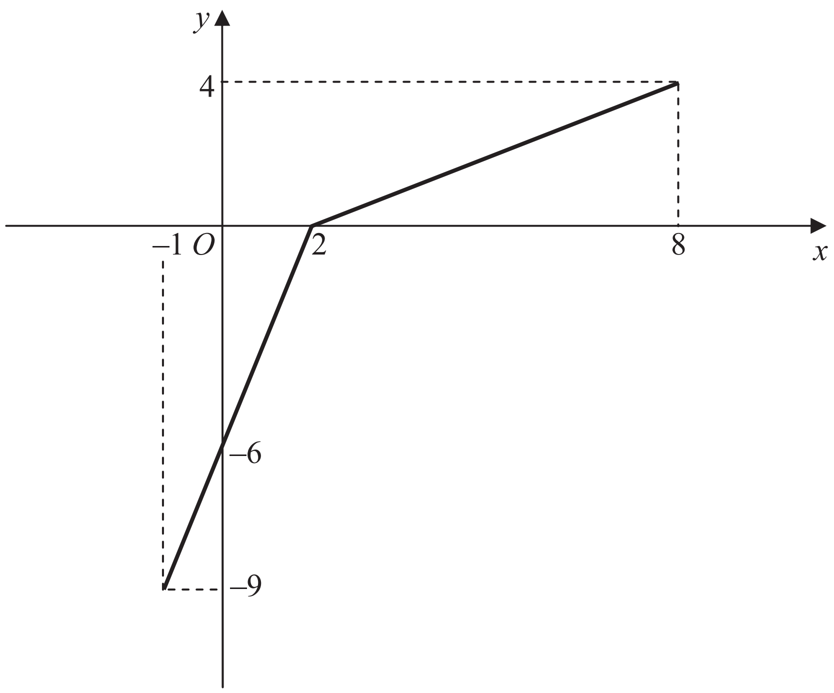


Figure 2

The function g has domain $-1 \leqslant x \leqslant 8$, and is linear from (–1, –9) to (2, 0) and from (2, 0) to (8, 4). Figure 2 shows a sketch of the graph of $y = g(x)$.

(b) Write down the range of g.

(1)

(c) Find gg(2).

(2)

(d) Find fg(8).

(2)

(e) On separate diagrams, sketch the graph with equation

(i) $y = |g(x)|$,

(ii) $y = g^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1}.

(1)

Leave blank

Question 174 continued

(Total 13 marks)

Leave blank

175. The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(b) Solve $f(x) = 15 + x$.

(3)

The function g is defined by

$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leqslant x \leqslant 5$$

(c) Find fg(2).

(2)

(d) Find the range of g.

(3)

Leave blank

Question 175 continued

(Total 10 marks)

176.

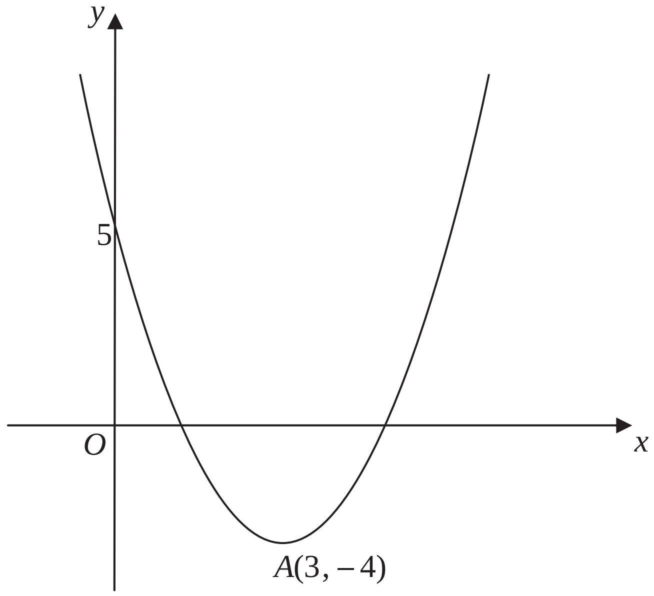


Figure 2

Figure 2 shows a sketch of the curve with the equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.
The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

(a) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |\mathrm{f}(x)|$,

(ii) $y = 2\mathrm{f}(\frac{1}{2}x)$.

(4)

(b) Sketch the curve with equation

$$y = \mathrm{f}(|x|)$$

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y-axis.

(3)

The curve with equation $y = \mathrm{f}(x)$ is a translation of the curve with equation $y = x^2$.

(c) Find $\mathrm{f}(x)$.

(2)

(d) Explain why the function f does not have an inverse.

(1)

Leave blank

Question 176 continued

(Total 10 marks)

Leave blank

177. Express

$$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$$

as a single fraction in its simplest form.

(4)

(Total 4 marks)

Leave blank

178. Sketch the graph of $y = \ln|x|$, stating the coordinates of any points of intersection with the axes.

(3)

(Total 3 marks)

EXPERT TUITION

Leave blank

179.

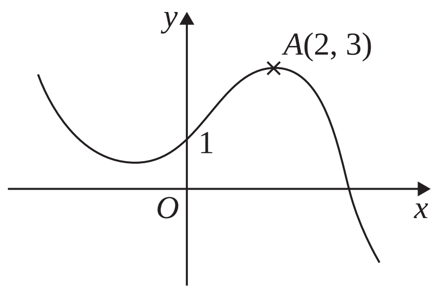


Figure 1

Figure 1 shows a sketch of the graph of $y = \mathrm{f}(x)$.

The graph intersects the y-axis at the point (0, 1) and the point $A(2, 3)$ is the maximum turning point.

Sketch, on separate axes, the graphs of

(i) $y = \mathrm{f}(-x) + 1$,

(ii) $y = \mathrm{f}(x + 2) + 3$,

(iii) $y = 2\mathrm{f}(2x)$.

On each sketch, show the coordinates of the point at which your graph intersects the y-axis and the coordinates of the point to which A is transformed.

(9)

Leave blank

Question 179 continued

(Total 9 marks)

Leave blank

180. The functions f and g are defined by

$$f(x) = e^{2x} + 3, \qquad x \in \mathbb{R}$$

$$g(x) = \ln(x - 1), \qquad x \in \mathbb{R},\ x > 1$$

(a) Find f^{-1} and state its domain. **(4)**

(b) Find fg and state its range. **(3)**

EXPERT TUITION

Leave blank

Question 180 continued

(Total 7 marks)

181.

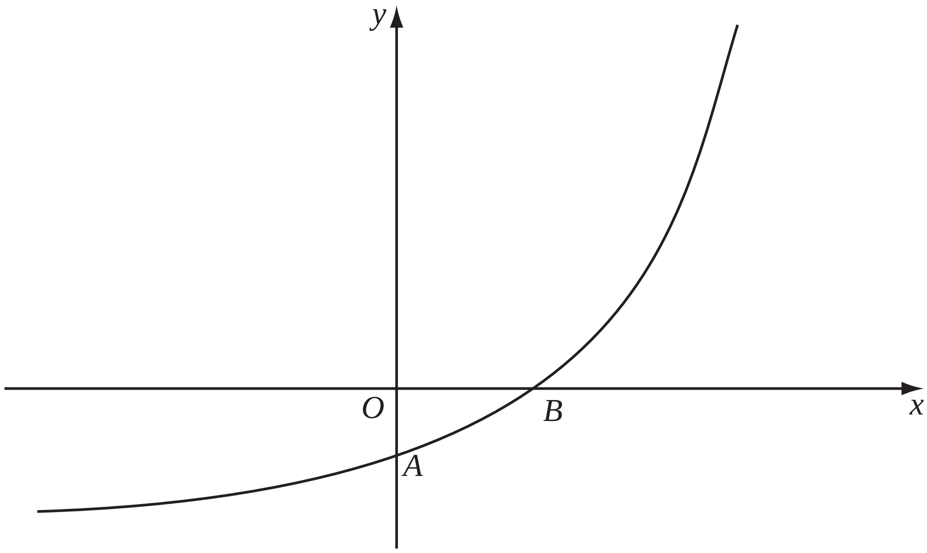


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2}\ln k, 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$,

(3)

(b) $y = \mathrm{f}^{-1}(x)$.

(2)

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $\mathrm{f}(x) = \mathrm{e}^{2x} - k$,

(c) state the range of f ,

(1)

(d) find $\mathrm{f}^{-1}(x)$,

(3)

(e) write down the domain of f^{-1}.

(1)

Leave blank

Leave blank

Question 181 continued

(Total 10 marks)

Leave blank

182. $$f(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$$

Express $f(x)$ as a single fraction in its simplest form.

(4)

(Total 4 marks)

183.

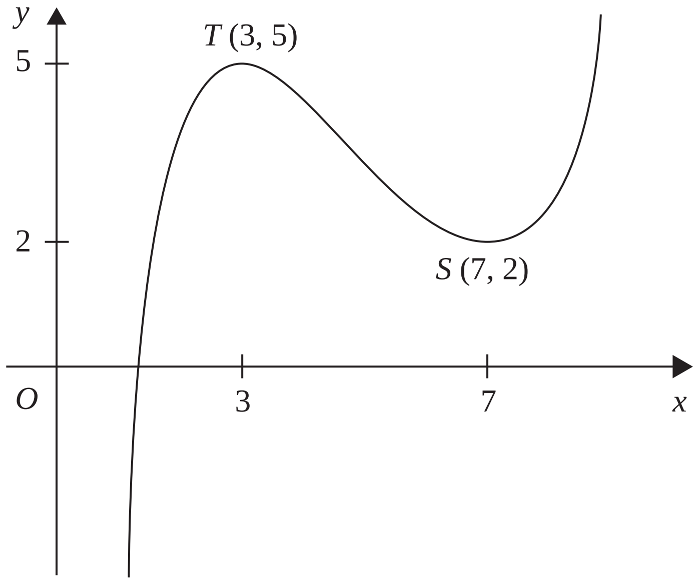


Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $1 < x < 9$.
The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 2\mathrm{f}(x) - 4$,

(3)

(b) $y = |\mathrm{f}(x)|$.

(3)

Indicate on each diagram the coordinates of any turning points on your sketch.

(Total 6 marks)

Leave blank

Leave blank

184. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(6)

(Total 6 marks)

EXPERT TUITION

Leave blank

185. The functions f and g are defined by

$$f : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R}$$

$$g : x \mapsto e^{x^2}, \quad x \in \mathbb{R}$$

(a) Write down the range of g.

(1)

(b) Show that the composite function fg is defined by

$$fg : x \mapsto x^2 + 3e^{x^2}, \quad x \in \mathbb{R}\ .$$

(2)

(c) Write down the range of fg.

(1)

(d) Solve the equation $\frac{d}{dx}\left[fg(x)\right] = x(xe^{x^2} + 2)$.

(6)

Leave blank

Question 185 continued

(Total 10 marks)

Leave blank

186.

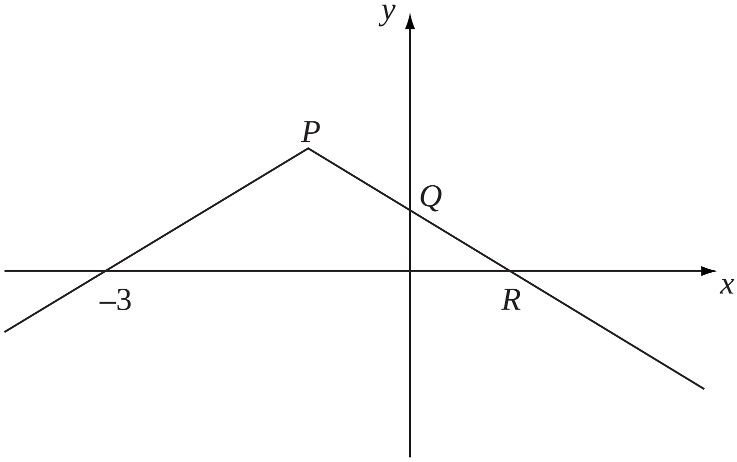


Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x),\ x \in \mathbb{R}$.
The graph consists of two line segments that meet at the point P.
The graph cuts the y-axis at the point Q and the x-axis at the points (–3, 0) and R.
Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$,

(2)

(b) $y = \mathrm{f}(-x)$.

(2)

Given that $\mathrm{f}(x) = 2 - |x+1|$,

(c) find the coordinates of the points P, Q and R,

(3)

(d) solve $\mathrm{f}(x) = \frac{1}{2}x$.

(5)

Leave blank

Question 186 continued

(Total 12 marks)

Leave blank

187. The function f is defined by

$$f : x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \qquad x > 3.$$

(a) Show that $f(x) = \frac{1}{x+1}, \qquad x > 3.$

(4)

(b) Find the range of f.

(2)

(c) Find $f^{-1}(x)$. State the domain of this inverse function.

(3)

The function g is defined by

$$g : x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $fg(x) = \frac{1}{8}$.

(3)

Leave blank

Question 187 continued

(Total 12 marks)

Leave blank

188. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a, b, c, d and e.

(4)

(Total 4 marks)

Leave blank

189.

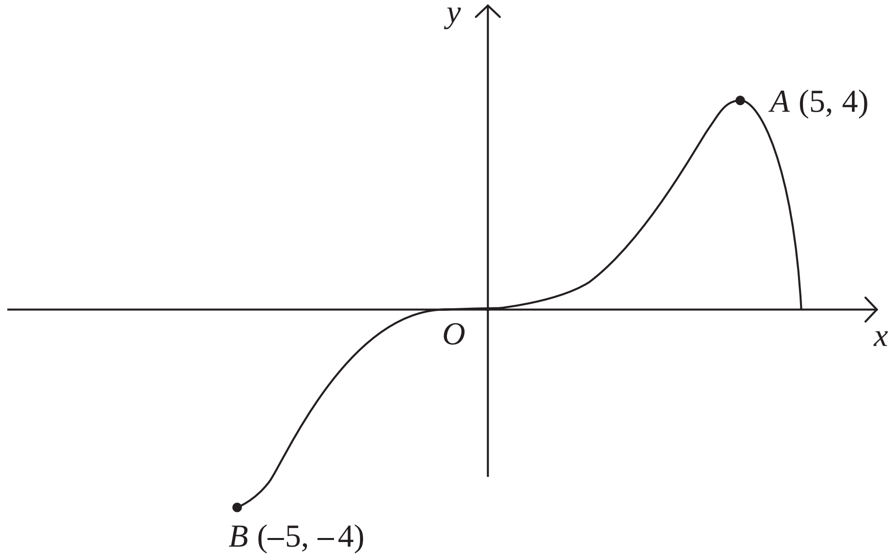


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$.
The curve passes through the origin O and the points $A(5, 4)$ and $B(-5, -4)$.

In separate diagrams, sketch the graph with equation

(a) $y = |\mathrm{f}(x)|$,

(3)

(b) $y = \mathrm{f}(|x|)$,

(3)

(c) $y = 2\mathrm{f}(x+1)$.

(4)

On each sketch, show the coordinates of the points corresponding to A and B.

Leave blank

Question 189 continued

(Total 10 marks)

Leave blank

190. The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}$$

$$g : x \mapsto \frac{3}{x} - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Find the inverse function f^{-1}.

(2)

(b) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve $gf(x) = 0$.

(2)

Leave blank

Question 190 continued

(Total 8 marks)

Leave blank

191. A curve C has parametric equations

$$x = 2\sin t, \quad y = 1 - \cos 2t, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find a cartesian equation for C in the form

$$y = \mathrm{f}(x), \quad -k \leqslant x \leqslant k,$$

stating the value of the constant k.

(3)

(b) Write down the range of $\mathrm{f}(x)$.

(2)

(Total 5 marks)

Leave blank

192. Express in partial fractions

$$\frac{5x + 3}{(2x + 1)(x + 1)^2}$$

(4)

(Total 4 marks)

Leave blank

193. Express $\dfrac{9x^2 + 20x - 10}{(x + 2)(3x - 1)}$ in partial fractions.

(4)

(Total 4 marks)

Leave blank

194. $$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}$$

Find the values of the constants A, B and C.

(4)

(Total 4 marks)

Leave blank

195.

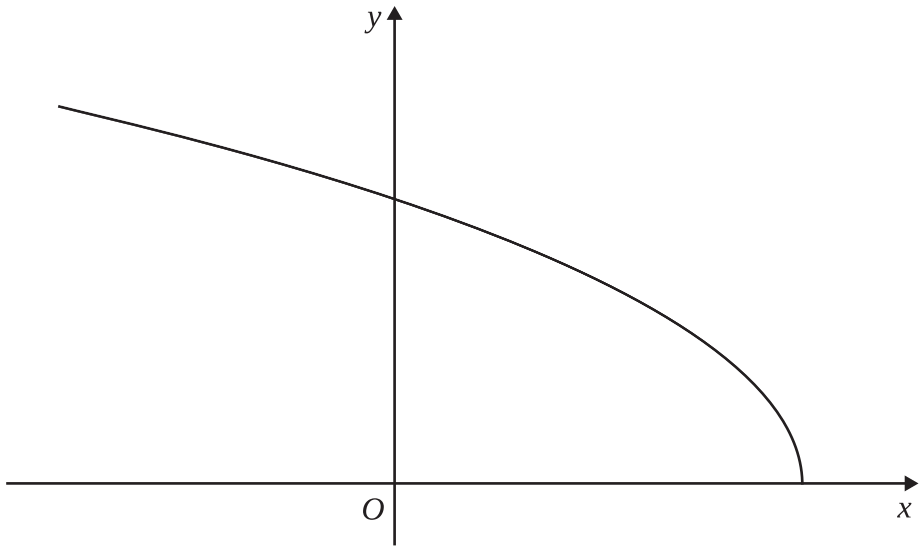


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \quad y = 6\sin t, \quad 0 \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find a cartesian equation of the curve in the form

$$y = \mathrm{f}(x), \quad -k \leqslant x \leqslant k,$$

stating the value of the constant k.

(4)

(b) Write down the range of $\mathrm{f}(x)$.

(2)

Leave blank

Question 195 continued

(Total 6 marks)

Leave blank

196. A curve is described by the equation

$$x^3 - 4y^2 = 12xy$$

Find the coordinates of the two points on the curve where $x = -8$.

(3)

(Total 3 marks)

Leave blank

197.

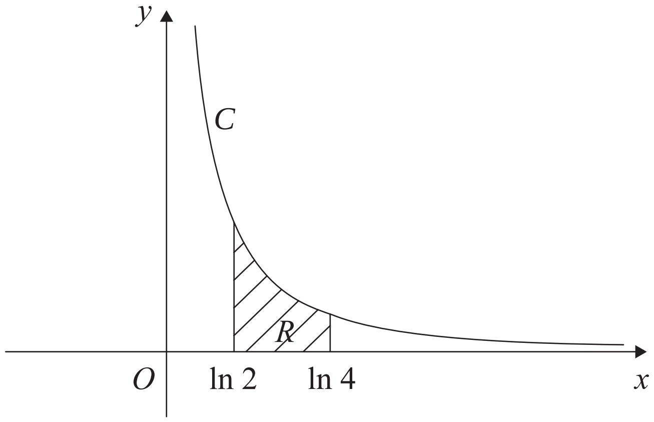


Figure 3

The curve C has parametric equations

$$x = \ln(t+2), \qquad y = \frac{1}{(t+1)}, \qquad t > -1$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Find a cartesian equation of the curve C, in the form $y = \text{f}(x)$. **(4)**

(b) State the domain of values for x for this curve. **(1)**

Leave blank

Question 197 continued

(Total 5 marks)